Light tetraquark states with exotic quantum numbers $J^{PC} = 2^{+-}$

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(Received 2 May 2024; accepted 16 July 2024; published 15 August 2024)

We study the masses of light tetraquark states $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ with exotic quantum numbers $J^{PC} = 2^{+-}$ using the method of QCD sum rules. It is found that there is no tetraquark operator with two Lorentz indices coupling to the 2^{+-} quantum numbers. To investigate such tetraquark states, we construct the interpolating tetraquark currents with three Lorentz indices and without a derivative operator. We calculate the correlation functions up to dimension-ten condensates and extract the 2^{+-} invariant functions via the projector operator. Our results show that the masses of the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 2^{+-}$ are about 3.3–3.5, 3.5–3.7, and 3.67 GeV, respectively. We further discuss the strong decays of these light tetraquarks into the two-meson and baryon-antibaryon final states and suggest to search for them in the $\rho\pi$, $\omega\pi$, $\phi\pi$, $b_1\pi$, $h_1\pi$, $K\bar{K}^*$, $K\bar{K}_1$, $\Delta\bar{\Delta}$, $\Sigma^*\bar{\Sigma}^*$, $\Xi^*\bar{\Xi}^*$, and $\Omega\bar{\Omega}$ channels in the future.

DOI: 10.1103/PhysRevD.110.034022

I. INTRODUCTION

Quantum chromodynamics (QCD) is the fundamental theory to describe the strong interaction among quarks and gluons in an SU(3) gauge symmetry. In the traditional quark model, hadrons are $q\bar{q}$ mesons and qqq baryons [1–3]. However, QCD allows for the existence of hadron states beyond the quark model, such as tetraquarks, pentaquarks, hexaquarks, dibaryons, hybrid mesons, glueballs, and so on [4–11].

A compact tetraquark is made of a diquark and antidiquark pair. In QCD, the light scalar mesons $\sigma(600)$, $\kappa(800)$, $a_0(980)$, and $f_0(980)$ have been considered as light tetraquark state candidates [12–16]. In 2006, the *BABAR* Collaboration observed $\phi(2170)$ in the $e^-e^+ \to \phi f_0(980)$ process [17], which was confirmed by later experiments [18–24]. Since its observation, this vector resonance with $J^{PC} = 1^{--}$ was considered as the candidate for a fully strange $ss\bar{s}\bar{s}$ tetraquark state [25–28], although some other interpretations have not been excluded [29,30]. Similarly, the $X(2239)$ structure observed by BESIII [31] was also interpreted as a light tetraquark state [32,33].

The conventional $\bar{q}q$ mesons are forbidden to carry exotic quantum numbers, such as $J^{PC} = 0^{--}$, even^{+-}, odd^{-+}. However, they can be reached in the tetraquark and hybrid meson configurations. To date, the only observed exotic J^{PC} quantum numbers appear for the isovector $\pi_1(1400)$ [34], $\pi_1(1600)$ [35], and $\pi_1(2015)$ [36] with $I^GJ^{PC} = 1^-1^{-+}$ and the isoscalar $\eta_1(1855)$ with $I^GJ^{PC} = 0^+1^{-+}$ [37,38], in which $\pi_1(1400)$ and $\pi_1(1600)$ were also considered to be the same state [39,40]. Over the past several decades, these exotic structures have been extensively investigated as the best candidates for hybrid mesons [4,10,41–47]. Nevertheless, they can also be interpreted as light compact tetraquarks and hadronic molecules [48–58]. Light tetraquark states have also been studied for the $J^{PC} = 0^{--}$ [59,60] and 0^{+-} [61–64] exotic channels.

There have been recent theoretical investigations on the exotic hadrons with $J^{PC} = 2^{+-}$. In Ref. [64], the fully strange $ss\bar{s}\bar{s}$ tetraquark state with such quantum numbers was studied using QCD sum rules and its mass was calculated to be around 3.1 GeV. The nonstrange and strangeonium light one-gluon hybrid mesons with $J^{PC} = 2^{+-}$ were studied using lattice QCD and QCD sum rules to give a mass prediction of 2.4–2.7 GeV [65–67], which is much heavier than the results in the flux tube model [68]. In Refs. [69–72], the exotic 2^{+-} glueballs were investigated using lattice QCD and QCD sum rules with diverse mass predictions. A new type of double-gluon hybrid mesons with exotic quantum numbers was proposed recently using QCD sum rules in Refs. [73–78]. In this work, we further study the mass spectra of the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark states with exotic quantum numbers $J^{PC} = 2^{+-}$ by constructing the interpolating tetraquark currents with three Lorentz indices using the method of QCD sum rules.

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This paper is organized as follows. In Sec. II we construct the three-Lorentz-indices interpolating tetraquark currents that can couple to physical hadron states with $J^{PC}=2^{+-}$ and compose the projector operator to extract the invariant functions. In Sec. III we calculate the correlation functions and spectral densities for these interpolating currents and establish the mass sum rules for the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark systems. In Sec. V we perform numerical analyses to obtain the tetraquark mass spectra. The last section contains a brief summary and discussion.

II. INTERPOLATING CURRENTS AND PROJECTORS

In this section, we construct the diquark-antidiquark interpolating currents coupling to the light tetraquark states with $J^{PC}=2^{+-}$. As a matter of fact, one cannot construct a $J^{PC}=2^{+-}$ current with two Lorentz indices by using the Dirac gamma matrices only. To construct a current without a derivative operator in such a channel, one needs to consider the interpolating current with more than two Lorentz indices [67]. We consider the diquark fields $q_a^TC\gamma_5q_b$, $q_a^TCq_b$, $q_a^TC\gamma_\mu\gamma_5q_b$, $q_a^TC\gamma_\mu q_b$, $u_a^TC\sigma_{\mu\nu}q_b$, $q_a^TC\sigma_{\mu\nu}\gamma_5q_b$ and the corresponding antidiquark fields, where a, b are color indices and T is the transpose of the matrices.

The spin parities of the diquark fields with various Lorentz structures are shown in Table I. One can construct a tetraquark operator as

$$O_{ij}=(q_a^TC\Gamma_iq_b)(\bar{q}_a\Gamma_jC\bar{q}_b^T), \tag{1}$$

where the color structures of the diquark and antidiquark fields depend on their Lorentz structures. It is easy to find the following identity under the charge-conjugation transform [79]:

TABLE I. Spins and parities of the diquark fields.

$q^TC\Gamma q$	J^P	States
$q_a^TC\gamma_5q_b$	0^+	$^1S_0$
$q_a^TCq_b$	0^-	$^3P_0$
$q_a^TC\gamma_\mu q_b$	1^+	$^3S_1$
$q_a^TC\gamma_\mu\gamma_5q_b$	0^+ if $\mu=0$	$^1S_0$
	1^- if $\mu=1,2,3$	$^3P_1$
$q_a^TC\sigma_{\mu\nu}q_b$	1^- if $\mu,\nu=1,2,3$	$^1P_1$
	1^+ if $\mu=0,\nu=1,2,3$	$^3S_1$
$q_a^TC\sigma_{\mu\nu}\gamma_5q_b$	1^+ if $\mu,\nu=1,2,3$	$^3S_1$
	1^- if $\mu=0,\nu=1,2,3$	$^1P_1$

$$\mathbb{C}O_{ij}\mathbb{C}^{-1}=O_{ij}^T. \tag{2}$$

One can find the tetraquark operators with even and odd C parities as

$$S=O_{ij}+O_{ij}^T,\quad A=O_{ij}-O_{ij}^T. \tag{3}$$

Considering a tetraquark operator with two symmetric Lorentz indices $O_{ij}=O_{\{\mu,\nu\}}$, it can couple to the spin-2 hadron state with the following possible Lorentz structures without derivatives:

$$\begin{aligned}\{\Gamma_i,\Gamma_j\}&=\{\gamma_\mu,\gamma_\nu\},\{\gamma_\mu\gamma_5,\gamma_\nu\gamma_5\},\{\sigma_{\mu\alpha},\sigma_{\alpha\nu}\},\\&\times\{\sigma_{\mu\alpha}\gamma_5,\sigma_{\alpha\nu}\gamma_5\},\{\gamma_\mu\gamma_5,\gamma_\nu\},\{\sigma_{\mu\alpha}\gamma_5,\sigma_{\alpha\nu}\}.\end{aligned} \tag{4}$$

