Dimension-eight operator basis for universal standard model effective field theory

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We present the basis of dimension-eight operators associated with universal theories. We first derive a complete list of independent dimension-eight operators formed with the Standard Model bosonic fields characteristic of such universal new physics scenarios. Without imposing C or P symmetries the basis contains 175 operators—that is, the assumption of universality reduces the number of independent Standard Model effective field theory (SMEFT) coefficients at dimension eight from 44807 to 175. 89 of the 175 universal operators are included in the general dimension-eight operator basis in the literature. The 86 additional operators involve higher derivatives of the Standard Model bosonic fields and can be rotated in favor of operators involving fermions using the Standard Model equations of motion for the bosonic fields. By doing so we obtain the allowed fermionic operators generated in this class of models which we map into the corresponding 86 independent combinations of operators in the dimension-eight basis of [C. W. Murphy, Dimension-8 operators in the standard model effective field theory, J. High Energy Phys. 10 (2020) 174.].

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I. INTRODUCTION

The Standard Model (SM) based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry has been extensively tested at the Large Hadron Collider (LHC) and so far, no deviation of its predictions [1] or new heavy state have been observed [2]. The natural conclusion is that there must be a mass gap between the electroweak scale and the beyond the Standard Model (BSM) physics required to address the well-known shortcomings of the SM. In this scenario, precision measurements of SM processes are an important tool to probe BSM physics and effective field theory (EFT) [3–5] has become the standard tool employed to search for hints of new physics.

The paradigmatic advantage of EFTs for BSM searches is its model–independence since they are based exclusively on the low-energy accessible states and symmetries. Assuming that the scalar particle observed in 2012 [6,7] belongs to an electroweak doublet, the $SU(2)_L \otimes U(1)_Y$ gauge symmetry can be realized linearly at low energies. The resulting model is the so-called Standard Model EFT (SMEFT) which can be written as

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{j=1} \sum_n \frac{f_n^{(j)}}{\Lambda^j} \mathcal{O}_n^{(j)}, \qquad (1)$$

where the higher-dimension operators $\mathcal{O}_n^{(j)}$ involve gauge–boson, Higgs–boson and/or fermionic fields with Wilson coefficients f_n and Λ is a characteristic scale.

There is a plethora of analyses of the LHC data in terms of the SMEFT up to dimension-six; see for instance [8–21] and references therein. In order to assess the importance of the different contributions in the $1/\Lambda$ expansion in such analysis, as well as avoid the appearance of phase space regions where the cross section is negative [13], one is required in many cases to perform the full calculation at order $1/\Lambda^4$. As is well known the consistent calculation at order $1/\Lambda^4$ requires the introduction of the contributions stemming from dimension-eight operators.

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At this point the advantage of the model-independent approach mentioned above becomes a limitation due to the large number of Wilson coefficients. Already at dimension-six there are 2499 possible operators when taking flavor into account [22,23]. At dimension-eight the number grows to 44,807 [24,25]. Clearly such large number of operators precludes a complete general analysis at any order beyond $1/\Lambda$ and we are forced to reintroduce some model dependent hypothesis. In this realm, identifying physically motivated hypothesis able to capture a large class of BSM theories becomes the new paradigm.

One such well-motivated hypothesis is that of *Universality*, which in brief refers to BSM scenarios where the new physics (NP) dominantly couples to the gauge bosons of the Standard Model. It was first put forward in the context of the analysis of electroweak precision data from LEP and low energy experiments, with the introduction of the oblique parameters S, T, U [26,27] (or ϵ_1, ϵ_2, ϵ_3 [28]) which captured the dominant NP effects in the observables. In the context of the SMEFT, Universality formally refers to BSM models for which the low-energy effects can be parametrized in terms of operators involving exclusively the SM bosons, hereon referred to as *bosonic operators* [29]. Ultraviolet (UV) completions that satisfy this specific definition of universal theories include theories in which the new states couple only to the bosonic sector, as in composite Higgs models [30], as well as models where the SM fermions are coupled to new states via SM-like currents [31,32] like in type I two-Higgs-doublet models [33].

In the EFT framework not all operators at a given order are independent as operators related by local changes of variables in quantum field theories possessing a jacobian determinant equal to one at the origin exhibit the same S–matrix elements [34,35]. In particular, operators connected by the use of the classical equations of motion (EOM) of the SM fields lead to the same S–matrix elements [36–39].[1] In general, a given SMEFT basis trades some of the bosonic operators for other bosonic operators and operators involving fermions, hereon called *fermionic operators*, in order to keep only independent operators. Therefore, the action of a rotated operator is equivalent to a relation between the Wilson coefficients in the basis. These relations for universal dimension-six operators were obtained in Ref. [29].

This work represents the next step in the exploration of the BSM effects for universal theories by presenting the SMEFT operator basis and relations implied by the universality hypothesis at dimension-eight. As a first step we search for a complete list of independent dimension-eight operators composed exclusively with SM bosons before the use of EOM. A large fraction of these operators involve higher derivatives of the gauge bosons and/or the Higgs field and therefore, in the existing dimension-eight basis [24,25], they have been generically eliminated in favor of fermionic operators. Consequently, in universal theories only a subset of the fermionic operators of the general dimension-eight operator basis are generated and, furthermore, their Wilson coefficients are related. In this work we use, for concreteness, the basis presented by Murphy in Ref. [24] which we refer to as M8B. Thus, the program at hand is first to identify a suitable basis of independent bosonic operators at dimension-eight and then by application of EOM to identify the combination of fermionic operators of M8B associated with universal theories.

The relevance of constructing the most general EFT within a minimal set of assumptions—such as that of Universality—is precisely to provide a tool for phenomenological studies as model independent as possible within that assumption. On this front, it is important to stress that the universality assumption allows us to perform detailed studies at $1/\Lambda^4$ without resorting to very simplified hypothesis where just one dimension-eight operator is considered, or to specific UV completions. For instance, working in the framework of universal models, Ref. [42] studies the impact of dimension-eight operators on the experimental analysis of anomalous triple gauge couplings by combining the available electroweak precision data and electroweak diboson (W^+W^-, $W^\pm Z$, $W^\pm\gamma$) productions. It is interesting to notice that the inclusion of dimension-eight operators breaks the relation $\lambda_\gamma = \lambda_Z$ that holds for the dimension-six operators. Another possible application is the complete $1/\Lambda^4$ analysis of Drell-Yan processes [43] that goes beyond the S, T, W, and Y oblique parameter analysis [44] with the introduction of further contributions to the electroweak gauge boson propagators.

For the sake of illustration we also present in Sec. VI a few simple UV completions of the SM that give rise only to bosonic operators when heavy states are integrated out at tree level. As expected, once a specific UV model is specified, only a subset of the possible dimension-eight universal operators is generated, and its number grows with the complexity of the UV completion and its mass spectrum. Thus the results in this paper can be generically utilized in two different approaches. Firstly, as mentioned above, it allows to perform a $1/\Lambda^4$ complete analysis in a totally model agnostic way by considering all universal dimension-six and -eight operators which contribute to the process of interest. Alternatively, it can be of practical use when working within a specific universal UV completion matched to the SMEFT by integrating out the heavy states to obtain the generated bosonic effective operators up to dimension eight. In this case the results in appendix A can be used to rotate these generated bosonic operators to M8B without

[1]When considering higher orders in the $1/\Lambda$ expansion one needs to take care when applying the EOM. While they are consistent when at the highest order in the expansion considered, at lower orders one needs to include terms "beyond linear order." Alternatively, the application of field redefinitions is always consistent [40,41].

having to do each time the exercise of applying the equivalence of operators by integration by parts, Fierz identities or equations of motion because it has been already taken care of.

The work is organized as follows. Sec. II contains our notation and framework. In Sec. II we present our notation and framework. Section III is dedicated to presenting our basis of independent dimension-eight universal bosonic operators while in Sec. IV we construct the Lorentz structures involving fermions associated with the product of SM currents, which are used in Sec. V to obtain the basis of universal fermionic operators. In Sec. VI we introduce a few simple bosonic UV completions and the corresponding low-energy operators, while we present our final remarks in Sec. VII. The work is complemented with three appendices. The full explicit expressions of the relations between the bosonic and fermionic operators for universal theories are presented in Appendix A. For convenience we include in Appendix B a compilation of the relations more frequently employed, and we reproduce in Appendix C the subset of M8B operators which appear in the universal operators.

II. NOTATION AND FRAMEWORK

Our conventions are such that the SM lagrangian reads

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_\mu H|^2 + \lambda v^2|H|^2 - \lambda|H|^4 + \sum_{f\in\{q,\ell,u,d,e\}} i\bar{f}\not{D}f - [(\tilde{H}^\dagger\bar{u}y^{u\dagger}q + \bar{q}y^d dH + \overline{\ell}y^e eH + \text{H.c.})], \tag{2}$$

where $G^A_{\mu\nu}, W^a_{\mu\nu}, B_{\mu\nu}$ stand for the field strength tensors of $SU(3)_c, SU(2)_L, U(1)_Y$ respectively. We denoted the quark and lepton doublets by q and ℓ while the $SU(2)_L$ singlets are u, d and e and the respective Yukawa couplings are $y^{u,d,e}$. We also define $\tilde{H}_j = \epsilon_{jk}H^{k\dagger}$ with $\epsilon_{12} = +1$.[2] The covariant derivative for objects in the fundamental representation reads $D_\mu = \partial_\mu - ig_s T^A G^A_\mu - ig\frac{\tau^a}{2}W^a_\mu - ig' Y B_\mu$ where Y is the hypercharge of the particle, T^A are the $SU(3)_c$ generators and τ^a stands for the Pauli matrices. On the other hand, the covariant derivatives for the field strengths are

$$D_\rho B^{\mu\nu} = \partial_\rho B^{\mu\nu}, \qquad D_\rho W^{a\mu\nu} = \partial_\rho W^{a\mu\nu} + g\epsilon^{abc}W^b_\rho W^{c\mu\nu}, \qquad D_\rho G^{A\mu\nu} = \partial_\rho G^{A\mu\nu} + g_s f^{ABC}G^B_\rho G^{C\mu\nu}, \tag{3}$$

where f^{ABC} are the $SU(3)_c$ structure constants. We denote the $SU(3)_c$ completely symmetric constants by d^{ABC}.

As mentioned above the first step in the program is to obtain the basis of independent dimension-eight operators consisting only of SM bosons. In order to do so we first obtained the number of independent operators belonging to each of the different bosonic classes before applying the EOM using available packages like BASISGEN [45], a modified version of ECO [46] given in Ref. [47] and GrIP [48]. Next, we wrote down all possible operators satisfying the SM gauge symmetry and Lorentz invariance. In this process, we worked with the irreducible Lorentz representation of the field strengths

$$X^{\mu\nu}_{L,R} = \frac{1}{2}(X^{\mu\nu} \mp i\tilde{X}^{\mu\nu}) \quad \text{with} \quad \tilde{X}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}X_{\rho\sigma}, \tag{4}$$

where we defined the Levi-Civita totally antisymmetric tensor $\epsilon_{0123} = -\epsilon^{0123} = +1$. The transformation properties of these fields under the Lorentz group are simple, $X_L \sim (1,0)$ and $X_R \sim (0,1)$ under $SU(2)_L \otimes SU(2)_R$. The Bianchi identity reads $D_\mu\tilde{X}^{\mu\nu} = 0$ implying that $D_\mu X^{\mu\nu}_L = D_\mu X^{\mu\nu}_R$. At this point, we obtained all possible linear relations between our set of operators using $SU(3)$ and $SU(2)$ Fierz transformations [49–51] summarized in Appendix B.

Further, linear relations between the effective operators in a given class can be obtained using integration by parts (IBP) for which we follow a procedure similar to the one described in Ref. [52]. In brief, given the field content and number of derivatives in a given class we obtain all operators invariant under gauge and Lorentz transformations. To obtain the relations among them implied by IBP we write all the vector structures y^ν_j that contain one less derivative than the operator class under consideration, then the IBP relations are obtained by setting $D_\nu y^\nu_j = 0$. At this point, we consider the Fierz and IBP linear relations and eliminate as many operators as there are independent relations. In order to apply the EOM more easily, we then express the final set of operators in terms of the field strengths $X^{\mu\nu}$ and their duals.

As illustration of the above procedure, let us consider the $D^2 B_L H^4$ operator class that contains eight members[3,4]:

[2] It should be noted, with our conventions for $\tilde{H}_j$ and ϵ_{jk} that assuming $y^{u\dagger}$ is diagonal will result in a wrong sign for the up-quark mass. Therefore if one neglects CKM considerations $y^{u\dagger}$ should be assumed to be proportional to $-\text{diag}(m_u, m_c, m_t)$.

[3] Terms like $D_\mu D_\nu B^{\mu\nu}_L$ give rise to operators in the $X_L B_L H^4$ class and were not considered for simplicity.

[4] Hereon $D^\nu H^\dagger$ stands for $(D^\nu H)^\dagger$ for the sake of simplicity.

$$x_1 = B_L^{\mu\nu}(D_\mu H^\dagger D_\nu H)(H^\dagger H), \tag{5}$$

$$x_2 = B_L^{\mu\nu}(D_\mu H^\dagger H)(H^\dagger D_\nu H), \tag{6}$$

$$x_3 = (D_\mu B_L^{\mu\nu})(D_\nu H^\dagger H)(H^\dagger H), \tag{7}$$

$$x_4 = (D_\mu B_L^{\mu\nu})(H^\dagger D_\nu H)(H^\dagger H), \tag{8}$$

$$x_5 = B_L^{\mu\nu}(D_\mu H^\dagger \tau^I D_\nu H)(H^\dagger \tau^I H), \tag{9}$$

$$x_6 = B_L^{\mu\nu}(D_\mu H^\dagger \tau^I H)(H^\dagger \tau^I D_\nu H), \tag{10}$$

$$x_7 = (D_\mu B_L^{\mu\nu})(D_\nu H^\dagger \tau^I H)(H^\dagger \tau^I H), \tag{11}$$

$$x_8 = (D_\mu B_L^{\mu\nu})(H^\dagger \tau^I D_\nu H)(H^\dagger \tau^I H). \tag{12}$$

At this stage, we consider operators and their Hermitian conjugates as different structures. In this example, linear Fierz relations can be obtained using Eq. (B2) leading to

$$x_5 = 2x_2 - x_1, \tag{13}$$

$$x_6 = 2x_1 - x_2, \tag{14}$$

$$x_7 = x_3, \tag{15}$$

$$x_8 = x_4. \tag{16}$$

We can see clearly from these relations that we can trade (x_5, x_6, x_7, x_8) for (x_1, x_2, x_3, x_4). Therefore, we focus on the latter operator set when obtaining the IBP relations which are derived from the following vector operators

$$y_1^\nu = B_L^{\mu\nu}(D_\mu H^\dagger H)(H^\dagger H), \tag{17}$$

$$y_2^\nu = B_L^{\mu\nu}(H^\dagger D_\mu H)(H^\dagger H), \tag{18}$$

$$y_3^\nu = (D_\mu B_L^{\mu\nu})(H^\dagger H)^2. \tag{19}$$

The IBP relations are, then, derived from $D_\nu y_j^\nu = 0$ and they read

$$x_1 + x_2 - x_3 = 0, \tag{20}$$

$$x_1 + x_2 + x_4 = 0, \tag{21}$$

$$x_3 + x_4 = 0. \tag{22}$$

Just two of the last relations are independent, so we have two independent operators that we can choose to be x_1 and x_3 since this choice renders the rotations of these operators into M8B straightforward.

Once the set of independent bosonic operators have been identified we apply the EOM to those with one or more derivatives acting on the gauge strength tensors and two or more acting on the Higgs field. With our conventions the EOM read

$$\begin{aligned} D_\mu G^{A\mu\nu} &= -J_G^{A\nu}, \\ D_\mu W^{I\mu\nu} &= -\frac{ig}{2} H^\dagger \overleftrightarrow{D}^{I\nu} H - J_W^{I\nu}, \\ D_\mu B^{\mu\nu} &= -\frac{ig'}{2} H^\dagger \overleftrightarrow{D}^{\nu} H - J_B^{\nu}, \\ (D^2 H^\dagger)^j &= \lambda v^2 H^{\dagger j} - 2\lambda (H^\dagger H) H^{\dagger j} - J_H^j, \end{aligned} \tag{23}$$

where $H^\dagger \overleftrightarrow{D}^{I\nu} H = H^\dagger \tau^I D^\nu H - D^\nu H^\dagger \tau^I H$ and we have defined the fermionic "currents"

$$\begin{aligned} J_G^{A\mu} &= g_s \sum_{f\in\{q,u,d\}} \sum_a \bar{f}_a \gamma^\mu T^A f_a, \\ J_W^{I\mu} &= \frac{g}{2} \sum_{f\in\{q,l\}} \sum_a \bar{f}_a \gamma^\mu \tau^I f_a, \\ J_B^{\mu} &= g' \sum_{f\in\{q,l,u,d,e\}} \sum_a Y_f \bar{f}_a \gamma^\mu f_a, \\ J_H^j &= \sum_{ab} \{ y_{ab}^{u\dagger} (\bar{u}_a q_{bk}) \epsilon^{jk} + y_{ab}^d, (\bar{q}_a^j d_b) + y_{ab}^e (\bar{l}_a^j e_b) \}, \\ J_{Hj}^\dagger &= \sum_{ab} \{ y_{ab}^u (\bar{q}_a^k u_b) \epsilon_{kj} + y_{ab}^{d\dagger}, (\bar{d}_a q_{bj}) + y_{ab}^{e\dagger} (\bar{e}_a l_{bj}) \}. \end{aligned} \tag{24}$$

Y_f are the fermionic hypercharges, $\{Y_q, Y_l, Y_u, Y_d, Y_e\} = \{\frac{1}{6}, -\frac{1}{2}, \frac{2}{3}, -\frac{1}{3}, -1\}$ and J_H^j, does not contain the CKM matrix because the fermion fields in these equations are in gauge eigenstates (labeled with the latin indexes a, b or c) and so are the Yukawa matrices y^f. In addition, we denote the $SU(2)_L$ indices as ijk.

Expressing the fermionic operators generated by products of these currents and their derivatives in terms of operators in the M8B basis requires in some cases trivial but lengthy field manipulations which make use of identities involving the $SU(2)$ and $SU(3)$ generators as well as Fierz field rearrangements [49–51]; see Appendix B for the more frequently employed relations. In addition, the simplification also involves the equations of motion for the fermions which in our notation read

$$\begin{aligned} i\not{D} l_{aj} &= \sum_b y_{ab}^e e_b H_j, \qquad i\not{D} e_a = \sum_b y_{ab}^{e\dagger} l_{bj} H^{\dagger,j}, \\ i\not{D} d_a &= \sum_b y_{ab}^{d\dagger} q_{bj} H^{\dagger,j}, \qquad i\not{D} u_a = \sum_b y_{ab}^{u\dagger} q_{bj} \tilde{H}^{\dagger,j}, \\ i\not{D} q_{aj} &= \sum_b [y_{ab}^d d_b H_j + y_{ab}^u u_b \tilde{H}_j], \end{aligned} \tag{25}$$

together with the covariant conservation of the gauge currents which imply that

$$D_\mu J_B^\mu = -i\frac{g'}{2} D_\mu (H^\dagger \overleftrightarrow{D}^\mu H), \qquad D_\mu J_W^{I,\mu} = -i\frac{g}{2} D_\mu (H^\dagger \overleftrightarrow{D}^{I\mu} H), \tag{26}$$

and the commutators of the covariant derivatives of the gauge currents are

$$[D_\alpha, D_\beta] J_B^\mu = 0, \qquad [D_\alpha, D_\beta] J_W^{I\nu} = g\epsilon^{IJK} W_{\alpha\beta}^J J_W^{K\nu},$$
$$[D_\alpha, D_\beta] J_G^{A\nu} = g_s f^{ABC} G_{\alpha\beta}^B J_G^{C\nu}. \tag{27}$$

III. INDEPENDENT BOSONIC OPERATORS

The building blocks of the operator basis for universal theories are the Higgs field H, the SM field strengths ($X_{L,R}^{\mu\nu} \sim B_{L,R}^{\mu\nu}, W_{L,R}^{a\mu\nu}, G_{L,R}^{A\mu\nu}$) and covariant derivatives D. As mentioned above we obtain the number of independent operators with this field content using the packages BASISGEN [45] and ECO [46,47]. Doing so one finds, prior to the application of the EOM and without imposing C and P symmetries, there are 175 independent bosonic operators at dimension-eight. Of those, 89 can be chosen to be those included in M8B, and which, for convenience, we list in Table I. They include all independent operators without derivatives acting on the gauge strength tensors and with up to one derivative acting on each Higgs field. They lead to a rich and well-known phenomenology. For example, the operators in the classes X^4, X^3X' and $X^2X'^2$ generate anomalous quartic and higher gauge self-couplings that have no triple gauge vertex associated to them [53,54]. The operator in the H^8 class modifies the Higgs self-couplings and the operatores in the X^3H^2 class give rise to multi H [55–58] and gauge boson [59,60] vertices, e.g., anomalous triple gauge couplings [42,61]. Furthermore, the operators in class X^2H^4 class give finite renormalization to the SM input parameters [42] and they also generate multi Higgs and gauge boson vertices [62,63].

The first task at hand is, therefore, to identify a suitable set for the remaining 86 operators following the procedure sketched in the previous section. Since our final objective is to find the corresponding combinations of fermionic operators generated after application of the EOM, we select the 86 operators for which the transformation can be more directly implemented. With this in mind, we make the following choice of operators.

A. Operators with Higgs fields and two or more derivatives

Prior to applying the EOM, the classes H^6D^2, H^4D^4 and H^2D^6 contain 18 independent bosonic operators of which five are those included in the corresponding classes in Table I. As for the remaining 13 independent bosonic operators, 2 of them are in the class H^6D^2 and we chose them as

$$R_{H^6D^2}^{(1)} = (D^2H^\dagger H)(H^\dagger H)(H^\dagger H), \qquad R_{H^6D^2}^{(2)} = (H^\dagger D^2 H)(H^\dagger H)(H^\dagger H). \tag{28}$$

In addition, there are 10 independent operators in the class H^4D^4 selected to be

$$\begin{aligned}
R_{H^4D^4}^{(1)} &= (D^2H^\dagger\tau^I H)(D^\mu H^\dagger \tau^I D_\mu H), & R_{H^4D^4}^{(2)} &= (D^2H^\dagger D_\mu H)(H^\dagger D^\mu H),\\
R_{H^4D^4}^{(3)} &= (D_\mu H^\dagger D^2 H)(D^\mu H^\dagger H), & R_{H^4D^4}^{(4)} &= (H^\dagger \tau^I D^2 H)(D_\mu H^\dagger \tau^I D^\mu H),\\
R_{H^4D^4}^{(5)} &= (D^2H^\dagger H)(D_\mu H^\dagger D^\mu H), & R_{H^4D^4}^{(6)} &= (H^\dagger D^2 H)(D^\mu H^\dagger D_\mu H),\\
R_{H^4D^4}^{(7)} &= (D^2H^\dagger D^2 H)(H^\dagger H), & R_{H^4D^4}^{(8)} &= (D^2H^\dagger H)(D^2H^\dagger H),\\
R_{H^4D^4}^{(9)} &= (D^2H^\dagger H)(H^\dagger D^2 H), & R_{H^4D^4}^{(10)} &= (H^\dagger D^2 H)(H^\dagger D^2 H),
\end{aligned} \tag{29}$$

while there is only one in the class H^2D^6

$$R_{H^2D^6}^{(1)} = (D^\mu D^2 H^\dagger D^\mu D^2 H). \tag{30}$$

As we will see upon application of EOM they generate combinations of fermionic operators with two fermions of classes ψ^2H^5 and $\psi^2H^3D^2$, and operators with four fermions in classes ψ^4H^2 and ψ^4D^2 with chiralities $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{L}R)$, and $(\bar{L}R)(\bar{R}L)$, with related Wilson coefficients. Explicit expressions for the relations can be found in Eqs. (A1)–(A13) of Appendix A.

B. Operators with gauge field strengths and derivatives

There are 19 independent operators in classes X^3D^2, $X^2X'D^2$ and X^2D^4 none of which is included in M8B. Four involve three powers of the W field strength tensor and another four three powers of the G tensor and we selected them to be

TABLE I. Independent bosonic operators belonging to M8B.

