Exploring the Efimov effect in the $D^*D^*D^*$ system

P. G. Ortega*

Departamento de Física Fundamental, Universidad de Salamanca, E-37008 Salamanca, Spain and Instituto Universitario de Física Fundamental y Matemáticas (IUFFyM), Universidad de Salamanca, E-37008 Salamanca, Spain

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The emergence of the Efimov effect in the $D^*D^*D^*$ system is explored under the assumption that the heavy partner of the T_{cc}^+ exists as a D^*D^* molecule with $(I)J^P = (0)1^+$. The three-to-three relativistic scattering amplitude is obtained from the ladder amplitude formalism, built from an energy-dependent contact two-body potential where the molecular component of the T_{cc}^* state can be varied. We find that $(I)J^P = (\frac{1}{2})0^-$ three-body bound states can be formed, with properties that suggest that the Efimov effect can be realized for reasonable values of the molecular probability and binding energy of the T_{cc}^*.

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When two particles form a nearly resonant bound state due to short-range attractive forces, an effective long-range three-body emerges giving rise to an infinite number of three-body bound states with a discrete scale invariance. This remarkable phenomena, called the *Efimov effect*, was first described in the 1970's by Efimov [1,2]. The Efimov effect has been mostly studied in atomic physics [3–7], due to its experimental observation in Cesium atoms in 2006 [8]. However, its relevance has also been explored in nuclear physics, e.g., in the ^{12}C three-α structure, the triton formation or the nuclear halo of ^{14}Be, ^{22}C, and ^{20}C nuclei [9–14].

For sufficiently shallow two-body states the system becomes universal, i.e., it is insensitive to the details of the short-range interaction and can be characterized by its S-wave scattering length $a_{\rm sc}$. In this limit, the three-body effective potential is proportional to $1/\rho^2$, where ρ is the hyperspherical radius related to the separation among the three particles [15–17]. For three identical bosons of mass m interacting via a short-range two-body potential the effective potential is attractive, so when $a_{\rm sc} \to \pm\infty$ an infinite family of three-body bound states appears with a scaling factor $\lambda = e^{\pi/|s_0|} \approx 22.7$. The binding energies of the trimer states scale as $\mathcal{B}_3 \to Q^{-2}\mathcal{B}_3$ with $a_{\rm sc} \to Q a_{\rm sc}$, where $Q \to \lambda$ in the unitarity limit, i.e.,

$$\frac{\mathcal{B}_3^{(n+1)}}{\mathcal{B}_3^{(n)}} \to \lambda^{-2} \approx \frac{1}{515}. \quad (1)$$

For large but finite scattering lengths the spectrum is not infinite, but few shallow Efimov states may emerge if some conditions are met [15,18]. In this case, the scaling law may also deviate from the universal value [19,20], so $Q \neq \lambda$.

The existence of three-body systems and the possible appearance of the Efimov effect in hadronic physics has been also suggested in the recent literature [16,21–24], specially since the discovery of the $X(3872)$ state [25], a loosely-bound $D^{*0}\bar{D}^0 + \mathrm{H.c.}$ molecule with quantum numbers $J^{PC} = 1^{++}$. The properties of the $X(3872)$, unfortunately, rule out the existence of the Efimov effect [16].

However, the recent discovery of the T_{cc}^+ [26,27] can renew this interest. In 2021, the LHCb Collaboration discovered a new tetraquarklike state in the $D^0D^0\pi^+$ invariant mass spectrum [26] with minimum quark content $cc\bar{u}\bar{d}$, named T_{cc}^+. The resonance is slightly below the $D^{*+}D^0$ threshold, with a binding energy estimated to be $\delta m_{\rm pole} = (360 \pm 40^{+4}_{-0})\ \mathrm{keV}/c^2$ [27]. Its scattering length has a value of $a_{\rm sc}^{\rm LHCb} = -7.15(51)$ fm and the experimental Weinberg factor[1] is $Z < 0.52(0.58)$ at 90 (95)% CL. These properties of the T_{cc}^+ are compatible with a state with a sizable DD^* molecular compound (for a review of the experimental and theoretical status of the T_{cc}^+ see, e.g., Ref. [28] and references therein).