However, all of these tetraquark operators cannot achieve the exotic quantum numbers $J^{PC}=2^{+-}$. In this work, we construct the following 2^{+-} interpolating tetraquark currents with three Lorentz indices:

$$\begin{aligned}
J^1_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T-\bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma_\alpha C\bar{d}_b^T-\bar{u}_b\gamma_\alpha C\bar{d}_a^T),\\
J^{1\prime}_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T+\bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma_\alpha C\bar{d}_b^T+\bar{u}_b\gamma_\alpha C\bar{d}_a^T),\\
J^2_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T-\bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma_\alpha\gamma_5 C\bar{d}_b^T-\bar{u}_b\gamma_\alpha\gamma_5 C\bar{d}_a^T),\\
J^{2\prime}_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}C\bar{d}_b^T+\bar{u}_b\sigma_{\mu\nu}C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}d_b(\bar{u}_a\gamma_\alpha\gamma_5 C\bar{d}_b^T+\bar{u}_b\gamma_\alpha\gamma_5 C\bar{d}_a^T),\\
J^3_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T-\bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma_\alpha C\bar{d}_b^T-\bar{u}_b\gamma_\alpha C\bar{d}_a^T),\\
J^{3\prime}_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T+\bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma_\alpha C\bar{d}_b^T+\bar{u}_b\gamma_\alpha C\bar{d}_a^T),\\
J^4_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T-\bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma_\alpha\gamma_5C\bar{d}_b^T-\bar{u}_b\gamma_\alpha\gamma_5C\bar{d}_a^T),\\
J^{4\prime}_{\alpha\mu\nu}&=u_a^TC\gamma_\alpha\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5C\bar{d}_b^T+\bar{u}_b\sigma_{\mu\nu}\gamma_5C\bar{d}_a^T)-u_a^TC\sigma_{\mu\nu}\gamma_5d_b(\bar{u}_a\gamma_\alpha\gamma_5C\bar{d}_b^T+\bar{u}_b\gamma_\alpha\gamma_5C\bar{d}_a^T),
\end{aligned} \tag{5}$$

in which the currents $J^1_{\alpha\mu\nu}$, $J^2_{\alpha\mu\nu}$, $J^3_{\alpha\mu\nu}$, and $J^4_{\alpha\mu\nu}$ have the color structure $\mathbf{3}\otimes\bar{\mathbf{3}}$, while $J^{1\prime}_{\alpha\mu\nu}$, $J^{2\prime}_{\alpha\mu\nu}$, $J^{3\prime}_{\alpha\mu\nu}$, and $J^{4\prime}_{\alpha\mu\nu}$ have the color structure $\bar{\mathbf{6}}\otimes\mathbf{6}$.

For the fully strange $ss\bar{s}\,\bar{s}$ tetraquark systems, only two interpolating currents survive since both of the symmetric flavor structures

$$J^{s1}_{\alpha\mu\nu}=s^T_aC\gamma_\alpha s_b\left(\bar{s}_a\sigma_{\mu\nu}C\bar{s}^T_b\right)-s^T_aC\sigma_{\mu\nu}s_b\left(\bar{s}_a\gamma_\alpha C\bar{s}^T_b\right),$$
$$J^{s3}_{\alpha\mu\nu}=s^T_aC\gamma_\alpha s_b\left(\bar{s}_a\sigma_{\mu\nu}\gamma_5C\bar{s}^T_b\right)-s^T_aC\sigma_{\mu\nu}\gamma_5s_b\left(\bar{s}_a\gamma_\alpha C\bar{s}^T_b\right)\quad(6)$$

have the antisymmetric color structure $\mathbf{3}\otimes\bar{\mathbf{3}}$.

To investigate the physical states with definite quantum numbers, we consider the couplings between the interpolating current and different hadron states as follows:

$$\langle 0|J_{\alpha\mu\nu}|0^{(-P)C}(p)\rangle=Z^0_1p_\alpha g_{\mu\nu}+Z^0_2p_\mu g_{\alpha\nu}+Z^0_3p_\nu g_{\alpha\mu}+Z^0_4p_\alpha p_\mu p_\nu,\quad(7)$$

$$\langle 0|J_{\alpha\mu\nu}|0^{PC}(p)\rangle=Z^0_5\varepsilon_{\alpha\mu\nu\tau}p^\tau,\quad(8)$$

$$\langle 0|J_{\alpha\mu\nu}|1^{PC}(p)\rangle=Z^1_1\epsilon_\alpha g_{\mu\nu}+Z^1_2\epsilon_\mu g_{\alpha\nu}+Z^1_3\epsilon_\nu g_{\alpha\mu}+Z^1_4\epsilon_\alpha p_\mu p_\nu+Z^1_5\epsilon_\mu p_\alpha p_\nu+Z^1_6\epsilon_\nu p_\alpha p_\mu,\quad(9)$$

$$\langle 0|J_{\alpha\mu\nu}|1^{(-P)C}(p)\rangle=Z^1_7\varepsilon_{\alpha\mu\nu\tau}\epsilon^\tau+Z^1_8\varepsilon_{\alpha\mu\tau\lambda}\epsilon^\tau p^\lambda p_\nu+Z^1_9\varepsilon_{\alpha\nu\tau\lambda}\epsilon^\tau p^\lambda p_\mu,\quad(10)$$

$$\langle 0|J_{\alpha\mu\nu}|2^{(-P)C}(p)\rangle=Z^2_1\epsilon_{\alpha\mu}p_\nu+Z^2_2\epsilon_{\alpha\nu}p_\mu+Z^2_3\epsilon_{\mu\nu}p_\alpha,\quad(11)$$

$$\langle 0|J_{\alpha\mu\nu}|2^{PC}(p)\rangle=Z^2_4\varepsilon_{\alpha\mu\tau\theta}\epsilon_\nu{}^\tau p^\theta+Z^2_5\varepsilon_{\alpha\nu\tau\theta}\epsilon_\mu{}^\tau p^\theta,\quad(12)$$

$$\langle 0|J_{\alpha\mu\nu}|3^{PC}(p)\rangle=Z^3_1\epsilon_{\alpha\mu\nu},\quad(13)$$

where ϵ_α, $\epsilon_{\alpha\mu}$, $\epsilon_{\alpha\mu\nu}$ are the polarization tensors for the spin-1, spin-2, and spin-3 states, respectively. It should be noted that the interpolating currents in Eqs. (5) and (6) cannot couple to any spin-3 state since their last two Lorentz indices are antisymmetric, while the spin-3 polarization tensor $\epsilon_{\alpha\mu\nu}$ is completely symmetric.

Since the parities for the currents $J^1_{\alpha\mu\nu}$, $J^{1\prime}_{\alpha\mu\nu}$, $J^4_{\alpha\mu\nu}$, $J^{4\prime}_{\alpha\mu\nu}$, $J^{s1}_{\alpha\mu\nu}$ and $J^2_{\alpha\mu\nu}$, $J^{2\prime}_{\alpha\mu\nu}$, $J^3_{\alpha\mu\nu}$, $J^{3\prime}_{\alpha\mu\nu}$, $J^{s3}_{\alpha\mu\nu}$ are opposite, they couple to the 2^{+-} tetraquark states via different coupling relations in Eqs. (11) and (12), respectively. For the currents $J^{2,2\prime,3,3\prime,s3}_{\alpha\mu\nu}$, we can rewrite the coupling in another way,

$$\begin{aligned}\langle 0|J^{2,2\prime,3,3\prime,s3}_{\alpha\mu\nu}|2^{+-}(p)\rangle&=Z^2_4\varepsilon_{\alpha\mu\tau\theta}\epsilon_\nu{}^\tau p^\theta+Z^2_5\varepsilon_{\alpha\nu\tau\theta}\epsilon_\mu{}^\tau p^\theta\\&=f^+\left(\varepsilon_{\alpha\mu\tau\theta}\epsilon_\nu{}^\tau p^\theta+\varepsilon_{\alpha\nu\tau\theta}\epsilon_\mu{}^\tau p^\theta\right)\\&\quad+f^-\left(\varepsilon_{\alpha\mu\tau\theta}\epsilon_\nu{}^\tau p^\theta-\varepsilon_{\alpha\nu\tau\theta}\epsilon_\mu{}^\tau p^\theta\right)\\&=f^-\left(\varepsilon_{\alpha\mu\tau\theta}\epsilon_\nu{}^\tau p^\theta-\varepsilon_{\alpha\nu\tau\theta}\epsilon_\mu{}^\tau p^\theta\right),\end{aligned}\quad(14)$$

in which the Lorentz indices $\mu\nu$ are antisymmetric in the last step to be consistent with those in the interpolating currents. One can construct the normalized projector operator for the 2^{+-} state,