$1:X^4, X^3X'$		$1:X^2X'^2$		$2:H^8$	
$Q_{G^4}^{(1)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{B\rho\sigma})$	$Q_{G^2W^2}^{(1)}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$	Q_{H^8}	$(H^\dagger H)^4$
$Q_{G^4}^{(2)}$	$(G^A_{\mu\nu}\tilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\tilde{G}^{B\rho\sigma})$	$Q_{G^2W^2}^{(2)}$	$(W^I_{\mu\nu}\tilde{W}^{I\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$3:H^6D^2$	
$Q_{G^4}^{(3)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}G^{B\rho\sigma})$	$Q_{G^2W^2}^{(3)}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}G^{A\rho\sigma})$	$Q_{H^6}^{(1)}$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$
$Q_{G^4}^{(4)}$	$(G^A_{\mu\nu}\tilde{G}^{B\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{B\rho\sigma})$	$Q_{G^2W^2}^{(4)}$	$(W^I_{\mu\nu}\tilde{G}^{A\mu\nu})(W^I_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$
$Q_{G^4}^{(5)}$	$(G^A_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\tilde{G}^{B\rho\sigma})$	$Q_{G^2W^2}^{(5)}$	$(W^I_{\mu\nu}\tilde{W}^{I\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$	$4:H^4D^4$	
$Q_{G^4}^{(6)}$	$(G^A_{\mu\nu}G^{B\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{B\rho\sigma})$	$Q_{G^2W^2}^{(6)}$	$(W^I_{\mu\nu}W^{I\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q_{G^4}^{(7)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}G^{D\rho\sigma})$	$Q_{G^2W^2}^{(7)}$	$(W^I_{\mu\nu}G^{A\mu\nu})(W^I_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q_{G^4}^{(8)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}\tilde{G}^{B\mu\nu})(G^C_{\rho\sigma}\tilde{G}^{D\rho\sigma})$	$Q_{G^2B^2}^{(1)}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$	$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$
$Q_{G^4}^{(9)}$	$d^{ABE}d^{CDE}(G^A_{\mu\nu}G^{B\mu\nu})(G^C_{\rho\sigma}\tilde{G}^{D\rho\sigma})$	$Q_{G^2B^2}^{(2)}$	$(B_{\mu\nu}\tilde{B}^{\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$5:X^3H^2$	
$Q_{W^4}^{(1)}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}W^{J\rho\sigma})$	$Q_{G^2B^2}^{(3)}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}G^{A\rho\sigma})$	$Q_{G^3H^2}^{(1)}$	$f^{ABC}(H^\dagger H)G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$
$Q_{W^4}^{(2)}$	$(W^I_{\mu\nu}\tilde{W}^{I\mu\nu})(W^J_{\rho\sigma}\tilde{W}^{J\rho\sigma})$	$Q_{G^2B^2}^{(4)}$	$(B_{\mu\nu}\tilde{G}^{A\mu\nu})(B_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{G^3H^2}^{(2)}$	$f^{ABC}(H^\dagger H)G^{A\nu}_\mu G^{B\rho}_\nu \tilde{G}^{C\mu}_\rho$
$Q_{W^4}^{(3)}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}W^{J\rho\sigma})$	$Q_{G^2B^2}^{(5)}$	$(B_{\mu\nu}\tilde{B}^{\mu\nu})(G^A_{\rho\sigma}G^{A\rho\sigma})$	$Q_{W^3H^2}^{(1)}$	$\epsilon^{IJK}(H^\dagger H)W^{I\nu}_\mu W^{J\rho}_\nu W^{K\mu}_\rho$
$Q_{W^4}^{(4)}$	$(W^I_{\mu\nu}\tilde{W}^{J\mu\nu})(W^I_{\rho\sigma}\tilde{W}^{J\rho\sigma})$	$Q_{G^2B^2}^{(6)}$	$(B_{\mu\nu}B^{\mu\nu})(G^A_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{W^3H^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger H)W^{I\nu}_\mu W^{J\rho}_\nu \tilde{W}^{K\mu}_\rho$
$Q_{W^4}^{(5)}$	$(W^I_{\mu\nu}W^{I\mu\nu})(W^J_{\rho\sigma}\tilde{W}^{J\rho\sigma})$	$Q_{G^2B^2}^{(7)}$	$(B_{\mu\nu}G^{A\mu\nu})(B_{\rho\sigma}\tilde{G}^{A\rho\sigma})$	$Q_{W^2BH^2}^{(1)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)B^\nu_\mu W^{J\rho}_\nu W^{K\mu}_\rho$
$Q_{W^4}^{(6)}$	$(W^I_{\mu\nu}W^{J\mu\nu})(W^I_{\rho\sigma}\tilde{W}^{J\rho\sigma})$	$Q_{W^2B^2}^{(1)}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$	$Q_{W^2BH^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)$ $(\tilde{B}^{\mu\nu}W^J_{\nu\rho}W^{K\rho}_\mu + B^{\mu\nu}W^J_{\nu\rho}\tilde{W}^{K\rho}_\mu)$
$Q_{B^4}^{(1)}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}B^{\rho\sigma})$	$Q_{W^2B^2}^{(2)}$	$(B_{\mu\nu}\tilde{B}^{\mu\nu})(W^I_{\rho\sigma}\tilde{W}^{I\rho\sigma})$		
$Q_{B^4}^{(2)}$	$(B_{\mu\nu}\tilde{B}^{\mu\nu})(B_{\rho\sigma}\tilde{B}^{\rho\sigma})$	$Q_{W^2B^2}^{(3)}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}W^{I\rho\sigma})$	$6:X^2H^4$	
$Q_{B^4}^{(3)}$	$(B_{\mu\nu}B^{\mu\nu})(B_{\rho\sigma}\tilde{B}^{\rho\sigma})$	$Q_{W^2B^2}^{(4)}$	$(B_{\mu\nu}\tilde{W}^{I\mu\nu})(B_{\rho\sigma}\tilde{W}^{I\rho\sigma})$	$Q_{G^2H^4}^{(1)}$	$(H^\dagger H)^2 G^A_{\mu\nu}G^{A\mu\nu}$
$Q_{G^3B}^{(1)}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q_{W^2B^2}^{(5)}$	$(B_{\mu\nu}\tilde{B}^{\mu\nu})(W^I_{\rho\sigma}W^{I\rho\sigma})$	$Q_{G^2H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{G}^A_{\mu\nu}G^{A\mu\nu}$
$Q_{G^3B}^{(2)}$	$d^{ABC}(B_{\mu\nu}\tilde{G}^{A\mu\nu})(G^B_{\rho\sigma}\tilde{G}^{C\rho\sigma})$	$Q_{W^2B^2}^{(6)}$	$(B_{\mu\nu}B^{\mu\nu})(W^I_{\rho\sigma}\tilde{W}^{I\rho\sigma})$	$Q_{W^2H^4}^{(1)}$	$(H^\dagger H)^2 W^I_{\mu\nu}W^{I\mu\nu}$
$Q_{G^3B}^{(3)}$	$d^{ABC}(B_{\mu\nu}\tilde{G}^{A\mu\nu})(G^B_{\rho\sigma}G^{C\rho\sigma})$	$Q_{W^2B^2}^{(7)}$	$(B_{\mu\nu}W^{I\mu\nu})(B_{\rho\sigma}\tilde{W}^{I\rho\sigma})$	$Q_{W^2H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{W}^I_{\mu\nu}W^{I\mu\nu}$
$Q_{G^3B}^{(4)}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G^B_{\rho\sigma}\tilde{G}^{C\rho\sigma})$			$Q_{W^2H^4}^{(3)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^J H)W^I_{\mu\nu}W^{J\mu\nu}$
$7:X^2H^2D^2$				$Q_{W^2H^4}^{(4)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^J H)\tilde{W}^I_{\mu\nu}W^{J\mu\nu}$
$Q_{G^2H^2D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)G^A_{\mu\rho}G^{A\rho}_\nu$	$Q_{B^2H^2D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho}B^\rho_\nu$	$Q_{B^2H^4}^{(1)}$	$(H^\dagger H)^2 B_{\mu\nu}B^{\mu\nu}$
$Q_{G^2H^2D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)G^A_{\nu\rho}G^{A\nu\rho}$	$Q_{B^2H^2D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)B_{\nu\rho}B^{\nu\rho}$	$Q_{WBH^4}^{(1)}$	$(H^\dagger H)(H^\dagger \tau^I H)W^I_{\mu\nu}B^{\mu\nu}$
$Q_{G^2H^2D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)G^A_{\nu\rho}\tilde{G}^{A\nu\rho}$	$Q_{B^2H^2D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)B_{\nu\rho}\tilde{B}^{\nu\rho}$	$Q_{WBH^4}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)\tilde{W}^I_{\mu\nu}B^{\mu\nu}$
$Q_{W^2H^2D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)W^I_{\mu\rho}W^{I\rho}_\nu$	$Q_{WBH^2D^2}^{(1)}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\nu\rho}W^{I\nu\rho}$	$Q_{B^2H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{B}_{\mu\nu}B^{\mu\nu}$
$Q_{W^2H^2D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)W^I_{\nu\rho}W^{I\nu\rho}$	$Q_{WBH^2D^2}^{(2)}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\nu\rho}\tilde{W}^{I\nu\rho}$	$8:XH^4D^2$	
$Q_{W^2H^2D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)W^I_{\nu\rho}\tilde{W}^{I\nu\rho}$	$Q_{WBH^2D^2}^{(3)}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu - B_{\nu\rho}W^{I\rho}_\mu)$	$Q_{WH^4D^2}^{(1)}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)W^I_{\mu\nu}$
$Q_{W^2H^2D^2}^{(4)}$	$i\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)W^J_{\mu\rho}W^{K\rho}_\nu$			$Q_{WH^4D^2}^{(2)}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)\tilde{W}^I_{\mu\nu}$
$Q_{W^2H^2D^2}^{(5)}$	$\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\tilde{W}^{K\rho}_\nu - \tilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$	$Q_{WBH^2D^2}^{(4)}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu)$	$Q_{WH^4D^2}^{(3)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H)W^K_{\mu\nu}$
				$Q_{WH^4D^2}^{(4)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H)\tilde{W}^K_{\mu\nu}$
$Q_{W^2H^2D^2}^{(6)}$	$i\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W^J_{\mu\rho}\tilde{W}^{K\rho}_\nu + \tilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$	$Q_{WBH^2D^2}^{(5)}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}\tilde{W}^{I\rho}_\nu - B_{\nu\rho}\tilde{W}^{I\rho}_\mu)$	$Q_{BH^4D^2}^{(1)}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H)B_{\mu\nu}$
				$Q_{BH^4D^2}^{(2)}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H)\tilde{B}_{\mu\nu}$
		$Q_{WBH^2D^2}^{(6)}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho}\tilde{W}^{I\rho}_\nu + B_{\nu\rho}\tilde{W}^{I\rho}_\mu)$		

$$R^{(1)}_{W^3D^2} = W^I_{\mu\nu}(D_\alpha W^{J,\alpha\mu})(D_\beta W^{K,\beta\nu})\epsilon^{IJK}, \quad R^{(1)}_{G^3D^2} = G^A_{\mu\nu}(D_\alpha G^{B,\alpha\mu})(D_\beta G^{C,\beta\nu})f^{ABC},$$
$$R^{(2)}_{W^3D^2} = \tilde{W}^I_{\mu\nu}(D_\alpha W^{J,\alpha\mu})(D_\beta W^{K,\beta\nu})\epsilon^{IJK}, \quad R^{(2)}_{G^3D^2} = \tilde{G}^A_{\mu\nu}(D_\alpha G^{B,\alpha\mu})(D_\beta G^{C,\beta\nu})f^{ABC},$$
$$R^{(3)}_{W^3D^2} = W^I_{\mu\nu}W^{J,\nu}_{\rho}(D^\mu D_\alpha W^{K,\alpha\rho})\epsilon^{IJK}, \quad R^{(3)}_{G^3D^2} = G^A_{\mu\nu}G^{B,\nu}_{\rho}(D^\mu D_\alpha G^{C,\alpha\rho})f^{ABC},$$
$$R^{(4)}_{W^3D^2} = W^I_{\mu\nu}\tilde{W}^{J,\nu}_{\rho}(D^\mu D_\alpha W^{K,\alpha\rho} - D^\rho D_\alpha W^{K,\alpha\mu})\epsilon^{IJK}, \quad R^{(4)}_{G^3D^2} = G^A_{\mu\nu}\tilde{G}^{B,\nu}_{\rho}(D^\mu D_\alpha G^{C,\alpha\rho} - D^\rho D_\alpha G^{C,\alpha\mu})f^{ABC}. \tag{31}$$

Eight operators contain two powers of $W^{\mu\nu}$ or $G^{\mu\nu}$ together with $B^{\mu\nu}$ which can be chosen as

$$R^{(1)}_{BW^2D^2} = (D^\mu B_{\mu\nu})W^{I,\nu\rho}(D^\alpha W^I_{\rho\alpha}), \quad R^{(1)}_{BG^2D^2} = G^A_{\mu\nu}(D_\alpha G^{A,\alpha\mu})(D_\beta B^{\beta\nu}),$$
$$R^{(2)}_{BW^2D^2} = (D^\mu B_{\mu\nu})\tilde{W}^{I,\nu\rho}(D^\alpha W^I_{\rho\alpha}), \quad R^{(2)}_{BG^2D^2} = \tilde{G}^A_{\mu\nu}(D_\alpha G^{A,\alpha\mu})(D_\beta B^{\beta\nu}),$$
$$R^{(3)}_{BW^2D^2} = B_{\mu\nu}W^{I,\nu}_{\rho}(D^\mu D_\alpha W^{I,\alpha\rho} - D^\rho D_\alpha W^{I,\alpha\mu}), \quad R^{(3)}_{BG^2D^2} = B_{\mu\nu}G^{A,\nu}_{\rho}(D^\mu D_\alpha G^{A,\alpha\rho} - D^\rho D_\alpha G^{A,\alpha\mu}),$$
$$R^{(4)}_{BW^2D^2} = B_{\mu\nu}\tilde{W}^{I,\nu}_{\rho}(D^\mu D_\alpha W^{I,\alpha\rho} - D^\rho D_\alpha W^{I,\alpha\mu}), \quad R^{(4)}_{BG^2D^2} = B_{\mu\nu}\tilde{G}^{A,\nu}_{\rho}(D^\mu D_\alpha G^{A,\alpha\rho} - D^\rho D_\alpha G^{A,\alpha\mu}). \tag{32}$$

These operators modify the triple (multi) gauge couplings. Upon application of the EOM they will lead to combinations of two-fermion operators in the classes ψ^2H^5, ψ^2H^4D, ψ^2XH^2D, ψ^2X^2H, and ψ^2X^2D, and uniquely generate four-fermion operators in the class ψ^4X [see Eqs. (A14)–(A29)].

Finally, there are three operators in X^2D^4, one per gauge boson,

$$R^{(1)}_{B^2D^4} = D^\rho D^\alpha B_{\alpha\mu}D_\rho D^\beta B^\mu_\beta, \quad R^{(1)}_{W^2D^4} = D^\rho D^\alpha W^I_{\alpha\mu}D_\rho D^\beta W^{I,\mu}_\beta,$$
$$R^{(1)}_{G^2D^4} = D^\alpha D^\mu G^A_{\mu\nu}D_\alpha D^\rho G^{A,\nu}_\rho. \tag{33}$$

They affect the gauge boson propagators and can give rise to ghosts [64] in addition to anomalous multigauge boson vertices. Equations of motion rotate these three operators to combinations of two-fermion operators in classes ψ^2H^5, ψ^2H^4D, $\psi^2H^2D^3$, and ψ^2XH^2D as well as four-fermion operators in classes ψ^4H^2—with chiralities $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{L}R)$ and $(\bar{L}R)(\bar{R}L)$—and ψ^4D^2 with chiralities $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{L}R)$, $(\bar{L}R)(\bar{R}L)$, and $(\bar{R}R)(\bar{R}R)$ which can be found in Eqs. (A30)–(A32).

C. Operators with field strengths, Higgs fields and derivatives

There are 62 independent bosonic operators in the class $X^2H^2D^2$ prior the use of EOM. M8B contains 18 operators in this class; see Table I. There are, therefore, 44 additional independent bosonic operators in class $X^2H^2D^2$ of which 9 contain two powers of the hypercharge field strength tensor and another 9 contain two powers of the gluon field strength tensor

$$R^{(1)}_{B^2H^2D^2} = B_{\mu\nu}B^{\mu\nu}(D^2H^\dagger H), \quad R^{(1)}_{G^2H^2D^2} = G^A_{\mu\nu}G^{A\mu\nu}(D^2H^\dagger H),$$
$$R^{(2)}_{B^2H^2D^2} = B_{\mu\nu}B^{\mu\nu}(H^\dagger D^2H), \quad R^{(2)}_{G^2H^2D^2} = G^A_{\mu\nu}G^{A\mu\nu}(H^\dagger D^2H),$$
$$R^{(3)}_{B^2H^2D^2} = B_{\mu\nu}\tilde{B}^{\mu\nu}(D^2H^\dagger H), \quad R^{(3)}_{G^2H^2D^2} = G^A_{\mu\nu}\tilde{G}^{A\mu\nu}(D^2H^\dagger H),$$
$$R^{(4)}_{B^2H^2D^2} = B_{\mu\nu}\tilde{B}^{\mu\nu}(H^\dagger D^2H), \quad R^{(4)}_{G^2H^2D^2} = G^A_{\mu\nu}\tilde{G}^{A\mu\nu}(H^\dagger D^2H),$$
$$R^{(5)}_{B^2H^2D^2} = (D^\mu B_{\mu\nu})B^{\alpha\nu}(D_\alpha H^\dagger H), \quad R^{(5)}_{G^2H^2D^2} = (D^\mu G^A_{\mu\nu})G^{A\alpha\nu}(D_\alpha H^\dagger H),$$
$$R^{(6)}_{B^2H^2D^2} = (D^\mu B_{\mu\nu})B^{\alpha\nu}(H^\dagger D_\alpha H), \quad R^{(6)}_{G^2H^2D^2} = (D^\mu G^A_{\mu\nu})G^{A\alpha\nu}(H^\dagger D_\alpha H),$$
$$R^{(7)}_{B^2H^2D^2} = (D^\mu B_{\mu\nu})\tilde{B}^{\alpha\nu}(D_\alpha H^\dagger H), \quad R^{(7)}_{G^2H^2D^2} = (D^\mu G^A_{\mu\nu})\tilde{G}^{A\alpha\nu}(D_\alpha H^\dagger H),$$
$$R^{(8)}_{B^2H^2D^2} = (D^\mu B_{\mu\nu})\tilde{B}^{\alpha\nu}(H^\dagger D_\alpha H), \quad R^{(8)}_{G^2H^2D^2} = (D^\mu G^A_{\mu\nu})\tilde{G}^{A\alpha\nu}(H^\dagger D_\alpha H),$$
$$R^{(9)}_{B^2H^2D^2} = (D^\mu B_{\mu\alpha})(D_\nu B^{\nu\alpha})(H^\dagger H), \quad R^{(9)}_{G^2H^2D^2} = (D^\mu G^A_{\mu\alpha})(D_\nu G^{A\nu\alpha})(H^\dagger H), \tag{34}$$

while 13 contain two powers of the W field strength tensor and another 13 contain the product of the hypercharge and W field strength tensors

$$
\begin{aligned}
R^{(1)}_{W^2H^2D^2} &= W^I_{\mu\nu}W^{I,\mu\nu}(D^2H^\dagger H), & R^{(1)}_{BWH^2D^2} &= B_{\mu\nu}W^{I,\mu\nu}(H^\dagger\tau^I D^2H),\\
R^{(2)}_{W^2H^2D^2} &= W^I_{\mu\nu}W^{I,\mu\nu}(H^\dagger D^2H), & R^{(2)}_{BWH^2D^2} &= B_{\mu\nu}W^{I,\mu\nu}(D^2H^\dagger\tau^I H),\\
R^{(3)}_{W^2H^2D^2} &= W^I_{\mu\nu}\tilde{W}^{I,\mu\nu}(D^2H^\dagger H), & R^{(3)}_{BWH^2D^2} &= B_{\mu\nu}\tilde{W}^{I,\mu\nu}(H^\dagger\tau^I D^2H),\\
R^{(4)}_{W^2H^2D^2} &= W^I_{\mu\nu}\tilde{W}^{I,\mu\nu}(H^\dagger D^2H), & R^{(4)}_{BWH^2D^2} &= B_{\mu\nu}\tilde{W}^{I,\mu\nu}(D^2H^\dagger\tau^I H),\\
R^{(5)}_{W^2H^2D^2} &= (D^\mu W^I_{\mu\nu})W^{I,\alpha\nu}(D_\alpha H^\dagger H), & R^{(5)}_{BWH^2D^2} &= (D^\mu B_{\mu\alpha})W^{I,\alpha\nu}(D_\nu H^\dagger\tau^I H),\\
R^{(6)}_{W^2H^2D^2} &= (D^\mu W^I_{\mu\nu})W^{I,\alpha\nu}(H^\dagger D_\alpha H), & R^{(6)}_{BWH^2D^2} &= (D^\mu B_{\mu\alpha})W^{I,\alpha\nu}(H^\dagger\tau^I D_\nu H),\\
R^{(7)}_{W^2H^2D^2} &= (D^\mu W^I_{\mu\nu})\tilde{W}^{I,\alpha\nu}(D_\alpha H^\dagger H), & R^{(7)}_{BWH^2D^2} &= (D^\mu B_{\mu\alpha})\tilde{W}^{I,\alpha\nu}(D_\nu H^\dagger\tau^I H),\\
R^{(8)}_{W^2H^2D^2} &= (D^\mu W^I_{\mu\nu})\tilde{W}^{I,\alpha\nu}(H^\dagger D_\alpha H), & R^{(8)}_{BWH^2D^2} &= (D^\mu B_{\mu\alpha})\tilde{W}^{I,\alpha\nu}(H^\dagger\tau^I D_\nu H),\\
R^{(9)}_{W^2H^2D^2} &= (D^\mu W^I_{\mu\alpha})(D_\nu W^{I,\nu\alpha})(H^\dagger H), & R^{(9)}_{BWH^2D^2} &= (D^\mu W^I_{\mu\nu})B^{\nu\alpha}(D_\alpha H^\dagger\tau^I H),\\
R^{(10)}_{W^2H^2D^2} &= \epsilon^{IJK}(D^\mu W^I_{\mu\nu})W^{J,\rho\nu}(D_\rho H^\dagger\tau^K H), & R^{(10)}_{BWH^2D^2} &= (D^\mu W^I_{\mu\nu})B^{\nu\alpha}(H^\dagger\tau^I D_\alpha H),\\
R^{(11)}_{W^2H^2D^2} &= \epsilon^{IJK}(D^\mu W^I_{\mu\nu})W^{J,\rho\nu}(H^\dagger\tau^K D_\rho H), & R^{(11)}_{BWH^2D^2} &= (D^\mu W^I_{\mu\nu})\tilde{B}^{\nu\alpha}(D_\alpha H^\dagger\tau^I H),\\
R^{(12)}_{W^2H^2D^2} &= \epsilon^{IJK}(D^\mu W^I_{\mu\nu})\tilde{W}^{J,\rho\nu}(D_\rho H^\dagger\tau^K H), & R^{(12)}_{BWH^2D^2} &= (D^\mu W^I_{\mu\nu})\tilde{B}^{\nu\alpha}(H^\dagger\tau^I D_\alpha H),\\
R^{(13)}_{W^2H^2D^2} &= \epsilon^{IJK}(D^\mu W^I_{\mu\nu})\tilde{W}^{J,\rho\nu}(H^\dagger\tau^K D_\rho H), & R^{(13)}_{BWH^2D^2} &= (D^\mu B_{\mu\alpha})(D_\nu W^{I,\nu\alpha})(H^\dagger\tau^I H).
\end{aligned}
\tag{35}
$$

Generically, operators in this class modify the gauge couplings of the Higgs boson and vertices with two scalars and two or more gauge bosons. As we will see upon application of EOM they generate combinations of fermionic operators with two fermions belonging to the classes ψ^2H^5, ψ^2H^4D, ψ^2X^2H, and ψ^2XH^2D, and also operators with four fermions in classes ψ^4H^2 involving chiralities $(\bar{L}L)(\bar{R}R)$, $(\bar{L}R)(\bar{L}R)$, $(\bar{L}R)(\bar{R}L)$, and $(\bar{R}R)(\bar{R}R)$. Explicit expressions for the relations can be found in Eqs. (A33)–(A76) of Appendix A.

Class XH^4D^2 contains 10 independent operators, six of them in M8B and another four which we chose as

$$
\begin{aligned}
R^{(1)}_{BH^4D^2} &= (D_\alpha B^{\alpha\mu})(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H), & R^{(1)}_{WH^4D^2} &= (D^\mu W^I_{\mu\nu})(H^\dagger \overleftrightarrow{D}^{I\nu} H)(H^\dagger H),\\
R^{(2)}_{WH^4D^2} &= \epsilon^{IJK}(H^\dagger\tau^I H)(D^\nu H^\dagger\tau^J H)(D^\mu W^K_{\mu\nu}), & R^{(3)}_{WH^4D^2} &= \epsilon^{IJK}(H^\dagger\tau^I H)(H^\dagger\tau^J D^\nu H)(D^\mu W^K_{\mu\nu}).
\end{aligned}
\tag{36}
$$

As seen in Eqs. (A77)–(A80), these four bosonic operators are rotated by EOM to combinations of two-fermion operators in classes ψ^2H^5 and ψ^2H^4D.

Finally, there are six independent operators in class XH^2D^4, none of which are in M8B, and that we write as

$$
\begin{aligned}
R^{(1)}_{BH^2D^4} &= (D^\mu H^\dagger D^2H)(D^\nu B_{\mu\nu}), & R^{(1)}_{WH^2D^4} &= (D^\mu H^\dagger\tau^I D^2H)(D^\nu W^I_{\mu\nu}),\\
R^{(2)}_{BH^2D^4} &= (D^2H^\dagger D^\mu H)(D^\nu B_{\mu\nu}), & R^{(2)}_{WH^2D^4} &= (D^2H^\dagger\tau^I D^\mu H)(D^\nu W^I_{\mu\nu}),\\
R^{(3)}_{BH^2D^4} &= (D^\mu H^\dagger D^\alpha H - D^\alpha H^\dagger D^\mu H)(D_\alpha D^\nu B_{\mu\nu}), & R^{(3)}_{WH^2D^4} &= (D^\mu H^\dagger\tau^I D^\alpha H - D^\alpha H^\dagger\tau^I D^\mu H)(D_\alpha D^\nu W^I_{\mu\nu}).
\end{aligned}
\tag{37}
$$

Application of EOM on these six operators will give two-fermion operators in classes ψ^2H^5, ψ^2H^4D and $\psi^2H^3D^2$, and four-fermion operators in classes ψ^4H^2 and ψ^4HD.

We finish this section by pointing out that an alternative basis of 86 dimension-eight purely bosonic operators has been presented in Refs. [65,66] motivated by the study of off-shell Green's functions. The universal basis presented here and that in these references are related by IBP and Bianchi identities. As mentioned above the basis of bosonic operators presented in this section was selected with the aim of allowing for a more direct implementation of the EOM and a more transparent identification of the resulting Lorentz structures involving fermions and the corresponding fermionic operator combinations associated with universal theories, as we discuss next.

IV. PRODUCTS OF FERMIONIC CURRENTS

In universal theories, fermionic operators are either generated involving the SM fermionic currents or originate through the use of EOM for the bosonic fields on purely bosonic operators. As such the only possible fermionic Lorentz structures are those listed in Eq. (24). Consequently, the Wilson coefficients of the possible fermionic operators in universal theories have well defined relations. At this point, it is interesting to identify the possible current combinations which are generated by the application of the EOM to the bosonic operators listed in Sec. III. These combinations contain two and four fermion fields.

Most of operators exhibiting two fermionic fields originate from direct contraction of the gauge and Higgs currents in Eq. (24) with dimension-five bosonic structures. In addition, some two-fermion operators contain derivatives of the fermionic currents in Eq. (24) contracted with dimension-four bosonic structures. The generated structures are

$$(\mathrm{D}\Psi^2_+)^{\mu\nu}_B \equiv D^\mu J^\nu_B + D^\nu J^\mu_B = g'\left\{\sum_a \sum_{f\in\{q,l,u,d,e\}} Y_f[D^\mu(\bar f_a\gamma^\nu f_a) + D^\nu(\bar f_a\gamma^\mu f_a)]\right\}, \tag{38}$$

$$(\mathrm{D}\Psi^2_+)^{K\mu\nu}_W \equiv D^\mu J^{K\nu}_W + D^\nu J^{K\mu}_W = \frac{g}{2}\left\{\sum_a \sum_{f\in\{q,l\}} [D^\mu(\bar f_a\gamma^\nu\tau^K f_a) + D^\nu(\bar f_a\gamma^\mu\tau^K f_a)]\right\}, \tag{39}$$

$$(\mathrm{D}\Psi^2_-)^{K\mu\nu}_W \equiv D^\mu J^{K\nu}_W - D^\nu J^{K\mu}_W = \frac{g}{2}\left\{-i\epsilon^{\mu\nu\sigma\alpha}\sum_{f\in\{q,l\}}\sum_a \bar f_a\gamma^\sigma \overleftrightarrow{D}^{K\alpha} f_a + \sum_{ab}\left[y^e_{ab}(\bar l_a\sigma^{\mu\nu}e_b)\tau^K H + y^d_{ab}(\bar q_a\sigma^{\mu\nu}d_b)\tau^K H + y^u_{ab}(\bar q_a\sigma^{\mu\nu}u_b)\tau^K\tilde H + \mathrm{H.c.}\right]\right\}, \tag{40}$$

$$(\mathrm{D}\Psi^2_-)^{A\mu\nu}_G \equiv D^\mu J^{A\nu}_G - D^\nu J^{A\mu}_G = g_s\left\{i\epsilon^{\mu\nu\sigma\alpha}\sum_a[\bar u_a\gamma^\sigma\overleftrightarrow{D}^\alpha T^A u_a + \bar d_a\gamma^\sigma\overleftrightarrow{D}^\alpha T^A d_a - \bar q_a\gamma^\sigma\overleftrightarrow{D}^\alpha T^A q_a] + 2\sum_{ab}\left[y^d_{ab}(\bar q_a\sigma^{\mu\nu}T^A d_b)H + y^u_{ab}(\bar q_a\sigma^{\mu\nu}T^A u_b)\tilde H + \mathrm{H.c.}\right]\right\}. \tag{41}$$

In order to facilitate the comparison with M8B we have transformed the last two equations using the relations in the Appendix B. In principle the same procedure could have been applied to the first two relations, however, we kept the form used in M8B.

