

Dark energy with the help of interacting dark sectors

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We analyze theories that do not have a de Sitter vacuum and cannot lead to slow-roll quintessence, but which nevertheless support a transient era of accelerated cosmological expansion due to interactions between a scalar ϕ and either a hidden sector thermal bath, which evolves as dark radiation, or an extremely light component of dark matter. We show that simple models can explain the present-day dark energy of the Universe consistently with current observations. This is possible both when ϕ 's potential has a hilltop form and when it has a steep exponential runaway, as might naturally arise from string theory. We also discuss a related theory of multifield quintessence, in which ϕ is coupled to a sector that sources a subdominant component of dark energy, which overcomes many of the challenges of slow-roll quintessence.

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I. INTRODUCTION

There is convincing evidence that the current energy budget of the Universe is dominated by a component, *dark energy*, which at redshift $z \lesssim 1$ has equation of state $w \lesssim -0.85$ [1,2]. Uncovering the nature of dark energy is one of the foremost problems in cosmology and, given that the vacuum energy density is sensitive to the details of physics at high energy scales, also fundamental particle physics [3].

Dark energy could simply be a cosmological constant that sources a de Sitter (dS) vacuum. However, indications are that dS vacua are at best difficult to obtain from string theory in the regimes that current techniques can access, see e.g. [4]. Moreover, it is unclear what the measurable observables of a theory of quantum gravity with never-ending accelerated expansion might be [5–7], e.g. an S matrix cannot be defined (such problems are especially sharp in asymptotic dS [8]). Motivated by these issues, it has been conjectured that no metastable dS vacua exist anywhere in the string landscape [9]. On the observational side, although cosmic microwave background (CMB) data generally show no evidence for physics beyond Lambda cold dark matter (Λ CDM), intriguingly, baryon acoustic oscillation (BAO), SNIa, and other non-CMB data might

hint towards favoring dynamical dark energy models over a cosmological constant (see e.g. [10–16]). Alternative possible sources of dark energy are therefore worth considering. The most studied of these is quintessence, which itself is not without challenges: The original quintessence tracking solutions of a single scalar field with a *steep* exponential or polynomial potential [17–19] are now in tension with observations for typical potentials¹ [20,22–24] and they anyway require extra model building if eternal acceleration is to be avoided. A scalar field with a sufficiently flat potential can lead to an era of slow-roll quintessence that is compatible with current data [22–24], but suitable potentials do not appear straightforward to realize within string theory in the absence of tuned initial conditions or super-Planckian field displacements [25].

In this paper we argue that there are other options to account for dark energy, in particular in extensions of the Standard Model of particle physics that contain *dark sectors* (i.e. sets of new particles that have sizable interactions among themselves but tiny couplings to visible matter) and ultralight fields that are weakly coupled to everything. Such modifications of the Standard Model are plausible given that they seem to commonly arise in string theory compactifications [26–30]. Theories of multifield quintessence, which can alleviate some of the challenges of the single field version, fall into this general framework and have been studied extensively, see e.g. [31–38].

¹For example, [20] constrains the inverse power in the polynomial potential $V \sim \phi^{-\alpha}$ to be $\alpha < 0.28$ (2σ limit) with TT + lowP + lensing + BAO and [21] finds that the exponential coefficient in $V \sim e^{-\lambda\phi}$ must be $\lambda \lesssim 0.54$ (2σ limit) using SN + CMB + BAO + H0.

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The main focus of our work is on scenarios that, unlike quintessence, do not rely on Hubble friction to sustain dark energy. In particular, we consider the possibility that interactions between a scalar and a field that either behaves as a component of dark matter or dark radiation can result in a transient era of dark energy domination, despite the Lagrangian potential having no stationary point that would correspond to a metastable dS vacuum. Both cases essentially consist of a moderately supercooled phase transition [39] currently taking place in a hidden sector. For completeness, we also consider the possibility of interactions between a scalar and a second field that itself acts as a quintessence field and provides a subdominant contribution to dark energy (this scenario is a version of multifield quintessence that, to our knowledge, has not been studied before).

We stress that we build on previous work that has proposed these types of theories to explain accelerated expansion in various contexts. We discuss this literature in detail as we go along, but (among other important works) we note that the dark matter assisted scenario is analogous to *new old inflation* [40–44], see also [45] for analysis of the same theory in the context of late-time dark energy (couplings between a quintessence field and dark matter have also been considered extensively in other settings, e.g. [46–49]). Meanwhile, the dark radiation assisted scenario is closely connected to *thermal inflation* [50,51] (and related ideas have been proposed to resolve the Hubble tension [52]), and a theory related to the quintessence assisted scenario has been considered in [53].

A. Setup

Throughout, we study a toy model that consists of two interacting hidden sector scalar fields with a low-energy effective Lagrangian

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + V(\phi, \psi), \quad (1)$$

comprising canonical kinetic terms and a scalar potential of the form

$$V(\phi, \psi) = V(\phi) + \frac{1}{2} m_\psi^2 \psi^2 + \frac{1}{2} \frac{m_{\text{int}}^2}{\Lambda^2} \phi^2 \psi^2 + \lambda \psi^4. \quad (2)$$

The field ϕ will, eventually, source dark energy via its potential $V(\phi)$, which we take to have a hilltop or exponential form

$$V_{\text{hill}}(\phi) \equiv \rho_{\text{de}} \left(\left(\frac{\phi}{\Lambda} \right)^2 - 1 \right)^2, \quad V_{\text{exp}}(\phi) \equiv \rho_{\text{de}} e^{-\phi/\Lambda}, \quad (3)$$

respectively. Note that our model thus has four additional parameters compared to the Λ CDM model: Λ , m_ψ , m_{int} , and λ ; plus the initial conditions for ϕ and ψ and their

respective velocities.² The scale Λ associated with the field range of ϕ is assumed to be $\lesssim M_{\text{Pl}}$ where $M_{\text{Pl}} = (8\pi G_{\text{N}})^{-1/2}$ is the reduced Planck mass. As a result, neither $V_{\text{hill}}(\phi)$ or $V_{\text{exp}}(\phi)$ could lead to dark energy in isolation without a fine-tuning of ϕ 's initial conditions. The typical values of ψ 's mass m_ψ and the quartic couplings $m_{\text{int}}^2/\Lambda^2$, λ , and $\rho_{\text{de}}/\Lambda^4$ vary between the different scenarios that we consider, but m_ψ is always at least sub-eV and the quartic couplings are all taken to be much smaller than 1. We assume throughout that ϕ and ψ , as well as the Standard Model particle content, are statistically spatially homogeneous and isotropic on cosmological scales, so the metric is of the Friedmann-Lemaître-Robertson-Walker (FLRW) form. We also fix that the contribution to the cosmological constant sourced by the Standard Model and any additional fields in the theory is zero (although a small negative cosmological constant $\ll \rho_{\text{de}}$ would not affect the dynamics or our key conclusions).

Our assumed initial conditions with ϕ spatially homogeneous naturally arise from an earlier era of primordial inflation provided that the Hubble parameter during inflation $H_I \ll \Lambda$ [such that the fluctuations in ϕ during inflation, which have size of order $H_I/(2\pi)$ are small compared to the typical field range]. For the values of Λ that we consider, not too far below M_{Pl} this condition is easily satisfied (in fact, almost automatically given observational constraints on H_I). Moreover, the initial velocity of the zero momentum modes after inflation are $\dot{\phi} \sim H_I^2 \ll \Lambda$ in the regime of interest. This small initial kinetic energy redshifts away fast $\dot{\phi}^2 \propto a^{-6}$, such that at the late-times when the dynamics we are interested in begin (not long before dark energy domination) we can expect initial conditions with $\dot{\phi}$ small. In the scenario that ψ acts as a component of dark matter or dark energy, we can likewise assume that ψ is initially spatially homogeneous and with $\dot{\psi} \ll H M_{\text{Pl}}$. Moreover, it is reasonable to expect that ψ starts away from its potential: provided $H_I^2 M_{\text{Pl}}^2 \gg m_\psi^2 M_{\text{Pl}}^2$ there is no reason to think that ψ will be close to the minimum of its potential (this is consistent with ψ being spatially homogeneous after inflation because we assume $m_\psi \ll M_{\text{Pl}}$). In the case that ψ acts as a component of dark radiation, an initial, close to spatially homogeneous, thermal bath might be populated by the decay of the inflaton. We do however note that in all the scenarios that we consider, small isocurvature fluctuations in ϕ and ψ , as well as the initially small adiabatic fluctuations in these fields that are inevitably present, could lead to interesting observational signals of our theories. We carry out an initial

²The several additional parameters will make it difficult to find statistical preference for these scenarios in cosmological data, however there may be other possible hints towards our models such as evidence for ultralight dark matter or dark radiation or further theoretical developments.

analysis of the late-time evolution of these perturbations in Appendix D leaving a full analysis to future work.

The point $\phi = 0$ is clearly special in the hilltop case, being a maximum of ϕ 's potential energy functional when $\psi = 0$. Moreover, for both $V_{\text{hill}}(\phi)$ and $V_{\text{eff}}(\phi)$, at $\phi = 0$ there is no contribution to ψ 's effective mass from ϕ . With the hilltop potential the theory equation (1), Eq. (2) has a global minimum at $\phi = \Lambda$ and $\psi = 0$ with vanishing vacuum energy, while in the case of the exponential potential there is a runaway towards vanishing vacuum energy $\phi \rightarrow \infty$ with $\psi = 0$. We note that the scalar potential $V(\phi, \psi)$ does not have any de Sitter vacuum, and as we will discuss, depending on the values of its parameters, it can be consistent with recent swampland conjectures.

Despite its simplicity, the model described above is sufficient to exhibit all the dynamics we are interested in: Depending on the values of the Lagrangian parameters and the initial conditions, ψ can provide a subdominant contribution to dark matter, dark radiation or, if it acts as a slow-rolling quintessence field, dark energy. We will show that in each regime the background energy in ψ can temporarily provide an effective stabilizing mass term for ϕ via the quartic $\phi - \psi$ interaction in Eq. (1), trapping ϕ at a field value much smaller than Λ where it sources a nonvanishing potential energy and leads to a transient era of dark energy [justifying our choice of notation, “ ρ_{de} ,” in Eq. (3)]. We explore the cosmological dynamics, constraints and signatures of such theories, including how the necessary field values can automatically arise at early cosmological times and how a graceful exit from the era of accelerated expansion occurs.

Our work is structured as follows. In Secs. II–IV we consider the dark matter, dark radiation and quintessence assisted scenarios in turn. Subsequently, in Secs. V and VI we make some general comments on fine-tuning and the relation with swampland conjectures. We end in Sec. VII with a comparison of the three scenarios and a discussion of observational prospects and directions for future work. Appendices A–C provide additional details on each of the scenarios, referred to in the main text, and in Appendix D we present out preliminary analysis of the evolution of cosmological perturbations.

II. DARK MATTER ASSISTED DARK ENERGY

In this section we consider the part of parameter space in which the mass of ψ is greater than today's Hubble parameter, $m_\psi \gtrsim H_0$, where the subscript 0 denotes quantities evaluated today, and we assume $\lambda \ll m_\psi^2/M_{\text{Pl}}^2$ in Eq. (2) so ψ 's quartic self-interaction can be neglected. Moreover, we suppose that both ψ and ϕ are initially homogeneous and isotropic with $\dot{\phi} = \dot{\psi} = 0$ (where a dot denotes a derivative with respect to cosmic time), for instance due to an earlier epoch of primordial inflation,

and that ψ starts away from the minimum of its potential with initial value $\psi_i \neq 0$. With these assumptions, in a FLRW background ψ classically oscillates with frequency m_ψ and amplitude falling as $a^{-3/2}$, where a is the scale factor. Such a field can be interpreted as a collection of coherent scalar particles, with energy density redshifting as matter [54]. We anticipate that, if the oscillations are sufficiently fast, we can replace ψ^2 in Eq. (2) by its time averaged value $\langle \psi^2 \rangle$, trapping ϕ until the amplitude of ψ 's oscillations decreases enough. We have checked numerically that the trapping of ϕ in a dS minimum turns out to be fairly independent of the initial conditions for ϕ , $\dot{\phi}$, and $\dot{\psi}$, requiring only that ψ starts sufficiently far away from its minimum, and that $\dot{\phi} \ll H\Lambda$ and $\dot{\psi} \ll HM_{\text{Pl}}$ when the dynamics begin.

This idea, with ϕ 's potential taking the hilltop form, has previously been considered as possible explanation for early Universe cosmic inflation and the late-time accelerated expansion, in scenarios named *locked inflation* [40,41] and *locked dark energy* [45]. We now extend these studies in the context of dark energy. We start with the relatively tractable case of a hilltop potential before turning to the perhaps more realistic possibility of an exponential potential.

A. Hilltop potential

1. Overview of dynamics and parameter space

We begin by analyzing a Universe containing only ϕ and ψ , with $V(\phi) = V_{\text{hill}}(\phi)$ and choosing ϕ 's initial field value $\phi_i \ll \Lambda$. The evolution of ψ is approximately independent of ϕ so long as³

$$m_{\text{int}} \frac{\phi}{\Lambda} \lesssim m_\psi, \quad (4)$$

in which case the oscillations in ψ are governed by the equation

$$\ddot{\psi} + 3H\dot{\psi} + m_\psi^2\psi = 0. \quad (5)$$

If we further assume that $\langle \psi^2 \rangle$ is sufficiently small that the total energy density is dominated by the contribution from ϕ 's potential energy $V_{\text{hill}}(\phi) \simeq V_{\text{hill}}(0) = \rho_{\text{de}}$, then the background FLRW metric takes an approximately dS form, with $H_0^2 \approx \rho_{\text{de}}/(3M_{\text{Pl}}^2)$ and Eq. (5) is solved by

³Numerical solutions of the equations of motion of particular theories show that an era of dark energy domination is possible even if Eq. (4) is violated, in which case our subsequent analysis replacing ψ by Eq. (6) is not accurate [typically with somewhat fewer e -folds of dark energy than predicted from Eqs. (10a) and (10b)]. We will show below that this regime is relevant for the case that ϕ has an exponential runaway potential.

$$\psi(t) = \psi_i e^{-3H_0 t/2} \cos(m_\psi t), \quad (6)$$

where we set the initial time $t_i = 0$.

The equation of motion for ϕ then becomes

$$\ddot{\phi} + 3H_0 \dot{\phi} + (m_{\text{int}}^2 \psi_i^2 e^{-3H_0 t} \cos^2(m_\psi t) - \rho_{\text{de}}) \frac{\phi}{\Lambda^2} \approx 0, \quad (7)$$

where we have expanded in $\phi/\Lambda \ll 1$. If ψ^2 is replaced by its average over timescales $\sim m_\psi^{-1}$, then ϕ has an effective mass squared

$$m_{\text{eff}}^2 \approx \frac{1}{2} \frac{m_{\text{int}}^2}{\Lambda^2} \psi_i^2 e^{-3H_0 t} - \frac{\rho_{\text{de}}}{\Lambda^2}. \quad (8)$$

We see that, provided $\langle \psi^2 \rangle > \psi_{\text{crit}}^2$, with

$$\psi_{\text{crit}} \equiv 2\sqrt{\rho_{\text{de}}}/m_{\text{int}}, \quad (9)$$

the term proportional to m_{int} in Eqs. (7) and (8) results in ϕ being temporarily held at the origin and sourcing dark energy. Intuitively, there is an energy cost to ϕ moving to larger field values because this would increase ψ 's effective mass.

This is only a transient de Sitter phase. To a first approximation, as the amplitude of the ψ oscillations falls, eventually the effective mass contribution to ϕ will be insufficient to hold the latter at the origin, which occurs at around $t = t_{\text{end}}$ [40,41],

$$m_{\text{eff}} \approx 0 \Rightarrow t_{\text{end}} H_0 \approx \frac{2}{3} \log \left(\frac{m_{\text{int}} \psi_i}{\sqrt{\rho_{\text{de}}}} \right). \quad (10a)$$

However, it is not sufficient that ϕ 's time averaged effective mass squared parameter is positive. During every oscillation ψ passes through a region in field space such that $|\psi| < \psi_{\text{crit}}$. If the time spent in this instability range, $2\psi_{\text{crit}}/(m_\psi \psi_i e^{-3H_0 t/2})$, is comparable to the time scale on which ϕ would roll due to its tachyonic mass near the hilltop, $1/m_{\text{tachyon}} \sim \Lambda/\sqrt{\rho_{\text{de}}}$, the point $\phi = 0$ is unstable. This happens around t_{inst} [40,41] where

$$t_{\text{inst}} H_0 = \frac{2}{3} \log \left(\frac{m_{\text{int}} \psi_i \Lambda m_\psi}{\rho_{\text{de}}} \right). \quad (10b)$$

Depending on the extra factor inside the logarithm in Eq. (10b), this can be a stronger or a weaker condition than Eq. (10a). An upper bound on the number of e -folds of dark energy domination N_{de} is thus obtained from these two conditions

$$N_{\text{de}} \leq \min \left[\frac{2}{3} \log \left(\frac{m_{\text{int}} \psi_i}{\sqrt{3} m_\psi M_{\text{pl}}} \right), \frac{2}{3} \log \left(\frac{m_{\text{int}} \Lambda \psi_i}{3 H_0 M_{\text{pl}} M_{\text{pl}}} \right) \right]. \quad (11)$$

2. Constraints from parametric resonance

The preceding analysis is not the end of the story. As pointed out for locked inflation in Ref. [41], the coupling of ϕ to the coherently oscillating ψ can cause resonant instabilities in ϕ , analogous to preheating at the end of inflation (where, however, it is the inflaton that coherently oscillates and matter fields that undergo resonant amplification). As we now show, following [41], this results in additional constraints on our theory's parameters.

By rescaling the dark energy field $\hat{\phi} = e^{3H_0 t/2} \phi$ and defining the new time variable $\tau = m_\psi t$, the equation of motion, Eq. (7), of ϕ is recast into the well-known Mathieu equation (a linear second order ordinary differential equation with periodic forcing of the stiffness coefficient)

$$\hat{\phi}'' + (c(\tau) + 2q(\tau) \cos(2\tau)) \hat{\phi} = 0, \quad (12)$$

albeit with time-dependent coefficients

$$c(\tau) = 2q(\tau) - b \quad \text{and} \quad q(\tau) = \frac{m_{\text{int}}^2 \psi_i^2}{4m_\psi^2 \Lambda^2} e^{-3H_0 \tau/m_\psi}, \quad (13)$$

where

$$b = \frac{H_0^2}{m_\psi^2} \left(\frac{3M_{\text{pl}}^2}{\Lambda^2} + \frac{9}{4} \right). \quad (14)$$

Because $H_0 \lesssim m_\psi$, $q(\tau)$ varies only slowly, and the evolution is well approximated by an ordinary Mathieu equation [55] at any given time. Floquet's theorem [56] then implies that the solutions are of the form

$$\hat{\phi}(\tau) = e^{s\tau} f(\tau) \quad \text{with periodic} \quad f(\tau + \pi) = f(\tau). \quad (15)$$

The Mathieu exponent $s(c, q)$ can be complex and its real part is always non-negative; when $\text{Re}(s(c, q)) = 0$, $|\hat{\phi}|$ is stable; when $\text{Re}(s(c, q)) > 0$, $|\hat{\phi}|$ is exponentially growing. One can solve for $s(c, q)$ numerically [56], mapping out a stability-instability chart with characteristic instability, or "resonance" bands.

Note that for sizable N_{de} , Eq. (10a) requires $1 \ll \log(m_{\text{int}} \psi_i / (H_0 M_{\text{pl}})) \lesssim \log(m_{\text{int}} \psi_i / (H_0 \Lambda))$ and we will see soon that $m_\psi / H_0 \lesssim 15$, so $q(\tau_0) \gg 1$. Then, as τ increases $q(\tau)$ falls, and $s(c, q)$ passes through the resonance bands on time scales $\approx t_{\text{inst}}$. In the parameter space of interest to us $q \gg \sqrt{b}$ for $H_0 \tau / m_\psi \lesssim 1$ (i.e. when ψ starts oscillating) and b is not far from $\mathcal{O}(1)$. In this regime the mean value of the Mathieu exponent, averaged over a range of q is $\bar{s} \approx 0.11$. Consequently, the full solution to Eq. (12) behaves as $\hat{\phi} \propto e^{\bar{s}\tau}$, so the resonance causes $\hat{\phi}$ to grow exponentially on a time scale for t set by $(\bar{s} m_\psi)^{-1}$. This instability in $\hat{\phi}$ is not disastrous provided the induced oscillations are damped sufficiently fast by the expansion of

the Universe: in terms of the original field, the solution to Eq. (7) behaves as

$$\phi(t) \propto e^{(5m_\psi - 3H_0/2)t}, \quad (16)$$

so for

$$m_\psi/H_0 \lesssim 15, \quad (17)$$

the resonant instability is evaded. Once the amplitude of ψ 's oscillations have dropped enough that $b \simeq \sqrt{q}$ the average \bar{s} increases and the resonance becomes more dangerous. However, this effect happens at (up to order-1 numerical factors that we do not have control of) the same time that the instability condition Eq. (10b) causes ϕ to roll away from the top of the potential anyway, so it does not lead to an additional bound on N_{de} .