$$\begin{aligned}\mathbb{P}(\alpha_1,\mu_1,\nu_1,\alpha_2,\mu_2,\nu_2)=\frac{1}{20}\sum&(\varepsilon_{\alpha_1\mu_1\tau_1\theta_1}\epsilon_{\nu_1}{}^{\tau_1}p^{\theta_1}\\&-\varepsilon_{\alpha_1\nu_1\tau_1\theta_1}\epsilon_{\mu_1}{}^{\tau_1}p^{\theta_1})(\varepsilon_{\alpha_2\mu_2\tau_2\theta_2}\epsilon_{\nu_2}{}^{\tau_2*}p^{\theta_2}\\&-\varepsilon_{\alpha_2\nu_2\tau_2\theta_2}\epsilon_{\mu_2}{}^{\tau_2*}p^{\theta_2})/p^2,\end{aligned}\quad(15)$$

where the summation over the polarization of the tensor $\epsilon_{\alpha\beta}$ is

$$\sum\epsilon_{\alpha_1\beta_1}\epsilon^*_{\alpha_2\beta_2}=\frac{1}{2}\left(\eta_{\alpha_1\alpha_2}\eta_{\beta_1\beta_2}+\eta_{\alpha_1\beta_2}\eta_{\beta_1\alpha_2}-\frac{2}{3}\eta_{\alpha_1\beta_1}\eta_{\alpha_2\beta_2}\right),\quad(16)$$

with

$$\eta_{\alpha\beta}=\frac{p_\alpha p_\beta}{p^2}-g_{\alpha\beta}.\quad(17)$$

One may wonder whether there should be a coupling corresponding to the Lorentz structure $\varepsilon_{\mu\nu\tau\theta}\epsilon^\tau_\alpha p^\theta$ in Eq. (12). As a matter of fact, the projector constructed using this structure is the same as that from the antisymmetric part in Eq. (14), so that there are only two independent tensor structures in Eq. (14).

For the currents $J^{1,1\prime,4,4\prime,s1}_{\alpha\mu\nu}$, the coupling relation in Eq. (11) can be rewritten as

$$\begin{aligned}\langle 0|J^{1,1\prime,4,4\prime,s1}_{\alpha\mu\nu}|2^{+-}(p)\rangle&=Z^2_1\epsilon_{\alpha\mu}p_\nu+Z^2_2\epsilon_{\alpha\nu}p_\mu+Z^2_3\epsilon_{\mu\nu}p_\alpha\\&=Z^2_1\epsilon_{\alpha\mu}p_\nu+Z^2_2\epsilon_{\alpha\nu}p_\mu\\&=f^{-\prime}(\epsilon_{\alpha\mu}p_\nu-\epsilon_{\alpha\nu}p_\mu),\end{aligned}\quad(18)$$

with the normalized projector operator

$$\mathbb{P}'(\alpha_1,\mu_1,\nu_1,\alpha_2,\mu_2,\nu_2)=\frac{1}{20}\sum(\epsilon_{\alpha_1\mu_1}p_{\nu_1}-\epsilon_{\alpha_1\nu_1}p_{\mu_1})\times(\epsilon_{\alpha_2\mu_2}p_{\nu_2}-\epsilon_{\alpha_2\nu_2}p_{\mu_2})/p^2.\quad(19)$$

In our calculations, we find that the 2^{+-} tetraquark states extracted from the currents $J^{1,1\prime,2,2\prime,s1}_{\alpha\mu\nu}$ are the same as those from $J^{3,3\prime,4,4\prime,s3}_{\alpha\mu\nu}$, respectively. In the following analyses, we consider only the interpolating currents $J^1_{\alpha\mu\nu}$, $J^{1\prime}_{\alpha\mu\nu}$, $J^2_{\alpha\mu\nu}$, $J^{2\prime}_{\alpha\mu\nu}$, and $J^{s1}_{\alpha\mu\nu}$ to investigate the mass spectra of the 2^{+-} tetraquark states.

III. FORMALISM OF QCD SUM RULES

The two-point correlation function of the current in Eq. (5) can be written as

$$\Pi_{\alpha_1\mu_1\nu_1,\alpha_2\mu_2\nu_2}(p^2)=i\int d^4xe^{ip\cdot x}\langle 0|T[J_{\alpha_1\mu_1\nu_1}(x)J^\dagger_{\alpha_2\mu_2\nu_2}(0)]|0\rangle.\quad(20)$$

We investigate only the $J^{PC} = 2^{+-}$ tetraquark states in this work, which can be extracted by applying the projection operator defined in Eq. (19),

$$\Pi_2(p^2) = \mathbb{P}^{(t)}(\alpha_1,\mu_1,\nu_1,\alpha_2,\mu_2,\nu_2)\Pi_{\alpha_1\mu_1\nu_1,\alpha_2\mu_2\nu_2}(p^2). \quad (21)$$

At the hadronic level, the correlation function $\Pi_2(p^2)$ can usually be described via the dispersion relation

$$\Pi_2(p^2) = \frac{(p^2)^N}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_2(s)}{s^N(s-p^2-i\epsilon)}ds + \sum_{n=0}^{N-1} b_n(p^2)^n, \quad (22)$$

where b_n is the subtraction constant. In QCD sum rules, the imaginary part of the correlation function is defined as the spectral function

$$\rho(s) \equiv \frac{1}{\pi}\mathrm{Im}\Pi_2(s) = f^2 m_H^2 \delta(s - m_H^2) + \text{QCD continuum and higher states}, \quad (23)$$

in which the "one pole plus continuum" parametrization assumption is used. The parameters f and m_H are the coupling constant and mass of the lowest-lying hadron state H, respectively.

To improve the convergence of the operator product expansion (OPE) series and suppress the contributions from continuum and higher states, a Borel transformation can be performed on the correlation functions at both the hadron and quark-gluon levels. The QCD sum rules are then obtained as

$$\Pi_2(s_0, M_B^2) = f^2 m_H^2 e^{-m_H^2/M_B^2} = \int_0^{s_0} ds e^{-s/M_B^2}\rho(s), \quad (24)$$

where M_B is the Borel mass introduced via the Borel transformation and s_0 is the continuum threshold. Since the Borel mass is an intermediate parameter, it should not be relevant to the physical state. These two parameters can be determined by requiring a suitable OPE convergence and a big enough pole contribution in the QCD sum rule analyses. Then, the hadron mass of the lowest-lying tetraquark state can be extracted as

$$m_H(s_0, M_B^2) = \sqrt{\frac{\frac{\partial}{\partial(-1/M_B^2)}\Pi_2(s_0, M_B^2)}{\Pi_2(s_0, M_B^2)}}. \quad (25)$$

We evaluate the correlation functions for the light tetraquark states with $J^{PC} = 2^{+-}$ up to dimension-ten condensates. The contribution from operators with higher dimensions are extremely small and we do not take them into consideration in this work. We list the results in the Appendix since these expressions are relatively complicated. For the nonstrange $ud\bar{u}\bar{d}$ systems, we neglect the masses of light quarks in the chiral limit so that there is no contribution from odd-dimensional condensates, such as the quark condensates and quark-gluon mixed condensates. For the $us\bar{u}\bar{s}$ and $ss\bar{s}\bar{s}$ systems, the strange quark mass is taken into consideration. For the high-dimension condensates, we use the factorization assumptions and set the factors to be 1 in our analyses.

IV. NUMERICAL ANALYSES AND MASS PREDICTIONS

In this section, we perform the QCD sum rule analyses for the light exotic tetraquark states with $J^{PC} = 2^{+-}$. We use the following values for various QCD parameters [3,80–82]:

$$\begin{aligned}
m_u &= m_d = m_q = 0, \\
m_s &= 93^{+11}_{-5}\ \text{MeV}, \\
\langle\bar{q}q\rangle &= -(0.24 \pm 0.01)^3\ \text{GeV}^3, \\
\langle\bar{s}s\rangle &= (0.8 \pm 0.1) \times \langle\bar{q}q\rangle, \\
\langle g_s\bar{q}\sigma Gq\rangle &= -(0.8 \pm 0.2) \times \langle\bar{q}q\rangle\ \text{GeV}^2, \\
\langle g_s\bar{s}\sigma Gs\rangle &= (0.8 \pm 0.2) \times \langle g_s\bar{q}\sigma Gq\rangle, \\
\langle\alpha_s GG\rangle &= (6.35 \pm 0.35) \times 10^{-2}\ \text{GeV}^4. \quad (26)
\end{aligned}$$

The above condensate values are taken at the energy renormalization scale $\mu = 1$ GeV, while the s-quark mass is taken at $\mu = 2$ GeV. To maintain energy-renormalization-scale consistency, we use the renormalization group to run the s-quark mass at $\mu = 1$ GeV, as adopted in Ref. [83].