The announcement of the T_{cc}^+ has already stimulated the study of three-body states containing charmed mesons. For example, in Ref. [29] the authors explore the DDD^*

*Contact author: pgortega@usal.es

[1]This factor estimates the probability of finding a compact component in the wave function of a particle, with $Z = 0$ for a pure molecular state and $Z = 1$ for a pure compact state.

system, finding a bound state of few hundred keV with $(I)J^P = (\frac{1}{2})1^-$. Reference [30] analyzes the existence of hadronic molecules composed of a $T_{cc}^{(*)}$ compact tetraquark and a $\bar{D}^{(*)}$ meson finding several candidates, while Refs. [31,32] study the $D^*D^*D^*$ and DD^*D^* systems finding many potential three-body candidates in the $J^P = 0^-$, 1^-, and 2^- with isospins $\frac{1}{2}$ and $\frac{3}{2}$. However, no exploration of the Efimov effect in these systems has been suggested yet.

In this study we will explore the universality of the $D^*D^*D^*$ meson system in the $J^P = 0^-$ sector with $I = \frac{1}{2}$, assuming that the isoscalar heavy partner of the T_{cc}^+, dubbed T_{cc}^*, exists close and below the D^*D^* threshold, which is a prediction of heavy-quark spin symmetry (HQSS) [33]. HQSS implies that the heavy-meson interactions are insensitive to the spin of the heavy quark, so the interaction of the D^*D^* system is identical to the DD^* one for the $(I)J^P = (0)1^+$ sector (up to $1/M_Q$ corrections). Actually, the T_{cc}^* state has been predicted by many groups [34–46], with a binding energy around few MeV.

The existence of the T_{cc}^* with a small binding energy would favor the appearance of Efimov states in this sector. If confirmed, it would be the first manifestation of the Efimov effect in hadronic physics, so it is worth exploring this system. The choice of the $D^*D^*D^*$ system instead of the DD^*D^* is that it allows us to work with identical bosons, simplifying the calculations. For three identical bosons, any two-body wave function must be symmetric. Then, the $J^P = 0^-$ sector is selected because all the symmetric D^*D^* pairs in S waves are in a relative $(I)J^P = (0)1^+$, whereas for $J^P = 1^-$ and 2^- other isospin-spin D^*D^* pairs are also allowed, such as the $(1)2^+$ and $(1)0^+$, adding repulsion to the three-body interaction [32]. Then, in this sector, all the allowed D^*D^* pairs interact via an attractive potential, condition needed for the Efimov effect to emerge.

The starting point is the analysis of the D^*D^* two-body system with a given binding energy $\mathcal{B}_2$, that is taken as a parameter. The two-body amplitude is obtained by solving the Bethe-Salpeter equation in the on shell approximation [47],

$$\mathcal{T}_2^{-1}(s) = \mathcal{V}^{-1}(s) - \mathcal{G}(s), \tag{2}$$

where $\mathcal{V}(s)$ is the two-meson interaction and $\mathcal{G}$ is the relativistic two-meson loop function

$$\mathcal{G}(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_1^2 + i\varepsilon}\frac{1}{(P-q)^2 - m_2^2 + i\varepsilon}, \tag{3}$$

being P the total initial four-momentum of the D^*D^* meson system. This loop function is regularized via a sharp cutoff [48], i.e., assuming a maximum trimomentum module $|\vec{q}| \leq \Lambda$. The value of the cutoff will be taken as $\Lambda = 0.7$ GeV. However, we will analyze the sensitivity of the results by varying the cutoff between $\Lambda = 0.5$ and 1 GeV.