So far we have only considered the zero mode in the Fourier expansion of $\phi(t, \vec{x})$ to $\phi_k(t)$ (and likewise ψ). In Appendix A 1, we show that in the parameter space where the zero mode is not exponentially growing, higher momentum modes—populated by quantum fluctuations or any small inhomogeneities e.g. from an earlier era of primordial inflation—are not amplified either.

We have confirmed with numerical solutions of the equations of motion that, provided Eq. (17) is satisfied, the number of e -folds of dark energy domination is reasonably well approximated by the upper bound in Eq. (11) over the majority of parameter space. The previously mentioned strengthening of parametric resonance at $t \simeq t_{\text{inst}}$ can affect N_{de} in some theories. However, the impact of this is relatively minor and models within the identified allowed parameter space still typically lead to viable cosmological histories. The end of the era of dark energy occurs via a second order phase transition.

3. Working example with realistic cosmological history

The same dynamics can occur in a realistic cosmological history that includes the Standard Model. Note that the condition Eq. (17) implies that ψ does not start oscillating until long after matter-radiation equality, so it cannot make up all (or the majority) of dark matter and a further dark matter component must be added. We assume that this and the Standard Model fields are totally decoupled from the $\psi - \phi$ sector.

Remarkably, the presence of the Standard Model and the dominant dark matter component, which as usual drive the evolution of the Universe at early times, allow the initial condition $\phi_i \ll \Lambda$ to be relaxed. Instead ϕ_i and ψ_i can be set to their “natural” values $\simeq \Lambda$ and $\simeq M_{\text{Pl}}$ respectively. Then, at early times all of the gradients from $V(\phi, \psi)$ are dominated by H and both ψ and ϕ are frozen. If $\Lambda \lesssim M_{\text{Pl}}$, $m_{\text{int}}\psi_i/\Lambda \gtrsim m_\psi$ and $m_{\text{int}}\psi_i \gtrsim \rho_{\text{de}}$, the first term from the potential to be cosmologically relevant is the interaction term in ϕ 's equation of motion

$\sim \partial_\phi(m_{\text{int}}^2\psi^2\phi^2/\Lambda^2)$, when $H \gtrsim m_{\text{int}}\psi_i/\Lambda$. At this stage ϕ evolves in a background of basically constant $\psi^2 = \psi_i^2$, and starts to oscillate around the minimum of its effective potential, which is at $\phi = 0$. These oscillations are damped by the expansion of the Universe, until the time when $3H \simeq m_\psi$ at which point ψ starts oscillating. In Appendix A 2, we show that ϕ/Λ is localized close to 0 before this time provided

$$\frac{m_\psi\Lambda}{m_{\text{int}}\psi_i} \ll 1 \quad \text{or} \quad (18a)$$

$$\frac{\Omega_r^{3/8}}{\Omega_m^{1/2}} \left(\frac{H_0\Lambda}{m_{\text{int}}\psi_i} \right)^{3/4} \left(\frac{m_\psi}{H_0} \right) \ll 1, \quad (18b)$$

depending on whether ϕ starts to oscillate during matter or radiation domination respectively (Ω_r and Ω_m are the present-day radiation and matter density parameters). In this way, ϕ is automatically driven to the required point in its potential prior to when dark energy domination must begin.

Subsequently, the evolution is similar to the system containing only ϕ and ψ , except that the main component of dark matter dominates the energy density of the Universe for a while until ϕ 's potential energy takes over and the dark energy era starts. As in a universe containing only ϕ and ψ , provided parametric resonance is ineffective, the dark energy epoch ends either when the time-averaged effective mass parameter for ϕ becomes tachyonic or when the time spent in the tachyonic region as ψ oscillates is comparable to the time scale of the hilltop roll. The expected number of e -folds of dark energy domination can be obtained from Eq. (11) replacing ψ_i with the amplitude of ψ 's oscillations at the time when dark energy domination starts (to account for earlier redshifting). Once ϕ becomes unlocked, it oscillates around the minimum of its potential at $\phi = \Lambda$. When this first happens ψ and ϕ are strongly coupled together and the system evolves non-linearly. Numerical solutions of the equations of motion show that the accelerated expansion of the Universe ends almost immediately at this time (see Fig. 5 in Appendix A 3). Eventually, once the amplitude of the oscillations of ϕ and ψ decrease sufficiently by redshifting, both evolve as matter.

We plot the evolution of ϕ and ψ in a particular theory that is consistent with observational constraints in Fig. 1. This is obtained by solving the equations of motion of the theory numerically including the Standard Model radiation and an additional dark matter component, accounting for the full contributions from the energy densities of ϕ and ψ to the expansion history of the Universe. The various stages of the evolution can be seen clearly, including the eras during which ϕ is locked and sources dark energy and the eventual end of dark energy at $a/a_0 \simeq 5$. Further analysis of this theory is given in Appendix A 3 where we show plots of the evolution of the energy density and equation of state parameter w , which match the Λ CDM predictions to within

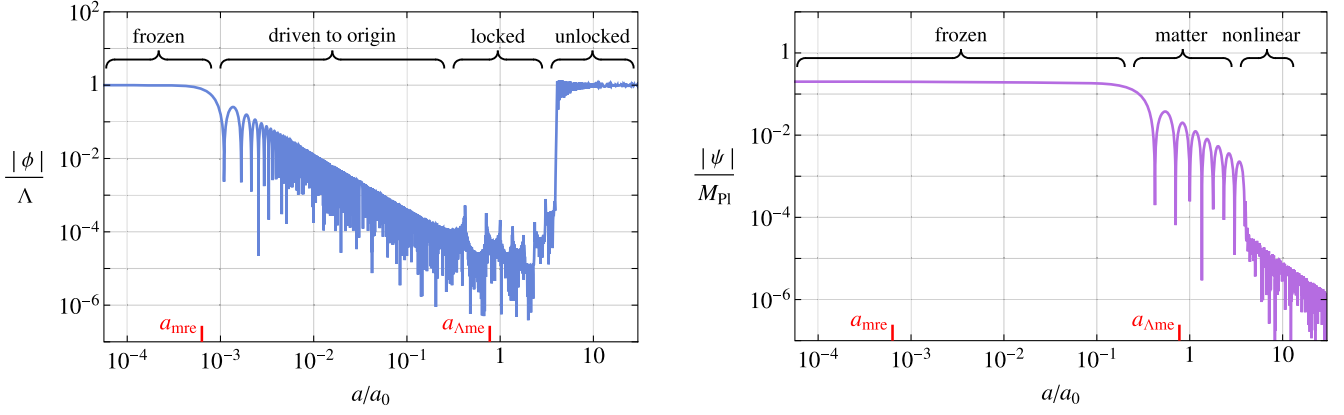


FIG. 1. The evolution of (left) ϕ and (right) ψ with scale factor a in a theory of dark matter assisted dark energy with a hilltop potential. The theory is given by Eqs. (1) and (2) with $V(\phi) = V_{\text{hill}}(\phi)$, with parameter values $m_\psi = 10H_0$, $m_{\text{int}} = 10^4 H_0$, $\Lambda = M_{\text{Pl}}/50$ and $\lambda = 0$ and initial field values $\phi_i = \Lambda$, $\psi_i = M_{\text{Pl}}/5$. We also include the Standard Model radiation and an additional (dominant) dark matter component that is assumed to be uncoupled to ϕ and ψ . The equations of motion of the theory and the Friedmann equations determining the expansion history are solved numerically. The various stages of ϕ and ψ 's dynamics, described in the main text, are labeled, and a_{mre} and $a_{\Lambda\text{me}}$ indicate the times of matter radiation and dark energy-matter equality respectively. Prior to $a/a_0 \simeq 5$ this theory matches the cosmological predictions of ΛCDM to an accuracy consistent with current observations (we give further details and show plots of the energy densities of ϕ and ψ and the equation of state parameter of the Universe in Appendix A 3).

percent level for $a/a_0 \lesssim 5$ and deviate dramatically after this.

4. Allowed parameter space

We now consider, in more generality, the constraints on the parameter space of dark matter assisted dark energy when ϕ has a hilltop potential. This consists of

$$m_{\text{int}}, m_\psi, \Lambda, \psi_i, \phi_i, \quad (19)$$

where we will assume $\Lambda \lesssim M_{\text{Pl}}$ and keep $\phi_i \lesssim \Lambda$ and $\psi_i \lesssim M_{\text{Pl}}$.

A number of conditions must be satisfied for the mechanism to work at all. We have already seen that we require $\langle \psi^2 \rangle > \psi_{\text{crit}}^2 \equiv 4\rho_{\text{de}}/m_{\text{int}}^2$ in order for the background ψ to generate an effective minimum. Then $\langle \psi^2 \rangle_0 > \psi_{\text{crit}}^2$ constrains

$$\psi_i^2 > \frac{4}{9} \frac{\rho_{\text{de}} m_\psi^2}{\Omega_{\text{m}} H_0^2 m_{\text{int}}^2}. \quad (20)$$

To obtain several e -folds of dark energy domination we need, from Eq. (11),

$$m_{\text{int}} > m_\psi, \quad (21)$$

and $m_\psi/H_0 \lesssim 15$. But we also require $m_\psi > H_0$ so that ψ oscillates, so overall

$$H_0 \lesssim m_\psi \lesssim 15H_0. \quad (22)$$

We must also impose observational limits on the energy density carried by extremely light scalar dark matter [57].

Equation (22) means that $m_\psi \sim 10^{-32}$ eV, and therefore the associated density parameter, $\Omega_\psi = m_\psi^2 \psi_0^2 / (6H_0^2 M_{\text{Pl}}^2)$, is bounded from above as $\Omega_\psi \lesssim 3 \times 10^{-2}$ [57], which implies that

$$\frac{3}{2} \frac{\psi_i^2 \Omega_{\text{m}}}{M_{\text{Pl}}^2} \lesssim 3 \times 10^{-2}, \quad (23)$$

where we used that ψ starts to oscillate during matter domination. The observational bound on Ω_ψ also implies $\rho_{\text{de}}/\rho_{\psi,0} \simeq 35$ (where $\rho_{\psi,0} \simeq m_\psi^2 \langle \psi^2 \rangle$ is the present-day energy density in ψ). In combination with the condition for a metastable minimum $\rho_{\text{de}} \ll m_{\text{int}}^2 \langle \psi^2 \rangle$, this demands a moderate hierarchy between the Lagrangian parameters m_ψ^2 and m_{int}^2 .

If we require ϕ to be driven close to the top of its potential beginning from $\phi_i \simeq \Lambda$ (as opposed to tuning its initial condition), we need Eq. (18a) or (18b) to be satisfied. We also need that ψ is still frozen when ϕ starts rolling towards $\phi = 0$, which is the case provided $\phi_i \ll \psi_i$, so $m_{\text{int}} \phi_i / \Lambda \ll m_{\text{int}} \psi_i / \Lambda$ and the effective ψ mass induced via m_{int} is negligible at these times (we also note that ψ 's Lagrangian mass is always cosmologically negligible when ϕ starts rolling in the relevant parameter space).

Further conditions on the theory's parameter space follow from the approximation we used, namely that ψ 's dynamics are linear when it starts oscillating, being dominated by its own mass term

$$m_{\text{int}} \phi_{\psi \text{ roll}} / \Lambda \ll m_\psi, \quad (24)$$

where $\phi_{\psi \text{ roll}}$ is the value of ϕ when $3H \simeq m_\psi$. If Eq. (24) is satisfied then ψ 's evolution remains linear until ϕ becomes

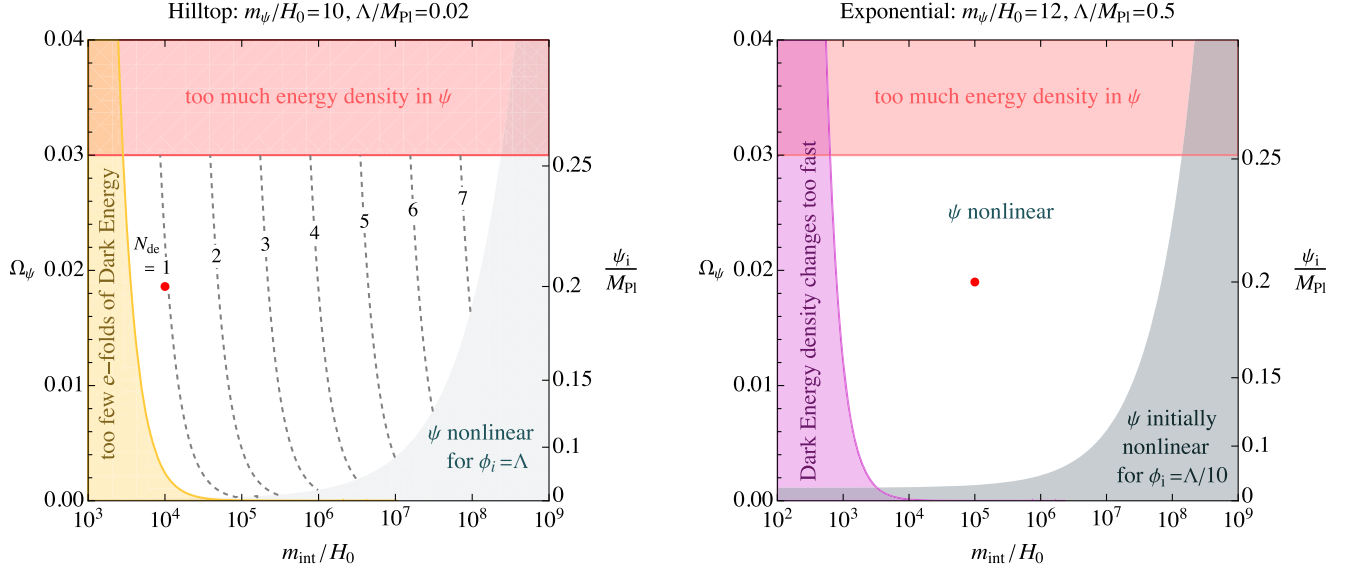


FIG. 2. Left: a slice of the parameter space for a dark matter assisted dark energy theory, with potential equation (2), in which ϕ 's potential has the hilltop form $V(\phi) = V_{\text{hill}}(\phi)$ in Eq. (3). Constraints come from the energy density in ψ (parametrized by Ω_ψ and determined by the initial field value ψ_i) exceeding current observational constraints on the fraction of extremely light dark matter (“too much energy density in ψ ”) and from the number of e -folds of dark energy domination not being sufficient to match observations (“too few e -folds of dark energy”). We also impose that ψ evolves linearly without backreaction from ϕ assuming that $\phi_i = \Lambda$, which allows analytic control but might not be essential for a viable theory (and can be relaxed if ϕ_i is assumed smaller). We indicate the number of e -folds of dark energy domination N_{de} expected from Eq. (11) (this can be somewhat altered by the nonlinear dynamics around the time when ϕ becomes unlocked). We also show the parameter point corresponding to the theory analyzed in Fig. 1 with a red dot. Right: the analogous plot for theories in which ϕ has a potential with an exponential runaway, $V(\phi) = V_{\text{exp}}(\phi)$ in Eq. (3), and larger fixed $\Lambda/M_{\text{Pl}} = 0.5$. In this case, ψ 's dynamics are necessarily nonlinear throughout the parameter space [see Eq. (33)] so we cannot reliably predict the number of e -folds of dark energy domination. However the resulting theories can still be consistent with observations especially if $\phi_i < \Lambda$ is assumed, e.g. the red dot indicates a theory analyzed in Appendix A 3 that is viable for $\phi_i = \Lambda/10$ leading to ≈ 0.5 e -folds of dark energy domination (i.e. dark energy domination ends at $a/a_0 \approx 1.25$). For such theories there is a constraint (absent in the case of a hilltop potential) from the minimum of ϕ 's potential (after time-averaging ψ) not moving too fast (“too large dark energy variation”). We also show the part of parameter space in which ψ 's evolution is nonlinear when it first starts oscillating, in which case an observationally viable era of dark energy domination is unlikely (this constraint depends on the value of ϕ_i , and for the plot we fix $\phi_i = \Lambda/10$).

unlocked and moves to large field values. Such a condition is not essential for a viable model, but the analytic control is appealing. If ϕ is driven to the top of its potential dynamically, $\phi_{\psi \text{ roll}}$ is small enough to satisfy Eq. (24) provided

$$\frac{\phi_i}{\psi_i} \ll 1 \quad \text{or} \quad (25a)$$

$$\frac{\Omega_r^{3/8}}{\Omega_m^{1/2}} \left(\frac{m_{\text{int}}}{H_0} \right)^{\frac{1}{4}} \frac{\phi_i}{\psi_i^{3/4} \Lambda^{1/4}} \ll 1, \quad (25b)$$

if ϕ becomes unfrozen during matter or radiation domination, respectively, see Eq. (A5a) or (A5b) in Appendix A 2.

We also note that Eq. (10b) requires that Λ is not too much smaller than M_{Pl} , and Eq. (25b) imposes that m_{int} is not too much larger than H_0 . As a result, the physical mass

of ϕ in the locked phase $\approx m_{\text{int}} \psi / \Lambda \gtrsim \rho_{\text{de}}^{1/2} / \Lambda$ [cf. Eq. (8)] is at most a few orders of magnitude larger than H_0 .

In Fig. 2 (left panel), we plot a slice of the allowed parameter space with m_ψ and Λ fixed and m_{int} and Ω_ψ (equivalently ψ_i) varying. We also set $\phi_i = \Lambda$, which affects the constraint from requiring ψ 's evolution be linear. Although the various constraints, especially the requirement that Ω_ψ is not too large, place important restrictions on the parameter space, a substantial region that leads to theories consistent with current observations remain. We also indicate the number of e -folds of dark energy domination predicted from Eq. (11); for the chosen m_ψ and Λ the condition from the time ψ spends in the instability range, Eq. (10b), is slightly stronger than the condition that ϕ 's time averaged mass squared is positive, Eq. (10a). A generic feature over the allowed region is that only a few e -folds of dark energy domination are obtained and $\Omega_\psi \gtrsim 10^{-3}$ (these are true also

for other values of m_ψ and Λ). A plot of another slice, varying m_ψ and Λ , can be found in Appendix A 3.

B. Exponential potential

We now show that coherent oscillations of ψ can source a transitory de Sitter vacuum and accelerated expansion even when the scalar potential equation (2) has no extremum, as happens for example for the exponential potential $V_{\text{exp}}(\phi)$ in Eq. (3). Such potentials occur very generically in string theory compactifications, where the leading perturbative corrections after supersymmetry breaking tend to lift flat directions in the moduli potential to steep runaways.

1. Overview of dynamics

We again begin with an analytical study of a theory containing only ψ and ϕ , before analyzing a full cosmological history numerically. Assuming that ψ 's oscillations are sufficiently fast to allow us to average over them, the potential for ϕ in Eq. (2) with $V(\phi) = V_{\text{exp}}(\phi)$ is extremized for

$$\frac{\partial V(\phi, \psi)}{\partial \phi} = 0 \Rightarrow m_{\text{int}}^2 \frac{\phi}{\Lambda^2} \langle \psi^2 \rangle = \frac{\rho_{\text{de}}}{\Lambda} e^{-\phi/\Lambda}, \quad (26)$$

which leads to a minimum at $\phi = \phi_{\text{min}}$

$$\frac{\phi_{\text{min}}}{\Lambda} = W_0 \left(\frac{\rho_{\text{de}}}{m_{\text{int}}^2 \langle \psi^2 \rangle} \right), \quad (27)$$

where W_0 is the principle branch of the Lambert W function [58]. Note that there is a minimum ϕ_{min} for any background value of $\langle \psi^2 \rangle$, however, $\langle \psi^2 \rangle \propto a^{-3}$ Eq. (6), so ϕ_{min} is time dependent. Assuming that ϕ follows its moving minimum, one can use $\partial W_0(x)/\partial x = W_0(x)/(x(1+W_0(x)))$ to show that the rate of change in the potential energy density goes as

$$\frac{\partial \log V(\phi_{\text{min}})}{\partial \log a} = -\frac{3\phi_{\text{min}}/\Lambda}{1 + \phi_{\text{min}}/\Lambda}. \quad (28)$$

Thus to meet the strong observational constraints on the time dependence of dark energy, we need

$$\phi_{\text{min}}/\Lambda \ll 1, \quad (29)$$

today, which replaces the condition for the existence of a minimum in the hilltop case, Eq. (10a). Expanding Eq. (27) in small $\rho_{\text{de}}/(m_{\text{int}}^2 \psi^2)$ gives $\phi_{\text{min}}/\Lambda \approx \rho_{\text{de}}/(m_{\text{int}}^2 \langle \psi^2 \rangle) \ll 1$, which implies $\rho_{\text{de}}/(m_{\text{int}}^2 \langle \psi^2 \rangle \phi_{\text{min}}^2/\Lambda^2) \gg 1$ so there is indeed a consistent solution in which dark energy dominates over the energy density in the $\psi - \phi$ interaction. Equation (29) leads to an upper bound on the number of e -foldings of dark energy domination analogous to Eq. (10)

$$N_{\text{de}} \ll \frac{2}{3} \log \left(\frac{m_{\text{int}} \psi_i}{\sqrt{\rho_{\text{de}}}} \right), \quad (30)$$

which can be larger than 1, suggesting that a transient dS era is plausible.

