Toward higher-spin symmetry breaking in the bulk

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(Received 10 April 2024; accepted 10 June 2024; published 9 July 2024)

We present a new vacuum of the bosonic higher-spin gauge theory in $d+1$ dimensions, which has leftover symmetry of the Poincaré algebra in d dimensions. Its structure is very simple: the space-time geometry is that of anti–de Sitter space, while the only nonzero field is a scalar. The scalar extends along the Poincaré radial coordinate z and is shown to be linearly exact for an arbitrary mixture of its two $\Delta = 2$ and $\Delta = d-2$ conformal branches. The obtained vacuum breaks the global higher-spin symmetry, leading to a broken phase that lives in the Minkowski space-time.

DOI: 10.1103/PhysRevD.110.026007

I. INTRODUCTION

Higher-spin (HS) gauge theories [1] are often thought of as underlying string theory in its allegedly unbroken symmetry phase [2]; see also [3,4] for further related ideas. A proposal in [5] suggested that superstrings propagate at the boundary of 11-dimensional anti–de Sitter (AdS) space[1] as a result of spontaneous HS symmetry breaking. The specific mechanism relating the two theories is not practically available for a number of reasons. For one, while an HS candidate rich enough to embrace arguably all stringy states was recently proposed [7], the analysis of this theory is still conceptually and technically challenging even at the linearized level (see Ref. [8] for a work in this direction). Second, HS theories formulated naturally in AdS space [9] have recently faced the locality problem [10,11] which still has not been fully resolved beyond cubic order (see Refs. [12–15] for various approaches at quartic order).

Some accessible nonlinear HS models suitable for symmetry-breaking studies are available in the form of Vasiliev's generating equations [16,17]. They describe interactions of totally symmetric gauge fields at the level of the classical equations of motion. Although the spectra of these models are much poorer than those that string theory suggests, the details of symmetry breaking that lead to massive states are not known even for these simpler models. It is curious to note in this regard how under an *ad hoc* assumption on the AdS symmetry breaking one arrives precisely at the stringy Regge leading trajectory [18].

In this paper, we attempt to make a step in a similar direction by addressing the following simple question. Is there an HS vacuum of the $(d+1)$-dimensional theory that has Poincaré algebra as the global space-time symmetry in d dimensions? Such a vacuum, if it exists, provides arguably a systematic way to analyze the HS broken phase, at least in some toy model examples. We answer this question in the affirmative by manifestly constructing the corresponding solution of the bosonic HS equations.

With such a vacuum, one can consider perturbation theory about it. Fluctuating fields naturally acquire dependence on the AdS boundary coordinates $\vec{x}$, as well as on the radial bulk direction z. This way, one arrives at d-dimensional (generally massive) excitations propagating in Minkowski space-time at a fixed slice z. Among these, of great interest are those that either depend on z trivially (for example, in a scaling fashion) or result in a reorganization of HS modules that effectively makes dynamics d-dimensional, along the lines of [19,20]. The latter correspond to the broken phase of $d+1$ HS theory in d dimensions.

In approaching this problem, we use the standard unfolded formalism of [21] (see also [22] for its quantum extension) that allows one to cast a highly nontrivial, many-derivative HS interaction into the form of first-order conditions at the cost of introducing infinitely many auxiliary fields. The schematic form of these equations is

$$d_x\omega + \omega * \omega = \Upsilon(\omega,\omega,C) + \Upsilon(\omega,\omega,C,C) + \cdots, \tag{1.1}$$

$$d_x C + \omega * C - C * \pi(\omega) = \Upsilon(\omega,C,C) + \Upsilon(\omega,C,C,C) + \cdots \tag{1.2}$$

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[1] Notice, however, that there are no simple supergroups in AdS in $d \geq 8$, [6].

Further details on the above system will be provided shortly. For now, we would like to focus on its general structure. The fields $\omega = \omega(Y|x)$ and $C = C(Y|x)$ are the generating functions of HS gauge fields and their field strengths, respectively. The generating variables, collectively called Y, encode spinning components and the necessary auxiliary fields organized in accordance with the HS algebra generated by the star product $*$. In particular, the field spectrum contains a scalar associated with the lowest component of C,

$$\phi(x) \coloneqq C(Y|x)|_{Y=0}. \tag{1.3}$$

Infinite series of Υ's govern nonlinear gauge-invariant field interactions. Their explicit form is at the core of the HS problem. These can be extracted from the Vasiliev equations modulo a field redefinition [17]. Although systematic, the procedure is substantially involved in practice and draws one into the order-by-order factorization of the trace ideal, a routine that sets the equations on shell. Given the highly nonlinear nature of the Vasiliev theory, it is not too surprising that there are only a handful of exact solutions available in the literature ([23,24] in three and [25–31] in four dimensions; see also the review [32]), while there are none in arbitrary d except for the trivial one corresponding to an empty AdS space,

$$\omega_0 = W_{\mathrm{AdS}}, \qquad C_0 = 0. \tag{1.4}$$

Even though Vasiliev's system describes full nonlinear dynamics, it is not yet clear which choice of field variables leads to the vertices Υ within a proper class of (non)local interactions. This problem is currently under active investigation; see, e.g., [33–35].

We do not pursue the analysis of the original equations from [17] in our work. Instead, we use the recently proposed Vasiliev-like system [36] specialized to HS interactions of symmetric fields in any dimensions [37]. The advantage of the latter approach is its manifest all-order (off-shell) locality, which clears the way for an unexpectedly simple nontrivial vacuum of the theory.

Let us briefly comment on the difference between the original generating equations of [17] and those of [37]. Both systems describe unconstrained, i.e., off-shell nonlinear bosonic HS fields in arbitrary dimensions. Both operate with the same set of fields governed by the off-shell HS algebra and as such result in the same unfolded equations (1.1)–(1.2). The key difference is the type of the large (z, Y) algebra featuring in the generating systems, which is responsible for the explicit form of the vertices that show up on the right-hand sides of (1.1)–(1.2). In the case of Vasiliev, the large algebra contains noncommuting z's, while in our case these z's commute, which is not feasible for generating the equations of [17] due to unavoidable star-product divergences. Nevertheless, the commuting z algebra has already effectively come out in the analysis of [38], where the requirement of locality for the Vasiliev vertices was imposed. To make it work within [37] required revising the basic elements of the original Vasiliev equations. In [36] it was shown that the modification of the Vasiliev construction that arises in the z-commuting limit is indeed possible for the $4d$ HS system. This result was then extended to any d in [37]. At the level of vertices in (1.1)–(1.2) we believe the two systems from [17,37] should reproduce identical results. This can be checked at the first few interaction orders, but not yet at higher orders, because the locality of the original equations of [17] is not yet settled at higher orders.