We take $J^1_{\alpha\mu\nu}$ as an example to show the details of our numerical analyses for the nonstrange $ud\bar{u}\bar{d}$ tetraquark state. To extract the output parameters, the Borel parameter M_B^2 should be large enough to guarantee the convergence of OPE series. We require the contribution from the high-dimension $D > 8$ condensates to be less than 1%, i.e.,

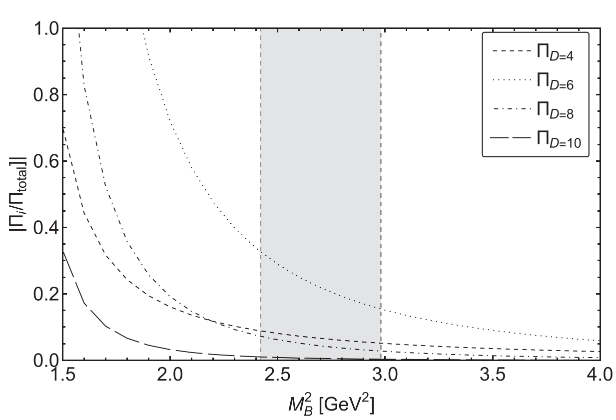


FIG. 1. OPE convergence for the $ud\bar{u}\bar{d}$ tetraquark state with $J^{PC} = 2^{+-}$ extracted from the current $J^1_{\alpha\mu\nu}$.

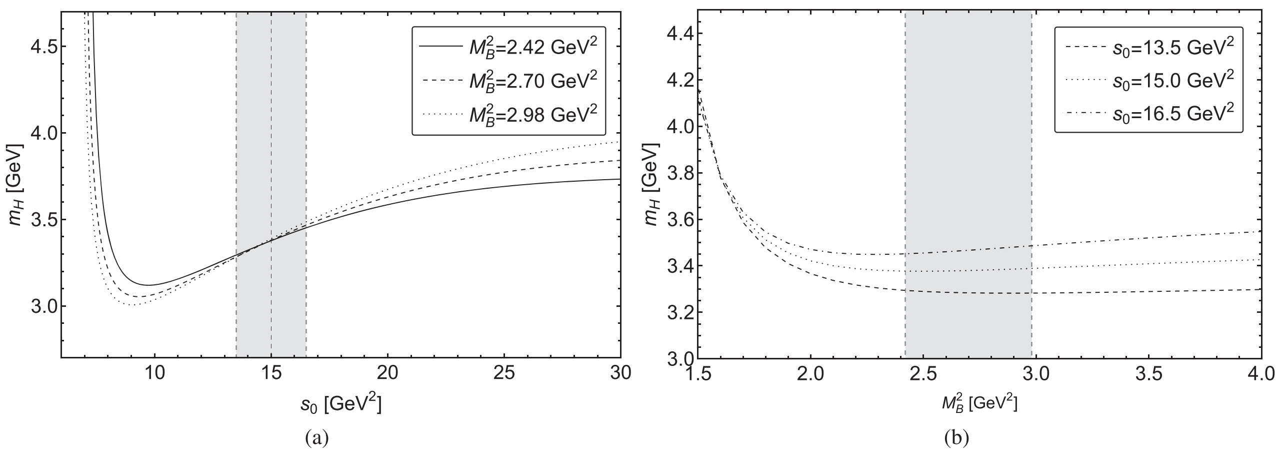


FIG. 2. Variation of the tetraquark mass m_H with (a) the continuum threshold s_0 and (b) the Brorel mass M_B^2, corresponding to the $J^{PC} = 2^{+-}$ $ud\bar{u}\bar{d}$ tetraquark state extracted from current $J^1_{\alpha\mu\nu}$.

$$R_{D>8} = \left| \frac{\Pi^{D>8}(M_B^2, \infty)}{\Pi^{\text{tot}}(M_B^2, \infty)} \right| < 1\%. \tag{27}$$

This requirement leads to the lower bound on the Borel parameter $M_B^2 \geq 2.42$ GeV^2. We show the contribution ratios from various condensates in Fig. 1, from which one finds that the convergence of OPE series can be well ensured. In this $ud\bar{u}\bar{d}$ tetraquark system, the dominant nonperturbative effect comes from the dimension-six four-quark condensate $\langle\bar{q}q\rangle^2$, since the contributions from the quark condensate $\langle\bar{q}q\rangle$ and quark-gluon mixed condensate $\langle g_s\bar{q}\sigma Gq\rangle$ vanish in the chiral limit. To get the upper bound on M_B^2, we first need to fix the value of s_0. As mentioned in Sec. III, the hadron mass m_H should be irrelevant to the intermediate parameter M_B^2. In Fig. 2(a), we show the variations of m_H with respect to the continuum threshold s_0 for various values of the Borel parameter M_B^2. It is shown that the variation of m_H with M_B^2 can be minimized in the working parameter region $13.5 \leq s_0 \leq 16.5$ GeV^2. Then, the upper bound on M_B^2 can be determined by requiring the following pole contribution to be larger than 50%:

$$\text{Pole Contribution} = \frac{\Pi(M_B^2, s_0)}{\Pi(M_B^2, \infty)} > 50\%. \tag{28}$$

Finally, the working region of the Borel parameter can be determined to be $2.42 \leq M_B^2 \leq 2.98$ GeV^2. We show the Borel curves in the above working parameter regions in Fig. 2(b), in which the QCD sum rules are stable enough to predict the hadron mass of the $ud\bar{u}\bar{d}$ tetraquark state as

$$m^1_{ud\bar{u}\bar{d}} = 3.38^{+0.13}_{-0.12}\ \text{GeV}. \tag{29}$$

The errors are mainly from the uncertainties of the continuum threshold s_0, various QCD condensates $\langle\bar{q}q\rangle$, $\langle\alpha_s GG\rangle$, and $\langle g_s\bar{q}\sigma Gq\rangle$. The error from the Borel mass is small enough to be neglected.

Replacing d with the s-quark field, one can perform similar QCD sum rule calculations and analyses for the hidden-strange $us\bar{u}\bar{s}$ tetraquark systems. As mentioned in Sec. IV, the correlation functions for the $us\bar{u}\bar{s}$ system contain the contributions from the dimension-three quark condensates and dimension-five quark-gluon mixed condensates. In Fig. 3, we show the OPE convergence for the $us\bar{u}\bar{s}$ tetraquark state from the current $J^1_{\alpha\mu\nu}$, from which one finds that the dominant nonperturbative effect is still from the four-quark condensates. However, the contribution from the quark condensates is significant and even larger than the four-quark condensates for large values of M_B^2. This is very different from the situation in the $ud\bar{u}\bar{d}$ system, where the quark condensates and quark-gluon mixed condensates give no contribution to the correlation function.

For the $us\bar{u}\bar{s}$ system with $J^1_{\alpha\mu\nu}$, the working parameter regions can be obtained as $2.51 \leq M_B^2 \leq 3.13$ GeV^2 and $14.5 \leq s_0 \leq 17.5$ GeV^2 after similar numerical analyses,

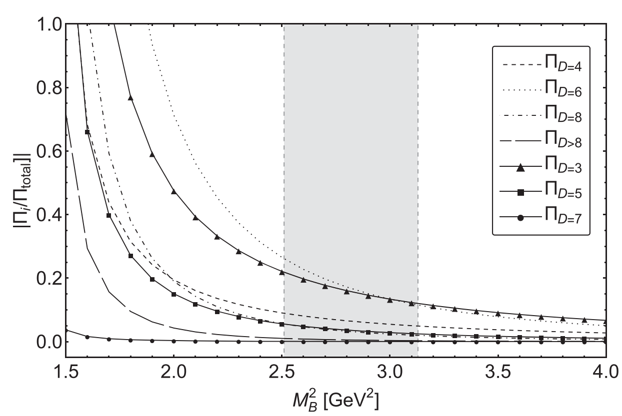


FIG. 3. OPE convergence for the $us\bar{u}\bar{s}$ tetraquark state with $J^{PC} = 2^{+-}$ extracted from the current $J^1_{\alpha\mu\nu}$.