Conversely, most operators containing four fermion fields originate from the product of two currents in Eq. (24) contracted with a field strength tensor, two Higgs fields or the derivative of a Higgs field. The operator rotations to M8B require the knowledge of sixteen current products. There are three structures coming from the product of two scalar J_H's

$$(\Psi^4)^{jk}_{HH} \equiv J^j_H J^k_H = \sum_{a,b,c,d}\left\{y^{u\dagger}_{ab}y^{u\dagger}_{cd}(\bar u_a q_{bn})\epsilon^{jn}(\bar u_c q_{dm})\epsilon^{km} + y^d_{ab}y^d_{cd}(\bar q^j_a d_b)(\bar q^k_c d_d) + y^e_{ab}y^e_{cd}(\bar l^j_a e_b)(\bar l^k_c e_d) + \left[y^e_{ab}y^{u\dagger}_{cd}(\bar l^j_a e_b)(\bar u_c q_{dm})\epsilon^{km} + y^d_{ab}y^{u\dagger}_{cd}(\bar q^j_a d_b)(\bar u_c q_{dm})\epsilon^{km} + y^e_{ab}y^d_{cd}(\bar l^j_a e_b)(\bar q^k_c d_d) + j\leftrightarrow k\right]\right\}, \tag{42}$$

$$
\begin{aligned}
(\Psi^4)_{HH} \equiv J_H^j J_{Hj}^\dagger = \sum_{a,b,c,d} \Big\{ & -\sum_{f\in\{u,d\}} y_{ab}^{f\dagger} y_{cd}^f \Big[\frac{1}{6}(\bar f_a \gamma^\mu f_d)(\bar q_c \gamma^\mu q_b) + (\bar f_a \gamma^\mu T^A f_d)(\bar q_c \gamma^\mu T^A q_b)\Big] \\
& -\frac{1}{2} y_{ab}^{e\dagger} y_{cd}^e (\bar e_a \gamma^\mu e_d)(\bar l_c \gamma^\mu l_b) + \Big[y_{ab}^e y_{cd}^u (\bar l_a^j e_b)\epsilon_{kj}(\bar q_c^k u_d) + y_{ab}^u y_{cd}^d (\bar q_a^j u_b)\epsilon_{jk}(\bar q_c^k d_d) \\
& + y_{ab}^e y_{cd}^{d\dagger}(\bar l_a e_b)(\bar d_c q_d) + \text{H.c.}\Big]\Big\},
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
[(\Psi^4)_{HH}]_k^j \equiv J_H^j J_{Hk}^\dagger = \frac{1}{2}\delta_k^j(\Psi^4)_{HH} + \frac{1}{2}(\tau^I)_k^j \sum_{a,b,c,d} \Big\{ & y_{ab}^{u\dagger} y_{cd}^u \Big[\frac{1}{6}(\bar u_a \gamma^\mu u_d)(\bar q_c \gamma^\mu \tau^I q_b) + (\bar u_a \gamma^\mu T^A u_d)(\bar q_c \gamma^\mu \tau^I T^A q_b)\Big] \\
& - y_{ab}^{d\dagger} y_{cd}^d \Big[\frac{1}{6}(\bar d_a \gamma^\mu d_d)(\bar q_c \gamma^\mu \tau^I q_b) + (\bar d_a \gamma^\mu T^A d_d)(\bar q_c \gamma^\mu \tau^I T^A q_b)\Big] \\
& -\frac{1}{2} y_{ab}^{e\dagger} y_{cd}^e (\bar e_a \gamma^\mu e_d)(\bar l_c \gamma^\mu \tau^I l_b) + \Big[-y_{ab}^e y_{cd}^u (\bar l_a^m e_b)(\tau^I \epsilon)_{mn}(\bar q_c^n u_d) \\
& - y_{ab}^u y_{cd}^d (\bar q_c^m d_d)(\tau^I\epsilon)_{mn}(\bar q_a^n u_b) + y_{ab}^e y_{cd}^{d\dagger}(\bar l_a e_b)\tau^I(\bar d_c q_d) + \text{H.c.}\Big]\Big\},
\end{aligned}
\tag{44}
$$

where, in writing the right-hand side of the above equations, we have made again use of the relations listed in Appendix B to express the fermion currents in the combinations appearing in M8B.

The product of two gauge currents $J^\mu_{B,W,G}$ gives rise to Lorentz scalar and tensor structures. The tensor ones related to bosonic operators are

$$
\begin{aligned}
(\Psi^4)_{WW}^{I\mu\nu} \equiv \epsilon^{IJK} J_W^{J\mu} J_W^{K\nu} = \frac{g^2}{4}\sum_{a,b}\Big\{ & 2\epsilon^{IJK}(\bar q_a \gamma^\mu \tau^J q_a)(\bar l_b \gamma^\nu \tau^K l_b) \\
& + \epsilon^{\mu\nu\rho\sigma}\Big[(\bar l_a \gamma_\rho \tau^I l_b)(\bar l_b \gamma_\sigma l_a) + \frac{1}{3}(\bar q_a \gamma_\rho \tau^I q_b)(\bar q_b \gamma_\sigma q_a) + 2(\bar q_a \gamma_\rho \tau^I T^A q_b)(\bar q_b \gamma_\sigma T^A q_a)\Big]\Big\},
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
(\Psi^4)_{GG}^{A\mu\nu} \equiv f^{ABC} J_G^{B\mu} J_G^{C\nu} = g_s^2 \sum_{a,b}\Big\{ & f^{ABC}\sum_{f\in\{q,u,d\}}\ \sum_{f'\in\{q,u,d\},f'\neq f}(\bar f_a \gamma^\mu T^B f_a)(\bar f'_b \gamma^\nu T^C f'_b) \\
& -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\Big[(\bar u_a \gamma_\rho T^A u_b)(\bar u_b \gamma_\sigma u_a) + (\bar d_a \gamma_\rho T^A d_b)(\bar d_b \gamma_\sigma d_a) \\
& -\frac{1}{2}(\bar q_a \gamma_\rho T^A q_b)(\bar q_b \gamma_\sigma q_a) - \frac{1}{2}(\bar q_a \gamma_\rho \tau^I T^A q_b)(\bar q_b \gamma_\sigma \tau^I q_a)\Big]\Big\},
\end{aligned}
\tag{46}
$$

$$
(\Psi^4)_{GB}^{A\mu\nu} \equiv J_G^{A\mu} J_B^\nu = g_s g' \sum_{a,b}\ \sum_{f\in\{q,u,d\}}\ \sum_{f'\in\{q,l,u,d,e\}} Y_{f'}(\bar f_a \gamma^\mu T^A f_a)(\bar f'_b \gamma^\nu f'_b),
\tag{47}
$$

$$
(\Psi^4)_{WB}^{I\mu\nu} \equiv J_W^{I\mu} J_B^\nu = \frac{g}{2} g' \sum_{a,b}\ \sum_{f\in\{q,l\}}\ \sum_{f'\in\{q,l,u,d,e\}} Y_{f'}(\bar f_a \gamma^\mu \tau^I f_a)(\bar f'_b \gamma^\nu f'_b),
\tag{48}
$$

where we have Fierz transformed the first two equations above for later convenience. On the other hand, the generated Lorentz scalar structures are

$$
(\Psi^4)_{BB} \equiv J_B^\mu J_{B\mu} = g'^2 \sum_{a,b}\ \sum_{f,f'\in\{q,l,u,d,e\}} Y_f Y_{f'}(\bar f_a \gamma^\mu f_a)(\bar f'_b \gamma_\mu f'_b),
\tag{49}
$$

$$
\begin{aligned}
(\Psi^4)_{WW} \equiv \sum_I J_W^{I\mu} J_{W\mu}^I = \frac{g^2}{4}\sum_{a,b}\{ & (\bar q_a \gamma^\mu \tau^I q_a)(\bar q_b \gamma_\mu \tau^I q_b) + 2(\bar q_a \gamma^\mu \tau^I q_a)(\bar l_b \gamma_\mu \tau^I l_b) \\
& + 2(\bar l_a \gamma^\mu l_b)(\bar l_b \gamma_\mu l_a) - (\bar l_a \gamma^\mu l_a)(\bar l_b \gamma_\mu l_b)\},
\end{aligned}
\tag{50}
$$

$$
\begin{aligned}
(\Psi^4)_{GG} \equiv \sum_A J_G^{A\mu} J_{G\mu}^A = g_s^2 \sum_{a,b} \Bigg\{ & \sum_{f\in\{q,u,d\}} \sum_{f'\in\{q,u,d\},f'\neq f} (\bar{f}_a\gamma^\mu T^A f_a)(\bar{f}'_b\gamma_\mu T^A f'_b) \\
& + \frac{1}{2}\sum_{f\in\{u,d\}} (\bar{f}_a\gamma^\mu f_b)(\bar{f}_b\gamma_\mu f_a) - \frac{1}{6}\sum_{f\in\{q,u,d\}} (\bar{f}_a\gamma^\mu f_a)(\bar{f}_b\gamma_\mu f_b) \\
& + \frac{1}{4}(\bar{q}_a\gamma^\mu q_b)(\bar{q}_b\gamma_\mu q_a) + \frac{1}{4}(\bar{q}_a\gamma^\mu\tau^I q_b)(\bar{q}_b\gamma_\mu\tau^I q_a)\Bigg\}, \qquad (51)
\end{aligned}
$$

$$
(\Psi^4)_{WB}^I \equiv J_W^{I\mu} J_{B\mu} = \frac{gg'}{2}\sum_{a,b}\sum_{f'\in\{q,l,u,d,e\}}\sum_{f\in\{q,l\}} Y_{f'}(\bar{f}_a\gamma^\mu\tau^I f_a)(\bar{f}'_b\gamma_\mu f'_b). \qquad (52)
$$

There are only two products of the scalar current J_H with a gauge current that are generated

$$
(\Psi^4)_{\mathrm{BH}}^{\mu j} \equiv J_B^\mu J_H^j = g'\sum_{a,b,c}\sum_{f\in\{q,l,u,d,e\}} \{Y_f y_{ab}^{u\dagger}(\bar{f}_c\gamma^\mu f_c)(\bar{u}_a q_{bk})\epsilon^{jk} + Y_f y_{ab}^d(\bar{f}_c\gamma^\mu f_c)(\bar{q}_a^j d_b) + Y_f y_{ab}^e(\bar{f}_c\gamma^\mu f_c)(\bar{l}_a^j e_b)\}, \qquad (53)
$$

$$
(\Psi^4)_{WH}^{I\mu j} \equiv J_W^{I\mu} J_H^j = \frac{g}{2}\sum_{a,b,c}\sum_{f\in\{q,l\}} \{y_{ab}^{u\dagger}(\bar{f}_c\gamma^\mu\tau^I f_c)(\bar{u}_a q_{bk})\epsilon^{jk} + y_{ab}^d(\bar{f}_c\gamma^\mu\tau^I f_c)(\bar{q}_a^j d_b) + y_{ab}^e(\bar{f}_c\gamma^\mu\tau^I f_c)(\bar{l}_a^j e_b)\}. \qquad (54)
$$

Finally, some operators with four fermions come from direct contraction of derivatives of two currents. They are

$$
(\mathrm{D}\Psi^4)_{BB} \equiv D^\alpha J_B^\mu D_\alpha J_{B\mu} = g'^2\sum_{a,b}\sum_{f,f'\in\{q,l,u,d,e\}} Y_f Y_{f'} D^\alpha(\bar{f}_a\gamma^\mu f_a) D_\alpha(\bar{f}'_b\gamma_\mu f'_b), \qquad (55)
$$

$$
\begin{aligned}
(\mathrm{D}\Psi^4)_{GG} \equiv \sum_A D^\alpha J_G^{A\mu} D_\alpha J_{G\mu}^A = g_s^2\sum_{a,b}\Bigg\{ & \sum_{f\in\{q,u,d\}}\sum_{f'\in\{q,u,d\},f'\neq f} D^\alpha(\bar{f}_a\gamma^\mu T^A f_a) D_\alpha(\bar{f}'_b\gamma^\nu T^A f'_b) \\
& + \frac{1}{2}\sum_{f\in\{u,d\}} D^\alpha(\bar{f}_a\gamma^\mu f_b) D_\alpha(\bar{f}_b\gamma_\mu f_a) - \frac{1}{6}\sum_{f\in\{q,u,d\}} D^\alpha(\bar{f}_a\gamma^\mu f_a) D_\alpha(\bar{f}_b\gamma_\mu f_b) \\
& + \frac{1}{4}D^\alpha(\bar{q}_a\gamma^\mu q_b) D_\alpha(\bar{q}_b\gamma_\mu q_a) + \frac{1}{4}D^\alpha(\bar{q}_a\gamma^\mu\tau^I q_b) D_\alpha(\bar{q}_b\gamma_\mu\tau^I q_a)\Bigg\}, \qquad (56)
\end{aligned}
$$

$$
\begin{aligned}
(\mathrm{D}\Psi^4)_{WW} \equiv \sum_I D^\alpha J_W^{I\mu} D_\alpha J_{W\mu}^I = \frac{g^2}{4}\sum_{a,b}\{ & D^\alpha(\bar{q}_a\gamma^\mu\tau^I q_a) D^\alpha(\bar{q}_b\gamma_\mu\tau^I q_b) + 2D^\alpha(\bar{q}_a\gamma^\mu\tau^I q_a) D_\alpha(\bar{l}_b\gamma_\mu\tau^I l_b) \\
& + 2D^\alpha(\bar{l}_a\gamma^\mu l_b) D_\alpha(\bar{l}_b\gamma_\mu l_a) - D^\alpha(\bar{l}_a\gamma^\mu l_a) D_\alpha(\bar{l}_b\gamma_\mu l_b)\}, \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
(\mathrm{D}\Psi^4)_{HH} \equiv D^\alpha J_H^j D_\alpha J_{Hj}^\dagger = \sum_{abcd}\Bigg\{ & [y_{ab}^e y_{cd}^u D^\mu(\bar{l}_a^j e_b)\epsilon_{kj} D_\mu(\bar{q}_c^k u_d) + y_{ab}^u y_{cd}^d D^\mu(\bar{q}_a^j u_b)\epsilon_{jk} D_\mu(\bar{q}_c^k d_d) \\
& + y_{ab}^e y_{cd}^{d\dagger} D^\mu(\bar{l}_a^j e_b) D_\mu(\bar{d}_c q_d) + \mathrm{H.c.}] - \frac{1}{2} y_{ab}^e y_{cd}^{e\dagger} D^\mu(\bar{l}_a\gamma^\nu l_d) D_\mu(\bar{e}_c\gamma_\nu e_b) \\
& - \sum_{f\in\{u,d\}} y_{ab}^f y_{cd}^{f\dagger}\left[\frac{1}{6}D^\mu(\bar{q}_a\gamma^\nu q_d) D_\mu(\bar{f}_c\gamma_\nu f_b) + D^\mu(\bar{q}_a\gamma^\nu T^A q_d) D_\mu(\bar{f}_c\gamma_\nu T^A f_b)\right]\Bigg\}. \qquad (58)
\end{aligned}
$$

Notice that these last structures do not need any further simplification as their present form appear in M8B.

V. FERMIONIC OPERATORS FOR UNIVERSAL THEORIES

We are now in position to present the combination of dimension-eight fermionic operators that are associated with universal theories. We call such combinations *universal fermionic operators* since they are the ones with independent couplings. That is, in universal theories the couplings of the fermionic operators must be linear combinations of the 86 independent couplings of the universal fermionic operators listed here.

The 86 universal fermionic operators are formed by the contraction of the fermionic Lorentz structures listed in Sec. IV with the remaining bosonic pieces of the universal operators listed in Sec. III. For convenience, we express them in terms of the fermionic operators in M8B and we employ the M8B naming and numbering of the operator classes. Also for convenience, we reproduce in Appendix C the subset of M8B operators which appear in the universal operators listed here. In addition, we have included a factor i to make the operators Hermitian whenever possible.

The full relation between the 86 bosonic operators in Sec. III, the universal fermionic operators, and the bosonic operators in M8B can be found in Appendix A.

A. Two-fermion operators

There are 62 independent universal combinations of two-fermion operators in the following classes:

(i) *Class* $9:\psi^2X^2H+\text{H.c.}$: there are 16 universal operators in this class arising from the direct contraction of the Higgs fermionic current Eq. (24) with two gauge boson strength tensors

$$
\begin{aligned}
Q^{(1)}_{\psi^2B^2H} &\equiv (J_H H)B_{\mu\nu}B^{\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(1)}_{quB^2H} + y^d_{pr}Q^{(1)}_{qdB^2H} + y^e_{pr}Q^{(1)}_{leB^2H}\Big],\\
Q^{(2)}_{\psi^2B^2H} &\equiv (J_H H)B_{\mu\nu}\tilde{B}^{\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(2)}_{quB^2H} + y^d_{pr}Q^{(2)}_{qdB^2H} + y^e_{pr}Q^{(2)}_{leB^2H}\Big],\\
Q^{(1)}_{\psi^2W^2H} &\equiv (J_H H)W^I_{\mu\nu}W^{I\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(1)}_{quW^2H} + y^d_{pr}Q^{(1)}_{qdW^2H} + y^e_{pr}Q^{(1)}_{leW^2H}\Big],\\
Q^{(2)}_{\psi^2W^2H} &\equiv (J_H H)W^I_{\mu\nu}\tilde{W}^{I\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(2)}_{quW^2H} + y^d_{pr}Q^{(2)}_{qdW^2H} + y^e_{pr}Q^{(2)}_{leW^2H}\Big],\\
Q^{(1)}_{\psi^2G^2H} &\equiv (J_H H)G^A_{\mu\nu}G^{A\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(1)}_{quG^2H} + y^d_{pr}Q^{(1)}_{qdG^2H} + y^e_{pr}Q^{(1)}_{leG^2H}\Big],\\
Q^{(2)}_{\psi^2G^2H} &\equiv (J_H H)G^A_{\mu\nu}\tilde{G}^{A\mu\nu} = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(2)}_{quG^2H} + y^d_{pr}Q^{(2)}_{qdG^2H} + y^e_{pr}Q^{(2)}_{leG^2H}\Big],\\
Q^{(1)}_{\psi^2WBH} &\equiv (J_H\tau^I H)B_{\mu\nu}W^{I\mu\nu} = \sum_{pr}\Big[-y^{u\dagger}_{rp}Q^{\dagger(1)}_{quWBH} + y^d_{pr}Q^{(1)}_{qdWBH} + y^e_{pr}Q^{(1)}_{leWBH}\Big],\\
Q^{(2)}_{\psi^2WBH} &\equiv (J_H\tau^I H)B_{\mu\nu}\tilde{W}^{I\mu\nu} = \sum_{pr}\Big[-y^{u\dagger}_{rp}Q^{\dagger(2)}_{quWBH} + y^d_{pr}Q^{(2)}_{qdWBH} + y^e_{pr}Q^{(2)}_{leWBH}\Big],
\end{aligned}
\tag{59}
$$

together with the Hermitian conjugates of the above operators. These universal fermionic operators are generated when applying EOM to some of the operators in class $X^2H^2D^2$ as can be seen in Eqs. (A33)–(A36), (A42)–(A45), (A55)–(A58), and (A64)–(A67),

(ii) *Hybrid class* $9\,\&\,14:\psi^2X^2H(D)$ contains 8 operators exhibiting specific combinations of operators in classes ψ^2X^2H and ψ^2X^2D originated from contraction of the fermionic structures in Eq. (40) and (41) with two gauge strength tensors

$$
\begin{aligned}
Q^{(1)}_{\psi^2WBH(D)} &\equiv (\mathrm{D}\Psi^2_-)^{I\mu\nu}_W B_{\mu\rho}W^{I\rho}_\nu = \frac{g}{2}\sum_{pr}\Big\{-\Big[y^u_{pr}Q^{(3)}_{quWBH} + y^d_{pr}Q^{(3)}_{qdWBH} + y^e_{pr}Q^{(3)}_{leWBH} + \text{H.c.}\Big]\\
&\quad - i\Big[Q^{(3)}_{q^2WBD} + Q^{(3)}_{l^2WBD}\Big]\delta_{pr}\Big\},\\
Q^{(2)}_{\psi^2WBH(D)} &\equiv (\mathrm{D}\Psi^2_-)^{I\mu\nu}_W B_{\mu\rho}\tilde{W}^{I\rho}_\nu = \frac{g}{2}\sum_{pr}\Big\{-\Big[iy^u_{pr}Q^{(3)}_{quWBH} + iy^d_{pr}Q^{(3)}_{qdWBH} + iy^e_{pr}Q^{(3)}_{leWBH} + \text{H.c.}\Big]\\
&\quad + i\Big[Q^{(1)}_{q^2WBD} + Q^{(1)}_{l^2WBD}\Big]\delta_{pr}\Big\},
\end{aligned}
$$

$$
\begin{aligned}
Q^{(1)}_{\psi^2 GBH(D)} &\equiv (\mathrm{D}\Psi^2_-)^{A\mu\nu}_G B_{\mu\rho} G^{A\rho}_\nu = g_s \sum_{pr} \Big\{ -2\Big[y^u_{pr} Q^{(3)}_{quGBH} + y^d_{pr} Q^{(3)}_{qdGBH} + \text{H.c.}\Big] \\
&\quad - i\Big[Q^{(3)}_{q^2GBD} - Q^{(3)}_{u^2GBD} - Q^{(3)}_{d^2GBD}\Big]\delta_{pr}\Big\}, \\
Q^{(2)}_{\psi^2 GBH(D)} &\equiv (\mathrm{D}\Psi^2_-)^{A\mu\nu}_G B_{\mu\rho} \tilde{G}^{A\rho}_\nu = g_s \sum_{pr} \Big\{ -2\Big[i y^u_{pr} Q^{(3)}_{quGBH} + i y^d_{pr} Q^{(3)}_{qdGBH} + \text{H.c.}\Big] \\
&\quad + i\Big[Q^{(1)}_{q^2GBD} - Q^{(1)}_{u^2GBD} - Q^{(1)}_{d^2GBD}\Big]\delta_{pr}\Big\}, \\
Q^{(1)}_{\psi^2 W^2 H(D)} &\equiv \epsilon^{IJK} (\mathrm{D}\Psi^2_-)^{I\mu\nu}_W W^J_{\mu\rho} W^{K\rho}_\nu = \frac{g}{2} \sum_{pr} \Big\{ \Big[y^u_{pr} Q^{(3)}_{quW^2H} + y^d_{pr} Q^{(3)}_{qdW^2H} + y^e_{pr} Q^{(3)}_{leW^2H} + \text{H.c.}\Big] \\
&\quad - i\Big[Q^{(4)}_{q^2W^2D} + Q^{(4)}_{l^2W^2D}\Big]\delta_{pr}\Big\}, \\
Q^{(2)}_{\psi^2 W^2 H(D)} &\equiv \epsilon^{IJK} (\mathrm{D}\Psi^2_-)^{I\mu\nu}_W W^J_{\mu\rho} \tilde{W}^{K\rho}_\nu = \frac{g}{2} \sum_{pr} \Big\{ + \Big[i y^u_{pr} Q^{(3)}_{quW^2H} + i y^d_{pr} Q^{(3)}_{qdW^2H} + i y^e_{pr} Q^{(3)}_{leW^2H} + \text{H.c.}\Big] \\
&\quad + 2i\Big[Q^{(2)}_{q^2W^2D} + Q^{(2)}_{l^2W^2D}\Big]\delta_{pr}\Big\}, \\
Q^{(1)}_{\psi^2 G^2 H(D)} &\equiv f^{ABC} (\mathrm{D}\Psi^2_-)^{A\mu\nu}_G G^B_{\mu\rho} G^{D\rho}_\nu = g_s \sum_{pr} \Big\{ 2\Big[y^u_{pr} Q^{(5)}_{quG^2H} + y^d_{pr} Q^{(5)}_{qdG^2H} + \text{H.c.}\Big] \\
&\quad - i\Big[Q^{(5)}_{q^2G^2D} - Q^{(5)}_{u^2G^2D} - Q^{(5)}_{d^2G^2D}\Big]\delta_{pr}\Big\}, \\
Q^{(2)}_{\psi^2 G^2 H(D)} &\equiv f^{ABC} (\mathrm{D}\Psi^2_-)^{A\mu\nu}_G G^B_{\mu\rho} \tilde{G}^{D\rho}_\nu = g_s \sum_{pr} \Big\{ +2\Big[i y^u_{pr} Q^{(5)}_{quG^2H} + i y^d_{pr} Q^{(5)}_{qdG^2H} + \text{H.c.}\Big] \\
&\quad + 2i\Big[Q^{(2)}_{q^2G^2D} - Q^{(2)}_{u^2G^2D} - Q^{(2)}_{d^2G^2D}\Big]\delta_{pr}\Big\}.
\end{aligned}
\tag{60}
$$

The universal fermionic operators in this hybrid class are generated when applying EOM to some of the operators in class X^3D^2 as can be seen in Eqs. (A16)–(A17), (A20)–(A21), (A24)–(A25), and (A28)–(A29).

(iii) *Class* $11: \psi^2 H^2 D^3$: the 2 operators in this class arise from the contraction of the structures in Eq. (38) and (39) with a current containing two symmetrized covariant derivatives acting on the Higgs field

$$
\begin{aligned}
Q^{(1)}_{\psi^2 H^2 D^3} &\equiv i(\mathrm{D}\Psi^2_+)^{\mu\nu}_B (D_{(\mu}D_{\nu)}H^\dagger H - H^\dagger D_{(\mu}D_{\nu)}H) \\
&= 2g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f \Big[Q^{(1)}_{f^2H^2D^3} - Q^{(2)}_{f^2H^2D^3} + \text{H.c.}\Big]\delta_{pr}, \\
Q^{(2)}_{\psi^2 H^2 D^3} &\equiv i(\mathrm{D}\Psi^2_+)^{I\mu\nu}_W (D_{(\mu}D_{\nu)}H^\dagger \tau^I H - H^\dagger \tau^I D_{(\mu}D_{\nu)}H) \\
&= g \sum_{pr} \sum_{f\in\{q,l\}} \Big[Q^{(3)}_{f^2H^2D^3} - Q^{(4)}_{f^2H^2D^3} + \text{H.c.}\Big]\delta_{pr}.
\end{aligned}
\tag{61}
$$

These operators are generated directly from application of the EOM of the Higgs field to the operators in class X^2D^4 as in Eqs. (A30) and (A31) and class XH^4D^2, see Eqs. (A83) and (A86).

(iv) *Class* $12: \psi^2 H^5 + \text{H.c.}$ contains 2 operators originating from the contraction of the Higgs fermionic currents in Eq. (24) directly with Higgs fields:

$$
Q^{(1)}_{\psi^2 H^5} \equiv (H^\dagger H)^2 (J_H H) = \sum_{pr} \Big[y^{u\dagger}_{rp} Q^\dagger_{quH^5} + y^d_{pr} Q_{qdH^5} + y^e_{pr} Q_{leH^5}\Big]
\tag{62}
$$

and its Hermitian conjugate. These operators appear directly in the application of the Higgs EOM to the two operators in class H^6D^2 Eqs. (A1)–(A2) but, as seen in Appendix A, they also arise in the rotation of a large fraction of the 86 bosonic operators. This is so, because these two operators in class H^6D^2 are generically generated when reducing the products of the Higgs-gauge currents introduced by the gauge boson EOM to the bosonic operators in M8B.

(v) *Class* 13: $\psi^2 H^4 D$: there are four universal fermionic operators in this class which appear in the contraction of the electroweak fermionic currents in Eq. (24) with the product of two Higgs pairs one of them containing one derivative.

$$\begin{aligned}
Q^{(1)}_{\psi^2 H^4 D} &\equiv i J^{\mu}_{B} (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H) = g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f Q^{(1)}_{f^2 H^4 D} \delta_{pr},\\
Q^{(2)}_{\psi^2 H^4 D} &\equiv i J^{I\mu}_{W} [(H^\dagger \overleftrightarrow{D}^I_\mu H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)] = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(2)}_{f^2 H^4 D} \delta_{pr},\\
Q^{(3)}_{\psi^2 H^4 D} &\equiv i \epsilon^{IJK} J^{I\mu}_{W} (H^\dagger \overleftrightarrow{D}^J_\mu H)(H^\dagger \tau^K H) = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(3)}_{f^2 H^4 D} \delta_{pr},\\
Q^{(4)}_{\psi^2 H^4 D} &\equiv \epsilon^{IJK} J^{I\mu}_{W} (H^\dagger \tau^J_\mu H) D_\mu (H^\dagger \tau^K H) = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(4)}_{f^2 H^4 D} \delta_{pr}.
\end{aligned} \tag{63}$$

Operators in this class are directly generated by application of the gauge-boson EOM in operators in class XH^4D^2 as seen in Eqs. (A77)–(A80). They also arise in the complete rotation to operators in M8B of some operators in classes X^3D^2 pEq. (A14)], X^2D^4 [Eqs. (A30)–(A31)], $X^2H^2D^2$ [Eqs. (A37), (A38), (A46), (A47), (A51), and (A52)], and XH^2D^4 [Eqs. (A83) and (A86)]. Notice that, for the sake of simplicity in writing the expressions above, in operators $Q^{(1)}_{f^2H^4D}$, where $f = u, d, e$, we have added a superscript of (1) to the M8B operators. This minimal change of labeling is reflected also when we list the operators in class 13 in Table III.