It should be stressed once again that we are dealing with the off-shell system here. The HS on-shell dynamics can be obtained using the factorization procedure, the details of which are currently under development. Taking the quotient comes along with the very definition of HS physical fields.

Our main finding is very simple. The HS theory of symmetric fields in $d+1$ dimensions parametrized by the Poincaré coordinates $x^\mu = (\vec{\mathrm{x}}, \mathrm{z})$ has the following exact solution of (1.1) and (1.2):

$$\omega_0 = W_{\mathrm{AdS}}, \quad \phi(\vec{\mathrm{x}}, \mathrm{z}) = \nu_1 \mathrm{z}^2 + \nu_2 \mathrm{z}^{d-2}, \tag{1.5}$$

where W_{AdS} is the appropriately chosen AdS_{d+1} connection and $\nu_{1,2}$ are arbitrary parameters. Unlike the standard HS vacuum (1.4), the vacuum (1.5) introduces a nonzero scalar profile, which is independent of the boundary coordinates $\vec{\mathrm{x}}$. It depends on the radial z satisfying the Klein-Gordon equation

$$\Box_{\mathrm{AdS}_{d+1}} \phi = m^2 \phi, \quad m^2 = 2(2-d), \tag{1.6}$$

where the mass-like term is given in terms of the negative cosmological constant. We thus show that the linearized approximation turns out to be all-order exact, leading to

$$\Upsilon(\omega_0, \omega_0, C_\phi, \ldots, C_\phi) = \Upsilon(\omega_0, C_\phi, \ldots, C_\phi) = 0. \tag{1.7}$$

The proposed vacuum breaks the global HS symmetry down to a subalgebra that has the Poincaré algebra as the space-time symmetry in d dimensions. The reason the scalar does not contribute to HS sectors is kinematical. Being $\vec{\mathrm{x}}$ independent, it is too symmetric, and thus offers no spin structure whatsoever. Less clear is the absence of its nonlinear self-interaction as the nature of the observed higher-order cancellations remains obscure to us. On the other hand, the solution obtained is remarkably simple and naturally suggests proceeding with the linearized analysis about it. The corresponding free theory arguably lives on the Minkowski background in d dimensions. We hope to report on progress in this direction elsewhere.

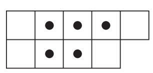
The remainder of this paper is structured as follows. In Sec. II we provide a brief review of Vasiliev's HS algebra in $d+1$ dimensions and present the generating equations of [37]. A suitable ansatz, as well as the solution of equations of motion, is given in Sec. III, where we also provide the on-shell condition used, elaborate on its global symmetries, and lay out a few basic properties of the obtained vacuum. Our conclusions are given in Sec. IV.

II. HS GENERATING EQUATIONS

Equations (1.1)–(1.2) contain the 1-form $\omega(Y|x)$ and 0-form $C(Y|x)$ valued in an HS algebra. Following [17], the bosonic HS algebra in $d+1$ dimensions can be generated using a set of oscillators,

$$Y = (y_\alpha, \mathbf{y}^a_\beta), \quad a = 0\ldots d, \quad \alpha, \beta = 1, 2, \tag{2.1}$$

where a is the $o(d,1)$ Lorentz index, while α and β are attributed to an $sp(2)$, which is designed to generate two-row Young diagrams. Indeed, as shown in [17], whenever the greek indices are contracted with the $sp(2)$ canonical form $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$ forming an $sp(2)$ singlet, the coefficients of a polynomial $f(\mathbf{y}, y)$ are nothing but a bunch of (Lorentz traceful) two-row diagrams. For example, $\omega(Y|x)$ generates the following set of HS fields of arbitrary spin $s \geq 1$:

$$\omega^{a(s-1),\,b(n)} = \mathrm{d}x^\mu \omega_\mu^{a(s-1),\,b(n)} =$$  $$, \qquad 0 \leq n \leq s-1\,, \tag{2.2}$$

where by $a(m)$ we traditionally denote the symmetrization over m indices. The Moyal star defines a product in the associative HS algebra,

$$(f * g)(y, \mathbf{y}) = \int f(y+u, \mathbf{y}+\mathbf{u}) \times g(y+v, \mathbf{y}+\mathbf{v}) e^{iu_\alpha v^\alpha + i\mathbf{u}^a_\alpha \mathbf{v}^b_\beta \eta_{ab}\epsilon^{\alpha\beta}}, \tag{2.3}$$

where the functions f and g are assumed to be $sp(2)$ singlets, while η_{ab} is the $o(d,1)$ metric.

The generating 0-form $C(Y|x)$ from (1.1)–(1.2) manifests the so-called twisted-adjoint module of the HS algebra, where the twist in (1.2) is defined as the following reflection:

$$\pi f(y, \mathbf{y}) = f(-y, \mathbf{y}). \tag{2.4}$$

The traceful two-row Young diagrams belong to what can be referred to as the *off-shell* HS algebra. It contains a greater set of fields than is required for the on-shell dynamics. Correspondingly, the system (1.1)–(1.2) does not describe dynamical evolution; rather, it offers a set of the generalized Bianchi consistency constraints and conditions that express any particular auxiliary field in terms of space-time derivatives of other fields. Such type of unfolded equations is usually called off shell (see, e.g., [39,40]). The on-shell spectrum contains fewer fields. Namely, those associated with the Lorentz traces of the two-row Young diagrams have to be consistently dismissed. A proper way of doing this is via factorization of the trace ideal. The reader may find more on this matter in [41]. Let us also add to this: star products in (1.1)–(1.2) do not respect a chosen on-shell field representative condition in general. Thus, Eqs. (1.1)–(1.2) should be treated modulo terms from the corresponding ideal.