However, as in the hilltop case, there are additional complications not captured when ψ^2 is time averaged. The equation of motion for ϕ , expanding in small ϕ/Λ is

$$\ddot{\phi} + 3H_0 \dot{\phi} + (m_{\text{int}}^2 \psi_i^2 e^{-3H_0 t} \cos^2(m_\psi t) + \rho_{\text{de}}) \frac{\phi}{\Lambda^2} \approx \frac{\rho_{\text{de}}}{\Lambda}, \quad (31)$$

where we again set $t_i = 0$. We assumed before that ϕ simply rolls with the minimum obtained after time-averaging ψ , but Eq. (31) actually implies that ϕ will undergo oscillations on time scales $\Delta t \sim 1/m_\psi$ as ψ moves through its field range and ϕ 's effective potential changes. In the limit $\phi \ll \Lambda$, the amplitude of ϕ 's oscillations can be estimated as (recalling that $m_\psi \gtrsim H_0$)

$$\dot{\phi} \approx \frac{\rho_{\text{de}}}{\Lambda} \Rightarrow \frac{\Delta \phi}{\Lambda} \approx \frac{\rho_{\text{de}}}{\Lambda^2 m_\psi^2}. \quad (32)$$

To have a theory that leads to an era of dark energy domination requires $\Delta \phi/\Lambda \ll 1$. Additionally, and importantly, Eq. (32) implies that the contribution from $m_{\text{int}}^2 \phi^2 \psi^2/\Lambda^2$ to the ψ equation of motion cannot be neglected [cf. Eqs. (4) and (5)] because the ratio between the induced mass and the Lagrangian mass, m_ψ , is

$$\frac{m_{\text{int}} \phi/\Lambda}{m_\psi} = \frac{m_{\text{int}} \rho_{\text{de}}}{\Lambda^2 m_\psi^3} = 3 \frac{m_{\text{int}} H_0^2 M_{\text{Pl}}^2}{m_\psi m_\psi^2 \Lambda^2}, \quad (33)$$

which is typically larger than 1. As a result, a numerical solution of the equations of motion is required to determine whether ϕ is trapped at a field value that leads to dark energy.

The same conclusion can be reached by noting that the Klein-Gordon equation for ϕ , Eq. (31), leads to an *inhomogeneous* Mathieu equation for $\hat{\phi}$, with an exponentially growing forcing term [cf. Eq. (12)], and the result of this forcing is that ϕ oscillates with an amplitude whose order matches Eq. (32). Then, as a result of Eq. (33), the starting assumption in deriving the Mathieu equation, that the backreaction on ψ can be neglected, fails.

Similarly to the case when ϕ has a hilltop potential, theories in which ϕ has an exponential runaway can lead to cosmological histories that are consistent with current observations once the Standard Model and a dominant dark matter (again assumed to be decoupled from ϕ and ψ) are included. The evolution of ϕ and ψ in such a theory is illustrated in Fig. 6 in Appendix A 3. The nonlinear behavior in ψ 's equation of motion makes the system

somewhat more delicate, and for many parts of the parameter space (including the example theory we plot) a mild fine-tuning of the initial conditions, $\phi_i/\Lambda \lesssim 1$, is required to obtain sufficient e -folds of accelerated expansion. With this, there is indeed an era during which ϕ is trapped and sources dark energy. As the background value of $\langle\psi^2\rangle$ falls, the minimum for ϕ moves out to larger values and the oscillations in ϕ increase in amplitude. Eventually, ϕ rolls down its runaway exponential potential basically unhindered with ψ 's energy density negligible. At this stage the dynamics of the system are the same as those of a single field with an exponential runaway potential and a background energy density in matter and radiation. The evolution of such theories has been studied in detail [59] (see [60] for a recent discussion). For our parameter space, with $\Lambda < M_{\text{Pl}}/\sqrt{3}$, there is a late-time, global attractor, non-accelerating scaling solution, for which the energy density in ϕ remains a fixed, small, fraction of the total energy. We also note that the validity of our effective theory is expected to break down around $\phi \gtrsim \Lambda$, when higher order terms would become unsuppressed and general ultraviolet considerations would suggest that new light states enter the theory.

2. Allowed parameter space

The allowed parameter space of theories in which ϕ has an exponential potential can be analyzed similarly to the hilltop case. One difference is that the conditions to ensure ϕ has an induced minimum that lasts long enough, Eqs. (20) and (21), no longer apply with an exponential potential. These can be replaced by the requirement that the time dependence of the minimum should be sufficiently mild. We estimate this by requiring that our dark energy density varies, over one e -fold, at most within the 2σ confidence range inferred by Planck (even if the latter is obtained by fitting the Λ CDM model). From Eq. (28), the resulting limit is

$$\frac{\delta\Omega_{\text{de}}}{\Omega_{\text{de}}} < 0.02 \Rightarrow \phi_{\text{min}}/\Lambda \lesssim 0.008, \quad (34)$$

where we have used the Planck 2018 [61] value $\Omega_\Lambda = 0.6834 \pm 0.0084$ and Eq. (28). The remaining constraints are Eq. (22) so that ψ oscillates but does not lead to parametric resonance too early (because we expect the resonant instability to be at least as bad as for the complementary function to the homogeneous Mathieu equation); Eq. (23) from observational limits on the energy density in an extremely light component of dark matter; and the typical amplitude of ϕ 's oscillations from Eq. (32) $\Delta\phi/\Lambda < 1$. Given the complicated nonlinear dynamics arising from the exponential potential, we allow ϕ_i/Λ to be chosen small rather than imposing Eq. (18a)/(18b), which would result in ϕ being driven to a small value starting from $\phi_i \simeq \Lambda$. An additional possible constraint can

be obtained by demanding that ψ at least evolves linearly at the time that it starts oscillating Eq. (24), which depends on ϕ_i . Numerical solutions of the equations of motion show that if this condition is violated there is unlikely to be an era of dark energy domination that lasts long enough to be consistent with observations. As in the hilltop case, the physical mass of ϕ in the locked phase is typically not too much larger than H_0 .

We plot a slice of the allowed parameter space in Fig. 2 (right panel), varying m_{int} and ψ_i .⁴ We set $m_\psi = 12H_0$ and fix $\Lambda = M_{\text{Pl}}/2$ to avoid large ϕ oscillations, cf. Fig. 7 in Appendix A 3 where we show the parameter space with Λ and m_ψ varying [for a hilltop potential such Λ would lead to ψ evolving nonlinearly, which is why we picked a smaller value $\Lambda/M_{\text{Pl}} = 0.02$ in Fig. 2 (left)]. We stress that not all of the parameter space that satisfies the preceding constraints leads to a cosmologically viable era of dark energy domination because with an exponential potential ϕ strongly affects ψ 's evolution during dark energy domination. In particular, there is a tension that smaller values of m_{int} tend to lead to ϕ being trapped less efficiently and for less long, see Eq. (30), but larger m_{int} tends to make the nonlinear effects on ψ stronger, see Eq. (33). Nevertheless, numerical investigation suggests that observationally viable theories with a few e -folds of dark energy domination can be obtained over substantial parts of what we identify as the allowed parameter space, even when $\phi_i/\Lambda \sim 1$.

III. DARK RADIATION ASSISTED DARK ENERGY

Suppose now that ψ is a light field behaving not as matter but as radiation in equilibrium with a thermal bath at a temperature T_h that is less than the visible temperature T_v to satisfy observational constraints [62]. Thermal equilibrium requires a sufficiently large interaction rate $\Gamma_1 \gg H$. For relativistic particles with $m_\psi \ll T_h$ as long as the relevant processes come from a renormalizable interaction in the low-energy effective Lagrangian typically $\Gamma_1 \sim g^n T_h$, where g is some dimensionless coupling constant, n depends on the details of the hidden sector and thermal equilibrium is generally easily achieved provided T_h is not too much smaller than T_v [e.g. in the present-day Universe for $g \gtrsim (10^{-30} T_v/T_h)^{1/n}$]. The potential of Eq. (2) can provide a minimal realization of this scenario with ψ 's quartic self-interaction maintaining thermal equilibrium ($\Gamma_1 \sim \lambda^4 T_h$ for the required number changing interactions), but more complex hidden sectors with additional fields are plausible and the details are unimportant for most of our purposes. Such a thermal population could be produced e.g. from the primordial inflaton's decay at very early times.

⁴For the purposes of the plot, we assume that ψ_i is related to Ω_ψ as if ψ evolved linearly, i.e. Eq. (23), which might lead to a slight inaccuracy in parts of parameter space.

We continue to assume that ϕ_i is initially homogeneous with only the zero momentum mode populated.⁵

Given the Lagrangian equations (1)–(3), a nonzero ϕ contributes to the mass of ψ . Consequently, because the ψ radiation bath interacts with a ϕ background, a thermal effective potential is produced for ϕ . At one-loop and for $T_h \gg m_{\text{int}}\phi/\Lambda$, ϕ 's corrected potential takes the form (see e.g. [63,64])⁶

$$V(\phi, T_h) = V(\phi) + \frac{1}{2}\kappa T_h^2 \phi^2 \quad \text{with} \quad \kappa = \frac{1}{12} \left(\frac{m_{\text{int}}}{\Lambda} \right)^2. \quad (35)$$

Therefore an effective mass term is generated, which can stabilize ϕ at a value where a positive potential energy density can source dark energy. The mechanism is very similar to the dark matter assisted case discussed in Sec. II, with, roughly, the amplitude of the background oscillations in ψ replaced by the temperature T_h when ψ is in thermal equilibrium. The stabilization of ϕ by finite temperature effects is again only transient, because the induced local minimum disappears below a critical temperature.

The dark radiation assisted scenario was called *thermal dark energy* in [64] (see also [50,51] for the related thermal inflation scenario and [52] for applications to early dark energy). The case with a Hilltop potential—the thermal analogy to Sec. II A—was studied in detail in [64] and so here we focus on the exponential runaway potential, which is well-motivated from string theory compactifications.

A. Overview of dynamics and parameter space

As before we start by considering a universe containing only ϕ and ψ . Assuming the high-temperature approximation Eq. (35), the condition for the thermally corrected potential to be extremized with respect to ϕ is

$$\kappa T_h^2 \phi = \frac{\rho_{\text{de}}}{\Lambda} e^{-\phi/\Lambda}, \quad (36)$$

which leads to a minimum at

$$\frac{\phi_{\text{min}}}{\Lambda} = W_0 \left(\frac{12\rho_{\text{de}}}{m_{\text{int}}^2 T_h^2} \right). \quad (37)$$

For this to be consistent, ϕ_{min} must be sufficiently small that the mass contribution to the dark radiation ψ

⁵A thermal population of ϕ is typically produced subsequently by $\psi - \psi$ scatterings, but this does not affect the dynamics that we consider.

⁶The thermal effective potential is $V(\phi_c, T_h) = V(\phi_c) + \frac{T_h^4}{2\pi^2} J_B((m_{\psi}^{\text{eff}}(\phi_c))^2/T_h^2)$ with $J_B(x^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12}x^2 - \frac{\pi}{6}x^3 + \dots$ for $x \ll 1$ [65].

$$\frac{m_{\text{int}}\phi}{\Lambda} \ll T_h. \quad (38)$$

Physically Eq. (38) corresponds to ψ being present in the thermal bath. While Eq. (38) is satisfied there is always a minimum for ϕ , given by Eq. (37). ϕ 's potential energy at $\phi = \phi_{\text{min}}$ dominates the energy density in dark radiation provided

$$\rho_{\text{de}} e^{-\phi_{\text{min}}/\Lambda} \gg \rho_{\psi} = \frac{\pi^2}{30} T_h^4. \quad (39)$$

Similarly to Eq. (28), as the temperature of the hidden sector falls the minimum equation (37) moves out to larger field values. Using $T_h \propto 1/a$, one can easily show that the time dependence of the dark energy density is

$$\frac{\partial \log V(\phi_{\text{min}})}{\partial \log a} = -\frac{2\phi_{\text{min}}/\Lambda}{1 + \phi_{\text{min}}/\Lambda}, \quad (40)$$

so to avoid too fast a change we again require

$$\phi_{\text{min}}/\Lambda \ll 1. \quad (41)$$

The high-temperature approximation to the thermal potential fails once the hidden sector temperature has decreased to $T_{\text{hend}} = m_{\psi}^{\text{eff}}$. If ϕ 's induced minimum equation (37) still satisfies $\phi_{\text{min}}/\Lambda \ll 1$ at this time then

$$T_{\text{hend}} = \left(\frac{\rho_{\text{de}}}{m_{\text{int}}} \right)^{\frac{1}{3}}. \quad (42)$$

Subsequently, thermal contributions to ϕ 's effective potential become exponentially suppressed, the induced minimum for ϕ disappears and ϕ begins to run away.

Equations (41) and (42) lead to upper bounds on the number of e -folds of dark energy domination with approximately constant energy density. Denoting the visible sector temperature when dark energy dominations starts as $T_{\text{v,de}}$, and defining $\xi_h = T_{\text{h,0}}/T_{\text{v,0}}$ with $T_{\text{h,0}}$ and $T_{\text{v,0}}$ the hidden sector and visible sector (photon) temperatures today respectively, we obtain

$$N_{\text{de}} \leq \min \left\{ \log \left(\frac{\xi_h T_{\text{v,de}} m_{\text{int}}}{\sqrt{12} \rho_{\text{de}}^{1/2}} \right), \log \left(\frac{\xi_h T_{\text{v,de}} m_{\text{int}}^{1/3}}{\rho_{\text{de}}^{1/3}} \right) \right\}. \quad (43)$$

Note that strongest bound typically comes from the second term on the right-hand side because $m_{\text{int}} \gtrsim \rho_{\text{de}}$ in our parameter space of interest.

Additionally, with its zero temperature potential having the assumed runaway form, ϕ 's thermally corrected potential always has a global minimum with vanishing potential energy out at $\phi \rightarrow \infty$ (in the hilltop case, such a minimum exists at $\phi = \Lambda$ for $T_h \ll m_{\text{int}}$). Therefore, an exit from the transient dS can take place by ϕ quantum [66] or thermal

tunneling [67] through the barrier in its finite temperature corrected potential leading to a first order phase transition, potentially shortening the era of dark energy domination. We analyze these processes in Appendix B 2 and show that, while the high-temperature approximation is valid, the rate of tunneling is negligible provided

$$\frac{32\pi^2}{3} \frac{\Lambda^4}{\rho_{\text{de}}} \frac{T_{\text{h}}^3}{m_{\text{int}}^3} \gg 1, \quad (44a)$$

and

$$\frac{8\pi\sqrt{3}}{5} \frac{\Lambda^2}{\rho_{\text{de}}^{1/2}} \frac{T_{\text{h}}^{3/2}}{m_{\text{int}}^{5/2}} \gg 1, \quad (44b)$$

for quantum and thermal fluctuations respectively. If these conditions are satisfied, tunneling only becomes significant immediately prior to when dark energy is predicted to end from Eq. (43) anyway (at which time the barrier in the potential is on the edge of vanishing). Meanwhile, there are no effects analogous to the instability time or parametric resonance that were relevant to the dark matter assisted scenario because the thermal fluctuations are fast and incoherent (with period $\sim 1/T_{\text{h}}$) compared to the time scale on which ϕ rolls, $\Delta t \sim \Lambda/\sqrt{\rho_{\text{de}}}$ (see Sec. IV A below).

As mentioned in the Introduction, the dark radiation assisted scenario requires a super-cooled phase transition in the hidden sector. In practice, this corresponds to $\phi_{\text{min}} \ll \Lambda$ in Eq. (37) in combination with ρ_{de} a few orders of magnitude larger than the energy density in hidden sector radiation equation (39), which together imposes

$$T_{\text{h}}^4 \ll \rho_{\text{de}} \ll m_{\text{int}}^2 T_{\text{h}}^2. \quad (45)$$

As a result, the coupling constant of the quartic ϕ self-interaction $\rho_{\text{de}}/\Lambda^4 \ll m_{\text{int}}^4/\Lambda^4 \ll 1$ (recall that $m_{\text{int}}^2/\Lambda^2$ is the coupling constant of the quartic $\phi - \psi$ interaction), and for the Λ and m_{int} we have in mind these values are tiny (similarly small couplings are also needed for the dark radiation assisted scenario with a hilltop potential). For comparison, in the dark matter assisted case the analogous conditions [discussed below Eq. (23)] require $m_{\text{int}}^2 \gg m_{\psi}^2$.

B. Working example with realistic cosmological history

Similarly to the dark matter assisted scenario, in a full cosmological history ϕ can be driven to the required field value (in this case its high-temperature minimum) at early times independently of its initial value, e.g. even if $\phi_i \approx \Lambda$. For this to occur, at some time after primordial inflation the hidden sector must be in thermal equilibrium with a temperature that satisfies $T_{\text{h}} > m_{\text{int}}$ so that the finite temperature correction to ϕ 's potential is relevant despite $\phi = \Lambda$ inducing a large ψ mass. Moreover, at the same time the resulting gradient in ϕ 's equation of motion must be

large enough to overcome Hubble friction. In Appendix B 1, we show that these two conditions are simultaneously satisfied, and ϕ evolves to $\phi/\Lambda \ll 1$, provided

$$\frac{\Lambda}{M_{\text{Pl}}} < \xi_{\text{h}}^2 \quad \text{and} \quad \frac{m_{\text{int}}}{M_{\text{Pl}}} < \lambda^4 \xi_{\text{h}}^2, \quad (46)$$

where we assume the hidden sector is kept in thermal equilibrium by interactions of typical rate $\Gamma_{\text{I}} \sim \lambda^4 T_{\text{h}}$ [which is appropriate to our minimal model equation (2) but can be relaxed in more complex theories]. Alternatively, it may simply be assumed that $\phi_i \ll \Lambda$.

In Fig. 3 we show an example of a theory of dark radiation assisted dark energy with ϕ 's potential having an exponential runaway that leads to a realistic cosmological history, with the evolution of ϕ obtained by numerically solving its equation of motion and the Friedmann equation. The theory includes the Standard Model and a separate source of dark matter, which, as usual, we assume are totally decoupled from ϕ and ψ . As expected, at times when the total mass of ψ (i.e. the combination of its bare mass and the mass induced by ϕ) is less than T_{h} , ϕ is trapped at a local minimum close to $\phi/\Lambda = 0$ where it sources dark energy. Once the mass of ψ is comparable to T_{h} , which is reached both directly due to T_{h} decreasing and also because ϕ_{min} increases, the thermal correction to ϕ 's potential becomes negligible. Subsequently, ϕ rolls down its zero temperature potential with its energy density dominated by kinetic energy, which therefore redshifts as a^{-6} . Similarly to the dark matter assisted exponential case, after the dark energy dominated epoch ends the system will approach the attractor, scaling solution with nonaccelerated expansion described in [59], although this happens beyond the range that we plot.

C. Allowed parameter space

For both the exponential and hilltop potentials, the parameter space of theories of dark radiation assisted dark energy consists of

$$\Lambda, \quad m_{\psi}, \quad m_{\text{int}}, \quad \xi_{\text{h}}, \quad \phi_i. \quad (47)$$

We assume that the number of relativistic degrees of freedom in the hidden sector is constant and that entropy in the hidden sector is conserved.⁷

The temperature of the hidden sector, i.e. ξ_{h} , is constrained by the observed expansion history of the Universe, parametrized by the effective number of neutrinos,

⁷The ratio of the hidden sector temperature to the visible sector temperature is not constant in the early Universe due to the change in the Standard Model number of degrees of freedom g_s , which depends on the temperature and has value $g_{s,0} = 3.909$ today; indeed, $T_{\text{h}}/T_{\text{v}} = \xi_{\text{h}}(g_s(T_{\text{v}})/g_{s,0})^{1/3}$.