(vi) *Class* 15: $\psi^2 X H^2 D$: It contains 24 operators generated by the contraction of fermionic gauge currents in Eq. (24) with a gauge field strength tensors and a pair of Higgs bosons with one derivative. In twelve operators the fermionic and Higgs currents are contracted with the $SU(2)_L$ field strength tensor

$$\begin{aligned}
Q^{(1)}_{\psi^2 WH^2 D} &\equiv J^{\nu}_{B} D^\mu (H^\dagger \tau^I H) W^I_{\mu\nu} = g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f Q^{(1)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(2)}_{\psi^2 WH^2 D} &\equiv J^{\nu}_{B} D^\mu (H^\dagger \tau^I H) \tilde{W}^I_{\mu\nu} = g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f Q^{(2)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(3)}_{\psi^2 WH^2 D} &\equiv i J^{\nu}_{B} (H^\dagger \overleftrightarrow{D}^I_\mu H) W^I_{\mu\nu} = i g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f Q^{(3)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(4)}_{\psi^2 WH^2 D} &\equiv i J^{\nu}_{B} (H^\dagger \overleftrightarrow{D}^I_\mu H) \tilde{W}^I_{\mu\nu} = i g' \sum_{pr} \sum_{f\in\{q,l,u,d,e\}} Y_f Q^{(4)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(5)}_{\psi^2 WH^2 D} &\equiv J^{I\nu}_{W} D^\mu (H^\dagger H) W^I_{\mu\nu} = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(5)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(6)}_{\psi^2 WH^2 D} &\equiv J^{I\nu}_{W} D^\mu (H^\dagger H) \tilde{W}^I_{\mu\nu} = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(6)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(7)}_{\psi^2 WH^2 D} &\equiv i J^{I\nu}_{W} (H^\dagger \overleftrightarrow{D}_\mu H) W^I_{\mu\nu} = i \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(7)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(8)}_{\psi^2 WH^2 D} &\equiv i J^{I\nu}_{W} (H^\dagger \overleftrightarrow{D}_\mu H) \tilde{W}^I_{\mu\nu} = i \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(8)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(9)}_{\psi^2 WH^2 D} &\equiv \epsilon^{IJK} J^{I\nu}_{W} D^\mu (H^\dagger \tau^J H) W^K_{\mu\nu} = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(9)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(10)}_{\psi^2 WH^2 D} &\equiv \epsilon^{IJK} J^{I\nu}_{W} D^\mu (H^\dagger \tau^J H) \tilde{W}^K_{\mu\nu} = \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(10)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(11)}_{\psi^2 WH^2 D} &\equiv i \epsilon^{IJK} J^{I\nu}_{W} (H^\dagger \overleftrightarrow{D}^J_\mu H) W^K_{\mu\nu} = i \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(11)}_{f^2 WH^2 D} \delta_{pr},\\
Q^{(12)}_{\psi^2 WH^2 D} &\equiv i \epsilon^{IJK} J^{I\nu}_{W} (H^\dagger \overleftrightarrow{D}^J_\mu H) \tilde{W}^K_{\mu\nu} = i \frac{g}{2} \sum_{pr} \sum_{f\in\{q,l\}} Q^{(12)}_{f^2 WH^2 D} \delta_{pr}.
\end{aligned} \tag{64}$$

They originate from direct application of EOM of the electroweak gauge bosons in operators in classes X^3D^2 [Eqs. (A14), (A15), (A22), and (A23)] as well as $X^2H^2D^2$ [Eqs. (A46)–(A49) and Eqs. (A51)–(A54)]. They are also generated in the rotation of operators $R^{(1)}_{W^2D^4}$ (A31), $R^{(3)}_{BH^2D^4}$ (A83), and $R^{(3)}_{WH^2D^4}$ (A86).

In eight operators in this class the fermionic structures couple to the hypercharge field strength tensor

$$\begin{aligned}
Q^{(1)}_{\psi^2BH^2D} &\equiv J^\nu_B D^\mu(H^\dagger H)B_{\mu\nu} = g'\sum_{pr}\left[\sum_{f\in\{u,d,e\}} Y_f Q^{(1)}_{f^2BH^2D} + \sum_{f\in\{q,l\}} Y_f Q^{(5)}_{f^2BH^2D}\right]\delta_{pr},\\
Q^{(2)}_{\psi^2BH^2D} &\equiv J^\nu_B D^\mu(H^\dagger H)\tilde{B}_{\mu\nu} = g'\sum_{pr}\left[\sum_{f\in\{u,d,e\}} Y_f Q^{(2)}_{f^2BH^2D} + \sum_{f\in\{q,l\}} Y_f Q^{(6)}_{f^2BH^2D}\right]\delta_{pr},\\
Q^{(3)}_{\psi^2BH^2D} &\equiv iJ^\nu_B (H^\dagger \overleftrightarrow{D}_\mu H)B_{\mu\nu} = ig'\sum_{pr}\left[\sum_{f\in\{u,d,e\}} Y_f Q^{(3)}_{f^2BH^2D} + \sum_{f\in\{q,l\}} Y_f Q^{(7)}_{f^2BH^2D}\right]\delta_{pr},\\
Q^{(4)}_{\psi^2BH^2D} &\equiv iJ^\nu_B (H^\dagger \overleftrightarrow{D}_\mu H)\tilde{B}_{\mu\nu} = ig'\sum_{pr}\left[\sum_{f\in\{u,d,e\}} Y_f Q^{(4)}_{f^2BH^2D} + \sum_{f\in\{q,l\}} Y_f Q^{(8)}_{f^2BH^2D}\right]\delta_{pr},\\
Q^{(5)}_{\psi^2BH^2D} &\equiv J^{I\nu}_W D^\mu(H^\dagger \tau^I H)B_{\mu\nu} = \frac{g}{2}\sum_{pr}\sum_{f\in\{q,l\}} Q^{(1)}_{f^2BH^2D}\delta_{pr},\\
Q^{(6)}_{\psi^2BH^2D} &\equiv J^{I\nu}_W D^\mu(H^\dagger \tau^I H)\tilde{B}_{\mu\nu} = \frac{g}{2}\sum_{pr}\sum_{f\in\{q,l\}} Q^{(2)}_{f^2BH^2D}\delta_{pr},\\
Q^{(7)}_{\psi^2BH^2D} &\equiv iJ^{I\nu}_W (H^\dagger \overleftrightarrow{D}^I_\mu H)B_{\mu\nu} = i\frac{g}{2}\sum_{pr}\sum_{f\in\{q,l\}} Q^{(3)}_{f^2BH^2D}\delta_{pr},\\
Q^{(8)}_{\psi^2BH^2D} &\equiv iJ^{I\nu}_W (H^\dagger \overleftrightarrow{D}^I_\mu H)\tilde{B}_{\mu\nu} = i\frac{g}{2}\sum_{pr}\sum_{f\in\{q,l\}} Q^{(4)}_{f^2BH^2D}\delta_{pr}.
\end{aligned} \tag{65}$$

They are generated by direct application of EOM of the electroweak gauge bosons in operators in class $X^2H^2D^2$ [Eqs. (A37)–(A40) and Eqs. (A72)–(A75)]. They are also generated in the rotation of operators $R^{(3)}_{BH^2D^4}$ (A83), and $R^{(3)}_{WH^2D^4}$ (A86).

And finally four operators involve the gluon strength tensor

$$\begin{aligned}
Q^{(1)}_{\psi^2GH^2D} &\equiv J^A_G D^\mu(H^\dagger H)G^A_{\mu\nu} = g_s\sum_{pr}\left[\sum_{f\in\{u,d\}} Q^{(1)}_{f^2GH^2D} + Q^{(5)}_{q^2GH^2D}\right]\delta_{pr},\\
Q^{(2)}_{\psi^2GH^2D} &\equiv J^A_G D^\mu(H^\dagger H)\tilde{G}^A_{\mu\nu} = g_s\sum_{pr}\left[\sum_{f\in\{u,d\}} Q^{(2)}_{f^2GH^2D} + Q^{(6)}_{q^2GH^2D}\right]\delta_{pr},\\
Q^{(3)}_{\psi^2GH^2D} &\equiv iJ^A_G (H^\dagger \overleftrightarrow{D}_\mu H)G^A_{\mu\nu} = ig_s\sum_{pr}\left[\sum_{f\in\{u,d\}} Q^{(3)}_{f^2GH^2D} + Q^{(7)}_{q^2GH^2D}\right]\delta_{pr},\\
Q^{(4)}_{\psi^2GH^2D} &\equiv iJ^A_G (H^\dagger \overleftrightarrow{D}_\mu H)\tilde{G}^A_{\mu\nu} = ig_s\sum_{pr}\left[\sum_{f\in\{u,d\}} Q^{(4)}_{f^2GH^2D} + Q^{(8)}_{q^2GH^2D}\right]\delta_{pr},
\end{aligned} \tag{66}$$

which stem from the direct application of the gluon EOM in operators in class $X^2H^2D^2$, as in Eqs. (A59)–(A62), and the operators $R^{(1)}_{BG^2D^2}$ (A26) and $R^{(2)}_{BG^2D^2}$ (A27) of class X^3D^2.

(vii) *Class* 17: $\psi^2H^3D^2$ + H.c. is generated by direct contraction of the Higgs fermionic current in Eq. (24) with one Higgs field and two derivatives of Higgs fields. There are six independent such contractions

$$Q^{(1)}_{\psi^2H^3D^2} \equiv (D_\mu H^\dagger D^\mu H)(J_H H) = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(1)}_{quH^3D^2} + y^d_{pr}Q^{(1)}_{qdH^3D^2} + y^e_{pr}Q^{(1)}_{leH^3D^2}\Big],$$
$$Q^{(2)}_{\psi^2H^3D^2} \equiv (D_\mu H^\dagger \tau^I D^\mu H)(J_H\tau^I H) = \sum_{pr}\Big[-y^{u\dagger}_{rp}Q^{\dagger(2)}_{quH^3D^2} + y^d_{pr}Q^{(2)}_{qdH^3D^2} + y^e_{pr}Q^{(2)}_{leH^3D^2}\Big],$$
$$Q^{(3)}_{\psi^2H^3D^2} \equiv (H^\dagger D_\mu H)(J_H D^\mu H) = \sum_{pr}\Big[y^{u\dagger}_{rp}Q^{\dagger(5)}_{quH^3D^2} + y^d_{pr}Q^{(5)}_{qdH^3D^2} + y^e_{pr}Q^{(5)}_{leH^3D^2}\Big], \tag{67}$$

and their Hermitian conjugates. These operators are generated directly from applying the Higgs field EOM to the operators in class H^4D^4 and H^2D^6 [see Eqs. (A3)–(A8) and (A13)]. In addition they also appear in the rotation of operators XH^2D^4, as in Eqs. (A81)–(A82) and (A84)–(A85), arising in the reduction of the products of the Higgs gauge currents introduced by the gauge boson EOM to the bosonic operators in M8B.

B. Four-fermion operators

We obtain 24 universal four-fermion operators in the following classes:

(i) *Class* $18{:}\psi^4H^2$: contains eight universal fermionic operators obtained from the product of the four-fermion currents in Eqs. (42)–(44) and Eqs. (49)–(52) with a pair Higgs fields

$$Q^{(1)}_{\psi^4H^2} \equiv (\Psi^4)_{HH}(H^\dagger H) = \sum_{prst}\Bigg\{-\sum_{f\in\{u,d\}} y^{f\dagger}_{sr}y^f_{pt}\left(\frac{1}{6}Q^{(1)}_{q^2f^2H^2} + Q^{(3)}_{q^2f^2H^2}\right) - \frac{1}{2}y^{e\dagger}_{sr}y^e_{pt}Q^{(1)}_{l^2e^2H^2}$$
$$+\Big[-y^e_{pr}y^u_{st}Q^{(1)}_{lequH^2} + y^u_{pr}y^d_{st}Q^{(1)}_{q^2udH^2} + y^e_{pr}y^{d\dagger}_{ts}Q^{(1)}_{leqdH^2} + \text{H.c.}\Big]\Bigg\},$$
$$Q^{(2)}_{\psi^4H^2} \equiv (\Psi^4)^{jk}_{HH}H_jH_k = \sum_{prst}\Bigg\{y^{u\dagger}_{rp}y^{u\dagger}_{ts}Q^{\dagger(5)}_{q^2u^2H^2} + y^d_{pr}y^d_{st}Q^{(5)}_{q^2d^2H^2} + y^e_{pr}y^e_{st}Q^{(3)}_{l^2e^2H^2}$$
$$+2\Big[y^e_{pr}y^{u\dagger}_{st}Q^{(5)}_{lequH^2} + y^d_{pr}y^{u\dagger}_{st}Q^{(5)}_{q^2udH^2} + y^e_{pr}y^d_{st}Q^{(3)}_{leqdH^2}\Big]\Bigg\},$$
$$Q^{(3)}_{\psi^4H^2} \equiv H^{\dagger k}\Big[(\Psi^4)_{HH}\Big]^j_k H_j - \frac{1}{2}Q^{(1)}_{\psi^4H^2}$$
$$= \frac{1}{2}\sum_{prst}\Bigg\{-y^{d\dagger}_{sr}y^d_{pt}\left(\frac{1}{6}Q^{(2)}_{q^2d^2H^2} + Q^{(4)}_{q^2d^2H^2}\right) + y^{u\dagger}_{sr}y^u_{pt}\left(\frac{1}{6}Q^{(2)}_{q^2u^2H^2} + Q^{(4)}_{q^2u^2H^2}\right) - \frac{1}{2}y^{e\dagger}_{sr}y^e_{pt}Q^{(2)}_{l^2e^2H^2}$$
$$+\Big[-y^e_{pr}y^u_{st}Q^{(2)}_{lequH^2} - y^u_{pr}y^d_{st}Q^{(2)}_{q^2udH^2} + y^e_{pr}y^{d\dagger}_{ts}Q^{(2)}_{leqdH^2} + \text{H.c.}\Big]\Bigg\},$$
$$Q^{(4)}_{\psi^4H^2} \equiv (\Psi^4)_{BB}(H^\dagger H) = g'^2\sum_{prst}\Bigg\{\sum_{f\in\{q,l,u,d,e\}}Y_f^2Q^{(1)}_{f^4H^2} + 2\sum_{f\in\{q,l\}}\sum_{f'\in\{u,d,e\}}Y_fY'_fQ^{(1)}_{f^2f'^2H^2}$$
$$+2Y_qY_lQ^{(1)}_{l^2q^2H^2} + 2\Big[Y_eY_uQ^{(1)}_{e^2u^2H^2} + Y_eY_dQ^{(1)}_{e^2d^2H^2} + Y_uY_dQ^{(1)}_{u^2d^2H^2}\Big]\Bigg\}\delta_{pr}\delta_{st},$$
$$Q^{(5)}_{\psi^4H^2} \equiv (\Psi^4)_{WW}(H^\dagger H) = \frac{g^2}{4}\sum_{prst}\Big\{\Big[Q^{(3)}_{q^4H^2} + 2Q^{(3)}_{l^2q^2H^2}\Big]\delta_{pr}\delta_{st} + Q^{(1)}_{l^4H^2}(2\delta_{pt}\delta_{sr} - \delta_{pr}\delta_{st})\Big\},$$
$$Q^{(6)}_{\psi^4H^2} \equiv (\Psi^4)_{GG}(H^\dagger H) = g_s^2\sum_{prst}\Bigg\{2\Big[Q^{(3)}_{q^2d^2H^2} + Q^{(3)}_{q^2u^2H^2} + Q^{(2)}_{u^2d^2H^2}\Big]\delta_{pr}\delta_{st}$$
$$+\frac{1}{2}\Big[Q_{u^4H^2} + Q_{d^4H^2}\Big]\left(\delta_{pt}\delta_{sr} - \frac{1}{3}\delta_{pr}\delta_{st}\right) + \frac{1}{2}Q^{(1)}_{q^4H^2}\left(\frac{1}{2}\delta_{pt}\delta_{sr} - \frac{1}{3}\delta_{pr}\delta_{st}\right) + \frac{1}{4}Q^{(3)}_{q^4H^2}\delta_{pt}\delta_{sr}\Bigg\},$$
$$Q^{(7)}_{\psi^4H^2} \equiv (\Psi^4)^I_{WB}(H^\dagger\tau^I H) = \frac{gg'}{2}\sum_{prst}\Bigg\{\sum_{f\in\{q,e,u,d\}}Y_f\Big[Q^{(2)}_{l^2f^2H^2} + Q^{(2)}_{q^2f^2H^2}\Big] + Y_l\Big[Q^{(2)}_{l^4H^2} + Q^{(4)}_{l^2q^2H^2}\Big]\Bigg\}\delta_{pr}\delta_{st}, \tag{68}$$

together with the Hermitian conjugate of $Q^{(2)}_{\psi^4H^2}$. The operators $Q^{(1)}_{\psi^4H^2}$–$Q^{(3)}_{\psi^4H^2}$ are generated by the use of the Higgs EOM directly in operators in class H^4D^4 and H^2D^6, see Eqs. (A9)–(A13), and in the rotation of operators in class X^2D^4, as in Eqs. (A30)–(A31). $Q^{(4)}_{\psi^4H^2}$–$Q^{(7)}_{\psi^4H^2}$ arise from direct application of the EOM for the gauge bosons in operators of class $X^2H^2D^2$, in particular the EOM of $B^{\mu\nu}$ in $R^{(9)}_{B^2H^2D^2}$ (A41), the EOM of $W^{\mu\nu}$ in $R^{(9)}_{W^2H^2D^2}$ (A50), the one for $G^{\mu\nu}$ in $R^{(9)}_{G^2H^2D^2}$ (A63), and the EOM's for $B^{\mu\nu}$ and $W^{\mu\nu}$ in $R^{(13)}_{BWH^2D^2}$ (A76). Here again, for convenience, when writing the expression of $Q^{(4)}_{\psi^4H^2}$ in terms of operators in M8B, we have added a superscript (1) in $Q^{(1)}_{f^4H^2}$, where $f = u, d, e$; and in $Q^{(1)}_{e^2u^2H^2}$ and $Q^{(1)}_{e^2d^2H^2}$. This minimal change of labeling is reflected also when we list the operators in class 18 in Table VI.

(ii) *Class* 19: ψ^4X: the eight universal operators in this class are formed by the contraction of the four-fermion tensor currents in Eqs. (45)–(48) with a gauge strength tensor

$$
\begin{aligned}
Q^{(1)}_{\psi^4W} &\equiv (\Psi^4)^{I\mu\nu}_{WW}W^I_{\mu\nu} = \frac{g^2}{2}\sum_{prst}\left\{Q^{(5)}_{l^2q^2W}\delta_{pr}\delta_{st} - \left[Q^{(2)}_{l^4W} + \frac{1}{3}Q^{(2)}_{q^4W} + 2Q^{(4)}_{q^4W}\right]\delta_{pt}\delta_{rs}\right\},\\
Q^{(2)}_{\psi^4W} &\equiv (\Psi^4)^{I\mu\nu}_{WW}\tilde{W}^I_{\mu\nu} = \frac{g^2}{2}\sum_{prst}\left\{Q^{(6)}_{l^2q^2W}\delta_{pr}\delta_{st} + \left[Q^{(1)}_{l^4W} + \frac{1}{3}Q^{(1)}_{q^4W} + 2Q^{(3)}_{q^4W}\right]\delta_{pt}\delta_{rs}\right\},\\
Q^{(3)}_{\psi^4W} &\equiv (\Psi^4)^{I\mu\nu}_{WB}W^I_{\mu\nu} = \frac{gg'}{2}\sum_{prst}\left\{-Y_l\left[Q^{(1)}_{l^4W} + Q^{(1)}_{l^2q^2W}\right] - Y_q\left[Q^{(1)}_{q^4W} - Q^{(3)}_{l^2q^2W}\right]\right.\\
&\quad\left. + \sum_{f\in\{u,d,e\}}Y_f\left[Q^{(1)}_{l^2f^2W} + Q^{(1)}_{q^2f^2W}\right]\right\}\delta_{pr}\delta_{st},\\
Q^{(4)}_{\psi^4W} &\equiv (\Psi^4)^{I\mu\nu}_{WB}\tilde{W}^I_{\mu\nu} = \frac{gg'}{2}\sum_{prst}\left\{-Y_l\left[Q^{(2)}_{l^4W} + Q^{(2)}_{l^2q^2W}\right] - Y_q\left[Q^{(2)}_{q^4W} - Q^{(4)}_{l^2q^2W}\right]\right.\\
&\quad\left. + \sum_{f\in\{u,d,e\}}Y_f\left[Q^{(2)}_{l^2f^2W} + Q^{(2)}_{q^2f^2W}\right]\right\}\delta_{pr}\delta_{st},\\
Q^{(1)}_{\psi^4G} &\equiv (\Psi^4)^{A\mu\nu}_{GG}G^A_{\mu\nu} = g_s^2\sum_{prst}\left\{2\left[Q^{(5)}_{q^2u^2G} + Q^{(5)}_{q^2d^2G} + Q^{(5)}_{u^2d^2G}\right]\delta_{pr}\delta_{st}\right.\\
&\quad\left. - \left[\frac{1}{2}Q^{(2)}_{q^4G} + \frac{1}{2}Q^{(4)}_{q^4G} - Q^{(2)}_{u^4G} - Q^{(2)}_{d^4G}\right]\delta_{pt}\delta_{sr}\right\},\\
Q^{(2)}_{\psi^4G} &\equiv (\Psi^4)^{A\mu\nu}_{GG}\tilde{G}^A_{\mu\nu} = g_s^2\sum_{prst}\left\{2\left[Q^{(6)}_{q^2u^2G} + Q^{(6)}_{q^2d^2G} + Q^{(6)}_{u^2d^2G}\right]\delta_{pr}\delta_{st}\right.\\
&\quad\left. + \left[\frac{1}{2}Q^{(1)}_{q^4G} + \frac{1}{2}Q^{(3)}_{q^4G} - Q^{(1)}_{u^4G} - Q^{(1)}_{d^4G}\right]\delta_{pt}\delta_{sr}\right\},\\
Q^{(3)}_{\psi^4G} &\equiv (\Psi^4)^{A\mu\nu}_{GB}G^A_{\mu\nu} = -g_s g'\sum_{prst}\left\{\sum_{f\in\{lq\}}\sum_{f'\in\{q,u,d\}}Y_fQ^{(1)}_{f^2f'^2G} + Y_e\left[-Q^{(1)}_{q^2e^2G} + Q^{(1)}_{e^2u^2G} + Q^{(1)}_{e^2d^2G}\right]\right.\\
&\quad\left. + Y_u\left[Q^{(1)}_{u^4G} - Q^{(3)}_{q^2u^2G} + Q^{(1)}_{u^2d^2G}\right] + Y_d\left[Q^{(1)}_{d^4G} - Q^{(3)}_{q^2d^2G} + Q^{(3)}_{u^2d^2G}\right]\right\}\delta_{pr}\delta_{st},\\
Q^{(4)}_{\psi^4G} &\equiv (\Psi^4)^{A\mu\nu}_{GB}\tilde{G}^A_{\mu\nu} = -g_s g'\sum_{prst}\left\{\sum_{f\in\{lq\}}\sum_{f'\in\{q,u,d\}}Y_fQ^{(2)}_{f^2f'^2G} + Y_e\left[-Q^{(2)}_{q^2e^2G} + Q^{(2)}_{e^2u^2G} + Q^{(2)}_{e^2d^2G}\right]\right.\\
&\quad\left. + Y_u\left[Q^{(2)}_{u^4G} - Q^{(4)}_{q^2u^2G} + Q^{(2)}_{u^2d^2G}\right] + Y_d\left[Q^{(2)}_{d^4G} - Q^{(4)}_{q^2d^2G} + Q^{(4)}_{u^2d^2G}\right]\right\}\delta_{pr}\delta_{st}.
\end{aligned}
\tag{69}
$$

They are all generated by the direct application of the EOM for the weak and strong gauge bosons in operators in class X^3D^2. In particular $Q^{(1)}_{\psi^4 W}$–$Q^{(4)}_{\psi^4 W}$ arise when using the EOM of $W^{\mu\nu}$ in the operators $R^{(1,2)}_{W^3D^2}$ in Eqs. (A14)–(A15), and $R^{(1,2)}_{BW^2D^2}$ in (A22)–(A23), respectively. Equivalently, $Q^{(1)}_{\psi^4 G}$–$Q^{(4)}_{\psi^4 G}$ arise when using the EOM of $G^{\mu\nu}$ in operators $R^{(1,2)}_{G^3D^2}$ [(A18)–(A19)] and $R^{(1,2)}_{BG^2D^2}$ [(A26)–(A27)].

(iii) *Class* 20:ψ^4HD: there are four universal four-fermion operators genrerated by the contraction of the gauge-Higgs fermion currents in Eqs. (53) and (54) with the derivative of a Higgs field

$$Q^{(1)}_{\psi^4HD} \equiv (\Psi^4)^{\mu j}_{\rm BH} D_\mu H_j = -ig' \sum_{prst} \sum_{f\in\{q,l,u,d,e\}} Y_f \Big\{ -y^{u\dagger}_{ts} Q^{\dagger(1)}_{f^2quHD} + y^d_{st} Q^{(1)}_{f^2qdHD}, + y^e_{st} Q^{(1)}_{f^2leHD} \Big\} \delta_{pr},$$

$$Q^{(2)}_{\psi^4HD} \equiv (\Psi^4)^{I\mu j}_{WH} (\tau^I)^k_j D_\mu H_k = -i\frac{g}{2} \sum_{prst} \Big\{ + y^{u\dagger}_{ts} \Big[Q^{\dagger(2)}_{q^3uHD} + Q^{\dagger(3)}_{l^2quHD} \Big]$$
$$+ y^d_{st} \Big[Q^{(2)}_{q^3dHD} + Q^{(3)}_{l^2qdHD} \Big] + y^e_{st} \Big[Q^{(2)}_{l^3eHD} + Q^{(3)}_{leq^2HD} \Big] \Big\} \delta_{pr}, \tag{70}$$

and their Hermitian conjugates. They are generated by applying the EOM for the electroweak gauge bosons and the Higgs in the four operators of class XH^2D^2: $R^{(1,2)}_{BH^2D^4}$ [(A81)–(A82)] and $R^{(1,2)}_{WH^2D^4}$ [(A84)–(A85)]. Notice that, to keep the notation compact, we took the liberty of reordering the fermion labeling for some operators in the first equation. In particular, in the case of $Q^{(1)}_{f^2leHD}$, when $f = q$, the operator needs to be identified with $Q^{(1)}_{leq^2HD}$ in Table VIII.