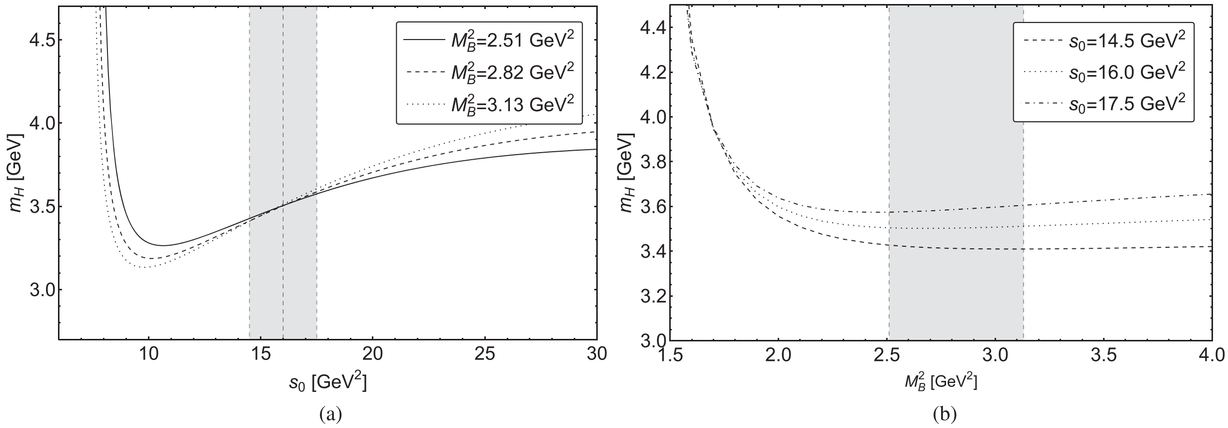


FIG. 4. Variation of the tetraquark mass m_H with (a) the continuum threshold s_0 and (b) the squared Brorel mass M_B^2, corresponding to the $J^{PC} = 2^{+-}$ $us\bar{u}\bar{s}$ tetraquark state extracted from current $J^1_{\alpha\mu\nu}$.

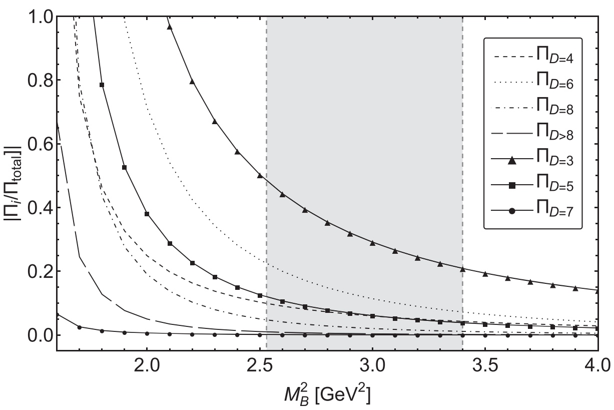


FIG. 5. OPE convergence for the $ss\bar{s}\bar{s}$ tetraquark state with $J^{PC} = 2^{+-}$ extracted from the current $J^{s1}_{\alpha\mu\nu}$.

where the Borel window is slightly broader than that for the $ud\bar{u}\bar{d}$ system. Then, the hadron mass can be predicted as

$$m^1_{us\bar{u}\bar{s}} = 3.50^{+0.13}_{-0.12}\ \text{GeV}, \tag{30}$$

which is about 100 MeV higher than the $ud\bar{u}\bar{d}$ tetraquark state. We show the corresponding mass curves in Fig. 4.

For the fully strange system, we show the OPE convergence for the $ss\bar{s}\bar{s}$ tetraquark state from the current $J^{s1}_{\alpha\mu\nu}$ in Fig. 5, from which one finds that the dominant nonperturbative effect is from the dimension-three quark condensate rather than from the four-quark condensates, and the dimension-five quark-gluon mixed condensate also plays an important role in the numerical analysis. For this current, the working parameter regions can be obtained as $2.53 \le M_B^2 \le 3.40$ GeV^2 and $16.0 \le s_0 \le 19.0$ GeV^2. After similar numerical analyses, the hadron mass can be predicted as

$$m_{ss\bar{s}\bar{s}} = 3.66^{+0.10}_{-0.09}\ \text{GeV}, \tag{31}$$

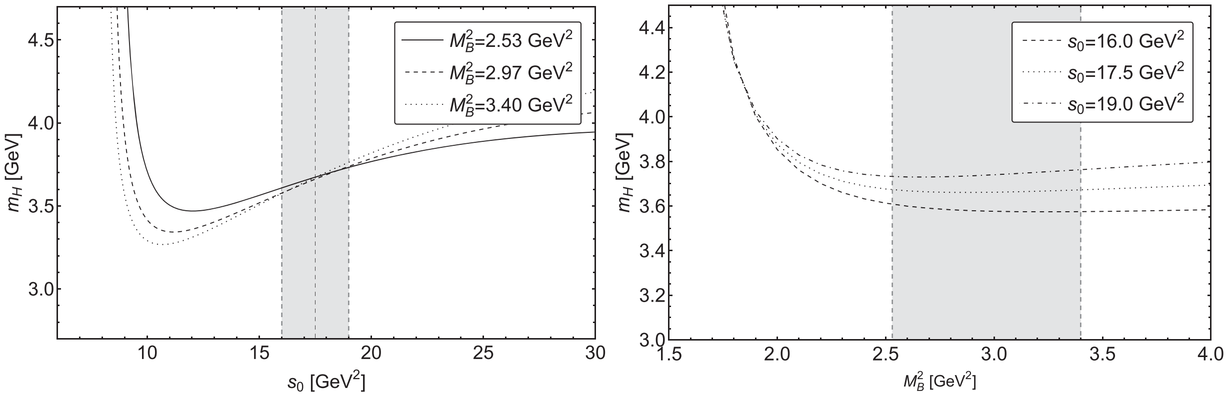
which is about 100 MeV higher than the $us\bar{u}\bar{s}$ tetraquark state. We show the corresponding mass curves in Fig. 6. For all interpolating currents in Eqs. (5) and (6), we collect the numerical results for the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 2^{+-}$ in Table II.

V. CONCLUSION AND DISCUSSION

We investigated the mass spectra of the light tetraquark states $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ with exotic quantum number $J^{PC} = 2^{+-}$ using QCD sum rules by constructing the interpolating currents with three Lorentz indices. We evaluated the correlation functions and spectral functions up to dimension-ten condensates. Our results show that the most important nonperturbative contributions come from the dimension-six four-quark condensates and dimension-three quark condensate for the $ud\bar{u}\bar{d}$ and $ss\bar{s}\bar{s}$ tetraquark systems, respectively. For the $us\bar{u}\bar{s}$ system, the contributions from the quark condensates and four-quark condensates are comparable.

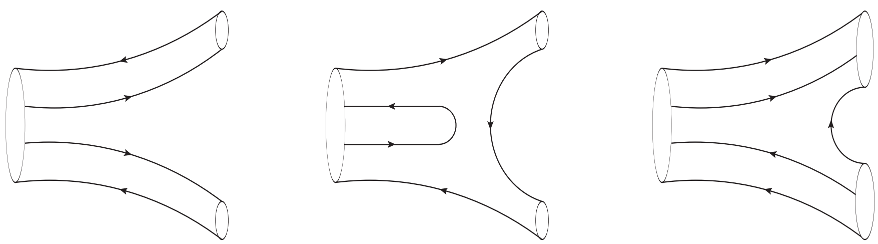
TABLE II. Predicted tetraquark masses and the corresponding parameters for all interpolating currents. The pole contributions are evaluated at the central values of s_0 and M_B^2.

	Current	s_0 (GeV^2)	M_B^2 (GeV^2)	Mass (GeV)	Pole contribution (%)
$ud\bar{u}\bar{d}$	$J^1_{\alpha\mu\nu}$	15.0 ± 1.5	2.42–2.98	$3.38^{+0.13}_{-0.12}$	57.9
	$J^{1\prime}_{\alpha\mu\nu}$	16.5 ± 1.5	2.21–3.33	$3.51^{+0.11}_{-0.10}$	64.7
	$J^2_{\alpha\mu\nu}$	15.0 ± 1.5	2.36–2.98	$3.39^{+0.13}_{-0.12}$	58.6
	$J^{2\prime}_{\alpha\mu\nu}$	14.5 ± 1.5	2.45–2.87	$3.34^{+0.14}_{-0.12}$	55.8
$us\bar{u}\bar{s}$	$J^1_{\alpha\mu\nu}$	16.0 ± 1.5	2.51–3.13	$3.50^{+0.13}_{-0.12}$	58.4
	$J^{1\prime}_{\alpha\mu\nu}$	18.0 ± 1.5	2.36–3.60	$3.66^{+0.11}_{-0.10}$	65.1
	$J^2_{\alpha\mu\nu}$	16.5 ± 1.5	2.46–3.25	$3.55^{+0.12}_{-0.11}$	60.3
	$J^{2\prime}_{\alpha\mu\nu}$	16.0 ± 1.5	2.53–3.14	$3.50^{+0.12}_{-0.12}$	58.2
$ss\bar{s}\bar{s}$	$J^{s1}_{\alpha\mu\nu}$	17.5 ± 1.5	2.53–3.40	$3.66^{+0.10}_{-0.09}$	60.9



(a) (b)

FIG. 6. Variation of the tetraquark mass m_H with (a) the continuum threshold s_0 and (b) the squared Brorel mass M_B^2, corresponding to the $J^{PC} = 2^{+-}$ $ss\bar{s}\bar{s}$ tetraquark state extracted from current $J^{s1}_{\alpha\mu\nu}$.