We consider the same mass for D^{*0} and $D^{*\pm}$, with $m = \frac{1}{2}(m_{D^{*0}} + m_{D^{*\pm}}) = 2008.55$ MeV. Then, for equal meson masses $m_1 = m_2 = m$, the loop function takes the form,

$$\mathcal{G}(s) = \frac{1}{(4\pi)^2}\left\{\sigma \log\frac{\sigma\sqrt{1+\frac{m^2}{\Lambda^2}}+1}{\sigma\sqrt{1+\frac{m^2}{\Lambda^2}}-1} - 2\log\left[\frac{\Lambda}{m}\left(1+\sqrt{1+\frac{m^2}{\Lambda^2}}\right)\right]\right\}, \tag{4}$$

with $\sigma = \sqrt{1-4m^2/s}$. The prescription for the logarithmic function is such that $\mathrm{Im}(\mathcal{G})$ is given by

$$\mathrm{Im}(\mathcal{G}) = -\frac{k}{8\pi\sqrt{s}}\Theta(s-4m^2), \tag{5}$$

with k the relativistic on shell momentum for the D^*D^* pair in its c.m. frame, $k = \frac{1}{2}\sqrt{s-4m^2}$.

The $\mathcal{V}$ potential for the D^*D^* pair is taken as a $I = 0$ S-wave interaction, neglecting the DD^*-D^*D^* coupled-channels effect and the finite width of the D^*. These effects can be modeled as a source of width for the T_{cc}^* and the $D^*D^*D^*$ trimers with little effect in the determination of the trimer masses [31,32,36,45].

In order to evaluate the effect of the T_{cc}^* inner composition, we follow Ref. [49] and consider the general energy-dependent contact potential

$$\mathcal{V}^{-1}(s) = C_0 - C_1\frac{1-\mathcal{P}}{\mathcal{P}}(s-m_*^2), \tag{6}$$

being C_0 and C_1 constants and $\mathcal{P}$ the molecular probability in the T_{cc}^* state, which ranges between 0 and 1. These C_0 and C_1 parameters are fixed in order to impose the existence of a pole below the D^*D^* threshold in the first Riemann sheet, with mass $m_* = 2m - \mathcal{B}_2$. Their values are related to the loop function as $C_0 \equiv \mathcal{G}(m_*^2)$ and $C_1 \equiv \mathcal{G}'(m_*^2)$, so there the only free parameters are $\mathcal{B}_2$ and $\mathcal{P}$. Depending on the molecular content in the T_{cc}^* the potential $\mathcal{V}$ changes its energy dependence, being constant in the case of a pure D^*D^* hadronic molecule ($\mathcal{P} = 1$) and behaving as $\mathcal{V} \sim \frac{1}{s-m_*^2}$ for a pure compact state ($\mathcal{P} \to 0$).

The relativistic three-to-three scattering amplitude is built using the so-called *ladder amplitude* formalism [50–52].

In this approach, the three-body amplitude is decomposed as[2]

$$\mathcal{M}_3(\vec{p}_i, \vec{p}_f) = \mathcal{D}(\vec{p}_i, \vec{p}_f) + \mathcal{M}_{df,3}(\vec{p}_i, \vec{p}_f), \qquad (7)$$

where $\mathcal{D}$ is called the ladder amplitude, which contains the sum over all possible pairwise interactions connected through a sequence of one-particle exchanges, and $\mathcal{M}_{df,3}$ encodes all the contributions that arise when short-range three-particle interactions are present. The latter amplitude $\mathcal{M}_{df,3}$ depends on the $\mathcal{K}_{df,3}$ matrix, which represents the short-range three-body interactions [20]. The $\vec{p}_i$ ($\vec{p}_f$) is the initial (final) momentum of one of the mesons, which is denoted as the *spectator*. The other two remaining mesons associated with the given spectator are, then, called a pair or *dimer*. If three-body short-range interactions are negligible, $\mathcal{K}_{df,3} = \mathcal{M}_{df,3} = 0$ and the ladder amplitude describes the full three-to-three scattering amplitude, $\mathcal{M}_3 = \mathcal{D}$. In Refs. [53,54] it was shown that there is a clean cancellation between the off shell parts of the two-body T matrix and the three-body forces in the framework of chiral Lagrangians. In this work we will make explicit use of this cancellation and neglect three-body forces, that is, $\mathcal{K}_{df,3} = 0$. The equivalence of this method with alternative three-particle scattering formalism, e.g., the nonrelativistic Faddeev equations, has been shown in Ref. [55].