(iv) *Class* 21:ψ^4D^2: Finally, there are four universal four-fermion operators generated directly by the contraction of the derivatives of two fermion currents in Eqs. (55)–(58)

$$Q^{(1)}_{\psi^4D^2} \equiv ({\rm D}\Psi^4)_{HH} = \sum_{prst} \Big\{ - \sum_{f\in\{u,d\}} y^{f\dagger}_{pt} y^f_{sr} \Big(\frac{1}{6} Q^{(1)}_{q^2f^2D^2} + Q^{(3)}_{q^2f^2D^2} \Big) - \frac{1}{2} y^{e\dagger}_{pt} y^e_{sr} Q^{(1)}_{l^2e^2D^2}$$
$$+ \Big[-y^e_{pr} y^u_{st} Q^{(1)}_{lequD^2} + y^u_{pr} y^d_{st} Q^{(1)}_{q^2udD^2} + y^e_{pr} y^{d\dagger}_{st} Q^{(1)}_{leqdD^2} + {\rm H.c.} \Big] \Big\},$$

$$Q^{(2)}_{\psi^4D^2} \equiv ({\rm D}\Psi^4)_{BB} = g'^2 \sum_{prst} \Big\{ \sum_{f\in\{q,l,u,d,e\}} Y_f^2 Q^{(1)}_{f^4D^2} + 2 \sum_{f\in\{q,l\}} \sum_{f\in\{u,d,e\}} Y_f Y'_f Q^{(1)}_{f^2f'^2D^2}$$
$$+ 2Y_q Y_l Q^{(1)}_{l^2q^2D^2} + 2\Big[Y_e Y_u Q^{(1)}_{e^2u^2D^2} + Y_e Y_d Q^{(1)}_{e^2d^2D^2} + Y_u Y_d Q^{(1)}_{u^2d^2D^2} \Big] \Big\} \delta_{pr}\delta_{st},$$

$$Q^{(3)}_{\psi^4D^2} \equiv ({\rm D}\Psi^4)_{WW} = \frac{g^2}{4} \sum_{prst} \Big\{ \Big[Q^{(3)}_{q^4D^2} + 2Q^{(3)}_{l^2q^2D^2} \Big] \delta_{pr}\delta_{st} + Q^{(1)}_{l^4D^2} (2\delta_{pt}\delta_{sr} - \delta_{pr}\delta_{st}) \Big\},$$

$$Q^{(4)}_{\psi^4D^2} \equiv ({\rm D}\Psi^4)_{GG} = g_s^2 \sum_{prst} \Big\{ 2\Big[Q^{(3)}_{q^2u^2D^2} + Q^{(3)}_{q^2d^2D^2} + Q^{(3)}_{u^2d^2D^2} \Big] \delta_{pr}\delta_{st}$$
$$+ \frac{1}{2} \Big[Q^{(1)}_{u^4D^2} + Q^{(1)}_{d^4D^2} \Big] \Big(\delta_{pt}\delta_{sr} - \frac{1}{3}\delta_{pr}\delta_{st} \Big) + \frac{1}{2} Q^{(1)}_{q^4D^2} \Big(\frac{1}{2}\delta_{pt}\delta_{sr} - \frac{1}{3}\delta_{pr}\delta_{st} \Big) + \frac{1}{4} Q^{(3)}_{q^4D^2} \delta_{pt}\delta_{sr} \Big\}. \tag{71}$$

They, respectively, originate from applying the EOM for the Higgs in $R^{(1)}_{H^2D^6}$ [(A13)], for the hypercharge gauge boson in $R^{(1)}_{B^2D^4}$ [(A30)], for the weak gauge boson in $R^{(1)}_{W^2D^4}$ [(A31)], and for the gluon in $R^{(1)}_{G^2D^4}$ [(A32)]. Notice that, for convenience, in the equation for $Q^{(2)}_{\psi^4D^2}$, we have a superscript of (1) in $Q^{(1)}_{e^4D^2}$ to the M8B $Q_{e^4D^2}$ operator. We have included this minimal change of labeling when listing the operators of this class in Table IX.

VI. SOME SIMPLE UNIVERSAL UV COMPLETIONS

Here we briefly discuss a few existing models which match onto the universal basis. For each model we point out the bosonic operators which are generated in the universal basis and specified those which are rotated to the fermionic basis in this work.

A straightforward way to construct universal extensions of the SM is to enlarge its scalar sector with new scalar fields which do not couple to the SM fermions. The simplest bosonic UV completion is therefore that of the SM supplemented by a real singlet scalar S

$$\Delta\mathcal{L} = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}M^2S^2 - A|H|^2S - \frac{1}{2}k|H|^2S^2 - \frac{1}{3!}\mu S^3 - \frac{1}{4!}\tilde{\lambda}_S S^4. \quad (72)$$

After tree level integration of the heavy S, the low energy effective theory contains the following operators up to dimension eight [41,67,68]

$$\Delta\mathcal{L}_{\rm eff} = \frac{A^2}{2M^2}|H|^4 + \frac{A^2}{2M^4}\partial_\mu|H|^2\partial^\mu|H|^2 + \frac{A^2}{2M^4}\left(\frac{A\mu}{3M^2} - k\right)|H|^6 + \frac{A^2}{2M^6}\left(k^2 - \frac{A^2\tilde{\lambda}_S}{12M^2} - \frac{A\mu k}{M^2}\right)Q_{H^8} + \frac{2A^2}{M^6}\left(\frac{A\mu}{2M^2} - k^2\right)|H|^2\partial_\mu|H|^2\partial^\mu|H|^2 + \frac{A^2}{2M^6}\Box|H|^2\Box|H|^2. \quad (73)$$

The last two dimension-eight operators can be written in our basis by the following relations:

$$|H|^2\partial_\mu|H|^2\partial^\mu|H|^2 = R^{(1)}_{H^6D^2} + R^{(2)}_{H^6D^2} + 2Q^{(1)}_{H^6}, \quad (74)$$

$$\Box|H|^2\Box|H|^2 = 4Q^{(3)}_{H^4} + 4(R^{(5)}_{H^4D^4} + R^{(6)}_{H^4D^4}) + R^{(8)}_{H^4D^4} + 2R^{(9)}_{H^4D^4} + R^{(10)}_{H^4D^4}. \quad (75)$$

From the results presented in Appendix A, we can see that the rotation of the operator in Eq. (74) to M8B only generates one fermionic universal operator, the real part of $Q^{(1)}_{\psi^2H^5}$ [see Eqs. (A1) and (A2)], while the operator in Eq. (75) generates a linear combination of five fermionic operators, the real parts of $Q^{(1)}_{\psi^2H^5}$, $Q^{(1)}_{\psi^2H^3D^2}$, and $Q^{(2)}_{\psi^4H^2}$, together with $Q^{(1)}_{\psi^4H^2}$, and $Q^{(3)}_{\psi^4H^2}$ [(see Eqs. (A5), (A6), and (A8)–(A10)].

Another possibility is the addition of a scalar $SU(2)_L$ triplet field ϕ^a with $Y = 0$. In this case, the new terms in Lagrangian read

$$\Delta\mathcal{L} = \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - \frac{1}{2}M^2(\phi^a)^2 + \kappa H^\dagger\sigma^a H\phi^a - \lambda_{\phi H}(\phi^a)^2|H|^2 - \lambda_\phi|\phi^a|^4. \quad (76)$$

The low energy effective theory of this model is, up to dimension-eight operators in terms of our basis can be written as [41,69,70],

$$\Delta\mathcal{L}_{\rm eff} = \frac{\kappa^2}{8M^2}|H|^4 - \frac{\lambda_{\phi H}\kappa^2}{4M^4}|H|^6 + \frac{\kappa^2}{8M^4}[(D_\mu H)^\dagger H(D_\mu H)^\dagger H + \mathrm{H.c.}] + \frac{\kappa^2}{2M^4}|H|^2|D_\mu H|^2 - \frac{\kappa^2}{4M^4}|(D_\mu H)^\dagger H|^2 - \frac{\kappa^2}{16M^6}\left(\frac{\lambda_\phi\kappa^2}{M^2} - 8\lambda^2_{\phi H}\right)Q_{H^8} + \frac{\lambda_{\phi H}\kappa^2}{2M^6}\left(R^{(1)}_{H^6D^2} + R^{(2)}_{H^6D^2} + 2Q^{(2)}_{H^6}\right) + \frac{\kappa^2}{8M^6}\left(8Q^{(1)}_{H^4} - 4Q^{(3)}_{H^4} + 4R^{(1)}_{H^4D^4} + 4R^{(4)}_{H^4D^4} + 4R^{(7)}_{H^4D^4} + R^{(8)}_{H^4D^4} - 2R^{(9)}_{H^4D^4} + R^{(10)}_{H^4D^4}\right) \quad (77)$$

In this case, from the Appendix A we note that the rotation of the operators to M8B generates the following fermionic operators, the real parts of $Q^{(1)}_{\psi^2H^5}$, $Q^{(2)}_{\psi^2H^3D^2}$, and $Q^{(2)}_{\psi^4H^2}$; together with $Q^{(1)}_{\psi^4H^2}$, and $Q^{(3)}_{\psi^4H^2}$ [see Eqs. (A1)–(A3), (A6), and (A9)–(A12)].

Next we consider a scenario presenting a hidden sector where its particles are not charged under the SM gauge group. In the kinetic mixing model, the hiden sector exhibits a $U(1)_X$ gauge symmetry and possesses a gauge boson V_μ which interacts with the SM via

$$\Delta\mathcal{L} = -\frac{1}{4}V_{\mu\nu}V^{\mu\nu} + \frac{1}{2}M^2V_\mu V^\mu - \frac{k}{2}B_{\mu\nu}V^{\mu\nu}. \quad (78)$$

For heavy V_μ we can integrate it out at tree level and obtain the following dimension-six and -eight operators [69,71]

$$\Delta\mathcal{L}_{\rm eff} = -\frac{k^2}{2M^2}(\partial_\nu B^{\mu\nu})(\partial^\alpha B_{\mu\alpha}) + \frac{k^2}{2M^4}(\partial_\nu B^{\mu\nu})(\partial^2\partial^\alpha B_{\mu\alpha}). \tag{79}$$

The dimension eight-operator is identified with the operator $R^{(1)}_{B^2D^4}$ in our basis, which is associated to seven fermionic universal operators in M8B: the real parts of $Q^{(1)}_{\psi^2H^5}$ and $Q^{(2)}_{\psi^4H^2}$, together with $Q^{(1)}_{\psi^4H^2}$, $Q^{(3)}_{\psi^4H^2}$, $Q^{(4)}_{\psi^4D^2}$, $Q^{(1)}_{\psi^2H^2D^3}$, and $Q^{(2)}_{\psi^2H^4,D}$, as can be seen in Eq. (A30).

At large energy, composite Higgs models possess a strongly interacting sector that is naturally connected to the SM bosons. Ergo, possible high-energy resonances can give rise to a plethora of bosonic low-energy effective operators depending on the spectrum in the UV region. As an illustration, let us consider the minimal model based on the coset $SO(5)/SO(4)$ [30,72,73] and consider a vector resonance (ρ_μ) that transforms in the adjoint of $SO(4)$. In this scenario many operators are generated and we will list a few of them.

The formalism develop by Coleman, Callan, Wess and Zumino [74] allow us to write down the allowed interactions of this resonance [75,76]. Denoting by $\Pi = h^a T^a$ where T^a are the $SO(5)$ broken generators and h^a the Goldstone bosons, we define $U = \exp(i\Pi)$. For simplicity, we assume that the SM gauge group satisfies $G_{\rm SM} \subset SO(4)$. In order to write down the ρ_μ interactions we need the building blocks $\mathcal{D}_\mu$ and $\mathcal{E}_\mu$:

$$U^\dagger(\partial_\mu + iA_\mu)U \equiv i\mathcal{D}^A_\mu T^A + i\mathcal{E}^a_\mu T^a = i\mathcal{D}_\mu + i\mathcal{E}_\mu, \tag{80}$$

with T^A being the unbroken generators and A_μ the SM gauge fields. The lowest order terms of $\mathcal{D}_\mu$ and $\mathcal{E}_\mu$ are

$$\mathcal{D}_\mu = D_\mu\Pi - \frac{1}{6}[\Pi, \Pi\overleftrightarrow{D}\Pi] + \cdots \tag{81}$$

$$\mathcal{E}_\mu = A_\mu - \frac{i}{2}\Pi\overleftrightarrow{D}\Pi + \cdots. \tag{82}$$

The most general two derivative $SO(5)$-invariant action for the ρ_μ is

$$m_\rho^4\Delta\mathcal{L}^{(2)} = m_\rho^2\mathcal{D}^A_\mu\mathcal{D}^\mu_A - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}m_\rho^2(\rho_\mu - \mathcal{E}_\mu)^2, \tag{83}$$

where $\rho_{\mu\nu} = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + i[\rho_\mu, \rho_\nu]$. Assuming that ρ_μ is heavy we can integrate it out to obtain [75]

$$\Delta\mathcal{L}_{\rm eff} = -\frac{1}{4g_\rho^2}\mathcal{E}_{\mu\nu}\mathcal{E}^{\mu\nu} - \frac{1}{2g_\rho^2}D^\mu\mathcal{E}_{\mu\nu}\frac{1}{\partial^2 + m_\rho^2}D_\alpha\mathcal{E}^{\alpha\nu} \tag{84}$$

where g_ρ is the coupling constant. Terms containing four or more derivatives and four or more field strength tensors have been omitted. The first term of the above equation leads to the dimension-six operators

$$(H^\dagger\tau^I\overleftrightarrow{D}_\mu H)D_\nu W^{I,\mu\nu} \quad \text{and} \quad (H^\dagger\overleftrightarrow{D}_\mu H)\partial_\nu B^{\mu\nu}. \tag{85}$$

On the other hand, the second term of Eq. (84) gives rise to the following dimension-six and -eight operators

$$D_\nu W^{I,\mu\nu}D^\alpha W^I\alpha\nu, \qquad \partial_\nu B^{\mu\nu}\partial^\alpha B_{\alpha\nu}, \tag{86}$$

$$R^{(1)}_{W^2D^4} = D_\nu W^{I,\mu\nu}D^\beta D_\beta D^\alpha W^I\alpha\mu,$$
$$R^{(1)}_{B^2D^4} = \partial_\nu B^{\mu\nu}\partial^\beta\partial_\beta\partial^\alpha B_{\alpha\nu}. \tag{87}$$

Upon rotation to M8B the operators in Eq. (87) give rise respectively to two linear combinations involving ten and seven universal operators respectively given in Eqs. (A31) and (A30).

In addition, possible four derivative ρ interactions can produce genuine anomalous quartic gauge couplings [53], i.e., anomalous quartic couplings that do not have triple couplings associated to them. For instance, the following ρ self-interaction

$$\frac{1}{m_\rho^4}(\rho_{\mu\nu}\rho^{\mu\nu})(\rho_{\alpha\beta}\rho^{\alpha\beta}) \tag{88}$$

generates the low-energy effective interaction

$$\frac{1}{m_\rho^4}(\mathcal{E}_{\mu\nu}\mathcal{E}^{\mu\nu})(\mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta}) \tag{89}$$

that contains the operators

$$Q^{(1)}_{W^4} = (W^I_{\mu\nu}W^{I,\mu\nu})(W^I_{\alpha\beta}W^{I,\alpha\beta}),$$
$$Q^{(1)}_{W^2B^2} = (W^I_{\mu\nu}W^{I,\mu\nu})(B_{\alpha\beta}B^{\alpha\beta}), \qquad Q^{(1)}_{B^4} = (B_{\mu\nu}B^{\mu\nu})(B_{\alpha\beta}B^{\alpha\beta}). \tag{90}$$

VII. FINAL REMARKS

In the absence of a smoking-gun signal for new physics at the LHC, EFTs, in particular the SMEFT, have become the standard tool for model-agnostic BSM explorations. Unfortunately, their power is in some sense also their weakness as, taken in their greatest generality, the number of parameters (i.e., the number of independent Wilson coefficients) is prohibitively large already at dimension-six. Identifying physically motivated hypotheses which reduce the EFT parameter space while still capturing a large class of BSM theories presents a motivated route to predictability. Universality, i.e., the assumption that the NP couples dominantly to the Standard Model bosons, is one such hypothesis. At dimension-six, universality reduces the dimensionality of the SMEFT parameter space from 2499 to 16, allowing for a constrained effective parametrization of NP effects [29].

In this work we have taken the next step by constructing the dimension-eight SMEFT operator basis which at the high energy matching scale can be used to encode the effects of universal extensions of the SM. To do so we have identified a suitable basis of independent operators formed with the Standard Model bosonic fields at dimension-eight. It contains 175 operators—that is, the assumption of Universality reduces the number of independent SMEFT coefficients at dimension eight from 44807 to 175. 89 of these operators are part of the general SMEFT dimension-eight basis in the literature; see Table I). Our choice of the additional 86 operators is presented in Eqs. (28)–(37). In the general dimension-eight SMEFT basis in the literature these 86 operators have been traded for combinations of the 89 bosonic operators in the basis and additional operators involving fermions. Thus in universal theories, only a subset of fermionic operators are generated (see Appendix C) and their Wilson coefficients have well defined relations: they must be linear combinations of the 86 independent couplings of the universal fermionic operators presented in Sec. V.

The drastic reduction of independent parameters implied by the Universality assumption opens up the possibility of employing it for quantitative phenomenology because just a few of them contribute to a specific reaction. For example, the direct effect of the dimension-eight universal operators can be seen in the production of multiple H, $W^\pm$ and Z. Operators in the classes X^3H^2 and X^3D^2 modify the triple gauge boson vertices, so contributing to diboson production at tree level. Moreover, many classes contribute to modifications of the quartic coupling among the SM gauge bosons, as well as to vertices with Higgs and gauge bosons. In addition, an interesting subset of operators from the rotated basis are those which include, after rotation, the Murphy basis operators $Q^{(i)}_{f^2VH^2D}$ for $i\in 1, 3, 5$ and $Q^{(i)}_{f^2H^2D^3}$ for $i\in 1, 3, 4$. These operators introduce novel kinematics to the Higgs associated production process [62]. The $Q^{(i)}_{f^2VH^2D}$ are generated by rotating $R^{(5,6)}_{B^2H^2D^2}$, $R^{(5,6)}_{BWH^2D}$, $R^{(3)}_{BH^2D^4}$, $R^{(1)}_{BW^2D^2}$, and $R^{(5,6)}_{W^2H^2D^2}$. Similarly $Q^{(i)}_{f^2H^2D^3}$ are generated by rotating $R^{(1)}_{B^2D^4}$ and $R^{(1)}_{W^2D^4}$. We notice in passing that the results in Sec. VI show that the simple composite Higgs model there presented generates both $R^{(1)}_{B^2D^4}$ and $R^{(1)}_{W^2D^4}$, while $R^{(1)}_{B^2D^4}$ also emerges in the minimal $U(1)_X$ kinetic-mixing scenario.

Of course, the phenomenological program first requires a careful accounting for the relevant field redefinitions and other finite renormalization effects since some of the operators give contributions to the definition of the SM parameters [42]. Moreover, even in a scenario where the high energy model is universal, there will be nonuniversal effects at the low energy EFT due to the renormalization group running [77] but controlled by the universal parameters at the matching scale. They should also be taken into account in phenomenological studies. Furthermore, the rich phenomenology of dimension-eight operators possesses many constraints originating from the causality and analyticity of the scattering amplitudes; see, for instance, Refs. [78–82]. These bounds define the regions of the parameter space associated with well-defined ultraviolet completions of the SM. We leave the quantitative exploration of the phenomenology of universal SMEFT at dimension-eight for future work.

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APPENDIX A: ROTATION OF THE HIGHER DERIVATIVE BOSONIC UNIVERSAL OPERATORS INTO FERMIONIC UNIVERSAL OPERATORS

This appendix contains the explicit expression of the 86 bosonic operators of universal theories not included in M8B in terms of the bosonic and fermionic operators in M8B as generated by the EOM. For the sake of convenience, we present in Appendix C the definition of all M8B fermionic operators involved. For some of the operators the final expressions in terms of operators in M8B basis may look rather cumbersome. This is particularly the case when the Higgs currents introduced by the application of the EOMs results in bosonic operators which are not included in M8B and which, therefore, need further simplification by reapplication of IBP, BI, and EOMs.

Notice that the mass term of the Higgs equation of motion Eq. (23) leads to the appearance of operators with dimension less than eight when we apply the EOM. For these terms we did not express the resulting operators in terms of any specific dimension-six operator basis of those in the literature: HISZ [83,84], Warsaw [85], EGGM [86], or SILH [87]. For convenience we define the following combinations involving terms arising from the Higgs mass part of the EOM and some Higgs operators which appear recurrently in the expressions

$$\Delta_0 \equiv \lambda[v^2(H^\dagger H) - 2(H^\dagger H)^2], \qquad \Delta_1 \equiv \lambda[v^2(H^\dagger H)^3 - 2Q_{H^8}],$$
$$\Delta_2 \equiv \lambda[v^2(H^\dagger H)^3 - 2Q_{H^8}] + 2Q_{H^6}^{(1)} + 2Q_{H^6}^{(2)}, \qquad \Delta_3 \equiv \lambda[v^2(H^\dagger H)^3 - 2Q_{H^8}] + 5Q_{H^6}^{(1)},$$
$$\Delta_4 \equiv \lambda[v^2(H^\dagger H)^3 - 2Q_{H^8}] + 4Q_{H^6}^{(1)} + Q_{H^6}^{(2)}, \qquad \Delta_5 \equiv \lambda[v^2(H^\dagger H)^3 - 2Q_{H^8}] + 3Q_{H^6}^{(1)} + 2Q_{H^6}^{(2)}.$$

Operators in the class H^6D^2:

$$R_{H^6D^2}^{(1)} = \Delta_1 - Q_{\psi^2 H^5}^{(1)}, \tag{A1}$$

$$R_{H^6D^2}^{(2)} = \Delta_1 - Q_{\psi^2 H^5}^{\dagger(1)}. \tag{A2}$$

Operators in the class H^4D^4:

$$R_{H^4D^4}^{(1)} = \lambda v^2(H^\dagger \tau^I H)(D^\mu H^\dagger)\tau^I(D_\mu H) - 2\lambda Q_{H^6}^{(2)} - Q_{\psi^2 H^3 D^2}^{(2)}, \tag{A3}$$

$$R_{H^4D^4}^{(2)} = \lambda\Delta_2 + \lambda v^2(H^\dagger(D^\mu H))^2 - \lambda Q_{\psi^2 H^5}^{\dagger(1)} - Q_{\psi^2 H^3 D^2}^{(3)}, \tag{A4}$$

$$R_{H^4D^4}^{(3)} = \lambda\Delta_2 + \lambda v^2(D^\mu H^\dagger H)^2 - \lambda Q_{\psi^2 H^5}^{(1)} - Q_{\psi^2 H^3 D^2}^{\dagger(3)}, \tag{A5}$$

$$R_{H^4D^4}^{(4)} = \lambda v^2(H^\dagger \tau^I H)(D^\mu H^\dagger)\tau^I(D_\mu H) - 2\lambda Q_{H^6}^{(2)} - Q_{\psi^2 H^3 D^2}^{\dagger(2)}, \tag{A6}$$

$$R_{H^4D^4}^{(5)} = \lambda v^2(H^\dagger H)(D_\mu H^\dagger)(D^\mu H) - 2\lambda Q_{H^6}^{(1)} - Q_{\psi^2 H^3 D^2}^{(1)}, \tag{A7}$$

$$R_{H^4D^4}^{(6)} = \lambda v^2(H^\dagger H)(D_\mu H^\dagger)(D^\mu H) - 2\lambda Q_{H^6}^{(1)} - Q_{\psi^2 H^3 D^2}^{\dagger(1)}, \tag{A8}$$

$$R_{H^4D^4}^{(7)} = (\Delta_0)^2 - \lambda v^2(H^\dagger J_H^\dagger + \text{H.c.})(H^\dagger H) + 2\lambda(Q_{\psi^2 H^5}^{(1)} + Q_{\psi^2 H^5}^{\dagger(1)}) + Q_{\psi^4 H^2}^{(1)}, \tag{A9}$$

$$R_{H^4D^4}^{(8)} = (\Delta_0)^2 - 2\lambda v^2(H^\dagger H)H^\dagger J_H^\dagger + 4\lambda Q_{\psi^2 H^5}^{(1)} + Q_{\psi^4 H^2}^{(2)}, \tag{A10}$$

$$R_{H^4D^4}^{(9)} = (\Delta_0)^2 - \lambda v^2(J_H H + \text{H.c.})(H^\dagger H) + 2\lambda(Q_{\psi^2 H^5}^{(1)} + Q_{\psi^2 H^5}^{\dagger(1)}) + Q_{\psi^4 H^2}^{(3)} + \frac{1}{2}Q_{\psi^4 H^2}^{(1)}, \tag{A11}$$

$$R_{H^4D^4}^{(10)} = (\Delta_0)^2 - 2\lambda v^2(H^\dagger H)J_H H + 4\lambda Q_{\psi^2 H^5}^{\dagger(1)} + Q_{\psi^4 H^2}^{\dagger(2)}. \tag{A12}$$

Operator in the class H^6D^2:

$$\begin{aligned} R_{H^2D^6}^{(1)} = {} & -8\lambda^2\Delta_1 + \lambda^2 v^4(D^\mu H^\dagger D_\mu H) - 2\lambda^2 v^2 D^\mu(H^\dagger H)D_\mu(H^\dagger H) - 4\lambda^2 v^2(H^\dagger H)(D^\mu H^\dagger D_\mu H) \\ & - \lambda v^2(D^\mu H^\dagger D_\mu J_H^\dagger + \text{H.c.}) - 6\lambda^2 v^2(H^\dagger H)(H^\dagger J_H^\dagger + \text{H.c.}) - 4\lambda^2 Q_{H^6}^{(1)} \\ & - 2\lambda(3Q_{\psi^2 H^3 D^2}^{(1)} + Q_{\psi^2 H^3 D^2}^{(2)} + 2Q_{\psi^2 H^3 D^2}^{(3)} + \text{H.c.}) + 16\lambda^2(Q_{\psi^2 H^5}^{(1)} + Q_{\psi^2 H^5}^{\dagger(1)}) \\ & + 2\lambda(3Q_{\psi^4 H^2}^{(1)} + Q_{\psi^4 H^2}^{(2)} + Q_{\psi^4 H^2}^{\dagger(2)} + 2Q_{\psi^4 H^2}^{(3)}) + Q_{\psi^4 D^2}^{(1)}. \end{aligned} \tag{A13}$$