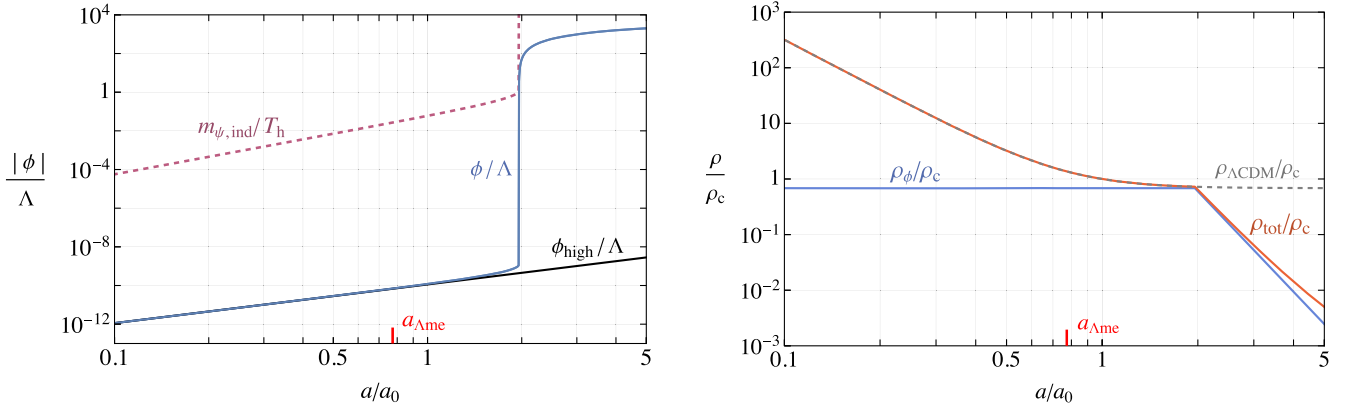


FIG. 3. Left: the evolution of ϕ with scale factor a in a theory of dark radiation assisted dark energy in which $V(\phi)$ takes the exponential runaway form of Eq. (3). We include the contributions of the Standard Model and dark matter to the expansion history assuming that these are decoupled from ϕ and ψ . The Lagrangian parameters are $\Lambda = 10^{-3}M_{\text{Pl}}$, $m_{\text{int}} = 10^8 T_{\text{v},0}$ and $m_\psi = 0$, and we set $\xi_h = 0.2$. We fix that at early times ϕ is in the minimum of its finite temperature-corrected potential (which can arise automatically from dynamics at much earlier times than shown). We also plot the ratio between the contribution to ψ 's mass from ϕ 's expectation value $m_{\psi,\text{ind}} \equiv m_{\text{int}}\phi/\Lambda$ and T_h . Finally, we plot the minimum of ϕ 's corrected potential assuming the high-temperature approximation Eq. (35), ϕ_{high} , which is tracked by ϕ while $m_{\psi,\text{ind}} \ll T_h$. Once $m_{\psi,\text{ind}}/T_h \simeq 1$ the finite temperature correction to ϕ 's potential is exponentially suppressed and ϕ rolls unhindered to large field values. Right: the energy density of ϕ (ρ_ϕ) and the total energy density including the Standard Model and dark matter (ρ_{tot}) of the theory plotted in the left panel, normalized to the critical energy density today (ρ_c). For comparison we also show the total energy density in Λ CDM ($\rho_{\Lambda\text{CDM}}$). While ϕ is trapped near $\phi/\Lambda \ll 1$ it sources dark energy and a Λ CDM cosmology is reproduced to better than % precision. After ϕ rolls down its zero temperature potential at $a/a_0 \simeq 2$, $\rho_\phi \propto a^{-6}$, because this is dominantly in the form of kinetic energy (at sufficiently late times, beyond the range of the plot, the theory will pick up the tracker solution for an exponential potential).

$$N_{\text{eff}} \approx 3.046 + \frac{4}{7} \left(\frac{11}{4} \right)^{4/3} g_h \xi_h^4, \quad (48)$$

where the first term corresponds to the effective number of neutrinos in the Standard Model accounting for noninstantaneous neutrino decoupling [68] and g_h counts the number of degrees of freedom in the hidden sector, weighted by 1 for bosons and $\frac{7}{8}$ for fermions (see e.g. [62] for more details). N_{eff} is bounded by cosmic microwave background observations to be $N_{\text{eff}} < 3.28$ [61] (the exact numerical value depends on which datasets are included in the fit and the value of the present-day Hubble parameter that is adopted). In our minimal model with only ψ and ϕ in the hidden sector thermal bath⁸ $g_h = 2$, and the corresponding constraint is

$$\xi_h \lesssim 0.48. \quad (49)$$

Big bang nucleosynthesis also leads to bounds on ξ_h that are similar to Eq. (49) [69].

In the case that ϕ has an exponential potential, further observational constraints arise from the time variation in dark energy. Using Eq. (40), asking that the change in $V(\phi)$

⁸As previously mentioned, a thermal population of ϕ is typically produced by $\psi\psi \rightarrow \phi\phi$ interactions.

across one e -fold stays within 2σ of the Planck 2018 [61] results on $\Omega_\Lambda = 0.6834 \pm 0.0084$ (albeit inferred by fitting the Λ CDM model), implies

$$\frac{\delta\Omega_{\text{de}}}{\Omega_{\text{de}}} < 0.02 \Rightarrow \phi_{\text{min}}/\Lambda \lesssim 0.01. \quad (50)$$

Additionally, for the high-temperature approximation to ψ 's contribution to ϕ 's thermally corrected potential to be valid today requires Eq. (42)

$$T_h > T_{\text{hend}} \equiv \left(\frac{\rho_{\text{de}}}{m_{\text{int}}} \right)^{\frac{1}{3}}, \quad (51)$$

which, as discussed around Eq. (43), is a stronger condition than that from the time variation of ρ_{de} , Eq. (50). To avoid nonperturbative decay through bubble nucleation, Eqs. (44a) and (44b) must be satisfied for the present-day hidden sector temperature $T_{h,0} = T_{\text{v},0}\xi_h$, which is easily achieved for the relatively large Λ that we have in mind. Additionally, if we require that ϕ is driven to the minimum of its finite temperature-corrected potential starting from $\phi_i \simeq \Lambda$ then Eq. (46) must be satisfied. Finally we note that the quartic interaction between ϕ and ψ has coupling constant $m_{\text{int}}^2/\Lambda^2$ and this must be $\lesssim 1$ for the theory to be weakly coupled (this constraint is

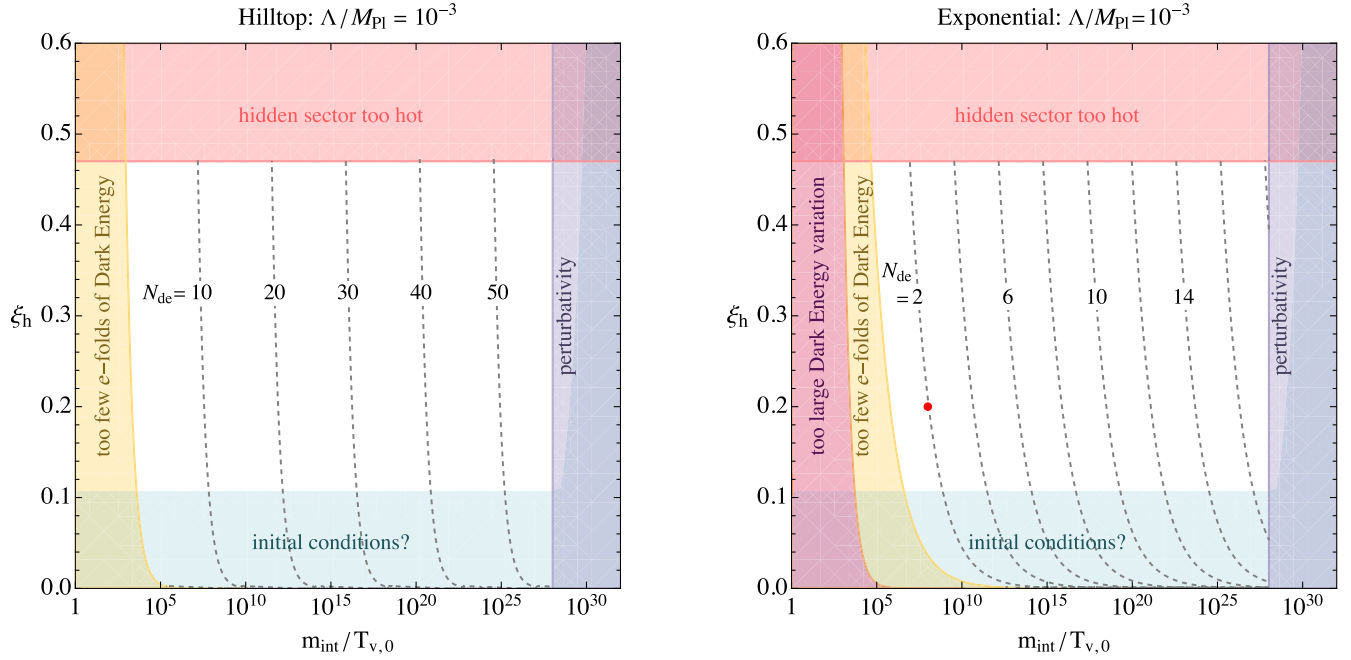


FIG. 4. Slices of the parameter space for dark radiation assisted dark energy, Left: when ϕ has a hilltop potential. Right: when ϕ 's potential has an exponential runaway [see Eqs. (2) and (3)]. The results are shown as a function of the ratio of the hidden sector and visible sector temperatures ξ_h and the Lagrangian parameter m_{int} with $T_{v,0}$ the visible sector temperature today. In both cases there are observational constraints from the hidden sector not containing too much energy density (“hidden sector too hot”) and from the era of dark energy domination not lasting long enough (“too few e -folds of dark energy”). In the case of an exponential potential there is an additional, weaker, constraint from the dark energy density varying too fast for observational limits (“too large dark energy variation”). We also impose that the quartic coupling between ϕ and ψ , $m_{\text{int}}^2/\Lambda^2$ is smaller than 1 (“perturbativity”). Finally, if we require that ϕ is dynamically driven to $\phi/\Lambda \simeq 0$ in the early Universe the region labeled “initial conditions?” is excluded, although such dynamics are not needed if $\phi_i \ll \Lambda$ is assumed. The number of e -folds of dark energy domination, N_{de} , is also shown. For both types of potential N_{de} can easily be sufficiently large for an observationally viable cosmological history, although in the case of an exponential potential there are typically fewer e -folds of dark energy domination in total. The red dot in the exponential potential plot corresponds to the theory shown in Fig. 3.

irrelevant for the typical parameter values we are interested in).

In Fig. 4 we plot a slice of the parameter space of dark radiation assisted dark energy theories, varying ξ_h and m_{int} with Λ fixed. Results are shown for theories in which ϕ 's potential takes the exponential form and, for comparison, also the case of a hilltop potential analyzed in detail in Ref. [64]. The constraints have similar origins for the two forms of the potential, except that with a hilltop potential the bound equation (50) from the time variation of ϕ_{min} is absent because as long as a metastable minimum exists it is at $\phi = 0$. We see that theories in which ϕ has an exponential runaway lead to fewer e -folds of dark energy than if ϕ had a hilltop potential. Nevertheless, in both cases, observationally viable theories are possible over large parts of the parameter space, with the constraints on the hidden sector temperature, i.e. ξ_h , being perhaps the most important.

Note that theories consistent with observations are possible for $\Lambda \ll M_{\text{Pl}}$. In this case, the physical mass of ϕ while in the metastable minimum $\simeq m_{\text{int}} T_h/\Lambda$ can be

large compared to H_0 and instead need only be smaller than T_h (so that the $\phi - \psi$ interaction is perturbative). Alternatively, if $\Lambda \simeq M_{\text{Pl}}$ is assumed then the mass of ϕ is similar to H_0 unless $m_{\text{int}} \gg T_{v,0}$.

IV. QUINTESSENCE ASSISTED DARK ENERGY

We now return to the possibility that, as in the dark matter assisted case, both ϕ and ψ are homogeneous and isotropic with $\psi_i \neq 0$ away from its minimum and, although not essential, $\dot{\phi} = \dot{\psi} = 0$ initially (and no thermal population), and we again assume $\lambda \ll m_\psi^2/M_{\text{Pl}}^2$ so that ψ self-interactions are negligible. However, here we suppose that $m_\psi \lesssim 3H_0$. As a result ψ can behave as a cosmologically frozen quintessence field sourcing a sub-dominant component of dark energy. We will show that in such theories the $\phi - \psi$ interaction circumvents many difficulties faced by single-field slow-roll quintessence and a transient era of dark energy domination sourced by $V(\phi)$ can easily be obtained. Moreover, there is an automatic end to accelerated expansion and the initial conditions, ϕ_i and

ψ_i , do not need to be tuned. Because many aspects of this regime are similar to the first phase of the dark matter assisted scenario before ψ begins to roll, we focus on overall physical intuition rather than detailed numerical analysis.

A. Challenges for slow-roll quintessence when $m_{\text{int}} = 0$

We know that when $m_{\text{int}} = 0$, that is ϕ and ψ are decoupled, ϕ cannot source accelerated expansion without fine-tuned initial conditions and/or large field displacements, $\gtrsim M_{\text{Pl}}$, which take us outside the regime of control in the effective field theory (because corrections of the form $\phi^n/M_{\text{Pl}}^{n-4}$ with $n > 4$ become important and moreover towers of states typically become light as predicted by the swampland distance conjecture [70]) [23,24]. Similarly, super-Planckian field displacements are necessary for ψ to source accelerated expansion.

In more detail, in the case of a hilltop potential the solution to ϕ 's equation of motion is, while $\phi \ll \Lambda$, approximately

$$\phi(t) \approx \phi_i e^{\sqrt{12}H_0 \frac{M_{\text{Pl}}}{\Lambda} t}, \quad (52)$$

assuming $M_{\text{Pl}} \gg \Lambda$ and fixing $\dot{\phi}_i = 0$ at $t_i = 0$. As a result, the number of e -folds of accelerated expansion generated by ϕ rolling from an initial value $\phi_i \ll \Lambda$ to the minimum at $\phi = \Lambda$ is

$$N_{\text{de}} \simeq H_0 t_\Lambda \simeq \frac{\Lambda}{\sqrt{12}M_{\text{Pl}}} \log\left(\frac{\Lambda}{\phi_i}\right). \quad (53)$$

Therefore, to have any significant number of e -folds ϕ_i must be fine-tuned to lie exponentially close to the top of its potential and/or $\Lambda \gg M_{\text{Pl}}$. This can also be seen from the slow-roll conditions near the hilltop:

$$\frac{M_{\text{Pl}}^2}{2} \left(\frac{V_\phi(\phi)}{V}\right)^2 \ll 1 \Rightarrow \frac{8M_{\text{Pl}}^2 \phi^2}{\Lambda^4} \ll 1, \quad (54a)$$

$$\text{and } M_{\text{Pl}}^2 \frac{|V_{\phi\phi}(\phi)|}{V} \ll 1 \Rightarrow \frac{4M_{\text{Pl}}^2}{\Lambda^2} \ll 1. \quad (54b)$$

The case that ϕ has an exponential potential likewise requires $\Lambda \gg M_{\text{Pl}}$ for the slow-roll conditions to be satisfied or fine-tuned of initial conditions, such that ϕ starts off rolling up its potential and comes momentarily to rest to drive a transient acceleration before rolling back down (see e.g. [71,72] for reviews on the observational and theoretical challenges in the latter scenario). Similarly, ψ can only source accelerated expansion if it takes a super-Planckian field value $\psi_i \gtrsim M_{\text{Pl}}$ (with $m_\psi \lesssim H_0$ to match the observed dark energy density).

It is worth noting that there are plenty of more involved models for slow-roll inflation/quintessence (see e.g.

[73–79]), but these usually require some fine-tuning between Lagrangian parameters to produce a sufficiently flat potential. An exception is assisted inflation/quintessence [76,77], which is related to the theory we consider and involves fields slow rolling due to the Hubble friction sourced by other fields. Assisted quintessence still requires fine-tuning of initial conditions and implies an equation of state too far from $w_{\text{de}} = -1$ to be compatible with recent observations; moreover, interactions between different fields—as are expected in ultraviolet models such as supergravity—are problematic [77] making such theories difficult to realize [80].

B. Slow-roll and a transient dS vacuum for $m_{\text{int}} \neq 0$

Remarkably, the problems of single field quintessence just discussed can be overcome simply by switching on the interaction term $m_{\text{int}} > 0$ in Eq. (2). Let us assume that m_ψ , m_{int} and ϕ_i are such that ψ is frozen at $\psi_i \lesssim M_{\text{Pl}}$. Then the contribution from ψ_i to the mass of ϕ via m_{int} can induce a minimum for ϕ . For a hilltop potential such a minimum exists provided

$$\psi_i > \psi_{\text{crit}} \equiv 2\sqrt{\rho_{\text{de}}}/m_{\text{int}}, \quad (55)$$

in which case $\phi_{\text{min}} = 0$. Meanwhile, if ϕ 's potential has the exponential form a minimum exists for any $\psi \neq 0$ with

$$\frac{\phi_{\text{min}}}{\Lambda} = W_0 \left(\frac{\rho_{\text{de}}}{m_{\text{int}}^2 \psi^2} \right), \quad (56)$$

but similarly to the hilltop case, $\phi_{\text{min}}/\Lambda \ll 1$ if $\psi_i \gtrsim \psi_{\text{crit}}$ defined in Eq. (55). For the remainder of this section we assume that ψ_i indeed satisfies this condition.

To begin with, let us also assume that ϕ_i is close to its induced minimum (we will relax this condition at the end of the section) and $\dot{\phi}(t_i) = 0$. We impose that the potential energy sourced by ϕ is greater than contributions from ψ and the $\phi - \psi$ interaction term

$$\rho_{\text{de}} \gg \frac{1}{2} \frac{m_{\text{int}}^2}{\Lambda^2} \phi_i^2 \psi_i^2 + \frac{1}{2} m_\psi^2 \psi_i^2. \quad (57)$$

For the case of a hilltop potential, this requirement is satisfied provided $m_\psi \psi_i / (H_0 M_{\text{Pl}}) \ll 1$ with ϕ_i/Λ assumed sufficiently close to the induced minimum at 0. Meanwhile, for an exponential potential Eq. (57) is consistent with Eq. (56) if $m_{\text{int}} \psi_i / (H_0 M_{\text{Pl}}) \gg 1$ and $m_\psi \psi_i / (H_0 M_{\text{Pl}}) \ll 1$ (which is possible for $m_{\text{int}} \gg m_\psi$).

The initial Hubble parameter in a theory that satisfied the preceding conditions is given by

$$H_i \equiv H(t_i) \approx \sqrt{\rho_{\text{de}} / (3M_{\text{Pl}}^2)}. \quad (58)$$

Then for $m_\psi \ll H_0$ ($\approx H_i$) and $m_{\text{int}} \phi_i / \Lambda \ll H_0$, $dV/d\psi \ll M_{\text{Pl}} H_0^2$ and ψ slowly rolls, even for sub-Planckian initial

field values, while ϕ remains trapped. The key change compared to single-field quintessence is that the slowly rolling field is distinct from the field sourcing dark energy; the slow-roll conditions thus reduce to

$$\frac{M_{\text{Pl}}^2}{2} \left(\frac{V_\psi(\psi)}{V(\phi)} \right)^2 \ll 1, \quad (59a)$$

$$\text{and } M_{\text{Pl}}^2 \frac{|V_{\psi\psi}(\psi)|}{V(\phi)} \ll 1, \quad (59b)$$

with $V(\psi) \approx m_\psi^2 \psi^2$. These can be more easily satisfied than the analogous conditions in a theory of single field quintessence because the scales in $V(\psi)$ and $V(\phi)$ can be separated.

C. The end of dark energy domination

The slowly rolling ψ evolves according to the Klein-Gordon equation (5) with

$$\psi \approx \psi_i \left(1 - \frac{m_\psi^2 t}{3H_0} \right). \quad (60)$$

In the hilltop case this background induces a minimum for ϕ at the origin until ψ falls below the critical value, $\psi_{\text{crit}} = 2\sqrt{\rho_{\text{de}}}/m_{\text{int}}$, at which point ϕ rolls away from the top of its potential according to Eq. (52). The time scale for ψ to reach ψ_{crit} (~ 0 assuming $2\sqrt{\rho_{\text{de}}}/m_{\text{int}} \ll \psi_i$) is $t_{\text{crit}} \sim 3H_0/m_\psi^2$, giving

$$N_{\text{de}} = H_0 t_{\text{crit}} = \frac{3H_0^2}{m_\psi^2}, \quad (61)$$

e -folds of dark energy domination. For $m_\psi \lesssim H_0$, the $N_{\text{de}} = \log(a_0/a_{\Lambda\text{me}}) \simeq 0.26$ e -folds observed in the real Universe thus far can easily be reached (where $a_{\Lambda\text{me}}$ is the scale factor at dark energy-matter equality).

In the exponential case ϕ tracks its induced minimum equation (56), which shifts to larger values as ψ slowly rolls down its potential. Using Eq. (60) to show that $\partial\psi/\partial a = -m_\psi^2\psi_i/(3H_0^2 a)$, one can deduce that the rate of change in the dark energy density $V \approx \rho_{\text{de}} e^{-\phi_{\text{min}}/\Lambda}$ is

$$\frac{\partial \log V}{\partial \log a} \approx -\frac{2m_\psi^2 \phi_{\text{min}}}{3H_0^2 \Lambda}, \quad (62)$$

which allows an extended era of dark energy domination with ρ_{de} approximately constant if $m_\psi \ll H_0$ and $\phi_{\text{min}}/\Lambda \ll 1$. Eventually, the value of ψ decreases so much that the minimum in ϕ moves too quickly outwards and ϕ effectively rolls away. Once ψ settles to its minimum at $\psi = 0$, the minimum of ϕ is out at infinity and the era with $w \simeq -1$ ends. If the exponential form of the potential

remains valid at $\phi \gtrsim \Lambda$ the system will approach the same nonaccelerating scaling solution described for the dark matter assisted scenario and in Ref. [59], but as discussed we expect the effective field theory to break down at this point anyway.