In this work we assume that all the two-body subsystems are in a partial S wave only, due to the proximity of the T_{cc}^* state to the D^*D^* threshold, which will suppress higher partial waves. At the same time, the dimer-spectator system is also assumed to be in $L = 0$, which is the expected dominant partial wave [31,32]. Indeed, in Ref. [31] the authors studied the $D^*D^*D^*$ system in all possible configurations with $L \leq 2$ and found a small effect of the S-D mixing compared to a S-wave only calculation.

The $\mathcal{D}$ ladder amplitude is defined by the integral equation,

$$\mathcal{D}(\vec{p}_i, \vec{p}_f) = -\mathcal{M}_2(p_i) G(\vec{p}_i, \vec{p}_f) \mathcal{M}_2(p_f) - \mathcal{M}_2(p_i) \int \frac{d^3\vec{q}}{(2\pi)^3 2\omega(q)} G(\vec{p}_i, \vec{q}) \mathcal{D}(\vec{q}, \vec{p}_f), \qquad (8)$$

which is diagrammatically shown in Fig. 1.

Here, G is the long-range interaction between the dimer and the spectator, mediated by a particle exchange, and $\mathcal{M}_2$ is the relativistic $2 \to 2$ scattering amplitude describing the meson-meson interaction in the dimer. The dimer energy is fixed by the momentum of the spectator, $s_2(p) = (\sqrt{s} - \omega(p))^2 - p^2$, with $\omega(p) = \sqrt{m^2 + p^2}$ and $p = |\vec{p}|$. The $\mathcal{M}_2$ amplitude is proportional to the two-body T matrix in Eq. (2) as $\mathcal{M}_2 = -2\mathcal{T}_2$.

[2]The full $3 \to 3$ scattering amplitude must be properly symmetrized, by summing over the nine possible spectator momenta (see, e.g., Ref. [51]), but in this work we refer to unsymmetrized amplitudes only.

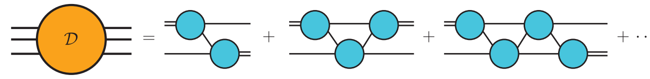

FIG. 1. Diagrammatic representation of the ladder amplitude $\mathcal{D}$ [Eq. (8)], where the blue circles represent the two-body amplitude $\mathcal{M}_2$ and the diagonal black lines connecting them are the one-particle exchange function G.

For all pairs in the S wave, the one-particle exchange propagator (OPE) G can be written as

$$G(p_i, p_f) \equiv -\frac{H(p_i, p_f)}{4 p_i p_f} \log\left(\frac{z(p_i, p_f) - 2p_i p_f + i\epsilon}{z(p_i, p_f) + 2p_i p_f + i\epsilon}\right), \qquad (9)$$

where $z(p_i, p_f) = (\sqrt{s} - \omega(p_i) - \omega(p_f))^2 - p_i^2 - p_f^2 - m^2$ and $H(p_i, p_f)$ is a cutoff function to ensure a finite integral in Eq. (8). For this cutoff function we use a sharp cutoff, which is one in the integration domain and zero elsewhere.

It is worth noticing that we are studying a system of three vector mesons with isospin $\frac{1}{2}$, different from the spinless case studied in Refs. [50–52]. The recoupling coefficient with particles with spin is more delicate than the spinless case [56], but for dimer-spectator systems in the S wave, the OPE of a D^* between two $(0)1^+$ dimers can be reduced to the Eq. (9) multiplied by the spin-isospin recoupling [57],

$$G(p_i, p_f) \longrightarrow \langle I_2, I | I_2', I \rangle \langle S_2, S | S_2', S \rangle G(p_i, p_f), \qquad (10)$$

where S (I) is the total three-body spin (isospin) and $S_2^{(\prime)}$ ($I_2^{(\prime)}$) is the spin (isospin) of the initial and final dimer.