A. Generating equations

Vertices on the right-hand sides of (1.1)–(1.2) can be generated order by order using equations from [37], which are based on the Vasiliev idea that $\omega(Y|x)$ can be embedded into a bigger space with the extra two coordinates $z_\alpha = (z_1, z_2)$,

$$W(z;Y|x) \coloneqq \omega(Y|x) + W_1(z;Y) + W_2(z;Y) + \cdots \tag{2.5}$$

The embedding is called *canonical* if

$$W(0;Y|x) = \omega(Y|x). \tag{2.6}$$

The generating equations of [37] read

$$\mathrm{d}_x W + W * W = 0, \tag{2.7}$$

$$\mathrm{d}_z W + \{W, \Lambda\}_* + \mathrm{d}_x \Lambda = 0, \tag{2.8}$$

$$\mathrm{d}_x C + \left(W(z';y,\mathbf{y}) * C - C * W(z';-y,\mathbf{y})\right)\big|_{z'=-y} = 0, \tag{2.9}$$

where $\mathrm{d}_z = \mathrm{d}z^\alpha \frac{\partial}{\partial z^\alpha}$, $C = C(y, \mathbf{y})$ is z independent, just as it appears in (1.1)–(1.2), and

$$\Lambda(z;y,\mathbf{y}) = \mathrm{d}z^\alpha z_\alpha \int_0^1 \mathrm{d}\tau \tau e^{i\tau z_\beta y^\beta} C(-\tau z, \mathbf{y}) \tag{2.10}$$

satisfies the condition

$$\mathrm{d}_z \Lambda = C(y, \mathbf{y}) * \gamma, \tag{2.11}$$

where

$$\gamma = \frac{1}{2} e^{iz_\alpha y^\alpha} \mathrm{d}z^\beta \mathrm{d}z_\beta, \tag{2.12}$$

while the star product $*$ extended to the $(z;Y)$ space has the form

$$(f * g)(z;Y) = \int f(z+u', y+u; \mathbf{y}) \star g(z-v, y+v+v'; \mathbf{y}) \times \exp(iu_\alpha v^\alpha + iu'_\alpha v'^\alpha), \tag{2.13}$$

where $\star$ is a part of the star product (2.3) that acts on $\mathbf{y}$ only,

$$(f\star g)(\mathbf{y}) = \int f(\mathbf{y}+\mathbf{u})g(\mathbf{y}+\mathbf{v})\exp(i\mathbf{u}_\alpha \mathbf{v}^\alpha). \tag{2.14}$$

From the above integrations, it is easy to derive

$$y* = y + i\frac{\partial}{\partial y} - i\frac{\partial}{\partial z}, \qquad z* = z + i\frac{\partial}{\partial y}, \tag{2.15}$$

$$*y = y - i\frac{\partial}{\partial y} - i\frac{\partial}{\partial z}, \qquad *z = z + i\frac{\partial}{\partial y}, \tag{2.16}$$

$$\mathbf{y}* = \mathbf{y} + i\frac{\partial}{\partial \mathbf{y}}, \qquad *\mathbf{y} = \mathbf{y} - i\frac{\partial}{\partial \mathbf{y}}. \tag{2.17}$$

In particular, one observes that z's commute,

$$[z_\alpha, z_\beta]_* = 0. \tag{2.18}$$

Equation (2.13) reduces to (2.3) for z-independent functions. Equations (1.1)–(1.2) result from (2.7)–(2.9) order by order upon solving for the z dependence of W using (2.8) and then substituting the result into (2.7) for (1.1) and into (2.9) for (1.2). However, the prescribed procedure leads to the unconstrained equations off the mass shell. To set them on shell, one has to choose representatives for ω and C and then strip the ideal contribution from (1.1)–(1.2) off [17]. The ideal is generated with the help of certain field-dependent $sp(2)$ generators found manifestly in [37].

Let us point to an unusual property of the system (2.7)–(2.9). The last equation (2.9) of the three is not independent. It follows from (2.8) via consistency. This fact is not quite manifest, however. To check it, one applies d_z to (2.8) and uses the following projective identity [36]:

$$\mathrm{d}_z\big(W(z;y,\mathbf{y}) * \Lambda\big) = \big(W(z';y,\mathbf{y}) * C\big)\big|_{z'=-y} * \gamma, \tag{2.19}$$

$$\mathrm{d}_z\big(\Lambda * W(z;y,\mathbf{y})\big) = -\big(C * W(z';-y,\mathbf{y})\big)\big|_{z'=-y} * \gamma. \tag{2.20}$$

Consistency of equations (2.19) and (2.20) is based on the specific star product (2.13), the precise form of Λ (2.10), and the functional class that evolves on (2.7) and (2.8), to which the field W belongs. For more details, we refer to [36,37]. Let us also note that the variable z' within the argument of W evades star multiplication and is set to $-y$, as prescribed above.

III. SOLUTION

The natural vacuum of HS theory is AdS_{d+1} space described by a z-independent bilinear in a Y flat connection W_0 satisfying (2.7). It is convenient to choose it using the Poincaré coordinates, in which the metric reads

$$ds^2 = \frac{1}{\mathrm{z}^2}(\mathrm{dz}^2 + \eta^{ij}\mathrm{dx}_i\mathrm{dx}_j), \tag{3.1}$$

where the radial coordinate z should not be confused with z_α, while $\mathrm{x}^j \coloneqq \vec{\mathrm{x}}$ are coordinates on the d-dimensional Minkowski boundary, $i,j = 0\ldots d-1$, with metric η_{ij}. Let us introduce the following notation for the split-component $\mathbf{y}^a$:

$$\mathbf{y}^a = \begin{cases} \mathbf{y}^j = \vec{\mathbf{y}}, & a = j < d, \\ i\bar{y}, & a = d, \end{cases} \tag{3.2}$$

where the imaginary i is introduced conventionally, while $\bar{y}$ is not complex conjugate to any $\mathbf{y}$'s but rather is an independent component. The commutation relations

$$[\mathbf{y}^i_\alpha, \mathbf{y}^j_\beta]_* = 2\eta^{ij}\epsilon_{\alpha\beta}, \quad [y_\alpha, y_\beta]_* = 2i\epsilon_{\alpha\beta}, \quad [\bar{y}_\alpha, \bar{y}_\beta]_* = -2i\epsilon_{\alpha\beta} \tag{3.3}$$

provide the following comprehensive set of $o(d,2)$ conformal algebra generators:

$$M_{ij} = \frac{1}{2}\mathbf{y}^\alpha_i \mathbf{y}_{j\alpha}, \quad P_i = \frac{1}{2}\mathbf{y}^\alpha_i (y-\bar{y})_\alpha,$$
$$K_i = \frac{1}{2}\mathbf{y}^\alpha_i (y+\bar{y})_\alpha, \quad D = -\frac{1}{2}y_\alpha \bar{y}^\alpha. \tag{3.4}$$