Operators in class X^3D^2:

$$R^{(1)}_{W^3D^2} = -\frac{g^3}{4}\Delta_3 + 2ig^2 Q^{(1)}_{WH^4D^2} + \frac{g^2}{4}\left(g' Q^{(1)}_{WBH^4} + g Q^{(1)}_{W^2H^4}\right) + \frac{g^3}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - \frac{g^2}{4}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D}\right) - gQ^{(11)}_{\psi^2WH^2D} + Q^{(1)}_{\psi^4W}, \tag{A14}$$

$$R^{(2)}_{W^3D^2} = 2ig^2 Q^{(2)}_{WH^4D^2} + \frac{g^2}{4}\left(g' Q^{(2)}_{WBH^4} + g Q^{(2)}_{W^2H^4}\right) - gQ^{(12)}_{\psi^2WH^2D} + Q^{(2)}_{\psi^4W}, \tag{A15}$$

$$R^{(3)}_{W^3D^2} = -gQ^{(4)}_{W^2H^2D^2} - \frac{g}{4}\left(g' Q^{(1)}_{W^2BH^2} + g Q^{(1)}_{W^3H^2}\right) - \frac{1}{2}Q^{(1)}_{\psi^2W^2H(D)}, \tag{A16}$$

$$R^{(4)}_{W^3D^2} = -gQ^{(6)}_{W^2H^2D^2} - \frac{g}{4}\left(-g' Q^{(2)}_{W^2BH^2} + 2g Q^{(2)}_{W^3H^2}\right) - Q^{(2)}_{\psi^2W^2H(D)}, \tag{A17}$$

$$R^{(1)}_{G^3D^2} = Q^{(1)}_{\psi^4G}, \tag{A18}$$

$$R^{(2)}_{G^3D^2} = Q^{(2)}_{\psi^4G}, \tag{A19}$$

$$R^{(3)}_{G^3D^2} = -\frac{1}{2}Q^{(1)}_{\psi^2G^2H(D)}, \tag{A20}$$

$$R^{(4)}_{G^3D^2} = -Q^{(2)}_{\psi^2G^2H(D)}, \tag{A21}$$

$$R^{(1)}_{BW^2D^2} = i\frac{gg'}{2}Q^{(3)}_{WH^4D^2} - \frac{g'}{2}Q^{(7)}_{\psi^2WH^2D} + \frac{g}{2}Q^{(3)}_{\psi^2WH^2D} - \frac{gg'}{8}Q^{(3)}_{\psi^2H^4D} + Q^{(3)}_{\psi^4W}, \tag{A22}$$

$$R^{(2)}_{BW^2D^2} = i\frac{gg'}{2}Q^{(4)}_{WH^4D^2} - \frac{g'}{2}Q^{(8)}_{\psi^2WH^2D} + \frac{g}{2}Q^{(4)}_{\psi^2WH^2D} + Q^{(4)}_{\psi^4W}, \tag{A23}$$

$$R^{(3)}_{BW^2D^2} = -gQ^{(3)}_{WBH^2D^2} + Q^{(1)}_{\psi^2WBH(D)}, \tag{A24}$$

$$R^{(4)}_{BW^2D^2} = -gQ^{(5)}_{WBH^2D^2} + Q^{(2)}_{\psi^2WBH(D)}, \tag{A25}$$

$$R^{(1)}_{BG^2D^2} = -\frac{g'}{2}Q^{(3)}_{\psi^2GH^2D} + Q^{(3)}_{\psi^4G}, \tag{A26}$$

$$R^{(2)}_{BG^2D^2} = -\frac{g'}{2}Q^{(4)}_{\psi^2GH^2D} + Q^{(4)}_{\psi^4G}, \tag{A27}$$

$$R^{(3)}_{BG^2D^2} = Q^{(1)}_{\psi^2GBH(D)}, \tag{A28}$$

$$R^{(4)}_{BG^2D^2} = Q^{(2)}_{\psi^2GBH(D)}. \tag{A29}$$

Operators in class X^2D^4:

$$R^{(1)}_{B^2D^4} = \frac{g^2g'^2}{4}\Delta_4 + g'^2\left(Q^{(1)}_{H^4} - Q^{(2)}_{H^4}\right) + \frac{g'^2}{8}\left(g'^2 Q^{(1)}_{B^2H^4} - g^2 Q^{(1)}_{W^2H^4}\right) + ig'^2\left(g' Q^{(1)}_{BH^4D^2} - g Q^{(1)}_{WH^4D^2}\right) - \frac{g'}{2}Q^{(1)}_{\psi^2H^2D^3} - \frac{g'^2 g}{4}Q^{(2)}_{\psi^2H^4D} - \frac{g^2g'^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - \frac{3g'^2}{4}\left(-Q^{(1)}_{\psi^4H^2} + Q^{(2)}_{\psi^4H^2} + Q^{\dagger(2)}_{\psi^4H^2} - 2Q^{(3)}_{\psi^4H^2}\right) + Q^{(2)}_{\psi^4D^2}, \tag{A30}$$

$$\begin{aligned} R^{(1)}_{W^2D^4} = {} & \frac{g^4}{4}\Delta_3 + \frac{g^2g'^2}{2}\Delta_5 - g^2\left(Q^{(1)}_{H^4} + Q^{(2)}_{H^4} - 2Q^{(3)}_{H^4}\right) \\ & - \frac{g^2}{8}\left(3g'^2 Q^{(1)}_{B^2H^4} + g^2 Q^{(1)}_{W^2H^4} + 4gg' Q^{(1)}_{WBH^4}\right) - ig^2\left(3g' Q^{(1)}_{BH^4D^2} + g Q^{(1)}_{WH^4D^2}\right) \\ & - \frac{g}{2} Q^{(2)}_{\psi^2H^2D^3} - g^2 Q^{(11)}_{\psi^2WH^2D} + \frac{g^2}{4}\left(g Q^{(2)}_{\psi^2H^4D} + g Q^{(4)}_{\psi^2H^4D} + 4g' Q^{(1)}_{\psi^2H^4D}\right) \\ & - \frac{g^2(g^2+2g'^2)}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - \frac{3g^2}{4}\left(-3Q^{(1)}_{\psi^4H^2} + Q^{(2)}_{\psi^4H^2} + Q^{\dagger(2)}_{\psi^4H^2} + 2Q^{(3)}_{\psi^4H^2}\right) + Q^{(3)}_{\psi^4D^2} \end{aligned} \tag{A31}$$

$$R^{(1)}_{G^2D^4} = Q^{(4)}_{\psi^4D^2}. \tag{A32}$$

Operators in class $X^2H^2D^2$:

$$R^{(1)}_{B^2H^2D^2} = \lambda v^2 B_{\mu\nu}B^{\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{B^2H^4} - Q^{(1)}_{\psi^2B^2H}, \tag{A33}$$

$$R^{(2)}_{B^2H^2D^2} = \lambda v^2 B_{\mu\nu}B^{\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{B^2H^4} - Q^{\dagger(1)}_{\psi^2B^2H}, \tag{A34}$$

$$R^{(3)}_{B^2H^2D^2} = \lambda v^2 B_{\mu\nu}\tilde{B}^{\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{B^2H^4} - Q^{(2)}_{\psi^2B^2H}, \tag{A35}$$

$$R^{(4)}_{B^2H^2D^2} = \lambda v^2 B_{\mu\nu}\tilde{B}^{\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{B^2H^4} - Q^{\dagger(2)}_{\psi^2B^2H}, \tag{A36}$$

$$\begin{aligned} R^{(5)}_{B^2H^2D^2} = {} & -\frac{g'^2}{8}\Delta_5 + i\frac{g'}{2} Q^{(1)}_{BH^4D^2} + \frac{g'}{8}\left(g' Q^{(1)}_{B^2H^4} + g Q^{(1)}_{WBH^4}\right) \\ & - \frac{1}{2}\left(Q^{(1)}_{\psi^2BH^2D} + iQ^{(3)}_{\psi^2BH^2D}\right) - \frac{g'}{4} Q^{(1)}_{\psi^2H^4D} + \frac{g'^2}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \end{aligned} \tag{A37}$$

$$\begin{aligned} R^{(6)}_{B^2H^2D^2} = {} & -\frac{g'^2}{8}\Delta_5 + i\frac{g'}{2} Q^{(1)}_{BH^4D^2} + \frac{g'}{8}\left(g' Q^{(1)}_{B^2H^4} + g Q^{(1)}_{WBH^4}\right) \\ & - \frac{1}{2}\left(Q^{(1)}_{\psi^2BH^2D} - iQ^{(3)}_{\psi^2BH^2D}\right) - \frac{g'}{4} Q^{(1)}_{\psi^2H^4D} + \frac{g'^2}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \end{aligned} \tag{A38}$$

$$R^{(7)}_{B^2H^2D^2} = i\frac{g'}{2} Q^{(2)}_{BH^4D^2} + \frac{g'}{8}\left(g' Q^{(2)}_{B^2H^4} + g Q^{(2)}_{WBH^4}\right) - \frac{1}{2}\left(Q^{(2)}_{\psi^2BH^2D} + iQ^{(4)}_{\psi^2BH^2D}\right), \tag{A39}$$

$$R^{(8)}_{B^2H^2D^2} = i\frac{g'}{2} Q^{(2)}_{BH^4D^2} + \frac{g'}{8}\left(g' Q^{(2)}_{B^2H^4} + g Q^{(2)}_{WBH^4}\right) - \frac{1}{2}\left(Q^{(2)}_{\psi^2BH^2D} - iQ^{(4)}_{\psi^2BH^2D}\right), \tag{A40}$$

$$R^{(9)}_{B^2H^2D^2} = \frac{g'^2}{4}\Delta_5 - \frac{g'^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) + g' Q^{(1)}_{\psi^2H^4D} + Q^{(4)}_{\psi^4H^2}, \tag{A41}$$

$$R^{(1)}_{W^2H^2D^2} = \lambda v^2 W^I_{\mu\nu}W^{I,\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{W^2H^4} - Q^{(1)}_{\psi^2W^2H}, \tag{A42}$$

$$R^{(2)}_{W^2H^2D^2} = \lambda v^2 W^I_{\mu\nu}W^{I,\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{W^2H^4} - Q^{\dagger(1)}_{\psi^2W^2H}, \tag{A43}$$

$$R^{(3)}_{W^2H^2D^2} = \lambda v^2 W^I_{\mu\nu}\tilde{W}^{I,\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{W^2H^4} - Q^{(2)}_{\psi^2W^2H}, \tag{A44}$$

$$R^{(4)}_{W^2H^2D^2} = \lambda v^2 W^I_{\mu\nu}\tilde{W}^{I,\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{W^2H^4} - Q^{\dagger(2)}_{\psi^2W^2H}, \tag{A45}$$

$$R^{(5)}_{W^2H^2D^2} = -\frac{g^2}{8}\Delta_3 + \frac{g}{2}\left(iQ^{(1)}_{WH^4D^2} - Q^{(3)}_{WH^4D^2}\right) + \frac{g}{8}\left(g'Q^{(1)}_{WBH^4} + gQ^{(1)}_{W^2H^4}\right) - \frac{1}{2}\left(Q^{(5)}_{\psi^2WH^2D} + iQ^{(7)}_{\psi^2WH^2D}\right) - \frac{g}{8}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} - iQ^{(3)}_{\psi^2H^4D}\right) + \frac{g^2}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A46}$$

$$R^{(6)}_{W^2H^2D^2} = -\frac{g^2}{8}\Delta_3 + \frac{g}{2}\left(iQ^{(1)}_{WH^4D^2} + Q^{(3)}_{WH^4D^2}\right) + \frac{g}{8}\left(g'Q^{(1)}_{WBH^4} + gQ^{(1)}_{W^2H^4}\right) - \frac{1}{2}\left(Q^{(5)}_{\psi^2WH^2D} - iQ^{(7)}_{\psi^2WH^2D}\right) - \frac{g}{8}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} + iQ^{(3)}_{\psi^2H^4D}\right) + \frac{g^2}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A47}$$

$$R^{(7)}_{W^2H^2D^2} = \frac{g}{2}\left(iQ^{(2)}_{WH^4D^2} - Q^{(4)}_{WH^4D^2}\right) + \frac{g}{8}\left(g'Q^{(2)}_{WBH^4} + gQ^{(2)}_{W^2H^4}\right) - \frac{1}{2}\left(Q^{(6)}_{\psi^2WH^2D} + iQ^{(8)}_{\psi^2WH^2D}\right), \tag{A48}$$

$$R^{(8)}_{W^2H^2D^2} = \frac{g}{2}\left(iQ^{(2)}_{WH^4D^2} + Q^{(4)}_{WH^4D^2}\right) + \frac{g}{8}\left(g'Q^{(2)}_{WBH^4} + gQ^{(2)}_{W^2H^4}\right) - \frac{1}{2}\left(Q^{(6)}_{\psi^2WH^2D} - iQ^{(8)}_{\psi^2WH^2D}\right), \tag{A49}$$

$$R^{(9)}_{W^2H^2D^2} = \frac{g^2}{4}\Delta_3 + \frac{g}{2}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D}\right) - \frac{g^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) + Q^{(5)}_{\psi^4H^2}, \tag{A50}$$

$$R^{(10)}_{W^2H^2D^2} = i\frac{g^2}{4}\Delta_3 - i\frac{g}{2}\left(4iQ^{(1)}_{WH^4D^2} - Q^{(3)}_{WH^4D^2}\right) - i\frac{g}{4}\left(g'Q^{(1)}_{WBH^4} + gQ^{(1)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(9)}_{\psi^2WH^2D} + iQ^{(11)}_{\psi^2WH^2D}\right) + i\frac{g}{4}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} - iQ^{(3)}_{\psi^2H^4D}\right) - i\frac{g^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A51}$$

$$R^{(11)}_{W^2H^2D^2} = -i\frac{g^2}{4}\Delta_3 + i\frac{g}{2}\left(4iQ^{(1)}_{WH^4D^2} + Q^{(3)}_{WH^4D^2}\right) + i\frac{g}{4}\left(g'Q^{(1)}_{WBH^4} + gQ^{(1)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(9)}_{\psi^2WH^2D} - iQ^{(11)}_{\psi^2WH^2D}\right) - i\frac{g}{4}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} + iQ^{(3)}_{\psi^2H^4D}\right) + i\frac{g^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A52}$$

$$R^{(12)}_{W^2H^2D^2} = \frac{g}{2}\left(4Q^{(2)}_{WH^4D^2} + iQ^{(4)}_{WH^4D^2}\right) - i\frac{g}{4}\left(g'Q^{(2)}_{WBH^4} + gQ^{(2)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(10)}_{\psi^2WH^2D} + iQ^{(12)}_{\psi^2WH^2D}\right), \tag{A53}$$

$$R^{(13)}_{W^2H^2D^2} = \frac{g}{2}\left(-4Q^{(2)}_{WH^4D^2} + iQ^{(4)}_{WH^4D^2}\right) + i\frac{g}{4}\left(g'Q^{(2)}_{WBH^4} + gQ^{(2)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(10)}_{\psi^2WH^2D} - iQ^{(12)}_{\psi^2WH^2D}\right), \tag{A54}$$

$$R^{(1)}_{G^2H^2D^2} = \lambda v^2 G^A_{\mu\nu}G^{A,\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{G^2H^4} - Q^{(1)}_{\psi^2G^2H}, \tag{A55}$$

$$R^{(2)}_{G^2H^2D^2} = \lambda v^2 G^A_{\mu\nu}G^{A,\mu\nu}(H^\dagger H) - 2\lambda Q^{(1)}_{G^2H^4} - Q^{\dagger(1)}_{\psi^2G^2H}, \tag{A56}$$

$$R^{(3)}_{G^2H^2D^2} = \lambda v^2 G^A_{\mu\nu}\tilde{G}^{A,\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{G^2H^4} - Q^{(2)}_{\psi^2G^2H}, \tag{A57}$$

$$R^{(4)}_{G^2H^2D^2} = \lambda v^2 G^A_{\mu\nu}\tilde{G}^{A,\mu\nu}(H^\dagger H) - 2\lambda Q^{(2)}_{G^2H^4} - Q^{\dagger(2)}_{\psi^2G^2H}, \tag{A58}$$

$$R^{(5)}_{G^2H^2D^2} = -\frac{1}{2}\left(Q^{(1)}_{\psi^2GH^2D} + iQ^{(3)}_{\psi^2GH^2D}\right), \tag{A59}$$

$$R^{(6)}_{G^2H^2D^2} = -\frac{1}{2}\left(Q^{(1)}_{\psi^2GH^2D} - iQ^{(3)}_{\psi^2GH^2D}\right), \tag{A60}$$

$$R^{(7)}_{G^2H^2D^2} = -\frac{1}{2}\left(Q^{(2)}_{\psi^2GH^2D} + iQ^{(4)}_{\psi^2GH^2D}\right), \tag{A61}$$

$$R^{(8)}_{G^2H^2D^2} = -\frac{1}{2}\left(Q^{(2)}_{\psi^2GH^2D} - iQ^{(4)}_{\psi^2GH^2D}\right), \tag{A62}$$

$$R^{(9)}_{G^2H^2D^2} = Q^{(6)}_{\psi^4H^2}, \tag{A63}$$

$$R^{(1)}_{BWH^2D^2} = \lambda v^2 B_{\mu\nu} W^{I,\mu\nu}(H^\dagger \tau^I H) - 2\lambda Q^{(1)}_{WBH^4} - Q^{(1)}_{\psi^2 WBH}, \tag{A64}$$

$$R^{(2)}_{BWH^2D^2} = \lambda v^2 B_{\mu\nu} W^{I,\mu\nu}(H^\dagger \tau^I H) - 2\lambda Q^{(1)}_{WBH^4} - Q^{\dagger(1)}_{\psi^2 WBH}, \tag{A65}$$

$$R^{(3)}_{BWH^2D^2} = \lambda v^2 B_{\mu\nu} \tilde{W}^{I,\mu\nu}(H^\dagger \tau^I H) - 2\lambda Q^{(2)}_{WBH^4} - Q^{(2)}_{\psi^2 WBH}, \tag{A66}$$

$$R^{(4)}_{BWH^2D^2} = \lambda v^2 B_{\mu\nu} \tilde{W}^{I,\mu\nu}(H^\dagger \tau^I H) - 2\lambda Q^{(2)}_{WBH^4} - Q^{\dagger(2)}_{\psi^2 WBH}, \tag{A67}$$

$$R^{(5)}_{BWH^2D^2} = -\frac{gg'}{8}\left(Q^{(1)}_{H^6} - Q^{(2)}_{H^6}\right) + \frac{g'}{2}\left(iQ^{(1)}_{WH^4D^2} - Q^{(3)}_{WH^4D^2}\right) + \frac{gg'}{16}\left(Q^{(1)}_{W^2H^4} - Q^{(3)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(1)}_{\psi^2 WH^2D} + iQ^{(3)}_{\psi^2 WH^2D}\right) - \frac{g'}{8}\left(Q^{(4)}_{\psi^2 H^4 D} - iQ^{(3)}_{\psi^2 H^4 D}\right), \tag{A68}$$

$$R^{(6)}_{BWH^2D^2} = -\frac{gg'}{8}\left(Q^{(1)}_{H^6} - Q^{(2)}_{H^6}\right) + \frac{g'}{2}\left(iQ^{(1)}_{WH^4D^2} + Q^{(3)}_{WH^4D^2}\right) + \frac{gg'}{16}\left(Q^{(1)}_{W^2H^4} - Q^{(3)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(1)}_{\psi^2 WH^2D} - iQ^{(3)}_{\psi^2 WH^2D}\right) - \frac{g'}{8}\left(Q^{(4)}_{\psi^2 H^4 D} + iQ^{(3)}_{\psi^2 H^4 D}\right), \tag{A69}$$

$$R^{(7)}_{BWH^2D^2} = \frac{g'}{2}\left(iQ^{(2)}_{WH^4D^2} - Q^{(4)}_{WH^4D^2}\right) + \frac{gg'}{16}\left(Q^{(2)}_{W^2H^4} - Q^{(4)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(2)}_{\psi^2 WH^2D} + iQ^{(4)}_{\psi^2 WH^2D}\right), \tag{A70}$$

$$R^{(8)}_{BWH^2D^2} = \frac{g'}{2}\left(iQ^{(2)}_{WH^4D^2} + Q^{(4)}_{WH^4D^2}\right) + \frac{gg'}{16}\left(Q^{(2)}_{W^2H^4} - Q^{(4)}_{W^2H^4}\right) + \frac{1}{2}\left(Q^{(2)}_{\psi^2 WH^2D} - iQ^{(4)}_{\psi^2 WH^2D}\right), \tag{A71}$$

$$R^{(9)}_{BWH^2D^2} = -\frac{gg'}{8}\Delta_5 + i\frac{3g}{2}Q^{(1)}_{BH^4D^2} + \frac{g}{8}\left(gQ^{(1)}_{WBH^4} + g'Q^{(1)}_{B^2H^4}\right) + \frac{1}{2}\left(Q^{(5)}_{\psi^2 BH^2D} + iQ^{(7)}_{\psi^2 BH^2D}\right) - \frac{g}{4}Q^{(1)}_{\psi^2H^4D} + \frac{gg'}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A72}$$

$$R^{(10)}_{BWH^2D^2} = -\frac{gg'}{8}\Delta_5 + i\frac{3g}{2}Q^{(1)}_{BH^4D^2} + \frac{g}{8}\left(gQ^{(1)}_{WBH^4} + g'Q^{(1)}_{B^2H^4}\right) + \frac{1}{2}\left(Q^{(5)}_{\psi^2 BH^2D} - iQ^{(7)}_{\psi^2 BH^2D}\right) - \frac{g}{4}Q^{(1)}_{\psi^2H^4D} + \frac{gg'}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A73}$$

$$R^{(11)}_{BWH^2D^2} = i\frac{3g}{2}Q^{(2)}_{BH^4D^2} + \frac{g}{8}\left(gQ^{(2)}_{WBH^4} + g'Q^{(2)}_{B^2H^4}\right) + \frac{1}{2}\left(Q^{(6)}_{\psi^2 BH^2D} + iQ^{(8)}_{\psi^2 BH^2D}\right), \tag{A74}$$

$$R^{(12)}_{BWH^2D^2} = i\frac{3g}{2}Q^{(2)}_{BH^4D^2} + \frac{g}{8}\left(gQ^{(2)}_{WBH^4} + g'Q^{(2)}_{B^2H^4}\right) + \frac{1}{2}\left(Q^{(6)}_{\psi^2 BH^2D} - iQ^{(8)}_{\psi^2 BH^2D}\right), \tag{A75}$$

$$R^{(13)}_{BWH^2D^2} = \frac{gg'}{4}\Delta_5 - \frac{gg'}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) + \frac{g}{2}Q^{(1)}_{\psi^2H^4D} + \frac{g'}{4}\left(Q^{(2)}_{\psi^2H^4D} - Q^{(4)}_{\psi^2H^4D}\right) + Q^{(7)}_{\psi^4H^2}. \tag{A76}$$

Operators in class XH^4D^2:

$$R^{(1)}_{BH^4D^2} = i\frac{g'}{2}\Delta_5 + iQ^{(1)}_{\psi^2H^4D} - i\frac{g'}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A77}$$

$$R^{(1)}_{WH^4D^2} = i\frac{g}{2}\Delta_3 + i\frac{1}{2}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D}\right) - i\frac{g}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right), \tag{A78}$$

$$R^{(2)}_{WH^4D^2} = -\frac{g}{2}\left(Q^{(1)}_{H^6} - Q^{(2)}_{H^6}\right) - \frac{1}{2}\left(Q^{(4)}_{\psi^2H^4D} - iQ^{(3)}_{\psi^2H^4D}\right), \quad \text{(A79)}$$

$$R^{(3)}_{WH^4D^2} = -\frac{g}{2}\left(Q^{(1)}_{H^6} - Q^{(2)}_{H^6}\right) - \frac{1}{2}\left(Q^{(4)}_{\psi^2H^4D} + iQ^{(3)}_{\psi^2H^4D}\right). \quad \text{(A80)}$$

Operators in class XH^2D^4:

$$\begin{aligned} R^{(1)}_{BH^2D^4} = &-i\frac{g'}{2}[\lambda\Delta_5 - \lambda v^2(D^\nu H^\dagger H)(H^\dagger \overleftrightarrow{D}_\nu H)] + \lambda v^2(D_\nu H^\dagger H)J^\nu_B \\ &- i\frac{g'}{4}\left(Q^{\dagger(1)}_{\psi^2H^3D^2} + Q^{\dagger(2)}_{\psi^2H^3D^2} - 2Q^{\dagger(3)}_{\psi^2H^3D^2}\right) - i\lambda Q^{(1)}_{\psi^2H^4D} + i\frac{g'\lambda}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - Q^{\dagger(1)}_{\psi^4HD}, \end{aligned} \quad \text{(A81)}$$

$$\begin{aligned} R^{(2)}_{BH^2D^4} = &\, i\frac{g'}{2}[\lambda\Delta_5 - \lambda v^2(H^\dagger D^\nu H)(H^\dagger \overleftrightarrow{D}_\nu H)] + \lambda v^2(H^\dagger D_\nu H)J^\nu_B \\ &+ i\frac{g'}{4}\left(Q^{(1)}_{\psi^2H^3D^2} + Q^{(2)}_{\psi^2H^3D^2} - 2Q^{(3)}_{\psi^2H^3D^2}\right) + i\lambda Q^{(1)}_{\psi^2H^4D} - i\frac{g'\lambda}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - Q^{(1)}_{\psi^4HD}, \end{aligned} \quad \text{(A82)}$$

$$\begin{aligned} R^{(3)}_{BH^2D^4} = &\, i\frac{g'g^2}{8}\Delta_4 + ig'\left(Q^{(1)}_{H^4} - Q^{(2)}_{H^4}\right) - \frac{g'}{2}\left(g'Q^{(1)}_{BH^4D^2} - gQ^{(1)}_{WH^4D^2}\right) \\ &- i\frac{gg'}{16}\left(gQ^{(1)}_{W^2H^4} + gQ^{(3)}_{W^2H^4} + 2g'Q^{(1)}_{WBH^4}\right) - i\frac{1}{4}Q^{(1)}_{\psi^2H^2D^3} + i\frac{1}{2}\left(g'Q^{(1)}_{\psi^2BH^2D} + gQ^{(1)}_{\psi^2WH^2D}\right) \\ &- i\frac{g^2+g'^2}{4}Q^{(1)}_{\psi^2H^4D} + i\frac{gg'}{8}Q^{(2)}_{\psi^2H^4D} - i\frac{g^2g'}{16}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) \\ &- i\frac{1}{2}\left[g'\left(-Q^{(1)}_{\psi^4H^2} + Q^{(2)}_{\psi^4H^2} + Q^{\dagger(2)}_{\psi^4H^2} - 2Q^{(3)}_{\psi^4H^2} + Q^{(4)}_{\psi^4H^2}\right) + gQ^{(7)}_{\psi^4H^2}\right], \end{aligned} \quad \text{(A83)}$$

$$\begin{aligned} R^{(1)}_{WH^2D^4} = &-i\frac{g}{2}\left[\lambda\Delta_3 - \lambda v^2(D^\nu H^\dagger\tau^I H)(H^\dagger \overleftrightarrow{D}^I{}_\nu H)\right] + \lambda v^2(D_\nu H^\dagger\tau^I H)J^{I\nu}_W \\ &- i\frac{g}{4}\left(3Q^{\dagger(1)}_{\psi^2H^3D^2} - Q^{\dagger(2)}_{\psi^2H^3D^2} - 2Q^{\dagger(3)}_{\psi^2H^3D^2}\right) - i\frac{\lambda}{2}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} - iQ^{(3)}_{\psi^2H^4D}\right) \\ &+ i\frac{g\lambda}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - Q^{\dagger(2)}_{\psi^4HD}, \end{aligned} \quad \text{(A84)}$$

$$\begin{aligned} R^{(2)}_{WH^2D^4} = &\, i\frac{g}{2}\left[\lambda\Delta_3 + \lambda v^2(H^\dagger\tau^I D^\nu H)(H^\dagger \overleftrightarrow{D}^I{}_\nu H)\right] + \lambda v^2(H^\dagger\tau^I D_\nu H)J^{I\nu}_W \\ &+ i\frac{g}{4}\left(3Q^{(1)}_{\psi^2H^3D^2} - Q^{(2)}_{\psi^2H^3D^2} - 2Q^{(3)}_{\psi^2H^3D^2}\right) + i\frac{\lambda}{2}\left(Q^{(2)}_{\psi^2H^4D} + Q^{(4)}_{\psi^2H^4D} + iQ^{(3)}_{\psi^2H^4D}\right) \\ &- i\frac{g\lambda}{4}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) - Q^{(2)}_{\psi^4HD}, \end{aligned} \quad \text{(A85)}$$

$$\begin{aligned} R^{(3)}_{WH^2D^4} = &\, i\frac{gg'^2}{4}\Delta_5 - ig\left(Q^{(1)}_{H^4} + Q^{(2)}_{H^4} - 2Q^{(3)}_{H^4}\right) - i\frac{gg'}{4}\left(g'Q^{(1)}_{B^2H^4} + gQ^{(1)}_{WBH^4}\right) - \frac{g}{2}\left(gQ^{(1)}_{WH^4D^2} - 3g'Q^{(1)}_{BH^4D^2}\right) \\ &- i\frac{1}{4}Q^{(2)}_{\psi^2H^2D^3} + i\frac{1}{2}\left(g'Q^{(5)}_{\psi^2BH^2D} + gQ^{(5)}_{\psi^2WH^2D} - 2gQ^{(11)}_{\psi^2WH^2D}\right) \\ &- i\frac{1}{8}\left((g^2+g'^2)Q^{(2)}_{\psi^2H^4D} + (g^2-g'^2)Q^{(4)}_{\psi^2H^4D} - 4gg'Q^{(1)}_{\psi^2H^4D}\right) - i\frac{gg'^2}{8}\left(Q^{(1)}_{\psi^2H^5} + Q^{\dagger(1)}_{\psi^2H^5}\right) \\ &- i\frac{1}{2}\left[g\left(-3Q^{(1)}_{\psi^4H^2} + Q^{(2)}_{\psi^4H^2} + Q^{\dagger(2)}_{\psi^4H^2} + 2Q^{(3)}_{\psi^4H^2} + Q^{(5)}_{\psi^4H^2}\right) + g'Q^{(7)}_{\psi^4H^2}\right]. \end{aligned} \quad \text{(A86)}$$

APPENDIX B: USEFUL RELATIONS

One useful tool to simplify the intermediate expressions are the Fierz transformations [49]. We made extensive use of the following relations:

(i) $SU(2)$ identities:

$$(\tau^I)^l_i(\tau^J)^j_l = \delta^{IJ}(\delta)^j_i + i\epsilon^{IJK}(\tau^K)^j_i, \quad \text{(B1)}$$

$$(\tau^I)^l_j(\tau^I)^n_m = 2\delta^n_j\delta^l_m - \delta^l_j\delta^n_m, \quad \text{(B2)}$$

$$2(\tau^I)^l_m\delta^n_j - (\tau^I)^n_m\delta^l_j - (\tau^I)^l_j\delta^n_m = +i\epsilon^{IJK}(\tau^J)^l_j(\tau^K)^n_m, \quad \text{(B3)}$$

$$(\tau^I)^n_m\delta^l_j - (\tau^I)^l_j\delta^n_m = +i\epsilon^{IJK}(\tau^J)^n_j(\tau^K)^l_m. \quad \text{(B4)}$$

(ii) $SU(3)$ identities:

$$(T^A)^b_a(T^B)^c_b = \frac{1}{6}\delta^{AB}\delta^c_a + \frac{1}{2}(d^{ABC} + if^{ABC})(T^C)^c_a, \quad \text{(B5)}$$

$$(T^A)^b_a(T^A)^d_c = \frac{1}{2}\delta^d_a\delta^b_c - \frac{1}{6}\delta^b_a\delta^d_c, \quad \text{(B6)}$$

$$(T^A)^d_a\delta^b_c - \frac{1}{3}((T^A)^b_a\delta^d_c + (T^A)^d_c\delta^b_a) - d^{ABC}(T^B)^b_a(T^C)^d_c = -if^{ABC}(T^B)^b_a(T^C)^d_c. \quad \text{(B7)}$$

(iii) Relations with γ matrices:

$$\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}, \quad \text{(B8)}$$

$$\gamma^\mu\gamma^\alpha\gamma^\rho = g^{\mu\alpha}\gamma^\rho + g^{\alpha\rho}\gamma^\mu - g^{\mu\rho}\gamma^\alpha - i\epsilon^{\mu\alpha\rho\sigma}\gamma_\sigma\gamma_5, \quad \text{(B9)}$$

$$\sigma^{\mu\nu} = -i\frac{1}{2}\epsilon^{\mu\nu\rho\eta}\sigma_{\rho\eta}\gamma^5, \quad \text{(B10)}$$

$$D^\mu(\bar{f}\gamma^\nu Mf) - D^\nu(\bar{f}\gamma^\mu Mf) = i[\bar{f}\sigma^{\mu\nu}M(\not{D}f) - (\overline{\not{D}f})\sigma^{\mu\nu}Mf \pm \epsilon^{\mu\nu\rho\eta}\bar{f}\gamma_\rho M\overleftrightarrow{D}_\eta f]. \quad \text{(B11)}$$

where M can be the identity, a Pauli matrix τ^I or a T^A matrix and the upper (lower) sign corresponds to right (left)-handed fermions.