(a) (b) (c)

FIG. 7. Three possible strong decay mechanisms of the 2^{+-} light tetraquark states: (a) spontaneous dissociation mechanism, (b) annihilation mechanism, (c) baryon-antibaryon decay channels.

The isospin can be $I = 0$, 1, 2 for the nonstrange $ud\bar{u}\bar{d}$ system, $I = 0$, 1 for the $us\bar{u}\bar{s}$ system, and $I = 0$ for the fully strange $ss\bar{s}\bar{s}$ system. In the SU(2) symmetry, we did not differentiate the up and down quarks in our calculations so that the states in the same tetraquark system with different isospins are degenerate. The predicted hadron masses for the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 2^{+-}$ are about 3.3–3.5, 3.5–3.7, and 3.67 GeV, respectively.

The 2^{+-} tetraquarks can decay into the two-meson final states via the strong interaction in the spontaneous dissociation mechanism and annihilation mechanism, as depicted in Figs. 7(a) and 7(b), respectively. In Table III, we list some possible two-meson decay modes for these $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquarks with different I^G quantum numbers. It is clear that all of the final states are P-wave mesons for the S-wave decay modes, while they are P-wave plus S-wave mesons for the P-wave decay modes. For the D-wave decay modes, the final states can be all S-wave mesons, such as the $\rho\pi, \omega\pi, \phi\pi$, and $K\bar{K}^*$ channels. Such peculiar decay properties may result in relatively narrow decay widths for these 2^{+-} tetraquark states.

TABLE III. Some possible two-meson decay modes for the tetraquarks with $I^G(J^{PC}) = 0^-(2^{+-})$, $1^+(2^{+-})$ and $2^-(2^{+-})$.

$I^G(J^{PC})$	$0^-(2^{+-})$	$1^+(2^{+-})$	$2^-(2^{+-})$
S-wave	$K_0^*\bar{K}_2^*$, $a_{1,2}b_1$, $f_{1,2}h_1$	$K_0^*\bar{K}_2^*$, $a_{1,2}h_1$, $f_{1,2}b_1$	$a_{1,2}b_1$
P-wave	$K\bar{K}_1$, $K\bar{K}_2^*$, $K^*\bar{K}_0^*$, $K^*\bar{K}_1$, $K^*\bar{K}_2^*$, $h_1\eta$, $b_1\pi$, $f_{0,1,2}\omega$, $f_{0,1,2}\phi$, $a_{0,1,2}\rho$	$K\bar{K}_1$, $K\bar{K}_2^*$, $K^*\bar{K}_0^*$, $K^*\bar{K}_1$, $K^*\bar{K}_2^*$, $b_1\eta^{(\prime)}$, $h_1\pi$, $f_{0,1,2}\rho$, $a_{0,1,2}\omega$, $a_{1,2}\pi$, $b_1\rho$	$b_1\pi$, $a_{0,1,2}\rho$
D-wave	$K\bar{K}^*$, $\rho\pi$, $\omega\eta^{(\prime)}$, $\phi\eta^{(\prime)}$	$K\bar{K}^*$, $\omega\pi$, $\rho\eta^{(\prime)}$, $\phi\pi$	$\rho\pi$

As shown in Fig. 7(c), the predicted tetraquark masses in Table II also allow some baryon-antibaryon decay channels by the creation of a light quark-antiquark pair, so that the $ud\bar{u}\bar{d}$, $us\bar{u}\bar{s}$, and $ss\bar{s}\bar{s}$ tetraquark states with $J^{PC} = 2^{+-}$ may be observed in the $\Delta\bar{\Delta}$, $\Sigma^*\bar{\Sigma}^*$, $\Xi^*\bar{\Xi}^*$, and $\Omega\bar{\Omega}$ decay modes. We suggest searching for these 2^{+-} light tetraquark states in the $\rho\pi, \omega\pi, \phi\pi$, $b_1\pi$, $h_1\pi$, $K\bar{K}^*, K\bar{K}_1$, $\Delta\bar{\Delta}$, $\Sigma^*\bar{\Sigma}^*$, $\Xi^*\bar{\Xi}^*$, and $\Omega\bar{\Omega}$ channels in future experiments such as BESIII, BelleII, GlueX, LHCb, and so on.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under Grants No. 12305147 and

No. 12175318, the National Key R&D Program of China under Contracts No. 2020YFA0406400, and the Natural Science Foundation of Guangdong Province of China under Grant No. 2022A1515011922.

APPENDIX: EXPRESSIONS OF CORRELATION FUNCTIONS

In this appendix, we show the expressions for the correlation functions for the interpolating currents $J^{1}_{\alpha\mu\nu}$, $J^{1\prime}_{\alpha\mu\nu}$, $J^{2}_{\alpha\mu\nu}$, $J^{2\prime}_{\alpha\mu\nu}$, and $J^{s1}_{\alpha\mu\nu}$. For the nonstrange $ud\bar{u}\,\bar{d}$ tetraquark system, the correlation functions after Borel transformation are

$$\Pi^{1}_{d}(M_B^2,s_0)=\int_0^{s_0}\left(\frac{s^4}{26880\pi^6}-\frac{\langle\alpha_sGG\rangle s^2}{1152\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{6\pi^2}-\frac{\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{9\pi^2}\right)e^{-\frac{s}{M_B^2}}ds+\frac{5\langle g_s\bar{q}\sigma Gq\rangle^2}{144\pi^2}+\frac{\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{27\pi},\tag{A1}$$

$$\Pi^{1'}_{d}(M_B^2,s_0)=\int_0^{s_0}\left(\frac{s^4}{13440\pi^6}-\frac{\langle\alpha_sGG\rangle s^2}{576\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{3\pi^2}-\frac{5\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{36\pi^2}\right)e^{-\frac{s}{M_B^2}}ds+\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{36\pi^2}+\frac{2\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{27\pi},\tag{A2}$$

$$\Pi^{2}_{d}(M_B^2,s_0)=\int_0^{s_0}\left(\frac{s^4}{26880\pi^6}-\frac{\langle\alpha_sGG\rangle s^2}{1152\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{6\pi^2}-\frac{7\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{72\pi^2}\right)e^{-\frac{s}{M_B^2}}ds+\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{36\pi^2}+\frac{\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{27\pi},\tag{A3}$$

$$\Pi^{2'}_{d}(M_B^2,s_0)=\int_0^{s_0}\left(\frac{s^4}{13440\pi^6}-\frac{\langle\alpha_sGG\rangle s^2}{576\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{3\pi^2}-\frac{17\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{72\pi^2}\right)e^{-\frac{s}{M_B^2}}ds+\frac{11\langle g_s\bar{q}\sigma Gq\rangle^2}{144\pi^2}+\frac{2\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{27\pi}.\tag{A4}$$

For the $us\bar{u}\,\bar{s}$ tetraquark system, the correlation functions after Borel transformation are

$$\begin{aligned}\Pi^{1}_{s}(M_B^2,s_0)=&\int_0^{s_0}\Bigg(\frac{s^4}{26880\pi^6}+\frac{11\langle\bar{s}s\rangle m_s s^2}{320\pi^4}-\frac{\langle\alpha_sGG\rangle s^2}{1152\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{12\pi^2}-\frac{\langle\bar{s}s\rangle^2 s}{12\pi^2}+\frac{\langle g_s\bar{q}\sigma Gq\rangle m_s s}{384\pi^4}+\frac{59\langle g_s\bar{s}\sigma Gs\rangle m_s s}{1152\pi^4}\\&-\frac{5\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{96\pi^2}-\frac{5\langle g_s\bar{s}\sigma Gs\rangle\langle\bar{s}s\rangle}{96\pi^2}-\frac{\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{288\pi^2}-\frac{\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{288\pi^2}+\frac{7\langle\alpha_sGG\rangle\langle\bar{q}q\rangle m_s}{1152\pi^3}\\&+\frac{\langle\alpha_sGG\rangle\langle\bar{s}s\rangle m_s}{1152\pi^3}\Bigg)e^{-\frac{s}{M_B^2}}ds+\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{64\pi^2}+\frac{\langle g_s\bar{s}\sigma Gs\rangle\langle g_s\bar{q}\sigma Gq\rangle}{288\pi^2}+\frac{\langle\alpha_sGG\rangle\langle g_s\bar{q}\sigma Gq\rangle m_s}{1152\pi^3}\\&-\frac{2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle m_s}{3}+\frac{\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{54\pi}+\frac{\langle\alpha_sGG\rangle\langle\bar{s}s\rangle^2}{54\pi}+\frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{64\pi^2}+\frac{5\langle\alpha_sGG\rangle\langle g_s\bar{s}\sigma Gs\rangle m_s}{3456\pi^3},\end{aligned}\tag{A5}$$