The connection

$$W_0 = \frac{i}{\mathrm{z}}(\mathrm{dx}^j P_j - \mathrm{dz} D) \tag{3.5}$$

can be shown to satisfy (2.7). We fix this vacuum in our analysis by assuming that it receives no correction even in the case of a nonzero field configuration of field C. In what follows, we also need the associated star commutators derived from (2.13),

$$[\vec{P}, \bullet]_* = i\vec{\mathbf{y}}^\alpha\left(\frac{\partial}{\partial \bar{y}^\alpha} + \frac{\partial}{\partial y^\alpha}\right) - i\left(y - \bar{y} - i\frac{\partial}{\partial z}\right)^\alpha \frac{\partial}{\partial \vec{\mathbf{y}}^\alpha}, \tag{3.6}$$

$$[D, \bullet]_* = -iy^\alpha \frac{\partial}{\partial \bar{y}^\alpha} - i\bar{y}^\alpha \frac{\partial}{\partial y^\alpha} + \epsilon^{\alpha\beta}\frac{\partial^2}{\partial z^\alpha \partial \bar{y}^\beta}. \tag{3.7}$$

A. Weyl module and T ansatz

The nontrivial part of the following analysis is to solve for the Weyl module C satisfying (2.8) and (2.9). Let us specify conditions that we impose to constrain our ansatz for C. First, as we have mentioned, Eqs. (2.7)–(2.9) result in no dynamics unless on-shell representatives are picked and the trace ideal is factored out. As shown in, e.g., [41], a convenient C-representative is *twisted* traceless [due to the twist (2.4)], rather than the usual AdS traceless typical of the 1-form $\omega(Y|x)$. In our notation, the former condition takes the form

$$\Delta_{\alpha\beta}C \coloneqq \left(\eta^{ij}\frac{\partial}{\partial\mathbf{y}^{i\alpha}}\frac{\partial}{\partial\mathbf{y}^{j\beta}} - \frac{\partial}{\partial\bar{y}^{\alpha}}\frac{\partial}{\partial\bar{y}^{\beta}} - y_{\alpha}y_{\beta}\right)C = 0. \tag{3.8}$$

The operator $\Delta_{\alpha\beta}$ can be shown to commute with the free equations arising from (2.9) upon substituting (3.5) in place of W:

$$\tilde{D}C = 0, \quad [\Delta_{\alpha\beta}, \tilde{D}] = 0, \tag{3.9}$$

where

$$\tilde{D} = \mathrm{d}_x + \frac{\mathrm{d}\vec{\mathrm{x}}}{\mathrm{z}}\cdot\left(i\vec{\mathbf{y}}^{\alpha}y_{\alpha} - i\frac{\partial}{\partial\vec{\mathbf{y}}^{\alpha}}\frac{\partial}{\partial y^{\alpha}} - \vec{\mathbf{y}}^{\alpha}\frac{\partial}{\partial\bar{y}^{\alpha}} - \bar{y}^{\alpha}\frac{\partial}{\partial\vec{\mathbf{y}}^{\alpha}}\right) + i\frac{\mathrm{dz}}{\mathrm{z}}\left(y_{\alpha}\bar{y}^{\alpha} + \epsilon^{\alpha\beta}\frac{\partial}{\partial y^{\alpha}}\frac{\partial}{\partial\bar{y}^{\beta}}\right). \tag{3.10}$$

In addition, representatives singled out by (3.8) respect the action of the spin operator of the twisted-adjoint module,

$$\hat{s} = \frac{1}{2}\left(\mathbf{y}^{a\alpha}\frac{\partial}{\partial\mathbf{y}^{a\alpha}} - y^{\alpha}\frac{\partial}{\partial y^{\alpha}}\right) = \frac{1}{2}(\vec{\mathbf{y}}\cdot\vec{\partial}_{\alpha} - \bar{y}^{\alpha}\bar{\partial}_{\alpha} - y^{\alpha}\partial_{\alpha}),$$
$$[\hat{s}, \Delta_{\alpha\beta}] = -\Delta_{\alpha\beta}. \tag{3.11}$$

Thus, Eq. (3.8) is consistent with a natural choice of having the scalar as the lowest component of the Weyl module $C(y,\bar{y};\vec{\mathbf{y}}|x)$ (see also [42]),

$$\phi(x) = C(0,0;\vec{0}|x). \tag{3.12}$$

Now, aiming at the exact solution, we would like C to be nonzero within the scalar sector only. This implies that the eigenvalue of spin operator is zero,

$$\hat{s}C = 0. \tag{3.13}$$

Even though the scalar sources higher spins in interactions, in general, this may not be the case for a highly symmetric scalar profile. Thus, we assume C to depend on the Poincaré radial z only; in other words, we take it to be $\vec{\mathrm{x}}$ independent.

With the above preparations, we propose the following ansatz:

$$C = \mathrm{z}^2 e^{iy_{\alpha}\bar{y}^{\alpha}}T(p,q;\mathrm{z}), \tag{3.14}$$

where

$$p = -\mathrm{z}^2\vec{\mathbf{y}}^{\alpha}\cdot\vec{\mathbf{y}}^{\beta}y_{\alpha}y_{\beta}, \quad q = 2i\mathrm{z}^2 y_{\alpha}\bar{y}^{\alpha}. \tag{3.15}$$

The normalization and z-scaling are chosen conveniently. Equation (3.13) is trivially satisfied, justifying the scalar structure of the module. The exponential factor in (3.14) although can be absorbed into T remains conveniently isolated. A similar exponential was already introduced in [19] as a kind of intertwining operator between free bulk fields and boundary currents. At the lowest interaction level, the same exponential was observed to commute with nonlinearities of HS equations in their local form [43,44]. In addition, being a star-product projector, it shares this characteristic feature with the HS bulk-to-boundary propagators [42],

$$e^{iy\bar{y}} * e^{iy\bar{y}} = \frac{1}{4}e^{iy\bar{y}}. \tag{3.16}$$

Given the universality of the above substitution in various applications, we refer to (3.14) as the T *ansatz*.