(iv) Lorentz scalar fermionic Fierz identities:

$$(\bar{l}_a, e_b)(\bar{e}_c l_d) = -\frac{1}{2}(\bar{l}_a\gamma^\mu l_d)(\bar{e}_c\gamma_\mu e_b), \quad \text{(B12)}$$

$$(\bar{l}^j_a, e_b)(\bar{e}_c l_{dk}) = -\frac{1}{4}[(\bar{l}_a\gamma^\mu l_d)(\bar{e}_c\gamma_\mu e_b)\delta^j_k + (\bar{l}_a\gamma^\mu\tau^I l_d)(\bar{e}_c\gamma_\mu e_b)(\tau^I)^j_k], \quad \text{(B13)}$$

$$(\bar{q}_a u_b)(\bar{u}_c q_d) = -\frac{1}{6}(\bar{q}_a\gamma^\mu q_d)(\bar{u}_c\gamma_\mu u_b) - (\bar{q}_a\gamma^\mu T^A q_d)(\bar{u}_c\gamma_\mu T^A u_b), \quad \text{(B14)}$$

$$(\bar{q}^j_a u_b)(\bar{u}_c q_{dk}) = -\frac{\delta^j_k}{2}\left[\frac{1}{6}(\bar{q}_a\gamma^\mu q_d)(\bar{u}_c\gamma_\mu u_b) + (\bar{q}_a\gamma^\mu T^A q_d)(\bar{u}_c\gamma_\mu T^A u_b)\right], \\ -\frac{(\tau)^j_k}{2}\left[\frac{1}{6}(\bar{q}_a\gamma^\mu\tau^I q_d)(\bar{u}_c\gamma_\mu u_b) + (\bar{q}_a\gamma^\mu T^A\tau^I q_d)(\bar{u}_c\gamma_\mu T^A u_b)\right], \quad \text{(B15)}$$

$$(\bar{f}_a\gamma^\mu f_a)(\bar{f}_b\gamma_\mu f_b) = (\bar{f}_a\gamma^\mu f_b)(\bar{f}_b\gamma_\mu f_a), \quad \text{for } f = q, l, u, d, e, \quad \text{(B16)}$$

$$(\bar{l}_a\gamma^\mu\tau^I l_a)(\bar{l}_b\gamma_\mu\tau^I l_b) = 2(\bar{l}_a\gamma^\mu l_b)(\bar{l}_b\gamma_\mu l_a) - (\bar{l}_a\gamma^\mu l_a)(\bar{l}_b\gamma_\mu l_b), \quad \text{(B17)}$$

$$(\bar{f}_a\gamma^\mu T^A f_a)(\bar{f}_b\gamma_\mu T^A f_b) = -\frac{1}{6}(\bar{f}_a\gamma^\mu f_a)(\bar{f}_b\gamma_\mu f_b) + \frac{1}{2}(\bar{f}_a\gamma^\mu f_b)(\bar{f}_b\gamma_\mu f_a), \quad \text{for } f = q, u, d. \quad \text{(B18)}$$

(v) Lorentz tensor fermionic Fierz identities: In the expressions below, $S^{\mu\nu}$ represent a piece which is symmetric under the exchange $\mu \leftrightarrow \nu$ and which will not contribute to the operators for which these relations are being used, so for simplicity we have not included their explicit forms. They can be found in Ref. [51].

$$(\bar{e}_a\gamma^\mu e_a)(\bar{e}_b\gamma^\nu e_b) = S_e^{\mu\nu} + \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma}(\bar{e}_a\gamma^\rho e_b)(\bar{e}_b\gamma^\sigma e_a), \tag{B19}$$

$$(\bar{l}_a\gamma^\mu l_a)(\bar{l}_b\gamma^\nu l_b) = S_l^{\mu\nu} - \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma}(\bar{l}_a\gamma^\rho l_b)(\bar{l}_b\gamma^\sigma l_a), \tag{B20}$$

$$(\bar{u}_a\gamma^\mu u_a)(\bar{u}_b\gamma^\nu u_b) = S_u^{\mu\nu} + \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma}\left[\frac{1}{3}(\bar{u}_a\gamma^\rho u_b)(\bar{u}_b\gamma^\sigma u_b) + 2(\bar{u}_a\gamma^\rho T^A u_b)(\bar{u}_b\gamma^\sigma T^A u_a)\right], \tag{B21}$$

$$(\bar{q}_a\gamma^\mu q_a)(\bar{q}_b\gamma^\nu q_b) = S_q^{\mu\nu} - \frac{1}{2}i\epsilon^{\mu\nu\rho\sigma}\left[\frac{1}{3}(\bar{q}_a\gamma^\rho q_b)(\bar{q}_b\gamma^\sigma q_a) + 2(\bar{q}_a\gamma^\rho T^A q_b)(\bar{q}_b\gamma^\sigma T^A q_a)\right], \tag{B22}$$

$$(\bar{l}_a\gamma^\mu\tau^I l_a)(\bar{l}_b\gamma^\nu\tau^J l_b)\epsilon^{IJK} = S_l^{K\mu\nu} + \epsilon^{\mu\nu\rho\sigma}(\bar{l}_a\gamma^\rho\tau^K l_b)(\bar{l}_b\gamma^\sigma l_b), \tag{B23}$$

$$(\bar{q}_a\gamma^\mu\tau^I q_a)(\bar{q}_b\gamma^\nu\tau^J q_b)\epsilon^{IJK} = S_q^{K\mu\nu} + \epsilon^{\mu\nu\rho\sigma}\left[\frac{1}{3}(\bar{q}_a\gamma^\rho\tau^K q_b)(\bar{q}_b\gamma^\sigma q_b) + 2(\bar{q}_a\gamma^\rho T^A\tau^K q_b)(\bar{q}_b\gamma^\sigma T^A q_b)\right], \tag{B24}$$

$$(\bar{u}_a\gamma^\mu T^A u_a)(\bar{u}_b\gamma^\nu T^B u_b)f^{ABC} = S_u^{C\mu\nu} - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\bar{u}_a\gamma^\rho T^C u_b)(\bar{u}_b\gamma^\sigma u_a), \tag{B25}$$

$$(\bar{q}_a\gamma^\mu T^A q_a)(\bar{q}_b\gamma^\nu T^B q_b)f^{ABC} = S_q^{C\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}(\bar{q}_a\gamma^\rho T^C q_b)(\bar{q}_b\gamma^\sigma q_a). \tag{B26}$$

APPENDIX C: RELEVANT FERMIONIC OPERATORS IN M8B

For convenience and reference, in Tables II, III, IV, V, VI, VII, VIII, and IX we list the operators in M8B that contain fermion and are in the rotation of the bosonic universal operators and, therefore, appear in Eqs. (59)–(71).

TABLE II. The dimension-eight operators in the M8B with particle content ψ^2X^2H and $\psi^2H^2D^3$ generated in universal theories. For the operators in the first two columns their Hermitian conjugates are *a priori* independent operators. For operators $\psi^2H^2D^3$ their Hermitian conjugates are not independent operators. The subscripts p, r are weak eigenstate indices.

$9: \psi^2X^2H + \text{H.c.}$	
$Q_{leG^2H}^{(1)}$	$(\bar{l}_pe_r)HG_{\mu\nu}^AG^{A\mu\nu}$
$Q_{leG^2H}^{(2)}$	$(\bar{l}_pe_r)H\tilde{G}_{\mu\nu}^AG^{A\mu\nu}$
$Q_{leW^2H}^{(1)}$	$(\bar{l}_pe_r)HW_{\mu\nu}^IW^{I\mu\nu}$
$Q_{leW^2H}^{(2)}$	$(\bar{l}_pe_r)H\tilde{W}_{\mu\nu}^IW^{I\mu\nu}$
$Q_{leW^2H}^{(3)}$	$\epsilon^{IJK}(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^IHW_{\mu\rho}^JW_\nu^{K\rho}$
$Q_{quG^2H}^{(1)}$	$(\bar{q}_pu_r)\tilde{H}G_{\mu\nu}^AG^{A\mu\nu}$
$Q_{quG^2H}^{(2)}$	$(\bar{q}_pu_r)\tilde{H}\tilde{G}_{\mu\nu}^AG^{A\mu\nu}$
$Q_{quG^2H}^{(3)}$	$d^{ABC}(\bar{q}_pT^Au_r)\tilde{H}G_{\mu\nu}^BG^{C\mu\nu}$
$Q_{quGBH}^{(3)}$	$(\bar{q}_p\sigma^{\mu\nu}T^Au_r)\tilde{H}G_{\mu\rho}^AB_\nu^\rho$
$Q_{quW^2H}^{(1)}$	$(\bar{q}_pu_r)\tilde{H}W_{\mu\nu}^IW^{I\mu\nu}$

(Table continued)

TABLE II. *(Continued)*

$9: \psi^2X^2H + \text{H.c.}$	
$Q_{quW^2H}^{(2)}$	$(\bar{q}_pu_r)\tilde{H}\tilde{W}_{\mu\nu}^IW^{I\mu\nu}$
$Q_{quW^2H}^{(3)}$	$\epsilon^{IJK}(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\tilde{H}W_{\mu\rho}^JW_\nu^{K\rho}$
$Q_{quWBH}^{(1)}$	$(\bar{q}_pu_r)\tau^I\tilde{H}W_{\mu\nu}^IB^{\mu\nu}$
$Q_{quWBH}^{(2)}$	$(\bar{q}_pu_r)\tau^I\tilde{H}\tilde{W}_{\mu\nu}^IB^{\mu\nu}$
$Q_{quWBH}^{(3)}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\tilde{H}W_{\mu\rho}^IB_\nu^\rho$
$Q_{quB^2H}^{(1)}$	$(\bar{q}_pu_r)\tilde{H}B_{\mu\nu}B^{\mu\nu}$
$Q_{quB^2H}^{(2)}$	$(\bar{q}_pu_r)\tilde{H}\tilde{B}_{\mu\nu}B^{\mu\nu}$

$9: \psi^2X^2H + \text{H.c.}$	
$Q_{leWBH}^{(1)}$	$(\bar{l}_pe_r)\tau^IHW_{\mu\nu}^IB^{\mu\nu}$
$Q_{leWBH}^{(2)}$	$(\bar{l}_pe_r)\tau^IH\tilde{W}_{\mu\nu}^IB^{\mu\nu}$
$Q_{leWBH}^{(3)}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^IHW_{\mu\rho}^IB_\nu^\rho$
$Q_{leB^2H}^{(1)}$	$(\bar{l}_pe_r)HB_{\mu\nu}B^{\mu\nu}$
$Q_{leB^2H}^{(2)}$	$(\bar{l}_pe_r)H\tilde{B}_{\mu\nu}B^{\mu\nu}$

(Table continued)

TABLE II. *(Continued)*

$9: \psi^2 X^2 H + \mathrm{H.c.}$	
$Q^{(1)}_{qdG^2H}$	$(\bar{q}_p d_r) H G^A_{\mu\nu} G^{A\mu\nu}$
$Q^{(2)}_{qdG^2H}$	$(\bar{q}_p d_r) H \tilde{G}^A_{\mu\nu} G^{A\mu\nu}$
$Q^{(3)}_{qdG^2H}$	$d^{ABC} (\bar{q}_p T^A d_r) H G^B_{\mu\nu} G^{C\mu\nu}$
$Q^{(3)}_{qdGBH}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H G^A_{\mu\rho} B^\rho_\nu$
$Q^{(1)}_{qdW^2H}$	$(\bar{q}_p d_r) H W^I_{\mu\nu} W^{I\mu\nu}$
$Q^{(2)}_{qdW^2H}$	$(\bar{q}_p d_r) H \tilde{W}^I_{\mu\nu} W^{I\mu\nu}$
$Q^{(3)}_{qdW^2H}$	$\epsilon^{IJK} (\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W^J_{\mu\rho} W^{K\rho}_\nu$
$Q^{(1)}_{qdWBH}$	$(\bar{q}_p d_r) \tau^I H W^I_{\mu\nu} B^{\mu\nu}$
$Q^{(2)}_{qdWBH}$	$(\bar{q}_p d_r) \tau^I H \tilde{W}^I_{\mu\nu} B^{\mu\nu}$
$Q^{(3)}_{qdWBH}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H W^I_{\mu\rho} B^\rho_\nu$
$Q^{(1)}_{qdB^2H}$	$(\bar{q}_p d_r) H B_{\mu\nu} B^{\mu\nu}$
$Q^{(2)}_{qdB^2H}$	$(\bar{q}_p d_r) H \tilde{B}_{\mu\nu} B^{\mu\nu}$

$11: \psi^2 H^2 D^3$	
$Q^{(1)}_{l^2H^2D^3}$	$i(\bar{l}_p \gamma^\mu D^\nu l_r)(D_{(\mu} D_{\nu)} H^\dagger H)$
$Q^{(2)}_{l^2H^2D^3}$	$i(\bar{l}_p \gamma^\mu D^\nu l_r)(H^\dagger D_{(\mu} D_{\nu)} H)$
$Q^{(3)}_{l^2H^2D^3}$	$i(\bar{l}_p \gamma^\mu \tau^I D^\nu l_r)(D_{(\mu} D_{\nu)} H^\dagger \tau^I H)$
$Q^{(4)}_{l^2H^2D^3}$	$i(\bar{l}_p \gamma^\mu \tau^I D^\nu l_r)(H^\dagger \tau^I D_{(\mu} D_{\nu)} H)$
$Q^{(1)}_{e^2H^2D^3}$	$i(\bar{e}_p \gamma^\mu D^\nu e_r)(D_{(\mu} D_{\nu)} H^\dagger H)$
$Q^{(2)}_{e^2H^2D^3}$	$i(\bar{e}_p \gamma^\mu D^\nu e_r)(H^\dagger D_{(\mu} D_{\nu)} H)$
$Q^{(1)}_{q^2H^2D^3}$	$i(\bar{q}_p \gamma^\mu D^\nu q_r)(D_{(\mu} D_{\nu)} H^\dagger H)$
$Q^{(2)}_{q^2H^2D^3}$	$i(\bar{q}_p \gamma^\mu D^\nu q_r)(H^\dagger D_{(\mu} D_{\nu)} H)$
$Q^{(3)}_{q^2H^2D^3}$	$i(\bar{q}_p \gamma^\mu \tau^I D^\nu q_r)(D_{(\mu} D_{\nu)} H^\dagger \tau^I H)$
$Q^{(4)}_{q^2H^2D^2}$	$i(\bar{q}_p \gamma^\mu \tau^I D^\nu q_r)(H^\dagger \tau^I D_{(\mu} D_{\nu)} H)$
$Q^{(1)}_{u^2H^2D^3}$	$i(\bar{u}_p \gamma^\mu D^\nu u_r)(D_{(\mu} D_{\nu)} H^\dagger H)$
$Q^{(2)}_{u^2H^2D^3}$	$i(\bar{u}_p \gamma^\mu D^\nu u_r)(H^\dagger D_{(\mu} D_{\nu)} H)$
$Q^{(1)}_{d^2H^2D^3}$	$i(\bar{d}_p \gamma^\mu D^\nu d_r)(D_{(\mu} D_{\nu)} H^\dagger H)$
$Q^{(2)}_{d^2H^2D^3}$	$i(\bar{d}_p \gamma^\mu D^\nu d_r)(H^\dagger D_{(\mu} D_{\nu)} H)$

TABLE III. The dimension-eight operators in the M8B with particle content $\psi^2 H^5$, $\psi^2 H^4 D$ and $\psi^2 H^3 D^2$ that are generated in universal theories. For the operators in the first column their Hermitian conjugates are *a priori* independent operators. Operators in class 13 are Hermitian. For operators $Q^{(1)}_{f^2H^4D}$, where $f = u,\ d,\ e$, we have added a superscript of (1) to the M8B operators. The subscripts p, r are weak eigenstate indices.

$12: \psi^2 H^5 + \mathrm{H.c.}$	
Q_{leH^5}	$(H^\dagger H)^2 (\bar{l}_p e_r H)$
Q_{quH^5}	$(H^\dagger H)^2 (\bar{q}_p u_r \tilde{H})$
Q_{qdH^5}	$(H^\dagger H)^2 (\bar{q}_p d_r H)$

$17: \psi^2 H^3 D^2 + \mathrm{H.c.}$	
$Q^{(1)}_{leH^3D^2}$	$(D_\mu H^\dagger D^\mu H)(\bar{l}_p e_r H)$
$Q^{(2)}_{leH^3D^2}$	$(D_\mu H^\dagger \tau^I D^\mu H)(\bar{l}_p e_r \tau^I H)$
$Q^{(5)}_{leH^3D^2}$	$(H^\dagger D_\mu H)(\bar{l}_p e_r D^\mu H)$
$Q^{(1)}_{quH^3D^2}$	$(D_\mu H^\dagger D^\mu H)(\bar{q}_p u_r \tilde{H})$
$Q^{(2)}_{quH^3D^2}$	$(D_\mu H^\dagger \tau^I D^\mu H)(\bar{q}_p u_r \tau^I \tilde{H})$
$Q^{(5)}_{quH^3D^2}$	$(D_\mu H^\dagger H)(\bar{q}_p u_r D^\mu \tilde{H})$
$Q^{(1)}_{qdH^3D^2}$	$(D_\mu H^\dagger D^\mu H)(\bar{q}_p d_r H)$
$Q^{(2)}_{qdH^3D^2}$	$(D_\mu H^\dagger \tau^I D^\mu H)(\bar{q}_p d_r \tau^I H)$
$Q^{(5)}_{qdH^3D^2}$	$(H^\dagger D_\mu H)(\bar{q}_p d_r D^\mu H)$

$13: \psi^2 H^4 D$	
$Q^{(1)}_{l^2H^4D}$	$i(\bar{l}_p \gamma^\mu l_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$Q^{(2)}_{l^2H^4D}$	$i(\bar{l}_p \gamma^\mu \tau^I l_r)[(H^\dagger \overleftrightarrow{D}^I_\mu H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)]$
$Q^{(3)}_{l^2H^4D}$	$i\epsilon^{IJK} (\bar{l}_p \gamma^\mu \tau^I l_r)(H^\dagger \overleftrightarrow{D}^J_\mu H)(H^\dagger \tau^K H)$
$Q^{(4)}_{l^2H^4D}$	$\epsilon^{IJK} (\bar{l}_p \gamma^\mu \tau^I l_r)(H^\dagger \tau^J H) D_\mu (H^\dagger \tau^K H)$
$Q^{(1)}_{e^2H^4D}$	$i(\bar{e}_p \gamma^\mu e_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$Q^{(1)}_{q^2H^4D}$	$i(\bar{q}_p \gamma^\mu q_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$Q^{(2)}_{q^2H^4D}$	$i(\bar{q}_p \gamma^\mu \tau^I q_r)[(H^\dagger \overleftrightarrow{D}^I_\mu H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)]$
$Q^{(3)}_{q^2H^4D}$	$i\epsilon^{IJK} (\bar{q}_p \gamma^\mu \tau^I q_r)(H^\dagger \overleftrightarrow{D}^J_\mu H)(H^\dagger \tau^K H)$
$Q^{(4)}_{q^2H^4D}$	$\epsilon^{IJK} (\bar{q}_p \gamma^\mu \tau^I q_r)(H^\dagger \tau^J H) D_\mu (H^\dagger \tau^K H)$
$Q^{(1)}_{u^2H^4D}$	$i(\bar{u}_p \gamma^\mu u_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$
$Q^{(1)}_{d^2H^4D}$	$i(\bar{d}_p \gamma^\mu d_r)(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$

TABLE IV. The dimension-eight operators in the M8B with particle content $\psi^2 X H^2 D$ generated in universal theories. All operators are either Hermitian or anti-Hermitian. Once again, the subscripts p, r are weak eigenstate indices.

$15: (\bar{R}R)XH^2D$	
$Q^{(1)}_{e^2WH^2D}$	$(\bar{e}_p\gamma^\nu e_r)D^\mu(H^\dagger\tau^I H)W^I_{\mu\nu}$
$Q^{(2)}_{e^2WH^2D}$	$(\bar{e}_p\gamma^\nu e_r)D^\mu(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{e^2WH^2D}$	$(\bar{e}_p\gamma^\nu e_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$
$Q^{(4)}_{e^2WH^2D}$	$(\bar{e}_p\gamma^\nu e_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{e^2BH^2D}$	$(\bar{e}_p\gamma^\nu e_r)D^\mu(H^\dagger H)B_{\mu\nu}$
$Q^{(2)}_{e^2BH^2D}$	$(\bar{e}_p\gamma^\nu e_r)D^\mu(H^\dagger H)\tilde{B}_{\mu\nu}$
$Q^{(3)}_{e^2BH^2D}$	$(\bar{e}_p\gamma^\nu e_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)B_{\mu\nu}$
$Q^{(4)}_{e^2BH^2D}$	$(\bar{e}_p\gamma^\nu e_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{B}_{\mu\nu}$
$Q^{(1)}_{u^2GH^2D}$	$(\bar{u}_p\gamma^\nu T^A u_r)D^\mu(H^\dagger H)G^A_{\mu\nu}$
$Q^{(2)}_{u^2GH^2D}$	$(\bar{u}_p\gamma^\nu T^A u_r)D^\mu(H^\dagger H)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{u^2GH^2D}$	$(\bar{u}_p\gamma^\nu T^A u_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)G^A_{\mu\nu}$
$Q^{(4)}_{u^2GH^2D}$	$(\bar{u}_p\gamma^\nu T^A u_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{u^2WH^2D}$	$(\bar{u}_p\gamma^\nu u_r)D^\mu(H^\dagger\tau^I H)W^I_{\mu\nu}$
$Q^{(2)}_{u^2WH^2D}$	$(\bar{u}_p\gamma^\nu u_r)D^\mu(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{u^2WH^2D}$	$(\bar{u}_p\gamma^\nu u_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$
$Q^{(4)}_{u^2WH^2D}$	$(\bar{u}_p\gamma^\nu u_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{u^2BH^2D}$	$(\bar{u}_p\gamma^\nu u_r)D^\mu(H^\dagger H)B_{\mu\nu}$
$Q^{(2)}_{u^2BH^2D}$	$(\bar{u}_p\gamma^\nu u_r)D^\mu(H^\dagger H)\tilde{B}_{\mu\nu}$
$Q^{(3)}_{u^2BH^2D}$	$(\bar{u}_p\gamma^\nu u_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)B_{\mu\nu}$
$Q^{(4)}_{u^2BH^2D}$	$(\bar{u}_p\gamma^\nu u_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{B}_{\mu\nu}$
$Q^{(1)}_{d^2GH^2D}$	$(\bar{d}_p\gamma^\nu T^A d_r)D^\mu(H^\dagger H)G^A_{\mu\nu}$
$Q^{(2)}_{d^2GH^2D}$	$(\bar{d}_p\gamma^\nu T^A d_r)D^\mu(H^\dagger H)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{d^2GH^2D}$	$(\bar{d}_p\gamma^\nu T^A d_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)G^A_{\mu\nu}$
$Q^{(4)}_{d^2GH^2D}$	$(\bar{d}_p\gamma^\nu T^A d_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{d^2WH^2D}$	$(\bar{d}_p\gamma^\nu d_r)D^\mu(H^\dagger\tau^I H)W^I_{\mu\nu}$
$Q^{(2)}_{d^2WH^2D}$	$(\bar{d}_p\gamma^\nu d_r)D^\mu(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{d^2WH^2D}$	$(\bar{d}_p\gamma^\nu d_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$
$Q^{(4)}_{d^2WH^2D}$	$(\bar{d}_p\gamma^\nu d_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{d^2BH^2D}$	$(\bar{d}_p\gamma^\nu d_r)D^\mu(H^\dagger H)B_{\mu\nu}$
$Q^{(2)}_{d^2BH^2D}$	$(\bar{d}_p\gamma^\nu d_r)D^\mu(H^\dagger H)\tilde{B}_{\mu\nu}$
$Q^{(3)}_{d^2BH^2D}$	$(\bar{d}_p\gamma^\nu d_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)B_{\mu\nu}$
$Q^{(4)}_{d^2BH^2D}$	$(\bar{d}_p\gamma^\nu d_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{B}_{\mu\nu}$

$15: (\bar{L}L)XH^2D$	
$Q^{(1)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu l_r)D^\mu(H^\dagger\tau^I H)W^I_{\mu\nu}$
$Q^{(2)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu l_r)D^\mu(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}$

(Table continued)

TABLE IV. *(Continued)*

$15: (\bar{L}L)XH^2D$	
$Q^{(3)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu l_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$
$Q^{(4)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu l_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(5)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger H)W^I_{\mu\nu}$
$Q^{(6)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger H)\tilde{W}^I_{\mu\nu}$
$Q^{(7)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)W^I_{\mu\nu}$
$Q^{(8)}_{l^2WH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(9)}_{l^2WH^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger\tau^J H)W^K_{\mu\nu}$
$Q^{(10)}_{l^2WH^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger\tau^J H)\tilde{W}^K_{\mu\nu}$
$Q^{(11)}_{l^2WH^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{J\mu}H)W^K_{\mu\nu}$
$Q^{(12)}_{l^2WH^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{J\mu}H)\tilde{W}^K_{\mu\nu}$
$Q^{(1)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger\tau^I H)B_{\mu\nu}$
$Q^{(2)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)D^\mu(H^\dagger\tau^I H)\tilde{B}_{\mu\nu}$
$Q^{(3)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)B_{\mu\nu}$
$Q^{(4)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{B}_{\mu\nu}$
$Q^{(5)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu l_r)D^\mu(H^\dagger H)B_{\mu\nu}$
$Q^{(6)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu l_r)D^\mu(H^\dagger H)\tilde{B}_{\mu\nu}$
$Q^{(7)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu l_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)B_{\mu\nu}$
$Q^{(8)}_{l^2BH^2D}$	$(\bar{l}_p\gamma^\nu l_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{B}_{\mu\nu}$

$15: (\bar{L}L)XH^2D$	
$Q^{(5)}_{q^2GH^2D}$	$(\bar{q}_p\gamma^\nu T^A q_r)D^\mu(H^\dagger H)G^A_{\mu\nu}$
$Q^{(6)}_{q^2GH^2D}$	$(\bar{q}_p\gamma^\nu T^A q_r)D^\mu(H^\dagger H)\tilde{G}^A_{\mu\nu}$
$Q^{(7)}_{q^2GH^2D}$	$(\bar{q}_p\gamma^\nu T^A q_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)G^A_{\mu\nu}$
$Q^{(8)}_{q^2GH^2D}$	$(\bar{q}_p\gamma^\nu T^A q_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu q_r)D^\mu(H^\dagger\tau^I H)W^I_{\mu\nu}$
$Q^{(2)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu q_r)D^\mu(H^\dagger\tau^I H)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu q_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$
$Q^{(4)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu q_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(5)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger H)W^I_{\mu\nu}$
$Q^{(6)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger H)\tilde{W}^I_{\mu\nu}$
$Q^{(7)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)W^I_{\mu\nu}$
$Q^{(8)}_{q^2WH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger\overleftrightarrow{D}^{\mu}H)\tilde{W}^I_{\mu\nu}$
$Q^{(9)}_{q^2WH^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger\tau^J H)W^K_{\mu\nu}$
$Q^{(10)}_{q^2WH^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger\tau^J H)\tilde{W}^K_{\mu\nu}$
$Q^{(11)}_{q^2WH^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger\overleftrightarrow{D}^{J\mu}H)W^K_{\mu\nu}$
$Q^{(12)}_{q^2WH^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger\overleftrightarrow{D}^{J\mu}H)\tilde{W}^K_{\mu\nu}$
$Q^{(1)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger\tau^I H)B_{\mu\nu}$

(Table continued)

TABLE IV. *(Continued)*

$15:(\bar{L}L)XH^2D$	
$Q^{(2)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)D^\mu(H^\dagger\tau^I H)\tilde{B}_{\mu\nu}$
$Q^{(3)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger \overleftrightarrow{D}^{I\mu} H)B_{\mu\nu}$
$Q^{(4)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu\tau^I q_r)(H^\dagger \overleftrightarrow{D}^{I\mu} H)\tilde{B}_{\mu\nu}$
$Q^{(5)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu q_r)D^\mu(H^\dagger H)B_{\mu\nu}$
$Q^{(6)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu q_r)D^\mu(H^\dagger H)\tilde{B}_{\mu\nu}$
$Q^{(7)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu q_r)(H^\dagger \overleftrightarrow{D}^\mu H)B_{\mu\nu}$
$Q^{(8)}_{q^2BH^2D}$	$(\bar{q}_p\gamma^\nu q_r)(H^\dagger \overleftrightarrow{D}^\mu H)\tilde{B}_{\mu\nu}$

TABLE V. The dimension-eight operators in the M8B with particle content ψ^2X^2D generated in universal theories. All operators are anti-Hermitian. As before, the subscripts p, r are weak eigenstate indices.