For the $D^*D^*D^*$ in $J^P = 0^-$, this implies $G(p_i, p_f) \to \frac{1}{2} G(p_i, p_f)$. In addition, an extra factor of 2 must be added in the $\mathcal{M}_2$ to account for the different isospin projections of the initial/final D^*D^* dimers, that's it, $\mathcal{M}_2(p) \to 2\mathcal{M}_2(p)$.

For three identical bosons, it is more convenient to work with the amputated amplitude d,

$$\mathcal{D}(\vec{p}_i, \vec{p}_f) \equiv \mathcal{M}_2(p_i) d(\vec{p}_i, \vec{p}_f) \mathcal{M}_2(p_f), \qquad (11)$$

which eliminates the singularities of the dimer $\mathcal{M}_2$ amplitude. With this definition, the S-wave amputated amplitude is given by

$$d(p_i, p_f) = -G(p_i, p_f) - \int \frac{dq q^2}{(2\pi)^3 2\omega(q)} G(p_i, q) \mathcal{M}_2(q) d(q, p_f), \qquad (12)$$

which will be numerically solved following Ref. [52].

As mentioned above, the main purpose of this work is to evaluate the existence of Efimov states in the $D^*D^*D^*$ in the $(I)J^P = (\frac{1}{2})0^-$ sector, given the existence of a bound $(I)J^P = (0)1^+$ D^*D^* state, T_{cc}^*, which has been predicted as the HQSS partner of the T_{cc}^+ state recently discovered. The advantage of using this system is that we can work with identical bosons, which favors the generation of Efimov states when the two-body pairs have attractive nearly resonant interactions. However, the main uncertainty are the properties of this hypothetical T_{cc}^*, i.e., how close we are to the resonant limit. For this reason we evaluate the possible $D^*D^*D^*$ trimer states assuming a selection of T_{cc}^* binding energies, $\mathcal{B}_2 = \{0.01, 0.5, 1.0, 5.0\}$ MeV. Then, we will study their properties as a function of the T_{cc}^* molecular content $\mathcal{P}$.

A first simple study that can provide useful insights into the problem is the analysis in the leading-order effective range expansion (ERE), that is,

$$\mathcal{T}_2(p) = \frac{8\pi\sqrt{s_2(p)}}{iq_2(p) + 1/a_{\rm sc}}, \tag{13}$$

with $q_2(p) = \sqrt{s_2(p)/4 - m^2}$ the relative momentum of the particles in the dimer. In this case, the phenomenology of the three-body states only depend on the D^*D^* scattering length $a_{\rm sc} = 2\hbar c/\sqrt{4m^2 - m_*^2}$. The results of this approach are given in Table I. For the four $\mathcal{B}_2$ under evaluation we find, at least, two bound-state trimers. For $\mathcal{B}_2 = 0.01$ MeV and 0.5 MeV a third bound and virtual trimer state is also found, respectively. Of course, the smaller the binding energy, the closer we are to the resonant limit. These results agree with the Efimov states analyzed in Ref. [20] for the general case of three identical bosons, both their energies and the ratios of subsequent binding energies. This suggests that, indeed, the Efimov effect can be present in the $D^*D^*D^*$ system.

The results are, though, more interesting when the full two-body potential introduced in Eq. (6) is used. In this case, the first three Efimov trimers are also found, but their masses depend on the molecular probability of the T_{cc}^*. The masses of the $D^*D^*D^*$ trimers for $\mathcal{B}_2 = 0.01$ MeV and 5 MeV are shown in the upper and lower panels of Fig. 2, respectively. Generally, the larger $\mathcal{P}$ and $\mathcal{B}_2$ in the T_{cc}^*, the deeper the binding energy of the trimers. For $\mathcal{P} > 20\%$ the first trimer emerges as a bound state, while the second emerges between 60% and 80%, depending on $\mathcal{B}_2$. Contrary to the ERE results, three Efimov states are found for all binding energies, but $\mathcal{P}$ values above 96% are needed in order to have three bound states, so it is unlikely that the third trimer will exist unless the T_{cc}^* is a pure molecule.