D. Relaxed initial conditions

Similarly to the dark matter and dark radiation assisted scenarios, in a full cosmological history the quintessence assisted scenario requires no fine-tuning of initial conditions. Indeed, as mentioned, the dynamics are identical to that in the first phase of the dark matter assisted case, before ψ has begun to roll; see the discussion around Eqs. (18a) and (18b) and in Appendix A 2. To briefly recap, assume that at early times $\phi_i \sim \Lambda$ and $\psi_i \sim M_{\text{Pl}}$, and both fields start off frozen by a high Hubble friction (even if $\dot{\phi}$ and $\dot{\psi}$ start off nonzero, they will quickly become frozen). Since $\psi_i \gg \phi_i$, the earliest effect of the interaction term is on the dynamics of ϕ ; when $H = H_{\phi\text{roll}} \equiv m_{\text{int}}\psi_i/\Lambda$, ϕ thaws and rolls towards its effective minimum, around which it oscillates and sources dark energy. The ϕ oscillations decrease as matter, $\phi = \phi_i(a_{\phi\text{roll}}/a)^{3/2}$, thus ϕ today is given by

$$\phi_0 = \phi_i \Omega_m^{1/2} \frac{H_0 \Lambda}{m_{\text{int}} \psi_i}, \quad (63)$$

where we assume that ϕ begins to roll during matter domination, as is the case in all of our parameter space of interest. Therefore, for $m_{\text{int}} \gg H_0$ and $\psi_i \gg \Lambda$, ϕ is driven to its effective minimum, close to the origin. At the same time, $m_{\text{int}}\phi/\Lambda \ll H$ throughout the evolution because $\phi \propto H$ during matter domination, which, together with $m_\psi < H_0$, ensures that ψ remains frozen.

To summarize, although quintessence assisted dark energy requires a small mass $m_\psi \lesssim H_0$ and a somewhat larger $m_{\text{int}} \gtrsim H_0$, it can lead to an epoch of accelerated expansion despite sub-Planckian field distances being traversed, thus staying within the regime of validity of the effective field theory. The physical mass of ϕ while trapped depends on Λ . Perhaps the most plausible possibility is to avoid large separations of scale with Λ not too much smaller than M_{Pl} and m_{int} not too much larger than H_0 , in which case the physical mass of ϕ is only somewhat larger than H_0 . As with our other assisted scenarios, these theories do not rely on fine-tuned initial conditions and also automatically lead to an end to accelerated expansion. A numerical solution of the equations of motion of a cosmologically viable example theory with ϕ having an exponential potential can be found in Fig. 9 in Appendix C. Interestingly, none of the parameters of that theory are particularly extreme, e.g. $m_\psi = H_0/5$ and $m_{\text{int}} = 40H_0$ are only separated by ~ 2 orders of magnitude.

V. FINE-TUNING

All three scenarios discussed involve light scalar fields: when ψ is dark matter we need $m_\psi \lesssim 15H_0$, when it is dark radiation we need $m_\psi \ll \rho_{\text{de}}^{1/4}$ and when it is quintessence we need $m_\psi \ll H_0$. Meanwhile, the dark energy field ϕ has an effective mass $m_{\text{phy}} \equiv m_{\text{int}} \sqrt{\langle \psi^2 \rangle} / \Lambda$, which must be larger than the Lagrangian mass scale $m_\phi \sim \rho_{\text{de}}^{1/2} / \Lambda$, so we require $m_{\text{int}} \gtrsim H_0$ for $\psi \lesssim M_{\text{Pl}}$. In fact, for a sustained epoch of dark energy domination $m_{\text{int}} \gtrsim 10^4 H_0$, $m_{\text{int}} \gtrsim 10^7 \rho_{\text{de}}^{1/4}$ and $m_{\text{int}} \gtrsim H_0$ are needed for the dark matter, dark radiation and quintessence assisted scenarios respectively.

These scalar masses would typically receive loop corrections that are sensitive to the scale of the UV completion of the theory, Λ_{UV} , which we may take to be somewhere below the string scale, and from loops of visible sector states that will typically interact with the dark sectors at least gravitationally. For instance, the key $\phi - \psi$ interaction term, $\frac{1}{2} \frac{m_{\text{int}}^2}{\Lambda^2} \phi^2 \psi^2$ in Eq. (2), itself leads to corrections to the mass-squared parameters of ϕ and ψ of

$$\Delta m_\psi^2 \approx \frac{m_{\text{int}}^2}{32\pi^2 \Lambda^2} \Lambda_{\text{UV}}^2 \quad \text{and} \quad \Delta m_\phi^2 \approx \frac{m_{\text{int}}^2}{32\pi^2 \Lambda^2} \Lambda_{\text{UV}}^2. \quad (64)$$

Such a radiative correction to the mass of ψ can be small, $\Delta m_\psi \ll m_\psi$, without fine-tuning in all three scenarios. In the dark matter and quintessence assisted cases, $\Delta m_\psi \ll m_\psi$ [$\lesssim H_0$ for quintessence and $\mathcal{O}(10)H_0$ for dark matter] provided that $\Lambda_{\text{UV}} \lesssim \Lambda$.⁹ In the dark radiation case, we need $\Delta m_\psi \ll m_\psi \lesssim T_{\text{h}} \lesssim \rho_{\text{de}}^{1/4}$, and we find again that $\Lambda_{\text{UV}} \lesssim \Lambda$. It is not a problem if the UV cutoff Λ_{UV} lies below Λ because all the energy densities in our theories remain far below Λ_{UV}^4 and the UV completion could correspond to physics that does not alter the dynamics described, e.g. the appearance of supersymmetric partners that cut off the UV divergence.

We also have to consider the loop contributions to the mass of ϕ , Δm_ϕ^2 , in Eq. (64). For this to correspond to a small correction $\Delta m_\phi^2 \ll |m_\phi^2| \lesssim m_{\text{phy}}^2$ requires that

$$\Lambda_{\text{UV}} \ll M_{\text{Pl}} H_0 / m_{\text{int}}. \quad (65)$$

In the dark matter and quintessence assisted cases, Eq. (65) can be achieved without fine-tuning simply with a UV cutoff somewhere below M_{Pl} because the condition that a minimum for ϕ is induced by the interaction term is $\psi_i m_{\text{int}} / \Lambda \gg \sqrt{\rho_{\text{de}}} / \Lambda$ with $\psi_i \lesssim M_{\text{Pl}}$. However, Eq. (65) is more problematic in the dark radiation case. Using $m_{\text{int}} \gtrsim \rho_{\text{de}}^{1/4}$, it requires a cutoff $\Lambda_{\text{UV}} \ll \rho_{\text{de}}^{1/4}$ below the energy scale of our effective theory. As a result, the theories

we have considered require some fine-tuning to keep scalar masses sufficiently small. It would be interesting to analyze whether this could be avoided in more complicated theories, e.g. whether a supersymmetric theory with breaking scale $m_{\text{soft hid}} \lesssim \rho_{\text{de}}^{1/4}$ can lead to the same dynamics. We note that in such a theory, the hidden sector would need to be sequestered from the visible sector supersymmetry breaking given that the visible sector soft terms are at least at the TeV scale. Within string theory, this could be achieved by some geometric separation between the dark sectors and supersymmetry breaking sectors within the extra dimensions, e.g. as in the constructions of Ref. [81].

In addition, there is an unavoidable interaction between the dark sectors and all other states, including e.g. visible sector states, via graviton exchange. This leads to contributions to the scalar mass (see e.g. [82])

$$\Delta m_\phi^2 \sim \frac{1}{(4\pi)^6} \frac{M^6}{M_{\text{Pl}}^4} \quad \text{and} \quad \Delta m_\psi^2 \sim \frac{1}{(4\pi)^6} \frac{M^6}{M_{\text{Pl}}^4}, \quad (66)$$

with M the mass of the additional states considered. In the dark matter and quintessence assisted cases, the mass of ψ is less than the mass of ϕ , so corrections to m_ψ are most dangerous and $\Delta m_\psi \ll m_\psi$ can be satisfied without fine-tuning only for $M < \text{GeV}$. In the dark radiation case the mass of ϕ , $\sim \rho_{\text{de}}^{1/2} / \Lambda$ is much smaller than the mass of ψ , $\sim T_{\text{h}}$, and $M < (M_{\text{Pl}} / \Lambda)^{1/3} \text{ GeV}$ is required to avoid fine-tuning from Eq. (66).

Finally, there is a danger ψ and ϕ might receive too large corrections due to additional couplings to heavy states at, say, the string scale. This UV sensitivity is potentially difficult, but no worse than normal quintessence, see e.g. [83]. Supersymmetry in the dark sector might help to suppress such corrections, provided that the supersymmetry breaking scale in the dark sector is sufficiently low. Similarly, sequestering between the dark sector and visible sector could help suppress portal couplings that lead to unobserved fifth forces, which, as we discuss in Sec. VII, are required by observations to be weaker than Planckian.

To summarize, the fine-tuning required to avoid UV sensitivity from the $\phi - \psi$ quartic interaction term is much milder in the dark matter and quintessence assisted scenarios compared to the dark radiation scenario. This is because for dark radiation assistance, one needs relatively strong couplings between the dark energy field and the thermal bath via $m_{\text{int}} \gtrsim \rho_{\text{de}}$, whereas for dark matter and quintessence assistance, ϕ and ψ couple relatively weakly $m_{\text{int}} \sim H_0$. On the other hand, the small value of m_ψ required for dark matter and quintessence assistance means that their interactions with other sectors present (including the Standard Model and e.g. a supersymmetry breaking sector) must be even more sequestered than for the dark radiation assisted case.

⁹This is better than the constraints found in locked inflation [40], where an order one quartic interaction was assumed.

VI. OUT OF THE SWAMPLAND

Conceptual issues, together with technical challenges in identifying parametrically controlled metastable de Sitter vacua within string theory, have led to the (controversial) conjecture that metastable de Sitter vacua are inconsistent with quantum gravity; in other words, they lie in the “string theory swampland.” The de Sitter swampland conjecture states that the scalar potential of an effective field theory that descends from quantum gravity must satisfy [9]

$$\text{either } \epsilon \equiv M_{\text{Pl}} \frac{|\nabla V|}{V} \geq \mathcal{O}(1) \quad (67a)$$

$$\text{or } \eta \equiv M_{\text{Pl}}^2 \frac{\min(\nabla_i \nabla_j V)}{V} \leq -\mathcal{O}(1), \quad (67b)$$

in parts of field space with $V > 0$. This means that any de Sitter solution must be unstable. The conjecture is widely believed to be true in the asymptotic regions of moduli space, where it is motivated by the swampland distance conjecture [70], and it has been proposed to be true even in the interior of the moduli space.

Equation (67) implies that there can be no extended epoch of slow-roll accelerated expansion, which is in strong tension with $60e$ -folds of early Universe inflation and in some tension with the less than $1e$ -fold of late-time accelerated expansion (see [22]). Interestingly, as we now show, the effective Lagrangians that we consider, Eqs. (1)–(3), can satisfy Eq. (67) while leading to accelerated expansion in basically all of the interesting dark radiation assisted parameter space and in parts of the viable parameter space for the dark matter assisted scenario. In contrast, the quintessence assisted scenario necessarily violates Eq. (67).

In the dark matter assisted scenario we evaluate ϵ and η for the Lagrangian equation (1) with the hilltop and exponential potentials equation (3). In particular, we require that at least one of Eqs. (67a) and (67b) is satisfied for all value of $\psi < \sqrt{2\langle\psi^2\rangle}$, i.e. the swampland conjecture must be satisfied throughout a full oscillation of ψ . For the hilltop potential, fixing $\phi = 0$ and recalling that $m_\psi > H_0$, Eq. (67a) reduces to

$$\psi > \psi_{\epsilon \text{ crit}} \approx 3M_{\text{Pl}} \frac{H_0^2}{m_\psi^2}, \quad (68a)$$

and Eq. (67b) becomes

$$\psi < \psi_{\eta \text{ crit}} \approx 2\sqrt{3}M_{\text{Pl}} \frac{H_0}{m_{\text{int}}}, \quad (68b)$$

where we used $M_{\text{Pl}} > \Lambda$ and $m_{\text{int}} > m_\psi$. The de Sitter conjecture is therefore satisfied provided $\psi_{\epsilon \text{ crit}} < \psi_{\eta \text{ crit}}$, that is

$$\frac{H_0}{m_\psi} < \frac{2}{\sqrt{3}} \frac{m_\psi}{m_{\text{int}}}. \quad (69)$$

This condition is somewhat restrictive because $m_\psi/H_0 \lesssim 15$ to avoid parametric resonance and is actually not satisfied for the theory plotted in Fig. 1. However, we have found cosmologically viable parameter points for which Eq. (69) is true (some of which require ϕ_i/Λ to be fixed small and which typically lead to a relatively short era of dark energy domination). The analysis is similar for theories in which ϕ has an exponential potential, except that this case only Eq. (67a) is relevant (because $\eta > 0$ always). For $\psi > \psi_{\epsilon \text{ crit}}$ Eq. (67a) is satisfied from the derivative of the potential with respect to ψ while for $\psi \lesssim M_{\text{Pl}}H_0/m_{\text{int}}$ the derivative with respect to ϕ (down its runaway potential, which is not stabilized for such ψ) is sufficiently large. Hence, up to numerical factors, Eq. (69) again guarantees the de Sitter conjecture is satisfied for $\phi = \phi_{\text{min}} = \Lambda W_0(\rho_{\text{de}}/(m_{\text{int}}^2 \langle\psi^2\rangle))$. So far we have only considered Eq. (67) with ϕ at the minimum of the induced potential, but we have also numerically analyzed the conditions for the full cosmological trajectories presented in preceding sections and confirmed that they remain satisfied for some viable points in parameter space.

Conversely, for the quintessence assisted scenario, $m_\psi < H_0$, so Eq. (68a) cannot be satisfied for $\psi < M_{\text{Pl}}$. Moreover, Eq. (68b) also cannot be satisfied for values of m_{int} such that ϕ is trapped [both for the hilltop and exponential potentials, cf. Eq. (55)]. This is to be expected since such models are examples of slow-roll quintessence. However, it is worth noting that both $V(\phi)$ and $\frac{1}{2}m_\psi^2\psi^2$ would satisfy the de Sitter conjecture alone and it is only $m_{\text{int}} \neq 0$ that prevents the full theory being compatible with this. It would be interesting to study whether such an interaction between scalars with otherwise unremarkable potentials is easier to obtain from string theory than a single scalar with a potential that violates Eqs. (68a) and (68b).

It is unclear how the swampland criteria should be applied to the dark radiation assisted scenario (further theoretical work on this, e.g. in the context of arguments for the dS conjecture connected to the distance conjecture and entropy bounds [9,84] would be worthwhile).¹⁰ Because the thermal corrections to the potential are a calculational tool to account for the net effect of the fast fluctuating background of low mass, relativistic, ψ particles at temperature T , we make the plausible guess that, if true, the de Sitter conjecture should apply only to the zero temperature potential equation (2). Moreover, we demand that the conditions are satisfied for ψ in the range $[0, T_h^2/m_\psi]$ such that the energy density in a constant ψ zero mode reaches that of the relativistic bath we consider.

¹⁰The compatibility of thermal inflation with the swampland conjectures was discussed in [85].

With such an extension to finite temperature, the de Sitter conjecture is satisfied over the required field range for both the hilltop and the exponential potentials for the vast majority of the interesting dark radiation assisted parameter space. For the hilltop potential with ϕ at the minimum of its finite temperature corrected potential $\phi_{\min} = 0$, Eq. (69) is true (because m_ψ is not much smaller than $\rho_{\text{de}}^{1/4}$ and m_ψ and m_{int} are similar). Therefore, as argued previously, at least one of Eqs. (68a) and (68b) is satisfied. Meanwhile, in the exponential case with ϕ at the high-temperature minima equation (37) $\phi_{\min} = \Lambda W_0 \left(\frac{12\rho_{\text{de}}}{m_{\text{int}}^2 T_h^2} \right)$ we find that Eq. (67a) is satisfied for the assumed range of ψ , again because $\Lambda < M_{\text{Pl}}$ and all the masses are not too far from $\rho_{\text{de}}^{1/4}$. We have also checked numerically that the de Sitter conjecture remains satisfied along full cosmological trajectories with $\phi_i \simeq \Lambda$.

Another related swampland conjecture is the trans-Planckian censorship conjecture [86], which starts from the assumption that any fluctuation that is sub-Planckian in length must remain forever hidden by cosmological horizons. It follows that both

$$\epsilon \geq \sqrt{\frac{2}{3}}, \quad (70a)$$

in the asymptotics of moduli space, that is, there are no asymptotic de Sitter vacua; and

$$\tau \leq \frac{1}{H} \log \frac{M_{\text{Pl}}}{H}, \quad (70b)$$

in the interior of moduli space, with τ the lifetime of any metastable de Sitter vacua and H its associated constant Hubble parameter. In other words, any metastable de Sitter vacua are short lived, with $N_{\text{de}} \lesssim 138$ for $H = H_0$. Because we always find only relatively few e -folds of dark energy domination, the trans-Planckian censorship conjecture is always satisfied for all of the dark matter, dark radiation and quintessence assisted theories we have considered.

VII. DISCUSSION

We have seen that coupled dark sectors open up interesting, underexplored routes to obtaining transient de Sitter vacua with numerous appealing features. The main ingredient is a coupling between a scalar ϕ that sources dark energy and another dark field, ψ . Depending on the size of its mass and interaction rates, ψ behaves as a frozen quintessence background, a component of dark matter or dark radiation. Although some of these ideas have been studied individually in the past, as locked dark energy [45] and thermal dark energy [64], we have provided several new insights, including strong constraints from parametric resonance for locked dark energy, and we have shown that runaway potentials can be stabilized.

In Table I we provide a comparison of the three scenarios including their parameter space, and theoretical and observational constraints. For $m_\psi \lesssim 3H_0$, ψ behaves as frozen quintessence; when $3H_0 \lesssim m_\psi \lesssim 15H_0$, it behaves as coherent classical oscillating dark matter component; and when in thermal equilibrium with a bath at temperature

TABLE I. Comparison of the dark matter (DM), dark radiation (DR), and quintessence (Q) assisted dark energy scenarios. We assume the Lagrangian equations (1)–(3), with $\Lambda \lesssim M_{\text{Pl}}$ and initial conditions $\psi_i \lesssim M_{\text{Pl}}$, $\phi_i \lesssim \Lambda$. Limits on m_ψ come from requiring ψ to behave as dark matter, dark radiation, or quintessence, with dynamics that induce a transient de Sitter minimum for ϕ . The physical mass of ϕ in the metastable vacuum, m_ϕ^{eff} , is necessarily larger than $\sqrt{\rho_{\text{de}}}/\Lambda$, and the values in the table are set by the allowed Λ . Constraints on m_{int} come from requiring a sufficient number of e -folds of dark energy domination ($N_{\text{de}} \gtrsim 0.26$). Constraints on λ are such that we can neglect ψ 's quartic interaction for the dark matter and quintessence assisted cases, and so that thermal equilibrium is maintained for the dark radiation case (in particular, for the dark radiation scenario we give the condition for thermal equilibrium to be reached early enough for ϕ to be driven from $\phi = \Lambda$ to $\phi/\Lambda \simeq 0$). The cases that $V(\phi)$ has a hilltop or exponential form are similar, although there is less analytic control for the dark matter assisted scenario with an exponential potential.