$14:\psi^2X^2D$	
$Q^{(2)}_{l^2W^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu l_r)W^J_{\mu\rho}W^{K\rho}_\nu$
$Q^{(4)}_{l^2W^2D}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu l_r)(W^J_{\mu\rho}\tilde{W}^{K\rho}_\nu+\tilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$Q^{(2)}_{q^2W^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu q_r)W^J_{\mu\rho}W^{K\rho}_\nu$
$Q^{(2)}_{q^2G^2D}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu q_r)G^B_{\mu\rho}G^{C\rho}_\nu$
$Q^{(4)}_{q^2W^2D}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu q_r)(W^J_{\mu\rho}\tilde{W}^{K\rho}_\nu+\tilde{W}^J_{\mu\rho}W^{K\rho}_\nu)$
$Q^{(5)}_{q^2G^2D}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu q_r)(G^B_{\mu\rho}\tilde{G}^{C\rho}_\nu+\tilde{G}^B_{\mu\rho}G^{C\rho}_\nu)$
$Q^{(2)}_{u^2G^2D}$	$f^{ABC}(\bar{u}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu u_r)G^B_{\mu\rho}G^{C\rho}_\nu$
$Q^{(5)}_{u^2G^2D}$	$f^{ABC}(\bar{u}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu u_r)(G^B_{\mu\rho}\tilde{G}^{C\rho}_\nu+\tilde{G}^B_{\mu\rho}G^{C\rho}_\nu)$
$Q^{(2)}_{d^2G^2D}$	$f^{ABC}(\bar{d}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu d_r)G^B_{\mu\rho}G^{C\rho}_\nu$
$Q^{(5)}_{d^2G^2D}$	$f^{ABC}(\bar{d}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu d_r)(G^B_{\mu\rho}\tilde{G}^{C\rho}_\nu+\tilde{G}^B_{\mu\rho}G^{C\rho}_\nu)$

$14:\psi^2X^2D$	
$Q^{(1)}_{l^2WBD}$	$(\bar{l}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu l_r)(B_{\mu\rho}W^{I\rho}_\nu-B_{\nu\rho}W^{I\rho}_\mu)$
$Q^{(3)}_{l^2WBD}$	$(\bar{l}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu l_r)(B_{\mu\rho}\tilde{W}^{I\rho}_\nu-B_{\nu\rho}\tilde{W}^{I\rho}_\mu)$
$Q^{(1)}_{q^2WBD}$	$(\bar{q}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu q_r)(B_{\mu\rho}W^{I\rho}_\nu-B_{\nu\rho}W^{I\rho}_\mu)$
$Q^{(3)}_{q^2WBD}$	$(\bar{q}_p\gamma^\mu\tau^I\overleftrightarrow{D}^\nu q_r)(B_{\mu\rho}\tilde{W}^{I\rho}_\nu-B_{\nu\rho}\tilde{W}^{I\rho}_\mu)$
$Q^{(1)}_{q^2GBD}$	$(\bar{q}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu q_r)(B_{\mu\rho}G^{A\rho}_\nu-B_{\nu\rho}G^{A\rho}_\mu)$
$Q^{(3)}_{q^2GBD}$	$(\bar{q}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu q_r)(B_{\mu\rho}\tilde{G}^{A\rho}_\nu-B_{\nu\rho}\tilde{G}^{A\rho}_\mu)$
$Q^{(1)}_{u^2GBD}$	$(\bar{u}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu u_r)(B_{\mu\rho}G^{A\rho}_\nu-B_{\nu\rho}G^{A\rho}_\mu)$
$Q^{(3)}_{u^2GBD}$	$(\bar{u}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu u_r)(B_{\mu\rho}\tilde{G}^{A\rho}_\nu-B_{\nu\rho}\tilde{G}^{A\rho}_\mu)$
$Q^{(1)}_{d^2GBD}$	$(\bar{d}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu d_r)(B_{\mu\rho}G^{A\rho}_\nu-B_{\nu\rho}G^{A\rho}_\mu)$
$Q^{(3)}_{d^2GBD}$	$(\bar{d}_p\gamma^\mu T^A\overleftrightarrow{D}^\nu d_r)(B_{\mu\rho}\tilde{G}^{A\rho}_\nu-B_{\nu\rho}\tilde{G}^{A\rho}_\mu)$

TABLE VI. The dimension-eight operators in the M8B with particle content ψ^4H^2 generated in universal theories. All operators are either Hermitian or anti-Hermitian. For operators $Q^{(1)}_{f^4H^2}$, where $f=u$, d, e; and for $Q^{(1)}_{e^2u^2H^2}$ and $Q^{(1)}_{e^2d^2H^2}$ we have added a superscript of (1) to the M8B operators. The subscripts p, r, s, t are weak eigenstate indices.

$18:(\bar{L}L)(\bar{L}L)H^2$	
$Q^{(1)}_{l^4H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma_\mu l_t)(H^\dagger H)$
$Q^{(2)}_{l^4H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma_\mu\tau^I l_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger H)$
$Q^{(2)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger H)$
$Q^{(1)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger H)$
$Q^{(2)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger H)$
$Q^{(4)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger\tau^I H)$

$18:(\bar{R}R)(\bar{R}R)H^2$	
$Q^{(1)}_{e^4H^2}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{e}_s\gamma_\mu e_t)(H^\dagger H)$
$Q^{(1)}_{u^4H^2}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger H)$
$Q^{(1)}_{d^4H^2}$	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger H)$
$Q^{(1)}_{e^2u^2H^2}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger H)$
$Q^{(1)}_{e^2d^2H^2}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger H)$
$Q^{(1)}_{u^2d^2H^2}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger H)$
$Q^{(2)}_{u^2d^2H^2}$	$(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma_\mu T^A d_t)(H^\dagger H)$

$18:(\bar{L}L)(\bar{R}R)H^2$	
$Q^{(1)}_{l^2e^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{e}_s\gamma_\mu e_t)(H^\dagger H)$
$Q^{(2)}_{l^2e^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{e}_s\gamma_\mu e_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{l^2u^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger H)$
$Q^{(2)}_{l^2u^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{l^2d^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger H)$
$Q^{(2)}_{l^2d^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{q^2e^2H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{e}_s\gamma_\mu e_t)(H^\dagger H)$
$Q^{(2)}_{q^2e^2H^2}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{e}_s\gamma_\mu e_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger H)$
$Q^{(2)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{u}_s\gamma_\mu u_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma_\mu T^A u_t)(H^\dagger H)$

(Table continued)

TABLE VI. *(Continued)*

$18:(\bar{L}L)(\bar{R}R)H^2$	
$Q^{(4)}_{q^2u^2H^2}$	$(\bar{q}_p\gamma^\mu T^A\tau^I q_r)(\bar{u}_s\gamma_\mu T^A u_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger H)$
$Q^{(2)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{d}_s\gamma_\mu d_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma_\mu T^A d_t)(H^\dagger H)$
$Q^{(4)}_{q^2d^2H^2}$	$(\bar{q}_p\gamma^\mu T^A\tau^I q_r)(\bar{d}_s\gamma_\mu T^A d_t)(H^\dagger\tau^I H)$

$18:(\bar{L}R)(\bar{L}R)H^2+\text{H.c.}$	
$Q^{(1)}_{q^2udH^2}$	$(\bar{q}^j_p u_r)\epsilon_{jk}(\bar{q}^k_s d_t)(H^\dagger H)$
$Q^{(2)}_{q^2udH^2}$	$(\bar{q}^j_p u_r)(\tau^I\epsilon)_{jk}(\bar{q}^k_s d_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{lequH^2}$	$(\bar{l}^j_p e_r)\epsilon_{jk}(\bar{q}^k_s u_t)(H^\dagger H)$
$Q^{(2)}_{lequH^2}$	$(\bar{l}^j_p e_r)(\tau^I\epsilon)_{jk}(\bar{q}^k_s u_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{l^2e^2H^2}$	$(\bar{l}_p e_r H)(\bar{l}_s e_t H)$
$Q^{(3)}_{leqdH^2}$	$(\bar{l}_p e_r H)(\bar{q}_s d_t H)$
$Q^{(5)}_{q^2u^2H^2}$	$(\bar{q}_p u_r\tilde{H})(\bar{q}_s u_t\tilde{H})$
$Q^{(5)}_{q^2d^2H^2}$	$(\bar{q}_p d_r H)(\bar{q}_s d_t H)$

$18:(\bar{L}R)(\bar{R}L)H^2+\text{H.c.}$	
$Q^{(1)}_{leqdH^2}$	$(\bar{l}^j_p e_r)(\bar{d}_s q_{tj})(H^\dagger H)$
$Q^{(2)}_{leqdH^2}$	$(\bar{l}_p e_r)\tau^I(\bar{d}_s q_t)(H^\dagger\tau^I H)$
$Q^{(5)}_{lequH^2}$	$(\bar{l}_p e_r H)(\tilde{H}^\dagger\bar{u}_s q_t)$
$Q^{(5)}_{q^2udH^2}$	$(\bar{q}_p d_r H)(\tilde{H}^\dagger\bar{u}_s q_t)$

TABLE VII. The dimension-eight operators in the M8B with particle content ψ^4X generated in universal theories. All operators are either Hermitian or anti-Hermitian. The subscripts p, r, s, t are weak eigenstate indices.

$19:(\bar{L}L)(\bar{L}L)X$	
$Q^{(1)}_{l^4W}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma^\nu\tau^I l_t)W^I_{\mu\nu}$
$Q^{(2)}_{l^4W}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma^\nu\tau^I l_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{q^4G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma^\nu T^A q_t)G^A_{\mu\nu}$
$Q^{(2)}_{q^4G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma^\nu T^A q_t)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{q^4G}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma^\nu T^A\tau^I q_t)G^A_{\mu\nu}$

(Table continued)

TABLE VII. *(Continued)*

$19:(\bar{L}L)(\bar{L}L)X$	
$Q^{(4)}_{q^4G}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma^\nu T^A\tau^I q_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{q^4W}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma^\nu\tau^I q_t)W^I_{\mu\nu}$
$Q^{(2)}_{q^4W}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma^\nu\tau^I q_t)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{q^4W}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{q}_s\gamma^\nu T^A\tau^I q_t)W^I_{\mu\nu}$
$Q^{(4)}_{q^4W}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{q}_s\gamma^\nu T^A\tau^I q_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{l^2q^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma^\nu T^A q_t)G^A_{\mu\nu}$
$Q^{(2)}_{l^2q^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma^\nu T^A q_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{l^2q^2W}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma^\nu\tau^I q_t)W^I_{\mu\nu}$
$Q^{(2)}_{l^2q^2W}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma^\nu\tau^I q_t)\tilde{W}^I_{\mu\nu}$
$Q^{(3)}_{l^2q^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma^\nu q_t)W^I_{\mu\nu}$
$Q^{(4)}_{l^2q^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma^\nu q_t)\tilde{W}^I_{\mu\nu}$
$Q^{(5)}_{l^2q^2W}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma^\nu\tau^J q_t)W^K_{\mu\nu}$
$Q^{(6)}_{l^2q^2W}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma^\nu\tau^J q_t)\tilde{W}^K_{\mu\nu}$

$19:(\bar{R}R)(\bar{R}R)X$	
$Q^{(1)}_{u^4G}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma^\nu T^A u_t)G^A_{\mu\nu}$
$Q^{(2)}_{u^4G}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma^\nu T^A u_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{d^4G}$	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_s\gamma^\nu T^A d_t)G^A_{\mu\nu}$
$Q^{(2)}_{d^4G}$	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_s\gamma^\nu T^A d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{e^2u^2G}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{u}_s\gamma^\nu T^A u_t)G^A_{\mu\nu}$
$Q^{(2)}_{e^2u^2G}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{u}_s\gamma^\nu T^A u_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{e^2d^2G}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{d}_s\gamma^\nu T^A d_t)G^A_{\mu\nu}$
$Q^{(2)}_{e^2d^2G}$	$(\bar{e}_p\gamma^\mu e_r)(\bar{d}_s\gamma^\nu T^A d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{u^2d^2G}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma^\nu T^A d_t)G^A_{\mu\nu}$
$Q^{(2)}_{u^2d^2G}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma^\nu T^A d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{u^2d^2G}$	$(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma^\nu d_t)G^A_{\mu\nu}$
$Q^{(4)}_{u^2d^2G}$	$(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma^\nu d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(5)}_{u^2d^2G}$	$f^{ABC}(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma^\nu T^B d_t)G^C_{\mu\nu}$
$Q^{(6)}_{u^2d^2G}$	$f^{ABC}(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma^\nu T^B d_t)\tilde{G}^C_{\mu\nu}$

$19:(\bar{L}L)(\bar{R}R)X$	
$Q^{(1)}_{l^2e^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{e}_s\gamma^\nu e_t)W^I_{\mu\nu}$
$Q^{(2)}_{l^2e^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{e}_s\gamma^\nu e_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{l^2u^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{u}_s\gamma^\nu T^A u_t)G^A_{\mu\nu}$
$Q^{(2)}_{l^2u^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{u}_s\gamma^\nu T^A u_t)\tilde{G}^A_{\mu\nu}$

(Table continued)

TABLE VII. *(Continued)*

19: $(\bar{L}L)(\bar{R}R)X$	
$Q^{(1)}_{l^2u^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{u}_s\gamma^\nu u_t)W^I_{\mu\nu}$
$Q^{(2)}_{l^2u^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{u}_s\gamma^\nu u_t)\tilde{W}^I_{\mu\nu}$

19: $(\bar{L}L)(\bar{R}R)X$	
$Q^{(1)}_{l^2d^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{d}_s\gamma^\nu T^A d_t)G^A_{\mu\nu}$
$Q^{(2)}_{l^2d^2G}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{d}_s\gamma^\nu T^A d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{l^2d^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{d}_s\gamma^\nu d_t)W^I_{\mu\nu}$
$Q^{(2)}_{l^2d^2W}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{d}_s\gamma^\nu d_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{q^2e^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{e}_s\gamma^\nu e_t)G^A_{\mu\nu}$
$Q^{(2)}_{q^2e^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{e}_s\gamma^\nu e_t)\tilde{G}^A_{\mu\nu}$
$Q^{(1)}_{q^2e^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{e}_s\gamma^\nu e_t)W^I_{\mu\nu}$
$Q^{(2)}_{q^2e^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{e}_s\gamma^\nu e_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{q^2u^2G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_s\gamma^\nu T^A u_t)G^A_{\mu\nu}$
$Q^{(2)}_{q^2u^2G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_s\gamma^\nu T^A u_t)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{q^2u^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma^\nu u_t)G^A_{\mu\nu}$
$Q^{(4)}_{q^2u^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma^\nu u_t)\tilde{G}^A_{\mu\nu}$
$Q^{(5)}_{q^2u^2G}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma^\nu T^B u_t)G^C_{\mu\nu}$
$Q^{(6)}_{q^2u^2G}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma^\nu T^B u_t)\tilde{G}^C_{\mu\nu}$
$Q^{(1)}_{q^2u^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{u}_s\gamma^\nu u_t)W^I_{\mu\nu}$
$Q^{(2)}_{q^2u^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{u}_s\gamma^\nu u_t)\tilde{W}^I_{\mu\nu}$
$Q^{(1)}_{q^2d^2G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_s\gamma^\nu T^A d_t)G^A_{\mu\nu}$
$Q^{(2)}_{q^2d^2G}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_s\gamma^\nu T^A d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(3)}_{q^2d^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma^\nu d_t)G^A_{\mu\nu}$
$Q^{(4)}_{q^2d^2G}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma^\nu d_t)\tilde{G}^A_{\mu\nu}$
$Q^{(5)}_{q^2d^2G}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma^\nu T^B d_t)G^C_{\mu\nu}$
$Q^{(6)}_{q^2d^2G}$	$f^{ABC}(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma^\nu T^B d_t)\tilde{G}^C_{\mu\nu}$
$Q^{(1)}_{q^2d^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{d}_s\gamma^\nu d_t)W^I_{\mu\nu}$
$Q^{(2)}_{q^2d^2W}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{d}_s\gamma^\nu d_t)\tilde{W}^I_{\mu\nu}$

TABLE VIII. The dimension-eight operators in the M8B with particle content $\psi^4 HD$ generated in universal theories. For all operators their Hermitian conjugates are *a priori* independent operators. The subscripts p, r, s, t are weak eigenstate indices.

20: $\psi^4 HD$ + H.c.	
$Q^{(1)}_{l^3eHD}$	$i(\bar{l}_p\gamma^\mu l_r)[(\bar{l}_s e_t)D_\mu H]$
$Q^{(2)}_{l^3eHD}$	$i(\bar{l}_p\gamma^\mu\tau^I l_r)[(\bar{l}_s e_t)\tau^I D_\mu H]$
$Q^{(1)}_{le^3HD}$	$i(\bar{e}_p\gamma^\mu e_r)[(\bar{l}_s D_\mu e_t)H]$
$Q^{(1)}_{leq^2HD}$	$i(\bar{q}_p\gamma^\mu q_r)[(\bar{l}_s e_t)D_\mu H]$
$Q^{(3)}_{leq^2HD}$	$i(\bar{q}_p\gamma^\mu\tau^I q_r)[(\bar{l}_s e_t)\tau^I D_\mu H]$
$Q^{(1)}_{leu^2HD}$	$i(\bar{u}_p\gamma^\mu u_r)[(\bar{l}_s e_t)D_\mu H]$
$Q^{(1)}_{led^2HD}$	$i(\bar{d}_p\gamma^\mu d_r)[(\bar{l}_s e_t)D_\mu H]$

20: $\psi^4 HD$ + H.c.	
$Q^{(1)}_{l^2quHD}$	$i(\bar{l}_p\gamma^\mu l_r)[(\bar{q}_s u_t)D_\mu\tilde{H}]$
$Q^{(3)}_{l^2quHD}$	$i(\bar{l}_p\gamma^\mu\tau^I l_r)[(\bar{q}_s u_t)\tau^I D_\mu\tilde{H}]$
$Q^{(1)}_{e^2quHD}$	$i(\bar{e}_p\gamma^\mu e_r)[(\bar{q}_s u_t)D_\mu\tilde{H}]$
$Q^{(1)}_{q^3uHD}$	$i(\bar{q}_p\gamma^\mu q_r)[(\bar{q}_s u_t)D_\mu\tilde{H}]$
$Q^{(2)}_{q^3uHD}$	$i(\bar{q}_p\gamma^\mu\tau^I q_r)[(\bar{q}_s u_t)\tau^I D_\mu\tilde{H}]$
$Q^{(1)}_{qu^3HD}$	$i(\bar{u}_p\gamma^\mu u_r)[(\bar{q}_s u_t)D_\mu\tilde{H}]$
$Q^{(1)}_{qud^2HD}$	$i(\bar{d}_p\gamma^\mu d_r)[(\bar{q}_s u_t)D_\mu\tilde{H}]$

20: $\psi^4 HD$ + H.c.	
$Q^{(1)}_{l^2qdHD}$	$i(\bar{l}_p\gamma^\mu l_r)[(\bar{q}_s d_t)D_\mu H]$
$Q^{(3)}_{l^2qdHD}$	$i(\bar{l}_p\gamma^\mu\tau^I l_r)[(\bar{q}_s d_t)\tau^I D_\mu H]$
$Q^{(1)}_{e^2qdHD}$	$i(\bar{e}_p\gamma^\mu e_r)[(\bar{q}_s d_t)D_\mu H]$
$Q^{(1)}_{q^3dHD}$	$i(\bar{q}_p\gamma^\mu q_r)[(\bar{q}_s d_t)D_\mu H]$
$Q^{(2)}_{q^3dHD}$	$i(\bar{q}_p\gamma^\mu\tau^I q_r)[(\bar{q}_s d_t)\tau^I D_\mu H]$
$Q^{(1)}_{qu^2dHD}$	$i(\bar{u}_p\gamma^\mu u_r)[(\bar{q}_s d_t)D_\mu H]$
$Q^{(1)}_{qd^3HD}$	$i(\bar{d}_p\gamma^\mu d_r)[(\bar{q}_s d_t)D_\mu H]$

TABLE IX. The dimension-eight operators in the M8B with particle content $\psi^4 HD$ generated in universal theories. All operators are either Hermitian or anti-Hermitian. For the operator $Q^{(1)}_{e^4D^2}$, we have added a superscript of (1) to the M8B operator. The subscripts p, r, s, t are weak eigenstate indices.

$21:(\bar{L}L)(\bar{L}L)D^2$	
$Q^{(1)}_{l^4D^2}$	$D^\nu(\bar{l}_p\gamma^\mu l_r)D_\nu(\bar{l}_s\gamma_\mu l_t)$
$Q^{(1)}_{q^4D^2}$	$D^\nu(\bar{q}_p\gamma^\mu q_r)D_\nu(\bar{q}_s\gamma_\mu q_t)$
$Q^{(3)}_{q^4D^2}$	$D^\nu(\bar{q}_p\gamma^\mu\tau^I q_r)D_\nu(\bar{q}_s\gamma_\mu\tau^I q_t)$
$Q^{(1)}_{l^2q^2D^2}$	$D^\nu(\bar{l}_p\gamma^\mu l_r)D_\nu(\bar{q}_s\gamma_\mu q_t)$
$Q^{(3)}_{l^2q^2D^2}$	$D^\nu(\bar{l}_p\gamma^\mu\tau^I l_r)D_\nu(\bar{q}_s\gamma_\mu\tau^I q_t)$
$21:(\bar{L}R)(\bar{L}R)D^2 + \text{H.c.}$	
$Q^{(1)}_{q^2udD^2}$	$D_\mu(\bar{q}^j_p u_r)\epsilon_{jk}D^\mu(\bar{q}^k_s d_t)$
$Q^{(1)}_{lequD^2}$	$D_\mu(\bar{l}^j_p e_r)\epsilon_{jk}D^\mu(\bar{q}^k_s u_t)$
$21:(\bar{R}R)(\bar{R}R)D^2$	
$Q^{(1)}_{e^4D^2}$	$D^\nu(\bar{e}_p\gamma^\mu e_r)D_\nu(\bar{e}_s\gamma_\mu e_t)$
$Q^{(1)}_{u^4D^2}$	$D^\nu(\bar{u}_p\gamma^\mu u_r)D_\nu(\bar{u}_s\gamma_\mu u_t)$

(Table continued)

TABLE IX. *(Continued)*

$21:(\bar{R}R)(\bar{R}R)D^2$	
$Q^{(1)}_{d^4D^2}$	$D^\nu(\bar{d}_p\gamma^\mu d_r)D_\nu(\bar{d}_s\gamma_\mu d_t)$
$Q^{(1)}_{e^2u^2D^2}$	$D^\nu(\bar{e}_p\gamma^\mu e_r)D_\nu(\bar{u}_s\gamma_\mu u_t)$
$Q^{(1)}_{e^2d^2D^2}$	$D^\nu(\bar{e}_p\gamma^\mu e_r)D_\nu(\bar{d}_s\gamma_\mu d_t)$
$Q^{(1)}_{u^2d^2D^2}$	$D^\nu(\bar{u}_p\gamma^\mu u_r)D_\nu(\bar{d}_s\gamma_\mu d_t)$
$Q^{(3)}_{u^2d^2D^2}$	$D^\nu(\bar{u}_p\gamma^\mu T^A u_r)D_\nu(\bar{d}_s\gamma_\mu T^A d_t)$
$21:(\bar{L}R)(\bar{R}L)D^2 + \text{H.c.}$	
$Q^{(1)}_{leqdD^2}$	$D_\mu(\bar{l}^j_p e_r)\epsilon_{jk}D^\mu(\bar{d}^k_s q_t)$
$21:(\bar{L}L)(\bar{R}R)D^2$	
$Q^{(1)}_{l^2e^2D^2}$	$D^\nu(\bar{l}_p\gamma^\mu l_r)D_\nu(\bar{e}_s\gamma_\mu e_t)$
$Q^{(1)}_{l^2u^2D^2}$	$D^\nu(\bar{l}_p\gamma^\mu l_r)D_\nu(\bar{u}_s\gamma_\mu u_t)$
$Q^{(1)}_{l^2d^2D^2}$	$D^\nu(\bar{l}_p\gamma^\mu l_r)D_\nu(\bar{d}_s\gamma_\mu d_t)$
$Q^{(1)}_{q^2e^2D^2}$	$D^\nu(\bar{q}_p\gamma^\mu q_r)D_\nu(\bar{e}_s\gamma_\mu e_t)$
$Q^{(1)}_{q^2u^2D^2}$	$D^\nu(\bar{q}_p\gamma^\mu q_r)D_\nu(\bar{u}_s\gamma_\mu u_t)$
$Q^{(3)}_{q^2u^2D^2}$	$D^\nu(\bar{q}_p\gamma^\mu T^A q_r)D_\nu(\bar{u}_s\gamma_\mu T^A u_t)$
$Q^{(1)}_{q^2d^2D^2}$	$D^\nu(\bar{q}_p\gamma^\mu q_r)D_\nu(\bar{d}_s\gamma_\mu d_t)$
$Q^{(3)}_{q^2d^2D^2}$	$D^\nu(\bar{q}_p\gamma^\mu T^A q_r)D_\nu(\bar{d}_s\gamma_\mu T^A d_t)$

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