TABLE I. Properties of the trimer states in the effective range expansion approach for the two-body amplitude. *First column:* Two-body binding energy, in MeV; *Second column:* Two-body scattering length, in fm; *Third to fifth columns:* Binding energies of the ith trimer state, $\mathcal{B}_3^{(i)} = 3m - E^{(i)}$, with $E^{(i)}$ the three-body mass of the ith trimer, in MeV; *Sixth column:* Ratio of the second to first trimer binding energies.

$\mathcal{B}_2$	$a_{\rm sc}$	$\mathcal{B}_3^{(1)}$	$\mathcal{B}_3^{(2)}$	$\mathcal{B}_3^{(3)}$	$\mathcal{B}_3^{(2)}/\mathcal{B}_3^{(1)}$
0.01	44.03	54.592	0.185	0.011	0.003
0.5	6.23	64.158	0.980	0.620[a]	0.015
1.0	4.40	69.099	1.557	⋯	0.023
5.0	1.97	91.365	5.521	⋯	0.060

[a]It indicates a virtual state (pole in the second Riemann sheet).

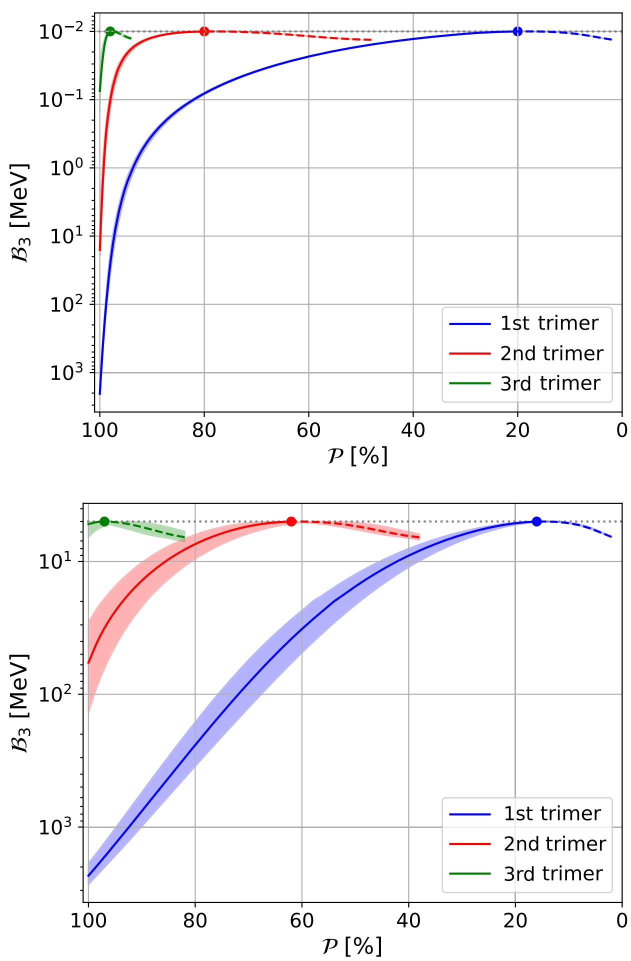

FIG. 2. Binding energies of the first $D^*D^*D^*$ trimers ($\mathcal{B}_3 = 3m - E_3$) for $\mathcal{B}_2 = 0.01$ MeV (upper panel) and $\mathcal{B}_2 = 5$ MeV (lower panel) as a function of the T_{cc}^* composition, ranging from a purely two-body molecular state ($\mathcal{P} = 100\%$) to a purely compact T_{cc}^* state ($\mathcal{P} = 0\%$). The central lines show the results for $\Lambda = 0.7$ GeV cutoff in the two-body amplitude. The color error bands indicate the results for the cutoff range $\Lambda = [0.5, 1]$ GeV. Solid lines represent bound states, whereas dashed lines represent virtual states. The dot marks the value of $\mathcal{P}$ where the pole changes the Riemann sheet. The dotted horizontal gray line shows the two-body binding energy $\mathcal{B}_2$, which acts as the threshold for the trimer states.