B. Obtaining the solution

All is set to proceed with solving (2.8) and (2.9). Plugging (3.5) into (2.8) and using that W_0 from (3.5) is z independent, we arrive at two conditions for Λ from the dz and $\mathrm{d}\vec{\mathrm{x}}$ 1-forms, correspondingly,

$$[D, \Lambda_{\alpha}]_* = -i\mathrm{z}\frac{\partial}{\partial\mathrm{z}}\Lambda_{\alpha}, \tag{3.17}$$

$$[P_i, \Lambda_{\alpha}]_* = 0. \tag{3.18}$$

Equivalently, using (3.6) and (3.7),

$$\left(\bar{y}^{\beta}\frac{\partial}{\partial y^{\beta}} + y^{\beta}\frac{\partial}{\partial\bar{y}^{\beta}} + i\epsilon^{\beta\gamma}\frac{\partial}{\partial z^{\beta}}\frac{\partial}{\partial\bar{y}^{\gamma}}\right)\Lambda_{\alpha} = \mathrm{z}\frac{\partial}{\partial\mathrm{z}}\Lambda_{\alpha}, \tag{3.19}$$

$$\left(y - \bar{y} - i\frac{\partial}{\partial z}\right)^{\beta}\frac{\partial}{\partial\vec{\mathbf{y}}^{\beta}}\Lambda_{\alpha} - \vec{\mathbf{y}}^{\beta}\left(\frac{\partial}{\partial y} + \frac{\partial}{\partial\bar{y}}\right)_{\beta}\Lambda_{\alpha} = 0. \tag{3.20}$$

Substituting (3.14) into (2.10),

$$\Lambda_{\alpha} = \mathrm{z}^2\int_0^1 \mathrm{d}\tau\tau z_{\alpha}e^{i\tau z(y-\bar{y})}T(\mathrm{p},\mathrm{q};\mathrm{z}), \tag{3.21}$$

where we introduced

$$\mathrm{p} = -\tau^2\mathrm{z}^2\vec{\mathbf{y}}_{\alpha}\cdot\vec{\mathbf{y}}_{\beta}z^{\alpha}z^{\beta}, \quad \mathrm{q} = -2i\tau\mathrm{z}^2 z_{\alpha}\bar{y}^{\alpha}. \tag{3.22}$$

Plugging (3.21) into (3.19) and (3.20) and after quite a lengthy calculation (see the Appendix for a sketch of the derivation) that repeatedly uses partial integration of the form

$$z^{\alpha}\frac{\partial}{\partial z^{\alpha}}\int_0^1 \mathrm{d}\tau\rho(\tau)f(\tau z) = \int_0^1 \mathrm{d}\tau\rho(\tau)\tau\partial_{\tau}f(\tau z) = \rho(1)f(z) - \rho(\tau)\tau f(\tau z)|_{\tau=0} - \int_0^1 \mathrm{d}\tau\partial_{\tau}(\tau\rho(\tau))f(\tau z), \tag{3.23}$$

we arrive at

$$\int_0^1 \mathrm{d}\tau\tau e^{i\tau z(y-\bar{y})}\left(T'_{\mathrm{z}} - 4\mathrm{z}\left(T'_{\mathrm{q}} + \mathrm{p}T''_{\mathrm{pq}} + \frac{\mathrm{q}}{2}T''_{\mathrm{qq}}\right)\right) = 0, \tag{3.24}$$

$$\int_0^1 \mathrm{d}\tau\tau^2 e^{i\tau z(y-\bar{y})}(T'_{\mathrm{q}} - 3T'_{\mathrm{p}} - 2\mathrm{p}T''_{\mathrm{pp}} - \mathrm{q}T''_{\mathrm{pq}}) = 0, \tag{3.25}$$

where $T'_s = \frac{\partial}{\partial s}T$. At this stage, the on-shell condition (3.8) has not been imposed yet. It gives us another constraint for our ansatz (3.14), namely,

$$d \cdot T'_p + 2pT''_{pp} - 2T'_q - 2\mathrm{z}^2 T''_{qq} = 0, \quad d = \delta_i{}^i. \tag{3.26}$$

Let us now set

$$T'_{\mathrm{z}} - 4\mathrm{z}\left(T'_q + pT''_{pq} + \frac{q}{2}T''_{qq}\right) = 0, \tag{3.27}$$

$$T'_q - 3T'_p - 2pT''_{pp} - qT''_{pq} = 0 \tag{3.28}$$

in order to satisfy (3.24) and (3.25). Substituting (3.14) into (2.9) gives no new conditions as it leads again to (3.27) and (3.28). This fact is not surprising because, as was stressed (see also [36]), (2.9) comes as a consistency condition of (2.8). Eventually, the solution we look for should satisfy the three differential equations (3.26)–(3.28). However, it should be analytic in p and q. The simplest solution of this system is

$$T = \nu = \mathrm{const}, \tag{3.29}$$

which is not trivial; see (3.14). However, there is another solution that can be found in terms of power series,

$$T = \sum_{m,n} \frac{f_{m,n}(\mathrm{z})}{m!n!} p^m q^n. \tag{3.30}$$

Substituting (3.30) into (3.26)–(3.28) and leaving the technical details for the Appendix, let us present the final result:

$$T = \mathrm{z}^{d-4} \sum_{m,n} \left(\frac{1}{\mathrm{z}^2}\right)^{m+n} \frac{p^m q^n}{m!n!\Gamma(2+2m+n)\Gamma(\frac{d-2}{2} - m - n)}. \tag{3.31}$$

The above power series can be summed up using the contour representation of the gamma function,

$$\frac{1}{\Gamma(k)} = \oint \mathrm{d}\rho \rho^{-k} e^{\rho}, \quad k \in \mathbb{N}, \tag{3.32}$$

leading eventually to the following final form of T:

$$T = \nu_1 + \nu_2 \oint \mathrm{d}\rho \frac{e^{\rho}}{\rho^2}\left(\mathrm{z}^2 + \frac{p}{\rho^2} + \frac{q}{\rho}\right)^{\frac{d-4}{2}}, \tag{3.33}$$

where ν_1 and ν_2 are arbitrary constants, while p and q are given by (3.15). The integration contour encircles the origin to avoid branch cuts. Thus, the obtained vacuum of the system (2.7)–(2.9) is described by the connection (3.5) and Weyl module (3.14) and (3.33), which we present here in terms of the original variables for convenience:

$$W_0 = \frac{i}{2\mathrm{z}}\left(\mathrm{d}\mathrm{x}^j \mathbf{y}_j^{\alpha}(y - \bar{y})_{\alpha} + \mathrm{d}\mathrm{z} y_{\alpha}\bar{y}^{\alpha}\right), \tag{3.34}$$

$$C = e^{iy_{\alpha}\bar{y}^{\alpha}}\left(\nu_1 \mathrm{z}^2 + \nu_2 \mathrm{z}^{d-2} \oint \mathrm{d}\rho \frac{e^{\rho}}{\rho^2}\left(1 + \frac{x_1}{\rho^2} + \frac{x_2}{\rho}\right)^{\frac{d-4}{2}}\right), \tag{3.35}$$

where

$$x_1 = -\vec{\mathbf{y}}^{\alpha} \cdot \vec{\mathbf{y}}^{\beta} y_{\alpha} y_{\beta}, \quad x_2 = 2iy_{\alpha}\bar{y}^{\alpha}. \tag{3.36}$$