	DM assisted	DR assisted	Q assisted
m_ψ	$H_0 \lesssim m_\psi \lesssim 15H_0$	$m_\psi \lesssim T_h \lesssim \rho_{\text{de}}^{1/4}$	$m_\psi \lesssim H_0$
m_ϕ^{eff}	$m_\phi^{\text{eff}} \gtrsim H_0$	$T_{\text{v},0} \gg m_\phi^{\text{eff}} \gtrsim H_0$	$m_\phi^{\text{eff}} \gtrsim H_0$
m_{int}	$m_{\text{int}} \gtrsim \frac{M_{\text{Pl}}}{\Lambda} \frac{M_{\text{Pl}}}{\psi_i} H_0$	$m_{\text{int}} \gtrsim \rho_{\text{de}}^{1/4}$	$m_{\text{int}} > \frac{M_{\text{Pl}}}{\psi_i} H_0$
λ	$\lambda \ll m_\psi^2 / M_{\text{Pl}}^2$	$\lambda \gtrsim (m_{\text{int}} / (M_{\text{Pl}} \xi_h^2))^{1/4}$	$\lambda \ll m_\psi^2 / M_{\text{Pl}}^2$
Parametric resonance?	Yes	No	No
Bubble nucleation?	No	Yes	No
No fine-tuning from quartic coupling?	$\Lambda_{\text{UV}} \ll \Lambda, M_{\text{Pl}} \frac{H_0}{m_{\text{int}}}$	$\Lambda_{\text{UV}} \lesssim \rho_{\text{de}}^{1/4}$	$\Lambda_{\text{UV}} \ll \Lambda, M_{\text{Pl}} \frac{H_0}{m_{\text{int}}}$
No fine-tuning from graviton exchange with other sectors?	$M < \text{GeV}$	$M < (M_{\text{Pl}}/\Lambda)^{1/3} \text{ GeV}$	$M < \text{GeV}$
No fine-tuning from $\mathcal{O}(1)$ couplings to other sectors?	$m_{\text{soft hid}} \ll H_0$	$m_{\text{soft hid}} \ll \rho_{\text{de}}^{1/4}$	$m_{\text{soft hid}} \ll H_0$
Sequestering of portal couplings, e.g. $\kappa\phi\mathcal{O}_{\text{SM}}$	$\kappa \lesssim 10^{-6} M_{\text{Pl}}^{-1}$	$\kappa \lesssim M_{\text{Pl}}^{-1}$	$\kappa \lesssim 10^{-6} M_{\text{Pl}}^{-1}$
Other potential signals?	$w(z), \Omega_\psi$	$w(z), N_{\text{eff}}$	$w(z)$
dS swampland constraint?	Satisfied for $\frac{H_0}{m_\psi} < \frac{m_\psi}{m_{\text{int}}}$	Satisfied	Violated

$T_h \gtrsim m_\psi$ it behaves as dark radiation. Outside of thermal equilibrium, and with $m_\psi > 15H_0$, parametric resonance destabilizes the induced de Sitter minimum too early. As a result, in the dark matter assisted scenario ψ behaves as an *extremely* light component of dark matter; because $m_\psi \lesssim 15H_0$ such a theory is not too far from the two field quintessence regime (but, given the de Sitter conjecture, the difference could be crucial). In the dark radiation case, for $m_\psi > T_h$ thermal effects are exponentially suppressed and there is again no de Sitter minimum. In each scenario, we need m_{int} to be sufficiently large for the background ψ to generate a transient minimum for ϕ , putting m_{int} somewhat above H_0 for dark matter and quintessence assisted cases and somewhat above $\rho_{\text{de}}^{1/4}$ for the dark radiation assisted case. Still, the quartic coupling can be sufficiently small that radiative corrections do not drive the scalar masses too high, especially in the dark matter and quintessence assisted scenarios, which do not require the Lagrangian parameters to be fine-tuned.

The theories that we have considered lead to a wide range of possible observational signals. These include deviations from Λ CDM due to a dynamical dark energy, coupled to another scalar field, at the edge of the allowed parameter space. Constraints on the effective dark energy equation of state parameter are an important target for current and future large-scale-structure surveys, including DESI (see e.g. [87]). The dark matter and quintessence assisted scenarios may also lead to some spatial variation in the effective w , due e.g. to adiabatic fluctuations in the initial conditions for ψ from primordial inflation, which is an interesting direction to explore in future work. The dark radiation case predicts deviations from the Standard Model value for N_{eff} ; the next-generation LSS and CMB observations, such as CMB-S4 (see e.g. [88]) could constrain $\Delta N_{\text{eff}} < 0.06$ [89], corresponding to $\xi_h < 0.35$ in our minimal model with $g_h = 2$, covering part of the parameter space in Fig. 4. Meanwhile, the dark matter assisted case predicts a potentially observable energy density in extremely light dark matter, which will be further constrained by future CMB observations and the Square Kilometre Array intensity mapping [57].

Additionally, both the light scalar fields ψ and ϕ would mediate fifth forces, which could be observed in future experiments. In fact, current fifth forces constraints impose strong upper bounds on possible portal couplings to the visible sector. For example, for interactions of the form $\kappa\phi\mathcal{O}_{\text{SM}}$, where κ is a coupling constant and \mathcal{O}_{SM} is a dimension four singlet under the Standard Model gauge group, scalars with masses $\ll 10^{-18}$ eV already require $\kappa \lesssim 10^{-6}M_{\text{Pl}}^{-1}$ [90–92]. When ψ behaves as dark matter or quintessence, with a mass around H_0 , a large suppression of portal couplings is therefore necessary to avoid unobserved fifth forces (comparable to that needed for standard quintessence [93]). This problem is greatly ameliorated in dark radiation assisted scenarios, for which the masses of ϕ and ψ

are typically much larger than H_0 , although $\kappa \lesssim M_{\text{Pl}}^{-1}$ is still needed [94]. On the other hand, in the dark radiation scenario, any portal couplings must still be tiny if fine-tuning is to be avoided because visible sector loops can generate a large ϕ tadpole and mass term (see [64] for further discussion). A sizeable suppression of portal couplings could be achieved within string constructions if visible and dark sectors are geometrically separated in the extra dimensions [93,95–100]. In our current work we have focused only on a simple toy model for clarity, and in the future it will be important to understand if more complete models can be constructed. From a purely field theory perspective, in the dark radiation case more complex (e.g. gauged) hidden sectors might naturally lead to the required super-cooling without small Lagrangian parameters. Meanwhile, in the dark matter case there could be theories in which the required dynamics are driven by the dominant dark matter component rather than the extremely light component that we have had to rely on. From a top-down perspective, the critical issue is whether viable models can be constructed from string compactifications. None of the essential features seem especially implausible; all that is needed are two interacting scalar fields, one with a mass term and another with a more complicated potential energy functional that increases as ϕ approaches 0 from the right.¹¹ It is also interesting to note that if either or both of our coupled dark sectors are pseudoscalar axions rather than scalars, then constraints from fifth forces are relaxed and apparent fine-tunings could be explained via the associated pseudo-Nambu-Goldstone shift symmetries. Given that the stabilization mechanisms work even when ϕ is otherwise a runaway modulus, it would also be interesting to explore whether a coupling to a dark matter or dark radiation sector could provide a dynamical mechanism to address the moduli stabilization problem in addition to accounting for dark energy. In this context, we highlight related previous analysis of the effects of thermal effects on the evolution of string moduli in the early Universe (which can sometimes lead to destabilization rather than stabilization) [101–109].

Given the richness of string theory hidden sectors, it is plausible that the dynamics we have described might be relevant not only in the present-day era, but could also have occurred in the cosmological past. For instance, the theories we have considered could provide models for a subdominant early dark energy [110], which has been proposed to resolve the growing tension between late-time direct measurements of H_0 and the value inferred from the CMB assuming Λ CDM. Potential signatures, in particular from the exit from past transient dark energy phases, could provide a target for future observations, e.g. bubble collisions at the end of a dark radiation assisted early dark energy might potentially produce detectable gravitational

¹¹For the induced transient dS minimum to be at small ϕ/Λ , the $\phi\psi^3$ interaction must be small or vanishing, which could be explained e.g. with a discrete symmetry under which $\phi \rightarrow -\phi$.

wave signatures. As mentioned, there could also be interesting late-time observational signals from the evolution of initially small perturbations in the fields ϕ and ψ , and it would be interesting to extend our initial investigation in Appendix D in future work.

Last but of course not least, it should be stressed that we have assumed some as yet unknown solution to the cosmological constant problem that precisely cancels all other contributions to the vacuum energy, leaving at most a small negative contribution.

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APPENDIX A: MORE DETAILS OF THE DARK MATTER ASSISTED SCENARIO

1. Analysis of the parametric resonance

Here we consider the effects of parametric resonance on the dark matter assisted scenario, discussed in Sec. II A 2, in more detail. We first note that for $b \gtrsim 1$ and $q \gtrsim b^2$, the value of s averaged over an $\mathcal{O}(1)$ range of q can be fitted numerically to

$$\bar{s} \simeq b/(4\sqrt{q}) + 0.11. \quad (\text{A1})$$

For the theories of interest, we do indeed have that b , given in Eq. (14), is roughly 1 because $\Lambda < M_{\text{pl}}$. As mentioned in the main text, this means that the resonance condition on m_ψ becomes stronger as time progresses and q decreases. However, $b/\sqrt{q} \simeq (H_0^2 M_{\text{pl}}^2)/(m_\psi \Lambda m_{\text{int}} \psi)$, so once $b/\sqrt{q} \simeq 1$ the era of dark energy is automatically at an end anyway by Eq. (10b) (up to a numerical factor inside the logarithm). Therefore, it is sufficient to impose the constraint, valid at $q \gtrsim b^2$, of $m_\psi/H_0 < \frac{3}{2}\bar{s}^{-1} \lesssim 15$.

As well as the zero momentum mode, ϕ also has small but nonzero initial occupation number in higher momentum modes (e.g. as a result of quantum fluctuations or due to an earlier era of primordial inflation). If such modes were resonantly amplified more strongly than the zero mode, this could lead to additional constraints despite their amplitude initially being suppressed. Actually ϕ modes with nonzero momentum grow at most as fast as the zero mode.

In particular, a mode of comoving momentum k follows Eq. (12) except with the replacement [41]

$$b \rightarrow b(k) \equiv b - \frac{k^2}{m_\psi^2} e^{-2H_0\tau/m_\psi}, \quad (\text{A2})$$

(where we set $a_i = 1$ for convenience). For $b(k)/\sqrt{q}$ small and negative, we again obtain $\bar{s} \simeq 0.11$ and \bar{s} is smaller for $b(k)/\sqrt{q} \lesssim -1$. Therefore, in the parameter space where the zero mode is not exponentially growing higher momentum modes are not amplified either.

2. Driving ϕ to the origin

In this appendix we provide more details on the dynamics that can lead ϕ to be driven to the origin in both the dark matter and quintessence assisted scenarios, in full cosmologies including the Standard Model and an additional dominant component of dark matter (both of which we assume are totally decoupled from ϕ and ψ). In the early Universe, ϕ will be frozen by Hubble friction until $3H \simeq m_{\text{int}}\psi_i/\Lambda$. This may happen in the cosmological era of radiation domination or later during matter domination. Using the appropriate regime of the Friedmann equation, $H^2/H_0^2 \approx \Omega_r a^{-4}$ or $H^2/H_0^2 \approx \Omega_m a^{-3}$, with Ω_m and Ω_r the matter and radiation density parameters, one can identify the scale factor when ϕ starts oscillating as

$$a_{\phi\text{roll}} = (9\Omega_m)^{1/3} \left(\frac{H_0\Lambda}{m_{\text{int}}\psi_i} \right)^{2/3} \quad \text{or} \quad (\text{A3a})$$

$$a_{\phi\text{roll}} = (9\Omega_r)^{1/4} \left(\frac{H_0\Lambda}{m_{\text{int}}\psi_i} \right)^{1/2}, \quad (\text{A3b})$$

if ϕ starts to oscillate during matter domination or radiation domination respectively. We note that even for the largest m_{int} and smallest Λ in our parameter space of interest, ϕ starts oscillating long after the temperature of the Universe is $\sim \text{MeV}$, i.e. after big bang nucleosynthesis (see Fig. 2). Observations constrain the reheating temperature of the Universe after primordial inflation to be above these temperatures so there is no additional constraint on the reheating temperature for ϕ to be driven to the origin.

The energy density in the ϕ oscillations redshifts as matter, with $\phi = \phi_i (a_{\phi\text{roll}}/a)^{3/2}$. Once ϕ is driven close to the top of its potential, ϕ 's energy density behaves as a component of dark energy owing to the potential energy being roughly constant throughout each oscillation [$V(0) = \rho_{\text{de}}$]. These dynamics continue until not long before dark energy domination when $3H \approx m_\psi$ and ψ unfreezes. At this time ψ starts to oscillate around its minimum, which happens at scale factor

$$a_{\psi\text{roll}} = (9\Omega_m)^{1/3} (H_0/m_\psi)^{2/3}. \quad (\text{A4})$$

At this time the energy density in ψ starts redshifting as matter, with $\psi = \psi_i (a_{\psi \text{ roll}}/a)^{3/2}$. Meanwhile, the amplitude of ϕ 's oscillations stops decreasing $\propto a^{-3/2}$, but ϕ is locked as a consequence of the coupled dynamics. We can estimate the value at which ϕ is locked as

$$\phi_{\psi \text{ roll}} = \phi_i \frac{m_\psi \Lambda}{m_{\text{int}} \psi_i} \quad \text{or} \quad (\text{A5a})$$

$$\phi_{\psi \text{ roll}} = \phi_i \frac{(9\Omega_r)^{3/8}}{(9\Omega_m)^{1/2}} \left(\frac{H_0 \Lambda}{m_{\text{int}} \psi_i} \right)^{3/4} \left(\frac{m_\psi}{H_0} \right), \quad (\text{A5b})$$

if ϕ starts to oscillate during matter or radiation domination, respectively. Thus, ϕ/Λ is small enough that ψ 's evolution is dominated by its bare mass m_ψ rather than $m_{\text{int}}\phi/\Lambda$ at the time ψ starts rolling (so its equation of motion is linear) if

$$\frac{\phi_i}{\psi_i} \ll 1 \quad \text{or} \quad (\text{A6a})$$

$$\frac{\Omega_r^{3/8}}{\Omega_m^{1/2}} \left(\frac{m_{\text{int}}}{H_0} \right)^{1/4} \frac{\phi_i}{\psi_i^{3/4} \Lambda^{1/4}} \ll 1, \quad (\text{A6b})$$

again if ϕ becomes unfrozen during matter or radiation domination, respectively. These are the conditions given in Eq. (25) in the main text.

3. Further analysis of the parameter space

In this appendix we provide further results from numerical solutions of the equations of motion of theories in the dark matter assisted scenario, supporting the analysis presented in Sec. II.

In Fig. 5 (left panel), we plot the evolution of the various energy densities with the scale factor for the same theory as in Fig. 1 in the main text (we include the energy density from the $\phi - \psi$ quartic interactions in ρ_ψ). Prior to $a/a_0 \simeq 0.7$, the total energy density of the Universe is dominated by (first) the energy density in the Standard Model radiation bath and (subsequently) the additional dark matter component that we include. At sufficiently early times, the energy density in the $\phi - \psi$ system is mainly in the interaction term. The energy density in ϕ initially increases as it is driven away from $\phi = \Lambda$ towards $\phi = 0$ (in the process decreasing the energy density in the $\phi - \psi$ interaction). Following this, part of ϕ 's energy density, corresponding to its oscillations around $\phi = 0$, redshifts away $\propto a^{-3}$ with the component corresponding to potential energy remaining. At $a/a_0 \simeq 0.7$ this potential energy comes to dominate the energy density of the Universe, acting as dark energy (meanwhile the energy density in ψ redshifts as matter). Once ϕ is unlocked at $a/a_0 \simeq 5$ the total energy density of the Universe decreases fast. In the right panel, we plot the evolution of the equation of state parameter of the Universe w in the same theory.

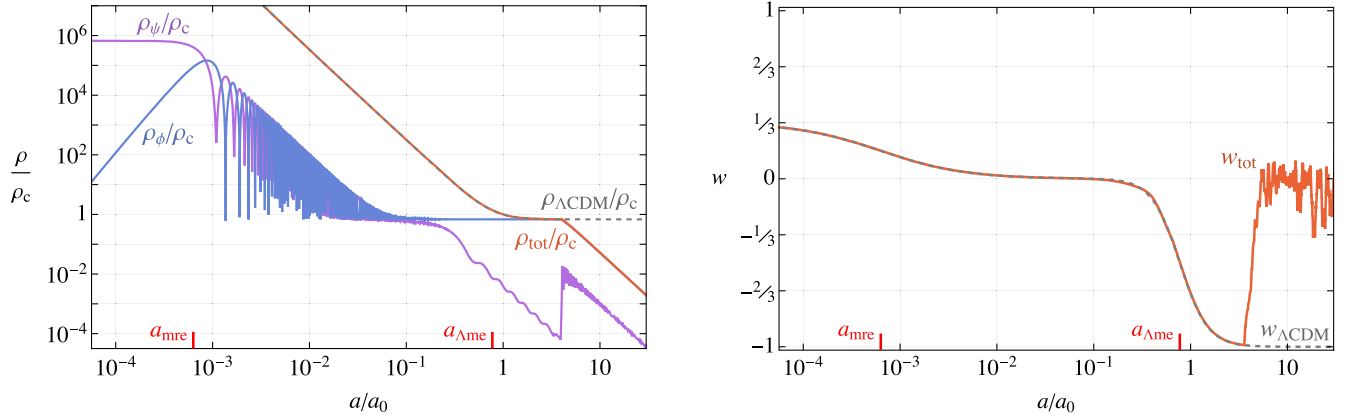


FIG. 5. Left: the energy densities in ϕ (ρ_ϕ), ψ (ρ_ψ) and in total, including the Standard Model and the additional dominant dark matter component, (ρ_{tot}) in the dark matter assisted theory plotted in Fig. 1 normalized to the present-day critical energy density of the Universe ρ_c (we choose to put the energy density from the $\phi - \psi$ interaction in ρ_ψ). For comparison, we also plot the total energy density in a Λ CDM theory ($\rho_{\Lambda\text{CDM}}$). At $a/a_0 \ll 10^{-3}$, the energy density of the $\phi - \psi$ system is dominantly in the interaction term $\rho \simeq m_{\text{int}}^2 \phi_i^2 \psi_i^2 / \Lambda^2$. This energy density is transferred to ϕ while $a/a_0 \lesssim 10^{-3}$ as ϕ is driven towards $\phi/\Lambda = 0$ (with ψ frozen). Most of ϕ 's energy density at these times is in the form of kinetic energy, which redshifts $\propto a^{-6}$. Starting from $a/a_0 \simeq 10^{-1}$, ϕ 's energy density is mostly potential energy, which comes to dominate the evolution of the Universe sourcing an era of dark energy domination (at these times ψ 's energy density decreases as matter $\propto a^{-3}$). After ϕ is unlocked at $a/a_0 \simeq 5$, its energy density decreases approximately like matter as it oscillates around the minimum of its potential. Right: the equation of state parameter $w_{\text{tot}} = p/\rho$, where p is pressure and ρ is the energy density of the Universe for the theory plotted in the left panel and in Fig. 1. At $a/a_0 < 5$ this agrees with the Λ CDM prediction ($w_{\Lambda\text{CDM}}$) to % level accuracy. Once ϕ is unfrozen and dark energy domination ends, w_{tot} deviates from $w_{\Lambda\text{CDM}}$; the theory is in a complex nonlinear regime at these times, but it will eventually settle down to $w_{\text{tot}} \simeq 0$ corresponding to matter domination as ϕ and ψ oscillate around the minimum of their potential.

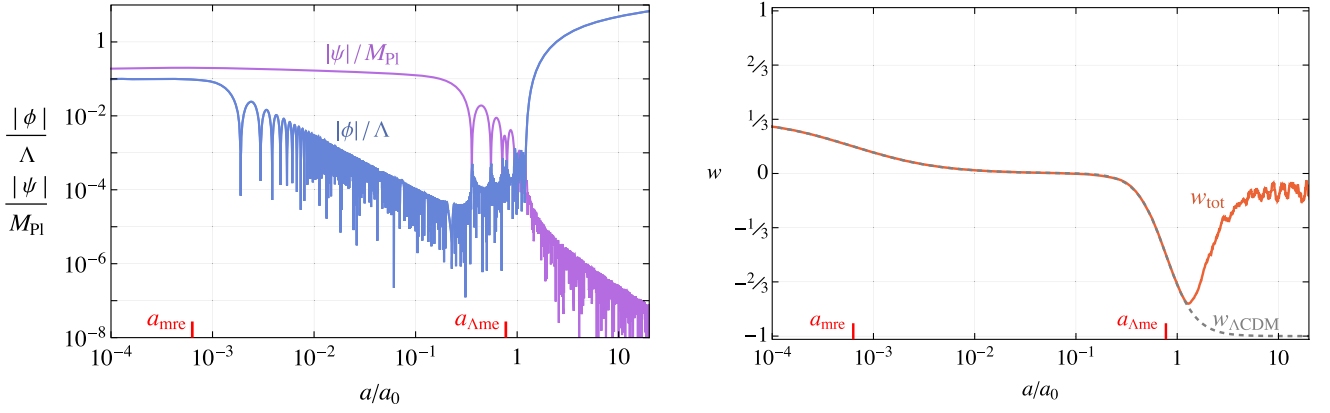


FIG. 6. Left: the evolution of ϕ and ψ in a theory of dark matter assisted dark energy in which ϕ has the exponential runaway potential of Eq. (3). The Lagrangian parameters are $m_\psi = 12H_0$, $m_{\text{int}} = 10^5 H_0$, $\Lambda = M_{\text{Pl}}/2$ and $\lambda = 0$ and initial field values $\phi_i = 10^{-1}\Lambda$, $\psi_i = M_{\text{Pl}}/5$. The dynamics are similar to the case that ϕ has a hilltop potential, described in Fig. 1, except that the era of ϕ being locked is less stable because the system is driven to a nonlinear regime [see Eq. (33)] soon after ϕ becomes unlocked. Nevertheless, ϕ remains close enough to the origin to source an era of dark energy domination that is consistent with current observations. Right: the equation of state parameter of the theory plotted in the left panel w_{tot} compared to the ΛCDM prediction, $w_{\Lambda\text{CDM}}$. Prior to ϕ becoming unlocked at $a/a_0 \simeq 2$, the theory matches the ΛCDM result to better than % level precision. Subsequently w_{tot} has a complex time dependence because the system is highly nonlinear, but it will eventually approach the tracker solution for an exponential potential discussed in the main text.