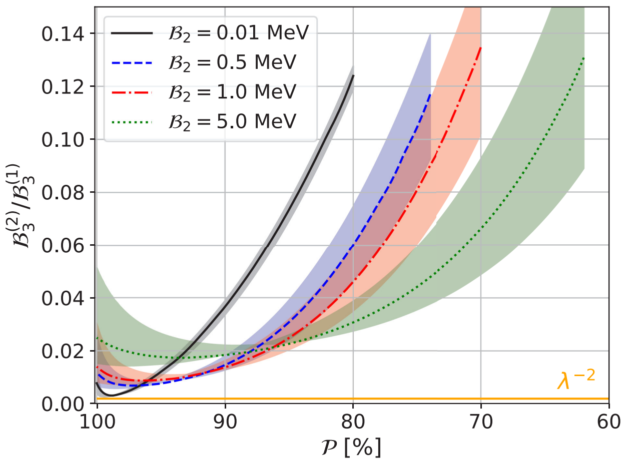

FIG. 3. Ratio of the second to first trimer binding energies for different $\mathcal{B}_2$ as a function of the T_{cc}^* composition, where $\mathcal{P} = 100\%$ denotes a pure two-body T_{cc}^* molecule and $\mathcal{P} = 0\%$ a pure compact state. The horizontal orange line represents the Efimov scaling factor $\lambda^{-2} \sim 1/515$ at the unitary limit $a_{\rm sc} \to \infty$. A cutoff of $\Lambda = 0.7$ GeV has been used in Eq. (4) for the central line, while the color error bands indicate the results for the cutoff range $\Lambda = [0.5, 1]$ GeV.

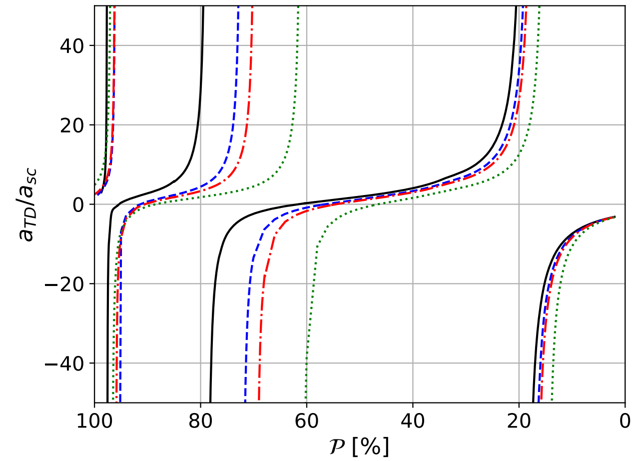

FIG. 4. $T_{cc}^* D^*$ scattering length normalized over the D^*D^* scattering length as a function of the T_{cc}^* composition for different binding energies $\mathcal{B}_2$, using $\Lambda = 0.7$ GeV. Same legend as in Fig. 3.

It should be recalled that the three-body binding energies are reduced with $\mathcal{P}$ because the coupling of the T_{cc}^* with the remaining D^* drops to zero as $\mathcal{P} \to 0$, as it does the scattering length $a_{\rm sc}$. This formalism does not include any interactions between a given compact component of the T_{cc}^* and the charmed meson, which could add further sources of attraction or repulsion as explored in Ref. [30]. Needless to say, if the T_{cc}^* is a compact tetraquark the $T_{cc}^* D^*$ system becomes a two-body problem and no Efimov state would manifest.

The ratios of the binding energies of the first and second trimer are shown in Fig. 3. The results indicate that the ratio increases as the T_{cc}^* becomes more compact, with a small valley that gets shallower as $\mathcal{B}_2$ increases. The bottom of this valley approaches the Efimov scaling law $\lambda^{-2} \sim 1/515 \sim 0.0019$ the smaller $\mathcal{B}_2$ and it is expected to reach it for $\mathcal{B}_2 \to 0$. This deviation from the λ^{-2} universal value appears because we are not exactly in the unitarity limit [20]. Furthermore, the addition of possible compact components in the T_{cc}^* wave function that mix with the D^*D^* pairs further modifies this scaling.