Assuming analyticity in p and q, the solution (3.33) is the only solution of the partial differential equations (3.26)–(3.28). This may sound surprising given that no boundary conditions were imposed, but in fact, as the analysis in the Appendix shows, this system is somewhat fine-tuned to have very few analytic solutions. For example, had the coefficient p in (3.27) been different, say, $2p$, there would be no analytic solutions at all, other than $T = \mathrm{const}$.

C. Basic properties

Let us recapitulate some salient features of the obtained background.

(1) The vacuum 1-form connection W_0 is given by the AdS bilinears in the Poincaré coordinates (3.5). It remains undeformed despite nontrivial scalar excitation in C. This in turn implies that the scalar itself satisfies free equations trivializing the nonlinear self-interaction. Its profile can be extracted from (3.12),

$$\phi(\vec{\mathrm{x}}, \mathrm{z}) = \nu_1 \mathrm{z}^2 + \nu_2 \mathrm{z}^{d-2}. \tag{3.37}$$

It does not depend on the boundary coordinates $\vec{\mathrm{x}}$ and offers an arbitrary mixture of its two branches[2] of conformal dimensions $\Delta_1 = 2$ and $\Delta_2 = d - 2$. Given that the radial coordinate z is dimensionless (in terms of the cosmological constant), the two constants ν_1 and ν_2 carry the standard dimension of a scalar in $d + 1$. The solution is on shell because it

[2]The case of $d = 4$ corresponding to $\Delta_1 = \Delta_2$ is exceptional as one loses the logarithmic scalar branch given by $\mathrm{z}^2 \log \mathrm{z}$. The missing branch cannot be captured by (3.33). It would be interesting to reconsider this case separately, in particular to see whether the free solution remains exact.

satisfies the chosen condition (3.8) from factor algebra. Since W gains no C corrections and since the linear C is exact, our background trivializes the higher-order vertices (1.7).

(2) The independence of Eq. (3.37) from $\vec{\mathrm{x}}$ suggests that the leftover global space-time symmetry of the vacuum is the Poincaré algebra in d dimensions spanned by the Lorentz generators M_{ij} and translations P_i. This is indeed the case, as can be seen from the analysis of the conditions $\delta_\epsilon W_0 = \delta_\epsilon C = 0$, which give

$$\mathrm{d}_x\epsilon + [W_0, \epsilon]_* = 0, \tag{3.38}$$

$$\mathrm{d}_z\epsilon + [\Lambda, \epsilon]_* = 0. \tag{3.39}$$

Taking ϵ to be z_α independent, we have from (3.39) that $[\Lambda, \epsilon]_* = 0$. Taking into account (3.18) as well as the simple observation that Lorentz generators commute with Λ, $[M_{ij}, \Lambda]_* = 0$, the space-time global symmetry parameter that satisfies (3.39) reads

$$\epsilon = \frac{1}{2}\xi^{ij}(\vec{\mathrm{x}}, \mathrm{z})M_{ij} + \xi^i(\vec{\mathrm{x}}, \mathrm{z})P_i, \tag{3.40}$$

where ξ are some x-dependent parameters. Plugging (3.40) into (3.38), it is easy to obtain that ξ_{ij} are arbitrary constants, while

$$\xi_i = \frac{1}{\mathrm{z}}(\xi_i^0 + \xi_{ji}\mathrm{x}^j), \qquad \xi_i^0 = \mathrm{const}. \tag{3.41}$$

Thus, (3.40) indeed parametrizes Poincaré algebra in d dimensions.

(3) The structure of the T module (3.33) is different in odd and even dimensions. In the latter case, T is always a polynomial, e.g.,

$$T_{d=8} = \nu_1 + \nu_2\mathrm{z}^4\left(1 + \frac{x_1^2}{120} + \frac{x_2^2}{6} + \frac{x_1}{3} + x_2 + \frac{x_1x_2}{12}\right), \tag{3.42}$$

while in the odd case it is not, as the integration in (3.33) brings all powers of p and q. Note also that the contour representation (3.33) was already introduced in [42] in a different context. This integral can be expressed in terms of the Gegenbauer polynomials $C_n^{(\alpha)}(x)$ as follows:

$$\oint \mathrm{d}\rho \frac{e^\rho}{\rho^2}\left(1 - \frac{2xy}{\rho} + \frac{y^2}{\rho^2}\right)^{-\alpha} = \sum_n \frac{y^n}{(n+1)!}C_n^{(\alpha)}(x). \tag{3.43}$$

The Gegenbauer polynomials are known to arise as generating functions of the conserved currents of the $O(N)$ model; see, e.g., [45]. The presence of these polynomials in the structure of the T module may be a manifestation of the Flato-Fronsdal theorem [46,47].

(4) One should be cautious about interpreting the fields (3.34) and (3.35) as proper physical fields of the on-shell system. Indeed, while they do enjoy the chosen representative conditions and therefore are on shell, their physical interpretation may not be straightforward, given that the factorization procedure that brings the off-shell system (2.7)–(2.9) on shell is not yet detailed. It is likely that the physical fields may acquire a form different from (3.34)–(3.35). That this might be the case is signaled by the lack of corrections to the space-time background from a scalar despite its nonzero stress tensor. Nevertheless, the Poincaré symmetry of physical fields is guaranteed due to the fact that the quotienting comes about in terms of the HS module C [37], which itself is Poincaré invariant in our case.

