

Model of the response of surface detectors to extensive air showers based on shower universality

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We present a full model of surface-detector responses to extensive air showers. The model is motivated by the principles of air-shower universality and can be applied to different types of surface detectors. Here we describe a parametrization for both water-Cherenkov detectors and scintillator surface detectors, as for instance employed by the upgraded detector array of the Pierre Auger Observatory. Using surface detector data, the model can be used to reconstruct with reasonable precision shower observables such as the depth of the shower maximum $X_{\max}$ and the number of muons R_{μ} .

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I. INTRODUCTION

Ultra-high-energy cosmic rays (UHECRs) are the highest energy particles known to humankind; they challenge the hypothesized limits for the acceleration of particles, while a single source of UHECRs has not yet been identified [1]. Although experimental evidence implies that the spectrum of cosmic rays is strongly dominated by ionized nuclei for energies greater than 10^{18} eV [2,3], the exact chemical composition of the UHECRs is still unknown.

Due to the low flux at the highest energies, the UHECRs are not observed directly, but indirectly by the extensive air showers (EASs) they induce in the Earth's atmosphere. These are cascades of secondary particles created when a UHECR interacts with an air nucleus at the top of the atmosphere. At the highest energies, the footprint of an EAS at the ground reaches several kilometers in diameter and can therefore be detected by a sparse array of surface detector (SD) stations.

To estimate the chemical composition (nuclear masses) of UHECRs, a precise understanding of the air shower phenomenon is crucial. Especially the depth $X_{\max}$ of the maximum of air-shower development and the relative number of muons R_{μ} in the shower are strong indicators of the primary mass of the cosmic ray. Studies

on both $X_{\max}$ and R_{μ} individually have been performed successfully [4–6].

$X_{\max}$ is an observable that is obtained from direct observation of the longitudinal profile of fluorescence light produced by a shower, which is, however, only possible during clear moonless nights. The shower observed in fluorescence light appears brightest at the point where its development reaches the maximum $X_{\max}$. Using only SD data, $X_{\max}$ can be estimated indirectly from the curvature and thickness of the shower front reaching the ground [7].

The relative¹ muon number R_{μ} can be estimated using the signal of water-Cherenkov detectors (WCDs) such as those employed by the Pierre Auger Observatory (Auger, in short), if the primary energy of the cosmic ray can be estimated independently. This can be achieved, for example, with a fluorescence detector or with the help of scintillator surface detectors (SSD), such as those deployed in the AugerPrime upgrade [9].

In this work, we present a model of the expected particle densities in air showers for given values of $X_{\max}$ and R_{μ} at a given primary energy. The model is based on the concept of “air-shower universality” [10], or in short Universality (for an overview see Ref. [11]). It describes the dependence of the expected signal in the SD as a function of the atmospheric distance to the shower maximum, which is otherwise considered only on average using the

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¹In this article, we will use the average muon content of showers induced by proton primary cosmic rays simulated with the EPOS-LHC [8] model of hadronic interactions as a reference. A quantitative definition will be given later in the text.

constant-intensity-cut method; it describes the lateral evolution of the shower particle densities; and it describes the temporal distribution of shower particles at the ground. The parameters of the model are determined using Monte Carlo simulations of the signal deposited in an Auger-like SD array using WCDs and SSDs. Using the observables $X_{\max}$ and R_{μ} as free parameters in a fit, the model can be used to describe the detector data and is thus able to estimate the mass of the primary particle.

This work builds on previous implementations of the universality model [12–18] in the *Offline* software framework [19] of the Pierre Auger Observatory. In contrast, the new implementation introduces physically motivated shower profile functions, as well as full detector simulations for the final parametrizations. The model developed in this work we will refer to as *Universality II*.

II. EXTENSIVE AIR SHOWERS

An EAS is a cascade of subatomic particles created upon impact of an UHECR with the Earth's atmosphere. The particles in an air shower multiply through collisions, decay, and radiation until the energy budget of the primary cosmic ray is converted entirely.

Most particles are created in the core of the EAS, which develops along the extended trajectory of the primary UHECR and which defines the shower axis as a line of reference; the plane that lies perpendicularly to the shower axis and that contains the current location of the shower core is defined as the plane front of the shower. The propagation of the plane front and the core is usually described as a function of slant atmospheric depth X (in units of g/cm^2). Shower plane is defined as the plane front that contains the impact point of the shower core on the ground. The positions of the detectors on the ground may be given in terms of their polar coordinates in the shower plane, defined by the distance r to the shower axis and the angle ψ between the station and the direction in which the shower axis is tilted. The inclination of the shower axis with respect to the zenith is described by the zenith angle θ . The geometry of the event is illustrated in Fig. 1.