As we have seen, the upper limit for the trimer states is the $T_{cc}^* D^*$ threshold. Actually, this channel can be a potential detection mechanism. Indeed, the existence of the T_{cc}^* allows us to evaluate the scattering length of the T_{cc}^* and D^*, which will be called $a_{\rm TD}$. This can be calculated by using the dimer-spectator scattering amplitude $\mathcal{M}_{\rm TD}$, which encodes the information of the $T_{cc}^* D^* \to T_{cc}^* D^*$ reaction and is obtained by expanding the three-body amplitude $\mathcal{M}_3$ in the vicinity of the two-body bound state m_* [51],

$$\mathcal{M}_{\rm TD}(s) = g^2 \lim_{s_2(p_i), s_2(p_f) \to m_*^2} d(p_i, p_f), \tag{14}$$

where g^2 is the residue of the two-body scattering amplitude $\mathcal{M}_2$ around the T_{cc}^* mass. Then, the $T_{cc}^* D^*$ scattering length can be calculated as

$$-\frac{1}{a_{\rm TD}} = \lim_{s \to m_{\rm TD}^2} 8\pi\sqrt{s}\,{\rm Re}(\mathcal{M}_{\rm TD}^{-1}(s)) \tag{15}$$

with $m_{\rm TD} = 3m - \mathcal{B}_2$ the mass of the $T_{cc}^* D^*$ threshold. Results for $a_{\rm TD}$ for different values of $\mathcal{B}_2$ and $\mathcal{P}$ are shown in Fig. 4. The crossing of the $T_{cc}^* D^*$ threshold by the $D^*D^*D^*$ states is shown as an infinite in $a_{\rm TD}$, similarly to a two-body state. Then, this parameter can give us information about the $T_{cc}^* D^*$ scattering and the closeness of a pole.

In this work we have analyzed the $D^*D^*D^*$ system in the $(I)J^P = (\frac{1}{2})0^-$ sector. The results indicate that the Efimov effect can indeed emerge in this system. That's it, we find a spectrum of trimer states bound due to long-range interactions as a consequence of a nearly-resonant two-body system, explored using a general energy-dependent two-body potential that models a mixed T_{cc}^* state composed of compact and molecular D^*D^* structures. At least one trimer can be formed, as predicted by some studies [31,32]. The emergence of a second and third trimer depends on the molecular percentage of the T_{cc}^* resonance and its binding energy. The third trimer is unlikely to be bound, but the second one can be formed for reasonable $\mathcal{P}$ and $\mathcal{B}_2$. Thus, this system deserves more experimental and theoretical studies to clarify this phenomena. Of course, the first step would be the experimental detection of the T_{cc}^* state.

We want to remark that we do not discard this effect in the DD^*D^* system. According to HQSS, the DD^* and D^*D^* systems have the same potential in the $(0)1^+$ sector.

Then, the scaling law of the Efimov effect in the case of two identical bosons M plus one particle m resonantly interacting with each other is also $\lambda \approx 22.7$. Actually, this ratio decreases with the mass ratio M/m, but for the DD^*D^* the factor is $m_{D^*}/m_D \approx 1.08$ and the effect will be small (See Fig. 12 of Ref. [15]). However, a more detailed calculation would be needed in order to fully clarify the existence of trimers in the $T_{cc}D^*$-T_{cc}^*D systems.

The realization of the Efimov effect with charmed mesons would be an exceptional discovery and a step forward in our understanding of multimeson states, and this system is a promising place to investigate it. In fact, an interesting framework could be the analysis of the $D^*D^*D^*$ system in nuclear medium, as some studies show significant modifications of masses and widths of the $T_{cc}^{(*)}$ state [49], so it is possible that the resonant limit can be modulated for specific nuclear densities.

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