$$\begin{aligned}\Pi^{1'}_{s}(M_B^2,s_0)=&\int_0^{s_0}\Bigg(\frac{s^4}{13440\pi^6}+\frac{11\langle\bar{s}s\rangle m_s s^2}{160\pi^4}-\frac{\langle\alpha_sGG\rangle s^2}{576\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{6\pi^2}-\frac{\langle\bar{s}s\rangle^2 s}{6\pi^2}-\frac{7\langle g_s\bar{q}\sigma Gq\rangle m_s s}{384\pi^4}+\frac{109\langle g_s\bar{s}\sigma Gs\rangle m_s s}{1152\pi^4}\\&-\frac{3\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{32\pi^2}-\frac{3\langle g_s\bar{s}\sigma Gs\rangle\langle\bar{s}s\rangle}{32\pi^2}+\frac{7\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{288\pi^2}+\frac{7\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{288\pi^2}-\frac{49\langle\alpha_sGG\rangle\langle\bar{q}q\rangle m_s}{1152\pi^3}\\&-\frac{19\langle\alpha_sGG\rangle\langle\bar{s}s\rangle m_s}{1152\pi^3}\Bigg)e^{-\frac{s}{M_B^2}}ds+\frac{5\langle g_s\bar{q}\sigma Gq\rangle^2}{192\pi^2}-\frac{7\langle g_s\bar{s}\sigma Gs\rangle\langle g_s\bar{q}\sigma Gq\rangle}{288\pi^2}-\frac{7\langle\alpha_sGG\rangle\langle g_s\bar{q}\sigma Gq\rangle m_s}{1152\pi^3}\\&-\frac{4\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle m_s}{3}+\frac{\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{27\pi}+\frac{\langle\alpha_sGG\rangle\langle\bar{s}s\rangle^2}{27\pi}+\frac{5\langle g_s\bar{s}\sigma Gs\rangle^2}{192\pi^2}+\frac{\langle\alpha_sGG\rangle\langle g_s\bar{s}\sigma Gs\rangle m_s}{3456\pi^3},\end{aligned}\tag{A6}$$

$$\begin{aligned}\Pi^{2}_{s}(M_B^2,s_0)=&\int_0^{s_0}\Bigg(\frac{s^4}{26880\pi^6}+\frac{11\langle\bar{s}s\rangle m_s s^2}{320\pi^4}-\frac{\langle\alpha_sGG\rangle s^2}{1152\pi^5}-\frac{\langle\bar{q}q\rangle^2 s}{12\pi^2}-\frac{\langle\bar{s}s\rangle^2 s}{12\pi^2}-\frac{\langle g_s\bar{q}\sigma Gq\rangle m_s s}{384\pi^4}+\frac{59\langle g_s\bar{s}\sigma Gs\rangle m_s s}{1152\pi^4}\\&-\frac{5\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{96\pi^2}-\frac{5\langle g_s\bar{s}\sigma Gs\rangle\langle\bar{s}s\rangle}{96\pi^2}+\frac{\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{288\pi^2}+\frac{\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{288\pi^2}-\frac{7\langle\alpha_sGG\rangle\langle\bar{q}q\rangle m_s}{1152\pi^3}\\&+\frac{\langle\alpha_sGG\rangle\langle\bar{s}s\rangle m_s}{1152\pi^3}\Bigg)e^{-\frac{s}{M_B^2}}ds+\frac{\langle g_s\bar{q}\sigma Gq\rangle^2}{64\pi^2}-\frac{\langle g_s\bar{s}\sigma Gs\rangle\langle g_s\bar{q}\sigma Gq\rangle}{288\pi^2}-\frac{\langle\alpha_sGG\rangle\langle g_s\bar{q}\sigma Gq\rangle m_s}{1152\pi^3}\\&-\frac{2\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle m_s}{3}+\frac{\langle\alpha_sGG\rangle\langle\bar{q}q\rangle^2}{54\pi}+\frac{\langle\alpha_sGG\rangle\langle\bar{s}s\rangle^2}{54\pi}+\frac{\langle g_s\bar{s}\sigma Gs\rangle^2}{64\pi^2}+\frac{5\langle\alpha_sGG\rangle\langle g_s\bar{s}\sigma Gs\rangle m_s}{3456\pi^3},\end{aligned}\tag{A7}$$

$$\Pi_s^{2'}(M_B^2, s_0) = \int_0^{s_0}\Big(\frac{s^4}{13440\pi^6} + \frac{11\langle\bar{s}s\rangle m_s s^2}{160\pi^4} - \frac{\langle\alpha_s GG\rangle s^2}{576\pi^5} - \frac{\langle\bar{q}q\rangle^2 s}{6\pi^2} - \frac{\langle\bar{s}s\rangle^2 s}{6\pi^2} + \frac{7\langle g_s\bar{q}\sigma Gq\rangle m_s s}{384\pi^4} + \frac{109\langle g_s\bar{s}\sigma Gs\rangle m_s s}{1152\pi^4}$$
$$- \frac{3\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{q}q\rangle}{32\pi^2} - \frac{3\langle g_s\bar{s}\sigma Gs\rangle\langle\bar{s}s\rangle}{32\pi^2} - \frac{7\langle\bar{q}q\rangle\langle g_s\bar{s}\sigma Gs\rangle}{288\pi^2} - \frac{7\langle g_s\bar{q}\sigma Gq\rangle\langle\bar{s}s\rangle}{288\pi^2} + \frac{49\langle\alpha_s GG\rangle\langle\bar{q}q\rangle m_s}{1152\pi^3}$$
$$- \frac{19\langle\alpha_s GG\rangle\langle\bar{s}s\rangle m_s}{1152\pi^3}\Big)e^{-\frac{s}{M_B^2}}ds + \frac{5\langle g_s\bar{q}\sigma Gq\rangle^2}{192\pi^2} + \frac{7\langle g_s\bar{s}\sigma Gs\rangle\langle g_s\bar{q}\sigma Gq\rangle}{288\pi^2} + \frac{7\langle\alpha_s GG\rangle\langle g_s\bar{q}\sigma Gq\rangle m_s}{1152\pi^3}$$
$$- \frac{4\langle\bar{q}q\rangle^2\langle\bar{s}s\rangle m_s}{3} + \frac{\langle\alpha_s GG\rangle\langle\bar{q}q\rangle^2}{27\pi} + \frac{\langle\alpha_s GG\rangle\langle\bar{s}s\rangle^2}{27\pi} + \frac{5\langle g_s\bar{s}\sigma Gs\rangle^2}{192\pi^2} + \frac{\langle\alpha_s GG\rangle\langle g_s\bar{s}\sigma Gs\rangle m_s}{3456\pi^3}. \quad \text{(A8)}$$

For the fully strange $ss\bar{s}\bar{s}$ tetraquark system, the correlation functions after Borel transformation are

$$\Pi_{ss}^{s1}(M_B^2, s_0) = \int_0^{s_0}\Big(\frac{s^4}{26880\pi^6} + \frac{11\langle\bar{s}s\rangle m_s s^2}{160\pi^4} - \frac{\langle\alpha_s GG\rangle s^2}{1152\pi^5} - \frac{\langle\bar{s}s\rangle^2 s}{6\pi^2} + \frac{31\langle g_s\bar{s}\sigma Gs\rangle m_s s}{288\pi^4} - \frac{\langle g_s\bar{s}\sigma Gs\rangle\langle\bar{s}s\rangle}{9\pi^2}$$
$$+ \frac{\langle\alpha_s GG\rangle\langle\bar{s}s\rangle m_s}{72\pi^3}\Big)e^{-\frac{s}{M_B^2}}ds - \frac{4\langle\bar{s}s\rangle^3 m_s}{3} + \frac{\langle\alpha_s GG\rangle\langle\bar{s}s\rangle^2}{27\pi} + \frac{5\langle g_s\bar{s}\sigma Gs\rangle^2}{144\pi^2} + \frac{\langle\alpha_s GG\rangle\langle g_s\bar{s}\sigma Gs\rangle m_s}{216\pi^3}. \quad \text{(A9)}$$