Even though the particle content of an EAS is heavily governed by the hadronization occurring in the first interactions, most of the particles in the cascade are created by electromagnetic processes and thus most of the energy is deposited in the atmosphere by electrons, positrons, and photons. Muons, which are mostly created from the decay of charged pions, play a significant role in the detection of UHECRs, too. First, they can be easily detected by surface detectors even in inclined events, because of their long lifetime and smaller energy losses. Second, they provide a measure for the amount of charged pions created in the shower, and thus of hadronization that took place in the first interactions [6,20]. Since the hadronization of the first interactions is enhanced by the number of nucleons taking

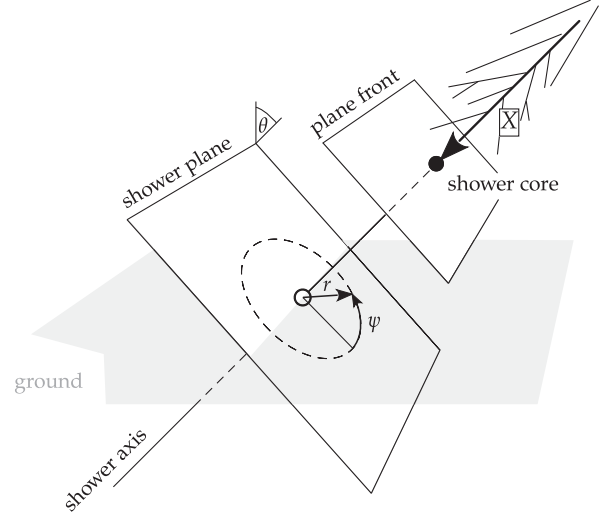


FIG. 1. Illustration of the event geometry. The shower core at the depth X is depicted as full black marker. The plane front of the shower contains the shower core and is perpendicular to the shower axis. The impact point of the shower core at the ground is given as hollow circular marker. The shower plane contains the shower core at the ground and is perpendicular to the shower axis. The zenith angle θ describes the inclination of the shower axis. Positions of points on the ground are expressed with their shower-plane coordinates r and ψ .

part in the first interaction, the amount of muons in an EAS is an indicator for the nuclear mass of the primary particle. The number of particles created by hadronic and electromagnetic processes is schematically described in the Heitler-Matthews model of hadronic showers [21]. In this model, a shower is initiated by a primary proton interacting hadronically with the atmosphere. In every hadronic interaction, charged and neutral pions² are created in a 2:1 ratio. In this model, charged pions will continue to interact hadronically, while neutral pions promptly decay into photons, fueling the hadronic and electromagnetic cascades. The energy is always distributed equally among all particles created in an interaction. Once the charged pions fall below a critical energy of $\epsilon_c^\pi \simeq 20 \text{ GeV}$, they stop fueling the cascade and decay into muons. The total number of muons N_μ created in a shower induced by a proton of energy E_0 thus scales as

$$N_\mu = \left(\frac{E_0}{\epsilon_c^\pi} \right)^\beta, \quad (1)$$

where the relative muon growth rate is $\beta \lesssim 1$.

According to the Heitler model [22] of electromagnetic cascades, the electromagnetic particles multiply without any hadronic interaction until a critical energy of

²The creation and interaction of heavier mesons are neglected.

$\epsilon_c^{\text{ey}} \simeq 87$ MeV is reached. At this point, the size of the shower, that is, the number of particles present, is at its maximum. The slant atmospheric depth of the position of the shower maximum is referred to as X_{max} .

Again according to the Heitler-Matthews model, in showers initiated by primary nuclei with nuclear mass A , all nucleons initiate individual and independent cascades, each of them with a $1/A$ fraction of the total energy of the primary nucleus, while ignoring shower-to-shower fluctuations. Thus, the critical energy ϵ_c^{ey} is reached after fewer interactions for primary nuclei with $A > 1$ than for protons ($A = 1$). This implies that on average the depth X_{max} is smaller for heavier primary particles. Furthermore, because the relative rate of muon growth β in Eq. (1) is less than 1, on average a factor of $A^{1-\beta}$ more muons are produced in showers induced by primary nuclei with mass A than in proton showers.

In the context of the mass composition of UHECRs, the depth of the shower maximum X_{max} and the number of muons N_μ created in a shower are thus important mass-sensitive observables.