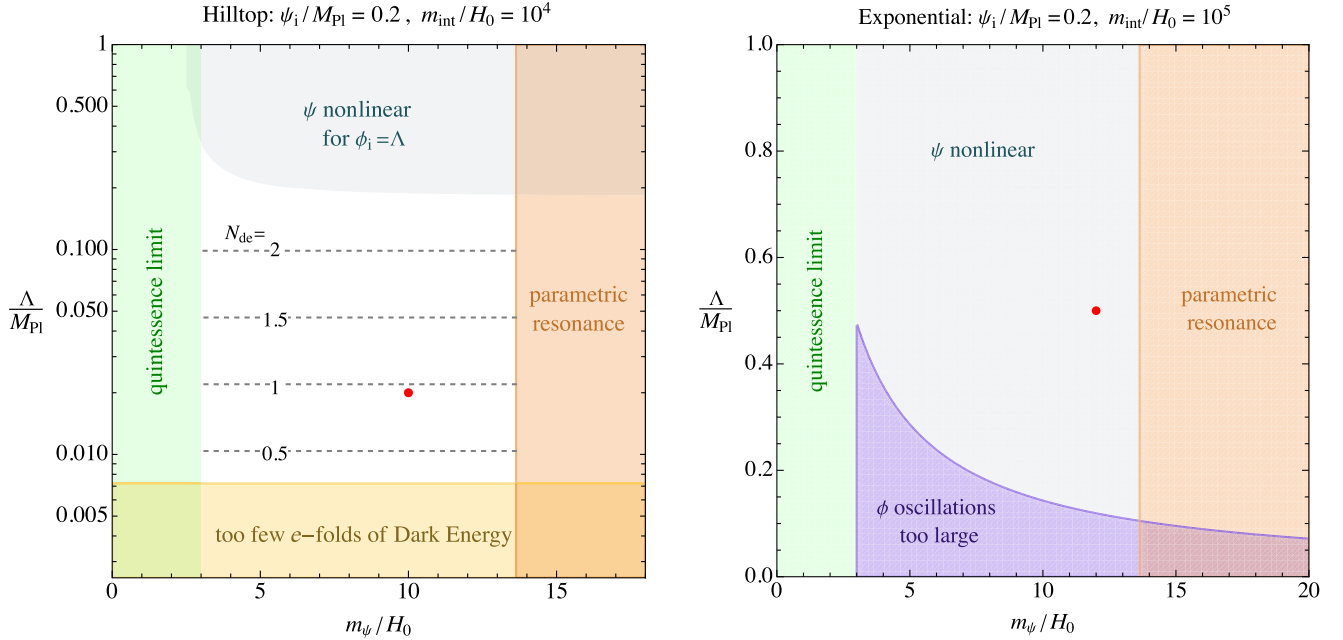


FIG. 7. Left: a slice of the parameter space for dark matter assisted dark energy in which ϕ has a hilltop potential, analogous to Fig. 2 (left panel), but varying Λ and m_ψ . The constraints come from parametric resonance causing ϕ to be unlocked if m_ψ is too large (“parametric resonance”); ψ evolving nonlinearly if Λ is too large (“ ψ nonlinear”) cf. Eq. (33); and not enough e -folds of dark energy (“too few e -folds of dark energy”) from Eq. (11). Additionally if $m_\psi < 3H_0$ the theory is in the quintessence assisted rather than dark matter assisted regime, i.e. it is a two field quintessence theory (“quintessence limit”). The theory plotted in Fig. 1 is again shown with a red dot and we denote the predicted number of e -folds of dark energy domination with N_{de} . Right: the analogous parameter space for when ϕ has an exponential potential. In this case ψ is nonlinear throughout the parameter space (so we cannot reliably predict N_{de}), but we still expect that $m_\psi \gtrsim 15H_0$ leads to dangerous parametric resonance similarly to the case of a hilltop potential. Additionally, if ϕ ’s oscillations in the locked regime are too large, cf. Eq. (32), there is no era of dark energy domination (“ ϕ oscillations too large”). For $\phi_i = \Lambda/10$ (as in the case of our example theory), ψ initially evolves linearly over all the slice plotted (for larger ϕ_i requiring this leads to a significant constraint). The red dot indicates the example theory plotted in Fig. 6; this leads to ~ 0.5 e -folds of dark energy domination consistent with observational data.

For $a/a_0 \leq 1$ this agrees with Λ CDM to a precision of better than %, comfortably within observational constraints. The Λ CDM prediction is followed until $a/a_0 \simeq 5$ when ϕ is unlocked. The subsequent dynamics of the system are initially nonlinear and complicated, but accelerated expansion ends soon after this time. Eventually $w = 0$ will be reached once both ϕ and ψ have only small oscillations around the minima of their potentials.

In Fig. 6 (left panel), we show the evolution of ϕ and ψ in the theory in which ϕ has an exponential runaway potential corresponding to the red dot in Fig. 2 (right panel) in the main text. As mentioned in the main text, we pick $\phi_i = \Lambda/10$, which leads to a slightly longer era of dark energy domination than if $\phi_i = \Lambda$. The dynamics are similar to the case of ϕ having a hilltop potential, except that the era of ϕ being trapped is less stable with the amplitude of ϕ 's oscillations gradually increasing [between $a/a_0 \simeq 0.3$ and 2 in the Fig. 6 (left)]. At $a/a_0 \simeq 2$, ϕ is unlocked and subsequently rolls to large field values. In Fig. 6 (right panel), we show the evolution of w in the same theory as in the left panel. As in the case of the hilltop potential theory in Fig. 5, w matches the Λ CDM prediction accurately until ϕ is unlocked and accelerated expansion ends.

Finally, in Fig. 7 we plot slices of the allowed dark matter assisted dark energy parameter space, similarly to Fig. 2 but varying Λ and m_ψ with ψ_i and m_{int} fixed. For both the cases of a hilltop potential and an exponential runaway, if $m_\psi \gtrsim 15H_0$ parametric resonance prevents ϕ remaining trapped at $\phi/\Lambda \ll 1$, while if $m_\psi \lesssim 3H_0$, ϕ remains frozen up to the present day and the theory is in the quintessence limit considered in Sec. IV. In the case of a hilltop potential, if Λ is too small there are not enough e -folds of dark energy to match observations, whereas if Λ is too large then ψ evolves nonlinearly and we cannot predict the number of e -folds of dark energy domination. Meanwhile, in the case of an exponential potential, ψ inevitably evolves nonlinearly so we cannot predict the number of e -folds of dark energy domination, although viable theories can be found numerically. Moreover, for an exponential potential, if Λ is too small then the expected amplitude of ϕ 's oscillations, given in Eq. (32), is larger than Λ and an extended era of dark energy domination is unlikely.

APPENDIX B: MORE DETAILS ON THE DARK RADIATION ASSISTED SCENARIO

1. Automatic initial conditions

In this appendix we provide more details about how ϕ can be driven from an initial condition $\phi_i \simeq \Lambda$ to close to $\phi/\Lambda \simeq 0$ in the dark radiation assisted scenario in a full cosmological theory. For this to occur, the hidden sector must be in thermal equilibrium and ψ itself must be present in the thermal bath such that the high-temperature limit, $T_{\text{h max}} \gg m_{\text{int}}$ for $\phi \simeq \Lambda$, applies (otherwise ψ decouples

from the thermal bath with only a freeze-out abundance remaining). Provided these conditions are satisfied, the scalar potential for ϕ receives an effective mass contribution with $(m_\phi^{\text{eff}})^2 \simeq T_{\text{h}}^2 m_{\text{int}}^2 / \Lambda^2$. However, ϕ is also subject to Hubble friction with $H^2 \sim T_{\text{v}}^4 / M_{\text{Pl}}^2$ during radiation domination (where T_{v} is the temperature of the visible sector), which, assuming a sufficiently large reheating temperature, will initially dominate and freeze the field. As the temperatures of the hidden and visible sectors fall, a critical temperature is reached, $T_{\text{h unfreeze}}$, when the potential gradient can beat the Hubble friction and succeed in pushing ϕ towards its effective minimum at $\phi/\Lambda \ll 1$; this occurs when $m_\phi^{\text{eff}} = H$, that is

$$T_{\text{h unfreeze}} = \frac{M_{\text{Pl}}}{\Lambda} m_{\text{int}} \xi_{\text{h}}^2, \quad (\text{B1})$$

where ξ_{h} is the ratio between hidden and visible sector temperatures, $\xi_{\text{h}} \equiv T_{\text{h}}/T_{\text{v}}$ (note that $T_{\text{h unfreeze}}$ is indeed before matter-radiation equality, as assumed).

We therefore require that there exists some hidden sector temperature

$$T_{\text{h roll}} < T_{\text{h unfreeze}}, \quad (\text{B2})$$

such that with $\phi = \Lambda$: (i) ψ is in thermal equilibrium, $\Gamma_{\text{I}} > H$. Assuming $\Gamma_{\text{I}} \sim \lambda^4 T_{\text{h}}$ as is the case for our minimal model of Eq. (2) (with a more complicated hidden sector there could be additional interactions such that thermalization need not be via λ and could involve different powers of different coupling constants), this leads to

$$\lambda^4 T_{\text{h roll}} > \frac{T_{\text{h roll}}^2}{\xi_{\text{h}}^2 M_{\text{Pl}}}, \quad (\text{B3})$$

which becomes easier to satisfy at late times; (ii) the high-temperature approximation for ψ 's contribution to ϕ 's potential is valid

$$T_{\text{h roll}} > m_{\text{int}}, \quad (\text{B4})$$

which, of course, is harder to satisfy at late times.

Equations (B2)–(B4) are simultaneously satisfied provided that first, from Eqs. (B2) and (B4),

$$\frac{\Lambda}{M_{\text{Pl}}} < \xi_{\text{h}}^2, \quad (\text{B5})$$

which can be a significant constraint in the parameter space we are interested in because typically Λ is not much smaller than M_{Pl} . Second, from Eqs. (B3) and (B4) we need

$$\frac{m_{\text{int}}}{M_{\text{Pl}}} < \lambda^4 \xi_{\text{h}}^2, \quad (\text{B6})$$

which in practice is easily satisfied provided λ is not tiny. This reproduces the conditions, Eq. (46), given in the main text.

Finally, we require that the reheating temperature after primordial inflation is sufficiently large that there is some post-inflationary era when these dynamics can take place. This is the case provided that the visible sector temperature after inflation $T_{v,\text{RH}} > m_{\text{int}}/\xi_{\text{h}}$ so that Eq. (B4) is satisfied and ψ is not decoupled from the thermal bath immediately after inflation. For the parameter space of interest, see e.g. Fig. 2, such a condition is easily satisfied for $T_{v,\text{RH}} \gtrsim \text{MeV}$ as is needed anyway to match the predictions of big bang nucleosynthesis.

2. Tunneling

As discussed in Sec. III A, in the dark radiation assisted scenario the de Sitter vacuum is unstable to tunneling by quantum and thermal fluctuations [39,66,67,111,112] through the finite temperature corrected potential barrier to a region in field space where thermal effects are exponentially suppressed. As a result, the exit from the dark radiation assisted dark energy epoch might take place via a first order phase transition, with nucleation of bubbles containing energetically preferred values of ϕ , which then expand at close to the speed of light. If such tunneling happens sufficiently early, it could lead to fewer e -folds of dark energy domination than predicted by Eq. (43). In this appendix, we analyze these processes focusing on the scenario that ϕ has an exponential runaway potential, in which case the full potential always has a global minimum out at $\phi \rightarrow \infty$. We will first compute the quantum tunneling rate, Γ_4 , and then the thermal decay rate, Γ_3 . For the radiatively generated minimum to source the current dark energy epoch, we require that vacuum decay occurs at a sufficiently slow rate compared to the lifetime of the Universe

$$\Gamma_4 < H_0^4 \quad \text{and} \quad \Gamma_3 < H_0^4. \quad (\text{B7})$$

The rate of quantum tunneling per unit volume \mathcal{V} depends on the Euclidean action, and is approximately [113,114]

$$\frac{\Gamma_4}{\mathcal{V}} = v^4 \left(\frac{S_4}{2\pi} \right)^2 e^{-S_4}, \quad (\text{B8})$$

where v is the width of the barrier. This rate is dominated by the classical $O(4)$ bounce solution with associated action S_4 [115]. With ϕ 's potential taking the exponential runaway form, the global minimum is infinitely far away in field space and the potential is such that the thin-wall approximation cannot be made. The tunneling will take ϕ not all the way to the true vacuum but to some energetically preferred value on the other side of the barrier, after which it will roll. The bounce solution can be found numerically

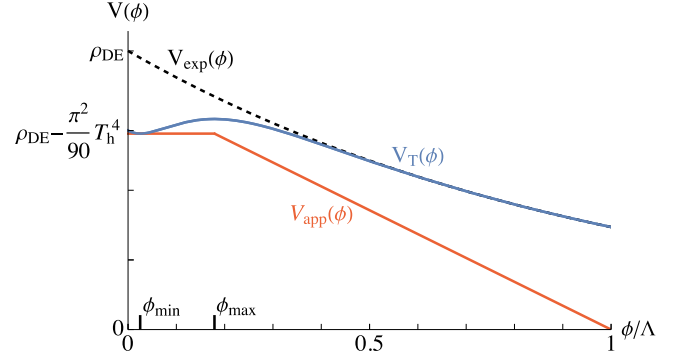


FIG. 8. The finite temperature corrected potential of ϕ , $V_T(\phi)$, and the approximation we make to obtain a lower bound on the tunneling rate, $V_{\text{app}}(\phi)$. We also indicate the zero temperature potential, $V_{\text{exp}}(\phi)$, and the metastable minimum ϕ_{min} and maximum ϕ_{max} of ϕ 's corrected potential. Note that the plot is not to scale for a realistic theory that can lead to an era of dark energy domination (for which $\phi_{\text{min}}/\Lambda$, the height of the barrier and $T_{\text{h}}^4/\rho_{\text{de}}$ would all be much smaller).

by the undershoot/overshoot method; however, we will make instead an analytical estimate. To do so, we approximate the potential as the no-barrier potential [116] shown in Fig. 8

$$V \approx V_{\text{app}}(\phi) = -k(|\phi| - |\phi_{\text{max}}|)\theta(|\phi| - |\phi_{\text{max}}|) + \rho_{\text{de}} e^{-\frac{\phi_{\text{min}}}{\Lambda}} - \frac{\pi^2}{90} T_{\text{h}}^4, \quad (\text{B9})$$

where we choose the point where the plateau ends as ϕ_{max} (the value of ϕ at the maximum of the full potential), and the slope to the right of the plateau to be given by $k = V(\phi_{\text{min}}, T_{\text{h}})/(\Lambda - \phi_{\text{max}})$. Given that for $\phi > \phi_{\text{max}}$, $\partial^2/\partial\phi^2 V(\phi, T) > 0$ we have that $V_{\text{app}}(\phi) < V(\phi, T)$ for all $\phi > \phi_{\text{min}}$. Further, we can estimate the value of ϕ_{max} by noting that the barrier in the full potential will occur around where the high-temperature approximation breaks down

$$\phi_{\text{max}}/\Lambda \sim T_{\text{h}}/m_{\text{int}}, \quad (\text{B10})$$

(i.e. $m_{\psi}^{\text{eff}} \sim T_{\text{h}}$, but still $\phi_{\text{max}} \ll \Lambda$).

Quantum tunneling in the no-barrier potential equation (B9) has been analyzed in [116]. The bounce solution to the Euclidean equations of motion,

$$\phi'' + \frac{3}{s}\phi' = \frac{\partial V}{\partial\phi}, \quad (\text{B11})$$

that satisfies the boundary conditions $\phi'(0) = 0$ and $\lim_{s \rightarrow \infty} \phi(s) = \phi_{\text{min}} \approx 12\Lambda\rho_{\text{de}}/(m_{\text{int}}^2 T_{\text{h}}^2) \approx 0$, and continuity across s_{max} , where $\phi = \phi_{\text{max}}$, is found to be

$$\phi(s) = \begin{cases} \phi_{\max} \left(2 - \frac{s^2}{s_{\max}^2} \right) & \text{for } s < s_{\max} \\ \phi_{\max} \frac{s_{\max}^2}{s^2} & \text{for } s > s_{\max}, \end{cases} \quad (\text{B12})$$

with $s_{\max} = \sqrt{8\phi_{\max}/k}$. The Euclidean action evaluated on this $O(4)$ bounce gives

$$S_4 = \frac{32\pi^2}{3} \frac{\phi_{\max}^3}{k}, \quad (\text{B13})$$

leading to a quantum tunneling rate

$$\frac{\Gamma_4}{\mathcal{V}} \sim v^4 \frac{T_h^6}{m_{\text{int}}^6} \frac{\Lambda^8}{\rho_{\text{de}}^2} e^{-\frac{32\pi^2 \Lambda^4 T_h^3}{3 \rho_{\text{de}} m_{\text{int}}^2}}. \quad (\text{B14})$$

The runaway part of ϕ 's finite temperature-corrected potential can also be reached via thermal fluctuations that roll ϕ up and over its barrier, with an associated decay rate per unit volume \mathcal{V} given approximately by [113,114]

$$\frac{\Gamma_3}{\mathcal{V}} = T_h^4 \left(\frac{S_3}{2\pi T_h} \right)^{3/2} e^{-S_3/T_h}. \quad (\text{B15})$$

This process is dominated by the $O(3)$ bounce solution with an associated action S_3 . The corresponding Euclidean equation of motion is

$$\phi'' + \frac{2}{r} \phi' = \frac{\partial V}{\partial \phi}, \quad (\text{B16})$$

with boundary conditions $\phi'(0) = 0$ and $\lim_{r \rightarrow \infty} \phi(r) = \phi_{\min} \approx 0$, and ϕ continuous across r_{\max} . The solution in this case is

$$\phi(r) = \begin{cases} \frac{1}{2} \phi_{\max} \left(3 - \frac{r^2}{r_{\max}^2} \right) & \text{for } r < r_{\max} \\ \phi_{\max} \frac{r_{\max}}{r} & \text{for } r > r_{\max}, \end{cases} \quad (\text{B17})$$

where $r_{\max} = \sqrt{3\phi_{\max}/k}$. The Euclidean action evaluated on the $O(3)$ bounce is

$$S_3 = \frac{8\sqrt{3}\pi}{5} \frac{\phi_{\max}^{5/2}}{k^{1/2}}, \quad (\text{B18})$$

thus giving a thermal decay rate

$$\frac{\Gamma_3}{\mathcal{V}} \sim \frac{\Lambda^{9/2} T_h^{25/4}}{m_{\text{int}}^{15/4} \rho_{\text{de}}^{3/4}} e^{-\frac{8\pi\sqrt{3}\Lambda^2 T_h^{3/2}}{5 \rho_{\text{de}}^{1/2} m_{\text{int}}^{5/2}}}. \quad (\text{B19})$$

Note that the action obtained from the full potential must be greater than the action obtained from the approximate no-barrier potential and, therefore, the actual decay rates will be slower than the estimates provided here. This can be

seen by noting that the full Euclidean action evaluated on its full bounce solution must be greater than the approximate no-barrier Euclidean action evaluated on the full bounce solution (since the full potential is always greater than or equal to the approximate no-barrier potential). Moreover, the approximate no-barrier Euclidean action evaluated on the full bounce solution must be greater than the approximate no-barrier Euclidean action evaluated on the no-barrier bounce solution because the no-barrier bounce solution minimizes the no-barrier Euclidean action. Therefore, the conditions in Eq. (B7), using the approximations equations (B14) and (B19), guarantee that the actual models are viable. The limits quoted in the main text, Eqs. (44a) and (44b), are obtained simply by insisting that Γ_3 and Γ_4 are strongly exponentially suppressed, which is sufficient precision for our purposes.

APPENDIX C: MORE DETAILS ON THE QUINTESSENCE ASSISTED SCENARIO

In Fig. 9 (left panel), we show the evolution of the fields ϕ and ψ in a theory of quintessence assisted dark energy in which ϕ has a potential with an exponential runaway. The dynamics of the theory are similar to the early stages of a dark matter assisted dark energy theory, e.g. as plotted in Fig. 6, except that ϕ sources dark energy while ψ is still frozen by Hubble friction. Once ψ is unfrozen, at around $a/a_0 \simeq 10$, ϕ is no longer trapped at $\phi/\Lambda \simeq 0$ by the $\phi - \psi$ interaction term and rolls down its runaway potential (for the parameters of the theory shown, there is no era in which ϕ is trapped while ψ behaves as matter). In the right panel we plot the equation of state parameter of the same theory. Prior to $a/a_0 \simeq 10$ this matches the Λ CMD prediction closely. Soon after ϕ is released from $\phi/\Lambda \simeq 0$ the theory is nonlinear and w evolves in a complicated way. However, eventually the system will approach the standard tracker solution for an exponential potential [59], which, for the value of Λ/M_{Pl} used, leads to nonaccelerating expansion.