(5) As a final remark, let us stress the importance of the Fock-type projector (3.16) in the construction of the Weyl module (3.14). It comes out in many HS applications. For example, it offers a “forgetful property” of the HS bulk-to-boundary propagators, making the HS N-point correlators calculable in [48]. It also appears within the structure of HS black holes; see, e.g., [28,29,44]. Most notably, in some cases it makes consideration of the projector-based solutions within the original Vasiliev framework of [16] problematic due to the artificial star-product divergences it elicits at the level of master fields. For example, in [31] a class of various exact solutions that admit six isometries was found. One of these (called type DW_0) is a four-dimensional analog of our solution. It is also based on the Fock-type projector which, however, develops singularities to the lowest interaction order. As a result, this particular solution of [31] is supplemented with a specific regularization prescription. Given that the vacuum obtained in this paper is free from any divergences, we may expect that the divergences of [31] are really spurious.

IV. DISCUSSION

We constructed a very simple vacuum (3.34)–(3.35) of the nonlinear bosonic HS theory in $d+1$ dimensions. All of its fields vanish except for the scalar, which spreads along the Poincaré radial direction z in AdS. Being highly symmetric, it respects the Poincaré algebra that naturally acts on AdS slices at fixed z as the global space-time symmetry. As a result, the obtained solution mildly breaks global HS symmetry.

In obtaining this vacuum, we chose a suitable AdS_{d+1} flat connection as a combination of translations and a dilatation from the algebra $o(d,2)$ using the standard Poincaré coordinates. As the HS equations (2.7)–(2.9) are off shell, we were forced to impose the extra condition (3.8) that selects the on-shell representative. To solve the system, the so-called T ansatz (3.14) based on the Fock projector was used. This particular choice, first introduced at the free level in [19], is motivated by the immunity of the Fock projector to HS nonlinearities [43,44]. The T module enjoys a system of partial differential equations that admits an explicit solution in terms of Gegenbauer polynomials. Quite remarkably, the solution we found trivializes the interacting HS vertices (1.7) in a given frame, which makes it linearly exact. Thus, the scalar profile features a superposition of its shadow $\Delta = 2$ and current $\Delta = d - 2$ branches that come with arbitrary dimensionful constants (3.37). In $d + 1 = 4$, the analogous vacuum was found as a solution of Vasiliev's equations in [31], modulo regularized divergencies. The formalism used in this paper features no divergences, thus, implying that the infinities Ref. [31] has clashed with are likely unphysical.

An intriguing problem for the future is to elaborate on the structure of the field spectrum about the proposed vacuum. Naturally, we expect the corresponding theory to live in d-dimensional flat space. It is conceivable that the spectra differ for $\Delta = 2$ and $\Delta = d - 2$ vacua, as well as for the mixture of the two. Given that the vacuum parameters $\nu_{1,2}$ [Eq. (3.37)] are dimensionful, one may expect the fluctuations on the Minkowski space to acquire ν-dependent masses, as an option. Another feasible option is that the spectrum is massless, while ν appears in interaction vertices. The latter case would relate HS theory in AdS to a hypothetical one in Minkowski space in a way that infers no flat limit. In any case, the free-field analysis does not promise to be immediately straightforward. Indeed, while the constructed vacuum is on shell, the generating system (2.7)–(2.9) is not. This implies that one has to factor out the ideal associated to the field traces to set free fields on their mass shell. The form of this ideal is driven by the field-dependent $sp(2)$ constructed in [37]. Unlike the case of the standard HS vacuum (1.4), which generates an ideal out of field traces at the free level, the new vacuum makes the $sp(2)$ generators depend on the vacuum structure of T from (3.33). This may offer a technical complication in the process of on-shell factorization.

The vacuum obtained may also play an important role in the HS AdS/CFT correspondence [49–52]. So, in $d = 3$, the case that sparked a surge of interest due to its conjectural relevance to the Wilson-Fisher model [49], it is the spinorial version of the solution (3.33) with $\nu_1 = 0$ that has emerged as an intertwiner of fields and currents within the Vasiliev equations at the free level [19]. It would be very interesting to extend this analysis to all orders. In particular, the holomorphic (chiral) generating equations of [36] are accessible to all-order analysis. It is also of interest to trace the $d = 3$ HS symmetry breaking from the boundary perspective in its massive state (see, e.g., [53]), where masses resulted from the breaking of scale invariance. It is conceivable that the vacuum parameters may be related to the mass of a scalar of the potential boundary dual Poincaré-invariant quantum field theory.

ACKNOWLEDGMENTS

We are grateful to Simone Giombi, Ruslan Metsaev, and especially Per Sundell and Mikhail Vasiliev for useful discussions. We are very thankful to the anonymous referee for very useful comments and suggestions. V. E. would like to thank Yasha Neiman for his warm hospitality at the OIST Quantum Gravity Unit while finishing this work. V. E. D. acknowledges the financial support from the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS".

APPENDIX: DERIVATION EXTRAS

Here we show that (3.21) solves Eq. (2.8) of the generating system, which for the chosen AdS connection (3.5) reduces to (3.19) and (3.20). To derive (3.19) and (3.20) from (2.8), the following formulas are very handy:

$$\begin{aligned}&\mathbf{y}^\alpha_j(y_\alpha-\bar{y}_\alpha)*f(z,y,\bar{y},\vec{\mathbf{y}})\\&=\left(\mathbf{y}^\alpha_j+i\epsilon^{\alpha\beta}\frac{\partial}{\partial\mathbf{y}^{\beta j}}\right)\left(y_\alpha+i\frac{\partial}{\partial y^\alpha}-\frac{\partial}{\partial z^\alpha}-\bar{y}_\alpha+i\frac{\partial}{\partial\bar{y}^\alpha}\right)\\&\quad\times f(z,y,\bar{y},\vec{\mathbf{y}}),\end{aligned}\tag{A1}$$

$$\begin{aligned}&f(z,y,\bar{y},\vec{\mathbf{y}})*\mathbf{y}^\alpha_j(y_\alpha-\bar{y}_\alpha)\\&=\left(\mathbf{y}^\alpha_j-i\epsilon^{\alpha\beta}\frac{\partial}{\partial\mathbf{y}^{\beta j}}\right)\left(y_\alpha-i\frac{\partial}{\partial z^\alpha}-i\frac{\partial}{\partial y^\alpha}-\bar{y}_\alpha-i\frac{\partial}{\partial\bar{y}^\alpha}\right)\\&\quad\times f(z,y,\bar{y},\vec{\mathbf{y}}),\end{aligned}\tag{A2}$$