[1] M. Gell-Mann, Phys. Lett. **8**, 214 (1964).
[2] G. Zweig, An SU(3) model for strong interaction symmetry and its breaking. Version 2, in *Developments in the Quark Theory of Hadrons* (1964), pp. 22–101, 10.17181/CERN-TH-412.
[3] R. L. Workman *et al.*, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[4] C. Meyer and E. Swanson, Prog. Part. Nucl. Phys. **82**, 21 (2015).
[5] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rep. **639**, 1 (2016).
[6] H. Clement, Prog. Part. Nucl. Phys. **93**, 195 (2017).
[7] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Rev. Mod. Phys. **90**, 015004 (2018).
[8] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Prog. Part. Nucl. Phys. **107**, 237 (2019).
[9] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, Phys. Rep. **873**, 1 (2020).
[10] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, Rep. Prog. Phys. **86**, 026201 (2023).
[11] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, arXiv:2404.06399.
[12] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
[13] D. Black, A. H. Fariborz, F. Sannino, and J. Schechter, Phys. Rev. D **59**, 074026 (1999).
[14] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. Lett. **93**, 212002 (2004).
[15] H.-X. Chen, A. Hosaka, and S.-L. Zhu, Phys. Rev. D **76**, 094025 (2007).
[16] S. Prelovsek and D. Mohler, Phys. Rev. D **79**, 014503 (2009).
[17] B. Aubert *et al.*, Phys. Rev. D **74**, 091103 (2006).
[18] M. Ablikim *et al.*, Phys. Rev. Lett. **100**, 102003 (2008).
[19] M. Ablikim *et al.*, Phys. Rev. D **91**, 052017 (2015).
[20] M. Ablikim *et al.*, Phys. Rev. D **99**, 012014 (2019).
[21] B. Aubert *et al.*, Phys. Rev. D **76**, 012008 (2007).
[22] B. Aubert *et al.*, Phys. Rev. D **77**, 092002 (2008).
[23] J. P. Lees *et al.*, Phys. Rev. D **86**, 012008 (2012).
[24] C. P. Shen *et al.*, Phys. Rev. D **80**, 031101 (2009).
[25] C. Deng, J. Ping, F. Wang, and T. Goldman, Phys. Rev. D **82**, 074001 (2010).
[26] Z.-G. Wang, Nucl. Phys. **A791**, 106 (2007).
[27] H.-W. Ke and X.-Q. Li, Phys. Rev. D **99**, 036014 (2019).
[28] H.-X. Chen, C.-P. Shen, and S.-L. Zhu, Phys. Rev. D **98**, 014011 (2018).
[29] J. Ho, R. Berg, T. G. Steele, W. Chen, and D. Harnett, Phys. Rev. D **100**, 034012 (2019).
[30] C. Deng, J. Ping, Y. Yang, and F. Wang, Phys. Rev. D **88**, 074007 (2013).
[31] M. Ablikim *et al.*, Phys. Rev. D **99**, 032001 (2019).
[32] Q.-F. Lü, K.-L. Wang, and Y.-B. Dong, Chin. Phys. C **44**, 024101 (2020).
[33] K. Azizi, S. S. Agaev, and H. Sundu, Nucl. Phys. **B948**, 114789 (2019).
[34] D. Alde *et al.*, Phys. Lett. B **205**, 397 (1988).
[35] E. I. Ivanov *et al.*, Phys. Rev. Lett. **86**, 3977 (2001).
[36] J. Kuhn *et al.*, Phys. Lett. B **595**, 109 (2004).
[37] M. Ablikim *et al.*, Phys. Rev. Lett. **129**, 192002 (2022).
[38] M. Ablikim *et al.*, Phys. Rev. D **106**, 072012 (2022); **107**, 079901(E) (2023).
[39] A. Rodas *et al.*, Phys. Rev. Lett. **122**, 042002 (2019).
[40] C. Adolph *et al.*, Phys. Lett. B **740**, 303 (2015); **811**, 135913(E) (2020).
[41] H.-X. Chen, N. Su, and S.-L. Zhu, Chin. Phys. Lett. **39**, 051201 (2022).
[42] B. Chen, S.-Q. Luo, and X. Liu, Phys. Rev. D **108**, 054034 (2023).

[43] F. Chen, X. Jiang, Y. Chen, M. Gong, Z. Liu, C. Shi, and W. Sun, Phys. Rev. D **107**, 054511 (2023).
[44] L. Qiu and Q. Zhao, Chin. Phys. C **46**, 051001 (2022).
[45] V. Shastry, Proc. Sci. ICHEP2022 (**2022**) 779.
[46] V. Shastry and F. Giacosa, Nucl. Phys. **A1037**, 122683 (2023).
[47] V. Shastry, C. S. Fischer, and F. Giacosa, Phys. Lett. B **834**, 137478 (2022).
[48] H.-X. Chen, A. Hosaka, and S.-L. Zhu, Phys. Rev. D **78**, 054017 (2008).
[49] H.-X. Chen, A. Hosaka, and S.-L. Zhu, Phys. Rev. D **78**, 117502 (2008).
[50] X. Zhang and J.-J. Xie, Chin. Phys. C **44**, 054104 (2020).
[51] Y. Huang and H. Q. Zhu, J. Phys. G **50**, 095002 (2022).
[52] F. Yang, H. Q. Zhu, and Y. Huang, Nucl. Phys. **A1030**, 122571 (2023).
[53] X.-K. Dong, Y.-H. Lin, and B.-S. Zou, Sci. China Phys. Mech. Astron. **65**, 261011 (2022).
[54] B.-D. Wan, S.-Q. Zhang, and C.-F. Qiao, Phys. Rev. D **106**, 074003 (2022).
[55] X.-Y. Wang, F.-C. Zeng, and X. Liu, Phys. Rev. D **106**, 036005 (2022).
[56] N. Su and H.-X. Chen, Phys. Rev. D **106**, 014023 (2022).
[57] Y. Yu, Z. Xiong, H. Zhang, B.-C. Ke, Y. Teng, Q.-S. Liu, and J.-W. Zhang, Phys. Lett. B **842**, 137965 (2023).
[58] M.-J. Yan, J. M. Dias, A. Guevara, F.-K. Guo, and B.-S. Zou, Universe **9**, 109 (2023).
[59] C.-K. Jiao, W. Chen, H.-X. Chen, and S.-L. Zhu, Phys. Rev. D **79**, 114034 (2009).
[60] Z.-R. Huang, W. Chen, T. G. Steele, Z.-F. Zhang, and H.-Y. Jin, Phys. Rev. D **95**, 076017 (2017).
[61] M.-L. Du, W. Chen, X.-L. Chen, and S.-L. Zhu, Chin. Phys. C **37**, 033104 (2013).
[62] Y.-C. Fu, Z.-R. Huang, Z.-F. Zhang, and W. Chen, Phys. Rev. D **99**, 014025 (2019).
[63] K. Ray, D. Harnett, and T. G. Steele, Phys. Rev. D **108**, 034001 (2023).
[64] H.-Z. Xi, Y.-W. Jiang, H.-X. Chen, A. Hosaka, and N. Su, Phys. Rev. D **108**, 094019 (2023).
[65] J. J. Dudek, Phys. Rev. D **84**, 074023 (2011).
[66] J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas, Phys. Rev. D **88**, 094505 (2013).
[67] Q.-N. Wang, D.-K. Lian, and W. Chen, Phys. Rev. D **108**, 114010 (2023).
[68] N. Isgur, R. Kokoski, and J. Paton, Phys. Rev. Lett. **54**, 869 (1985).
[69] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60**, 034509 (1999).
[70] Y. Chen *et al.*, Phys. Rev. D **73**, 014516 (2006).
[71] L. Tang and C.-F. Qiao, Nucl. Phys. **B904**, 282 (2016).
[72] H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **103**, L091503 (2021).
[73] H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **105**, L051501 (2022).
[74] C.-M. Tang, Y.-C. Zhao, and L. Tang, Phys. Rev. D **105**, 114004 (2022).
[75] N. Su, H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **107**, 034010 (2023).
[76] N. Su, W.-H. Tan, H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **107**, 114005 (2023).
[77] N. Su, H.-X. Chen, W. Chen, and S.-L. Zhu, Phys. Rev. D **109**, L011502 (2024).
[78] D.-K. Lian, Q.-N. Wang, X.-L. Chen, P.-F. Yang, W. Chen, and H.-X. Chen, J. High Energy Phys. 06 (2024) 173.
[79] W. Chen and S.-L. Zhu, Phys. Rev. D **83**, 034010 (2011).
[80] M. Jamin, Phys. Lett. B **538**, 71 (2002).
[81] S. Narison, Phys. Lett. B **706**, 412 (2012).
[82] S. Narison, Int. J. Mod. Phys. A **33**, 1850045 (2018); **33**, 1892004(A) (2018).
[83] Z.-G. Wang, Adv. High Energy Phys. **2020**, 6438730 (2020).