APPENDIX D: PRELIMINARY ANALYSIS OF COSMOLOGICAL PERTURBATIONS

The theories that we consider allow for additional cosmological perturbations compared to Λ CDM. Indeed, during primordial inflation both ψ and ϕ inevitably get a (approximately) flat spectrum of isocurvature perturbations of magnitude roughly $H_I/(2\pi)$, where H_I is the Hubble scale during inflation (in addition to the usual adiabatic perturbations). Depending on their size at different times during the cosmological history, such perturbations could lead to either constraints or new observational signals. For instance, these could arise from their impact on the cosmic microwave background [1], structure formation either at early times (as probed by e.g. Lyman- α observations [117]) or late-time observations at redshift $z \lesssim \text{few}$.

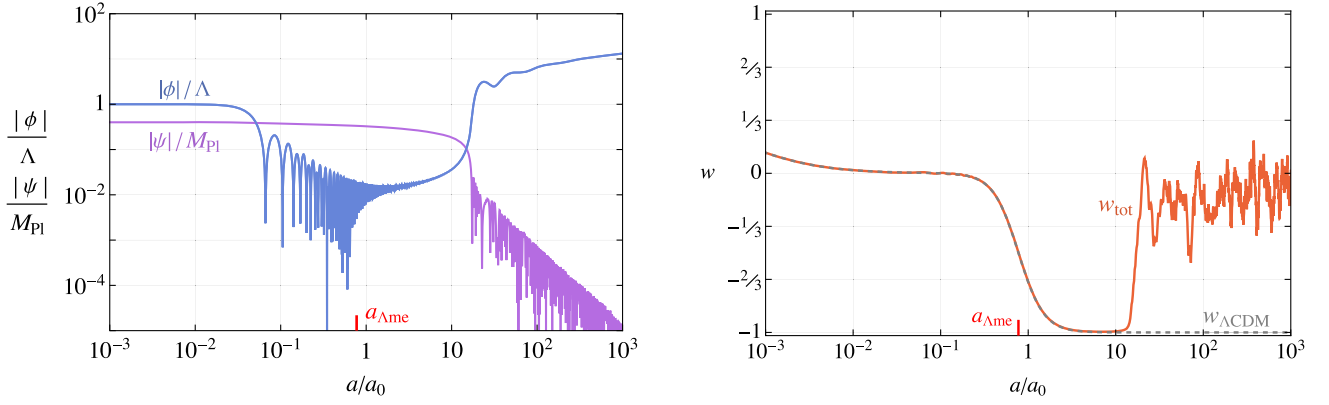


FIG. 9. Left: the evolution of ϕ and ψ in a theory of quintessence assisted dark energy, i.e. two field quintessence, in which ϕ has the exponential runaway potential of Eq. (3). The Lagrangian parameters are $m_\psi = H_0/5$, $m_{\text{int}} = 40H_0$, $\Lambda = M_{\text{Pl}}/10$ and $\lambda = 0$ and initial field values $\phi_i = \Lambda$, $\psi_i = 2M_{\text{Pl}}/5$. The dynamics are similar to the dark matter assisted scenario shown in Fig. 6, except that because of its small mass ψ does not roll down its potential until after the present day. While ψ is frozen, it first drives and then traps ϕ at a field value $\ll \Lambda$ where it sources dark energy. Once ψ rolls to sufficiently small values, ϕ rolls down its runaway potential to large field values (for the particular parameters chosen, there is not an era of dark matter assisted dark energy after ψ starts oscillating). Eventually the system will reach the standard tracker solution discussed in the main text. Right: the equation of state parameter of the theory plotted in the left panel w_{tot} compared to the Λ CDM prediction, $w_{\Lambda\text{CDM}}$. Until $a/a_0 \simeq 10$, the theory matches the Λ CDM result, and subsequently the era of accelerated expansion ends.

In this appendix, we carry out a preliminary analysis of the evolution of cosmological perturbations in our theories. We argue that if, as imposed in the main text, the initial perturbations in ϕ and ψ at the end of inflation are small then they do not grow exponentially and can therefore be safe from observational constraints.¹² In particular, we focus on the evolution of density perturbations during those eras that are potentially the most dangerous for the growth of beyond- Λ CDM perturbations. We leave the important work of a full analysis and comparison to observational data to future work.

We begin by considering the dark matter and quintessence assisted scenarios. We consider only times before when ψ starts oscillating in the dark matter assisted scenario (such analysis might miss interesting late-time observational signals; we will return to this in future work). In this regime, the evolution of initially small cosmological perturbations can be analyzed by considering the relativistic linear perturbation equations for the two coupled scalar fields ϕ and ψ and a perfect fluid, which represents either the dominant dark matter component or the SM radiation bath (which we assume to be only coupled to ϕ and ψ gravitationally). To do so, we follow Ref. [118] closely (see also [119] and e.g. [18,120,121]).

¹²Note that even if they do not grow, there can still be strong observational constraints that require the initial size of e.g. any isocurvature perturbations in even a subcomponent of dark matter (as ψ is in the matter dominated regime) to be a few orders of magnitude smaller than the adiabatic perturbations. This typically translates to an upper bound on H_I .

As usual in linear perturbation theory we split all quantities into a spatially homogeneous background (denoted with a subscript 0) and spacetime-dependent small perturbations. The perturbations consist of a perturbation for each scalar field (ϕ_1 and ψ_1) and the energy density perturbation ($\rho_1^{(f)}$) and velocity perturbation ($v^{(f)}$) in the perfect fluid (there is no anisotropic stress for perfect fluids or scalar fields [118,119]). In addition these perturbations couple to the gravity perturbations. The perturbed Friedmann-Lemaître-Robertson-Walker metric has the form [118]:

$$ds^2 = -(1 + 2\alpha)dt^2 - 2a\partial_i\beta dx^i dt + a^2[(1 + 2\varphi)\delta_{ij} + 2\partial_i\partial_j\gamma]dx^i dx^j, \quad (\text{D1})$$

where α , β , γ , and φ are the spacetime-dependent first order scalar-type metric perturbations, and a is the scale factor.

In momentum space, the equations of motion of the metric perturbations can be written as [118]

$$-\frac{k^2}{a^2}\varphi + H\kappa = -\frac{1}{2M_{\text{Pl}}^2}\rho_1, \quad (\text{D2a})$$

$$\kappa - \frac{k^2}{a^2}\chi = \frac{3}{2M_{\text{Pl}}^2}\frac{a}{k}(\rho_0 + p_0)v, \quad (\text{D2b})$$

$$\dot{\chi} + H\chi - \alpha - \varphi = 0, \quad (\text{D2c})$$

$$\dot{\kappa} + 2H\kappa + \left(3\dot{H} - \frac{k^2}{a^2}\right)\alpha = \frac{1}{2M_{\text{Pl}}^2}(\rho_1 + 3p_1), \quad (\text{D2d})$$

where $\kappa \equiv 3(-\dot{\phi} + H\alpha) + \frac{k^2}{a^2}\chi$, $\chi \equiv a(\beta + a\dot{\gamma})$, and k is the comoving wave number. Here ρ and p denote the total energy density and pressure respectively, which include contributions from the background fluid, ϕ and ψ ; v denotes the total velocity perturbation. Meanwhile, the perfect fluid perturbations satisfy [118]

$$\dot{\rho}_1^{(f)} + 3H\rho_1^{(f)}(1 + w^{(f)}) = \rho_0^{(f)}(1 + w^{(f)})\left(-\frac{k}{a}v^{(f)} + \kappa - 3H\alpha\right), \quad (\text{D3a})$$

$$\dot{v}^{(f)} + Hv^{(f)}(1 - 3w^{(f)}) = \frac{k}{a}\left(\alpha + \frac{w^{(f)}}{(1 + w^{(f)})}\frac{\rho_1^{(f)}}{\rho_0^{(f)}}\right), \quad (\text{D3b})$$

where for a noninteracting perfect fluid both background and perturbations satisfy $p^{(f)} = w^{(f)}\rho^{(f)}$, which defines $w^{(f)}$, since its nonadiabatic pressure vanished [119]. The equations of motion for the scalar field perturbations are given by [119]

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 + \frac{k^2}{a^2}\phi_1 + \partial_{\phi_0\psi_0}V_0\psi_1 + \partial_{\phi_0}^2V_0\phi_1 = \dot{\phi}_0(\dot{\alpha} + \kappa) + \alpha(2\ddot{\phi}_0 + 3H\dot{\phi}_0), \quad (\text{D4a})$$

$$\ddot{\psi}_1 + 3H\dot{\psi}_1 + \frac{k^2}{a^2}\psi_1 + \partial_{\phi_0\psi_0}V_0\phi_1 + \partial_{\psi_0}^2V_0\psi_1 = \dot{\psi}_0(\dot{\alpha} + \kappa) + \alpha(2\ddot{\psi}_0 + 3H\dot{\psi}_0), \quad (\text{D4b})$$

where ϕ_0 and ψ_0 represent the background scalar fields and V_0 is their potential.

Finally one needs to relate the metric perturbations with the matter content. The energy-momentum perturbations have contributions from the perfect fluid and from the two field system

$$\rho_1 = \rho_1^{(f)} + \rho_1^{(\phi\psi)}, \quad (\text{D5a})$$

$$p_1 = p_1^{(f)} + p_1^{(\phi\psi)}, \quad (\text{D5b})$$

$$v = v^{(f)} + v^{(\phi\psi)}. \quad (\text{D5c})$$

The perturbation of the scalars can be written in terms of field perturbations as [118,119]

$$\rho_1^{(\phi\psi)} = \dot{\phi}_0\dot{\phi}_1 + \dot{\psi}_0\dot{\psi}_1 - \alpha(\dot{\phi}_0^2 + \dot{\psi}_0^2) + \partial_{\phi_0}V_0\phi_1 + \partial_{\psi_0}V_0\psi_1, \quad (\text{D6a})$$

$$p_1^{(\phi\psi)} = \dot{\phi}_0\dot{\phi}_1 + \dot{\psi}_0\dot{\psi}_1 - \alpha(\dot{\phi}_0^2 + \dot{\psi}_0^2) - \partial_{\phi_0}V_0\phi_1 - \partial_{\psi_0}V_0\psi_1, \quad (\text{D6b})$$

$$(\rho_0^{(\phi\psi)} + p_0^{(\phi\psi)})v^{(\phi\psi)} = \frac{k}{a}(\dot{\phi}_0\phi_1 + \dot{\psi}_0\psi_1). \quad (\text{D6c})$$

We now have a complete set of equation for the perturbations in a ‘‘gauge-ready’’ form. In a full analysis, it would be convenient to translate these in terms of gauge invariant quantities as in [119], which would allow the evolution of adiabatic and entropic perturbations to be consistently isolated. Instead, for our purposes of seeing that the perturbations do not grow, it is sufficient to consider the synchronous gauge by setting $\alpha = \beta = 0$ [122]. This choice reduces the number of independent perturbations to

$$\ddot{\phi}_1 + 3H\dot{\phi}_1 + \frac{k^2}{a^2}\phi_1 + \partial_{\phi_0\psi_0}V_0\psi_1 + \partial_{\phi_0}^2V_0\phi_1 = \dot{\phi}_0\kappa, \quad (\text{D7a})$$

$$\ddot{\psi}_1 + 3H\dot{\psi}_1 + \frac{k^2}{a^2}\psi_1 + \partial_{\phi_0\psi_0}V_0\phi_1 + \partial_{\psi_0}^2V_0\psi_1 = \dot{\psi}_0\kappa, \quad (\text{D7b})$$

$$\dot{\rho}_1^{(f)} + 3H\rho_1^{(f)}(1 + w^{(f)}) = \rho_0^{(f)}(1 + w^{(f)})\left(-\frac{k}{a}v^{(f)} + \kappa\right), \quad (\text{D7c})$$

$$\dot{v}^{(f)} + Hv^{(f)}(1 - 3w^{(f)}) = \frac{k}{a}\frac{w^{(f)}}{(1 + w^{(f)})}\frac{\rho_1^{(f)}}{\rho_0^{(f)}}, \quad (\text{D7d})$$

$$\begin{aligned} \dot{\kappa} + 2H\kappa = & \frac{1}{2M_{\text{Pl}}^2}(\rho_1^{(f)}(1 + 3w^{(f)}) + 4\dot{\phi}_0\dot{\phi}_1 + 4\psi_0\dot{\psi}_1 \\ & - 2\partial_{\phi_0}V_0\phi_1 - 2\partial_{\psi_0}V_0\psi_1). \end{aligned} \quad (\text{D7e})$$

In both the dark matter and quintessence assisted scenarios, at early times, when $H \gg m_{\text{int}}\psi/\Lambda$, both ϕ_0 and ψ_0 are frozen. At such times, the perturbations ϕ_1 and ψ_1 are also frozen, and (given that ϕ and ψ contribute only a tiny contribution to the overall energy density of the Universe) no significant metric perturbations are induced. In typical theories allowed by existing constraints, this era continues until around the time of matter-radiation equality or later (cf. Figs. 1 and 9).

The subsequent evolution of perturbations during the era while ϕ is being driven to the origin due to its interaction with ψ (which remains frozen) is potentially more interesting. For simplicity, we will assume that this occurs during matter domination, as is the case of over the majority of the interesting parameter space. Consequently, the dominant background fluid has $w^{(f)} = 0$. Denoting quantities at the time when ϕ first starts being driven towards the origin by the subscript *osc*, we can write the scale factor and background energy density during this era as $a = a_{\text{osc}}(t/t_{\text{osc}})^{2/3}$ and $\rho_0(t) = \rho_{\text{osc}}(t_{\text{osc}}/t)^2$, and we note that t_{osc} is set by $3H_{\text{osc}} = m_{\text{int}}\psi_i/\Lambda$. The background fields satisfy

$$\phi_0(t) = \phi_{\text{osc}} \frac{t_{\text{osc}}}{t} \cos(m_{\text{int}}\psi_i(t - t_{\text{osc}})/\Lambda), \quad (\text{D8a})$$

$$\psi_0(t) = \psi_{\text{osc}}. \quad (\text{D8b})$$

It is then straightforward to solve the system of equations (D7) numerically, both for super- and subhorizon modes. As an example, in Fig. 10 we plot the evolution of

superhorizon perturbations ($k \ll aH$), starting from initial conditions with $\phi_0 = \Lambda$ and $\psi_0 = M_{\text{Pl}}/5$ as in Fig. 1 and small initial perturbations $\phi_1 = 10^{-3}\Lambda \ll \phi_0$ and $\psi_1 = 10^{-3}M_{\text{Pl}} \ll \psi_0$ (and with initial $\kappa_1 = v_1 = \rho_1 = 0$). We have checked that these results are independent of whether ϕ has a potential with a hilltop or an exponential form, as expected given that ϕ 's potential is subdominant at these times. Results are shown until $a/a_{\text{osc}} \simeq 300$, at which point ψ_0 starts oscillating for the parameters used. Note that for a matter dominated universe v is not sourced (and any initial value would decay as $v \propto t^{-2}$). The perturbation in ϕ grows by a factor of 10, but does not increase any further (and decreases towards the final times) and also has average value 0. Meanwhile ψ_1 is basically frozen. The induced perturbations in ρ and κ are tiny, which is consistent with the energy densities in ϕ and ψ being subdominant during this time [in particular, from Eq. (D7) the natural normalization for the induced κ is H_{osc} , and κ is suppressed relative to this by ρ_ψ/ρ_0 with ρ_ψ the energy density in ψ 's potential]. We checked numerically that the behavior in Fig. 10 is representative of superhorizon perturbations also for parameters corresponding to the quintessence assisted scenario. We have also checked that subhorizon ($k \gg aH$) perturbations oscillate with a decreasing amplitude. We therefore conclude that provided they are initially small, cosmological perturbations do not grow to magnitudes that are dangerously large for observations during this era.

Subsequently, in the dark matter assisted scenario, ψ starts oscillating and ϕ enters the locked regime. We have argued that during this era, subhorizon ϕ modes (corresponding to subhorizon perturbations) grow at most as fast as the zero mode (cf. Appendix A 1), so these remain small during dark energy domination assuming a homogeneous ψ background. It would be very interesting to investigate the

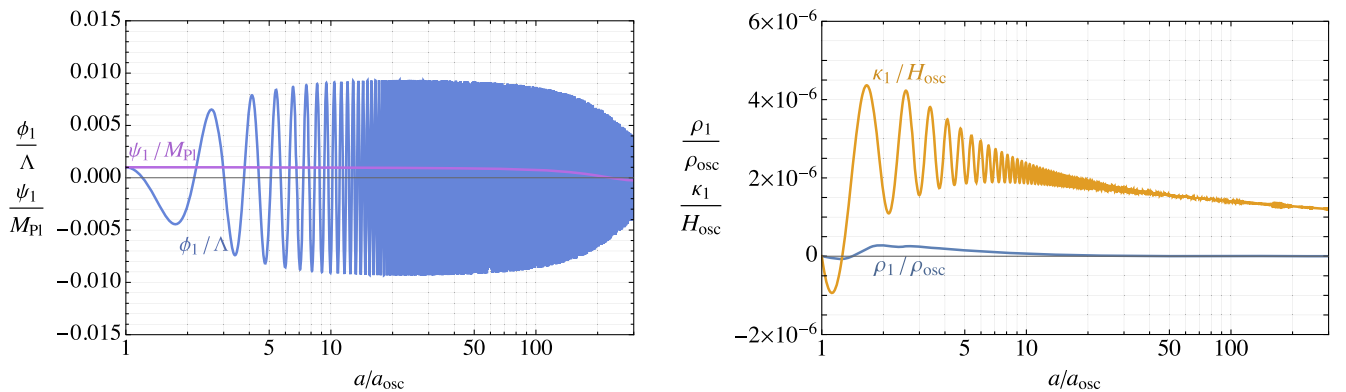


FIG. 10. The evolution of superhorizon perturbations (left) ϕ_1 and ψ_1 (right) ρ and κ (in the synchronous gauge) in a theory of dark matter assisted dark energy with ϕ having a hilltop potential. The theory is given by Eqs. (1) and (2) with $V(\phi) = V_{\text{hill}}(\phi)$, with parameter values $m_\psi = 10H_0$, $m_{\text{int}} = 10^4H_0$, $\Lambda = M_{\text{Pl}}/5$ and $\lambda = 0$ and initial field values $\phi_0 = \Lambda$, $\psi_0 = M_{\text{Pl}}/5$. The initial conditions for the perturbations are $\phi_1 = 10^{-3}\Lambda$, $\psi_1 = 10^{-3}M_{\text{Pl}}$ and $\rho_1 = v = \kappa = 0$. The evolution of the perturbations is obtained by numerically solving the equations of motion of the perturbations with a background with ϕ_0 oscillating towards the origin and ψ_0 frozen by Hubble friction in a matter dominated universe ($w = 0$). The initial time is set by the time when ϕ starts being driven towards $\phi = 0$, which occurs when $3H \simeq m_{\text{int}}\psi_i/\Lambda$.

evolution of metric perturbations and perturbations in ψ during this era as well, and any possible observational signatures, but we leave this for future work. Meanwhile, in quintessence assisted scenario the subsequent dynamics, during dark energy domination, consist of ϕ and ψ slow rolling with only small field displacements. We leave the interesting question of the determining the evolution of perturbations during dark energy domination and possible resulting late-time observational signals to future work (we do not expect the dynamics during this era, long after CMB formation, to lead to observational signals in the CMB).

Finally, we briefly comment on perturbations in the dark radiation assisted scenario, in which there is a thermal bath of ψ that acts as subdominant radiation component. In such theories it is natural to suppose that the ψ thermal bath is produced by the decay of the inflation, and as a result only inherits the usual initially small adiabatic fluctuations. We have checked numerically that initially small perturbations in ϕ with a homogeneous ψ background also do not grow in such theories (see also [120,121] for the related analysis of perturbation for a perfect fluid and a scalar field system)

while ϕ is being driven towards the origin (as in the dark matter and quintessence assisted scenarios, it might be interesting to consider the evolution of perturbations during dark energy domination). We have also checked that small perturbations in ψ (which result in ϕ having a slightly different thermal potential in different regions of space) do not induce growing perturbations in ϕ . Moreover, we expect that, given that the ψ thermal bath is relativistic today, it will free stream out of the gravitationally collapsed halos that form from adiabatic perturbations during usual structure formation, which means that large perturbations in ϕ would not be induced this way. From these results it is plausible that such theories with only small perturbations in ϕ and adiabatic fluctuations in the ψ thermal bath do not contradict observations. However, in the future it would be very interesting to analyze the dynamics of ϕ perturbations in the era of structure formation in detail, especially in the case that ϕ has an exponential potential, such that the minimum of its full thermally corrected potential varies continuously with temperature rather than always being at $\phi = 0$ while ϕ is trapped.

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