Combining these gives us (3.6). One derives (3.7) analogously. Substituting (3.21) into (3.19) yields

$$\begin{aligned}&[\mathbf{y}^\alpha_j(y_\alpha-\bar{y}_\alpha),\Lambda_\xi]_*\\&=4\mathsf{z}^2z_\xi z_\gamma\mathbf{y}^\gamma_j\int_0^1 d\tau\tau^2e^{i\tau z(y-\bar{y})}\left\{\frac{\partial T}{\partial\mathsf{q}}+\tau(1-\tau)\big(iz_\alpha(y^\alpha-\bar{y}^\alpha)\big)\right.\\&\left.\times\frac{\partial T}{\partial\mathsf{p}}-4\tau\frac{\partial T}{\partial\mathsf{p}}-2\tau\mathsf{p}\frac{\partial^2T}{\partial\mathsf{p}^2}-\mathsf{q}\tau\frac{\partial^2T}{\partial\mathsf{p}\partial\mathsf{q}}\right\}.\end{aligned}\tag{A3}$$

To proceed, we notice that for $T = T(\mathsf{p}, \mathsf{q})$ we have the identity

$$\tau\frac{\partial T}{\partial\tau}=2\mathsf{p}\frac{\partial T}{\partial\mathsf{p}}+\mathsf{q}\frac{\partial T}{\partial\mathsf{q}},\tag{A4}$$

which allows us to rewrite some terms as derivatives with respect to τ and then integrate by parts using (3.29). This way, we obtain

$$\begin{aligned}[\mathbf{y}^\alpha_j(y_\alpha-\bar{y}_\alpha),\Lambda_\xi]_*&=4\mathsf{z}^2z_\xi z_\gamma\mathbf{y}^\gamma_j\int_0^1 d\tau\tau^2e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)}\\&\quad\times\left\{\frac{\partial T}{\partial\mathsf{q}}-3\frac{\partial T}{\partial\mathsf{p}}-2\mathsf{p}\frac{\partial^2T}{\partial\mathsf{p}^2}-\mathsf{q}\frac{\partial^2T}{\partial\mathsf{p}\partial\mathsf{q}}\right\},\end{aligned}\tag{A5}$$

which equals zero due to (3.28).

In a similar way, the dz sector of (2.8) can be solved. Plugging (3.21) into (3.7) and using (A4) gives

$$\frac{i}{2\mathrm{z}}[y_\alpha\bar{y}^\alpha,\Lambda_\xi]_* = \mathrm{z}z_\xi\int_0^1 d\tau\tau^2\frac{\partial}{\partial\tau}(e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)})\left(1+2\mathrm{z}^2\frac{\partial}{\partial\mathrm{q}}\right)\times T-\mathrm{z}z_\xi\int_0^1 d\tau\tau e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)}\mathrm{q}\frac{\partial T}{\partial\mathrm{q}} -\mathrm{z}z_\xi\int_0^1 d\tau\frac{\partial}{\partial\tau}\left(\tau^3 e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)}\left(1+2\mathrm{z}^2\frac{\partial}{\partial\mathrm{q}}\right)T\right). \tag{A6}$$

Differentiation with respect to z then amounts to

$$\frac{\partial\Lambda_\xi}{\partial\mathrm{z}} = z_\xi\int_0^1 d\tau\tau e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)}\left(2\mathrm{z}+2\mathrm{z}\mathrm{p}\frac{\partial}{\partial\mathrm{p}}+2\mathrm{z}\mathrm{q}\frac{\partial}{\partial\mathrm{q}}+\mathrm{z}^2\frac{\partial}{\partial\mathrm{z}}\right)T. \tag{A7}$$

Combining (A6) and (A7), one arrives at

$$\frac{\partial\Lambda_\xi}{\partial\mathrm{z}}+\frac{i}{2\mathrm{z}}[y_\alpha\bar{y}^\alpha,\Lambda_\xi]_* = \mathrm{z}^2 z_\xi\int_0^1 d\tau\tau e^{i\tau z_\alpha(y^\alpha-\bar{y}^\alpha)}\left(\frac{\partial T}{\partial\mathrm{z}}-4\mathrm{z}\left(\frac{\partial T}{\partial\mathrm{q}}+\mathrm{p}\frac{\partial^2 T}{\partial\mathrm{p}\partial\mathrm{q}}+\frac{\mathrm{q}}{2}\frac{\partial^2 T}{\partial\mathrm{q}^2}\right)\right), \tag{A8}$$

which is again zero due to (3.27).

1. Solving equations on T

Plugging the power series ansatz (3.30) into (3.28) and (3.26), we obtain the following equations for coefficients, respectively:

$$f_{m,n+1} = (3+2m+n)f_{m+1,n}, \tag{A9}$$

$$(d+2m)f_{m+1,n}-2f_{m,n+1}-2\mathrm{z}^2 f_{m,n+2} = 0. \tag{A10}$$

Using (A9), one can reduce (A10) to

$$f_{m,n+1} = \frac{1}{\mathrm{z}^2}\frac{\frac{d}{2}-m-n-2}{2+2m+n}f_{m,n} \quad \text{for } n\geq 1. \tag{A11}$$

This equation can be easily solved as

$$f_{m,n}(\mathrm{z}) = \frac{\varphi(\mathrm{z})}{\mathrm{z}^{2(m+n)}}\frac{1}{\Gamma(2m+n+2)\Gamma(\frac{d}{2}-m-n-1)}, \tag{A12}$$

where $\varphi(\mathrm{z})$ is a yet undefined function. Equation (3.27) describes evolution with respect to z and gives the following equation for $\varphi(\mathrm{z})$:

$$\frac{\partial\varphi(\mathrm{z})}{\partial\mathrm{z}} = (d-4)\frac{\varphi(\mathrm{z})}{\mathrm{z}}. \tag{A13}$$

Thus, the general solution (up to an overall constant) of equations on $f_{m,n}(\mathrm{z})$ reads

$$f_{m,n}(\mathrm{z}) = \frac{\mathrm{z}^{d-4-2m-2n}}{\Gamma(2m+n+2)\Gamma(\frac{d}{2}-m-n-1)}. \tag{A14}$$

Plugging these coefficients back into the power series (3.30), one obtains (3.31).

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