III. A MODEL FROM AIR-SHOWER UNIVERSALITY

Air-shower universality, as first observed in simulations by Hillas [10], states that the electromagnetic particles in individual air showers initiated by primary particles with a large energy E_0 show the same energy spectrum, as well as radial and angular distributions, and that these distributions and spectra are well described by idealized models such as those given in Refs. [23–25]. Especially around the maximum of the shower, where the electromagnetic particles of a shower on average carry the same energy ϵ_c^{ey} , the longitudinal and lateral distributions of the particles (profiles), the energy spectra, and the angular distributions of particles are in a good approximation the same for all showers initiated with the same primary energy E_0 . This is true independent of the absolute value of X_{max} and of the type of primary particle. Simulated longitudinal profiles of the showers, which describe the number of particles N present at depth X , are depicted in Fig. 2. Profiles become universal when introducing a shower-depth parameter $\Delta X = X - X_{\text{max}}$, which describes the depth relative to the shower maximum, and when the number of particles is simultaneously scaled with the primary energy; 10^{19} eV is chosen as the reference energy E_{ref} , to which the profiles shown in Fig. 2 are scaled.

The longitudinal profile of air showers and the spectra of the corresponding electromagnetic particles are strongly related to each other. It was already demonstrated analytically in the 1930s [23] and later using intense simulations [25] that the spectra of electromagnetic particles in air showers can be well described as a function of the shower-age parameter s . The

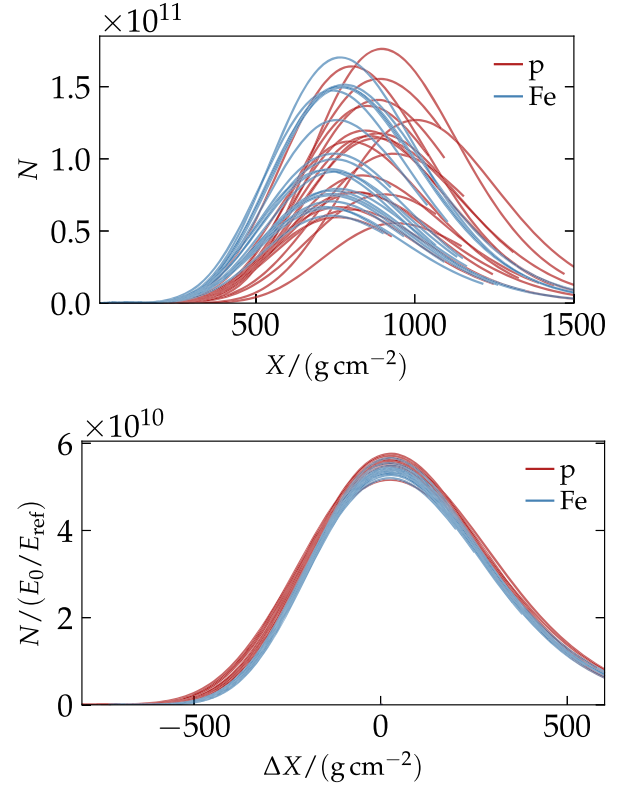


FIG. 2. Longitudinal number profiles $N(X)$ of simulated air showers from proton and iron primary particles, p and Fe, as a function of depth X . The showers are simulated with primary energies between $10^{19.5}$ and 10^{20} eV and with different zenith angles. Using the primary energy E_0 and reference energy $E_{\text{ref}} = 10^{19}$ eV, the $N(X)$ profiles in the second panel are scaled by E_0/E_{ref} in magnitude and shifted individually, $\Delta X = X - X_{\text{max}}$, to place the maximum at zero and align the various showers.

parameter s can be expressed as a function of depth X and depth of the shower maximum X_{max} approximately as $s \simeq 3X/(X + 2X_{\text{max}})$ [26]. The relative rate of change of the longitudinal profiles,³ g , of electromagnetic particles, $\lambda_1 = (\partial g/\partial X)/g$, is well described as a function⁴ of s [11,23], which unambiguously relates the longitudinal profile g to the shower age.

Furthermore, for depths X around X_{max} , the NKG function [27], which has been proven to successfully describe the lateral distribution of shower particles [28], is approximately constant in s . Therefore, around the maximum, the dependence of the particle distributions on energy, depth, and radius factorizes. Furthermore, the number of electromagnetic particles for any primary energy E_0 is well described by a power law, $(E_0/E_{\text{ref}})^\gamma$ with $\gamma \lesssim 1$ [21]. Thus, in general, the density of electromagnetic particles in the shower plane at the depth ΔX near the shower maximum and at the distance r to the shower axis

³An example function for g is given in Eq. (7).

⁴Possible functions for λ_1 are given in Appendix A.

can be factorized to a good approximation by a universal function of the form [26]

$$\rho \simeq (E_0/E_{\text{ref}})^\gamma f(r)g(\Delta X), \quad (2)$$

with a longitudinal profile function $g(\Delta X)$ and an NKG-like function $f(r)$; the zenith-angle dependence of the number of particles reaching the SD is effectively described by $g(\Delta X)$ with the distance ΔX to the shower maximum, cf. Eq. (C1). This concept has been extended to different particle components in air showers [13], so that not only electromagnetic, but also hadronic showers can be well described by a model based on Universality. For this purpose, the particles in air showers are subdivided into four components, for which the longitudinal and lateral profiles are parametrized individually using a function of the form given in Eq. (2). The number of particles in each component—except for the electromagnetic—scales approximately linearly with the relative amount of muons R_μ in the shower. In contrast, the size of the electromagnetic component is to a large extent independent of R_μ . This is discussed in Appendix B. The total shower content is then comprised by the sum of the four components. Since R_μ is in general an observable that depends on the primary particle, it is expected that three of the four particle components will be systematically improved in showers initiated by heavier primary particles.

The four-component model, as well as the longitudinal and lateral profiles used to parametrize the individual particle densities, will be discussed in the following.

A. The four-component shower model

We consider hadronic air showers to be constituted by the four particle components introduced in [13]. The resulting shower is disentangled into each of the four components with different longitudinal and lateral profiles. The electromagnetic component, $e\gamma$, contains electrons and positrons, as well as photons, all of them created by the electromagnetic cascade of the air shower. The muon component, μ , contains all muons and antimuons from the shower. The electromagnetic muon component, $e\gamma(\mu)$, contains electromagnetic particles that were created by decaying particles of the muon component in the first or second generation. The hadronic component, $e\gamma(\pi)$, contains all electromagnetic particles that were created in hadronic decays within two generations as well as all hadrons. The rest of the particles, such as neutrinos, which do not deposit a significant amount of signal in a surface detector, are neglected.

The number of particles N_i of the i th component is considered at a fixed depth ΔX_{ref} in the shower plane. The amount of particles relative to the respective average in a proton shower $\langle N_i^p \rangle$ is thus given by

$$R_i := \frac{N_i}{\langle N_i^p \rangle}, \quad (3)$$

which for $i = \mu$ defines the relative number of muons. For convenience, the size of all particle components is expressed as a function of R_μ , independent of the energy. By definition, the proton showers have on average $R_\mu = 1$. In the first order, the size of each component scales linearly with respect to R_μ , according to the relation [13]

$$R_i - 1 = a_i(R_\mu - 1), \quad (4)$$

with a constant a_i for each particle component. Since the number of particles in the electromagnetic component scales roughly independently of R_μ , $a_{e\gamma}$ is expected to be very close to 0, but slightly smaller than 0 due to energy conservation. For the $e\gamma(\mu)$ and $e\gamma(\pi)$ components values of $a_{e\gamma(\mu)} \simeq a_{e\gamma(\pi)} \simeq 1$ are expected. Except for their correlation according to Eq. (4), the four components are treated individually and independently.

For each component, the total number of particles as a function of the primary energy E_0 is given by a power law. For reference, the expected number of particles produced by a proton shower with $E_0 = 10^{19}$ eV is written as $N_{19,i}$. The expected number of particles in a shower of primary energy E_0 is thus given by

$$N_i(E_0) = N_{19,i} \left(\frac{E_0}{10^{19} \text{ eV}} \right)^{\gamma_i}. \quad (5)$$

For better readability we will in the following drop the component index i from all component-specific parameters.

B. The longitudinal profile

The Gaisser-Hillas profile function accurately describes the longitudinal development of EAS, as was directly measured in Ref. [29]. To parametrize the particle density in any point at a distance r and at a depth⁵ ΔX , the modified Gaisser-Hillas profile was introduced in Ref. [13]. A sketch of the geometry is given in Fig. 3(left). We use the modified Gaisser-Hillas function to describe the development of the particle density in the shower plane front along the shower axis. At the moment when the plane front intersects with a surface detector, the shower core is at the height h_{proj} [cf. Eq. (C2)] above the ground. The modified Gaisser-Hillas profile $g(\Delta X)$ is normalized to a fixed depth ΔX_{ref} after the maximum of the shower. For each of the four components of the shower, the particle density can be parametrized using $g(\Delta X)$ as

$$\rho = \rho_{\text{ref}}(r)g(\Delta X), \quad (6)$$

$$g(\Delta X) = \left[\frac{\Delta X - \Delta X_1}{\Delta X_{\text{ref}} - \Delta X_1} \right]^{\frac{\Delta X_{\text{max}} - \Delta X_1}{\lambda}} \exp \left[-\frac{\Delta X - \Delta X_{\text{ref}}}{\lambda} \right]. \quad (7)$$

⁵The relation between shower-plane coordinates of a detector and the depth ΔX is given in Eq. (C1).

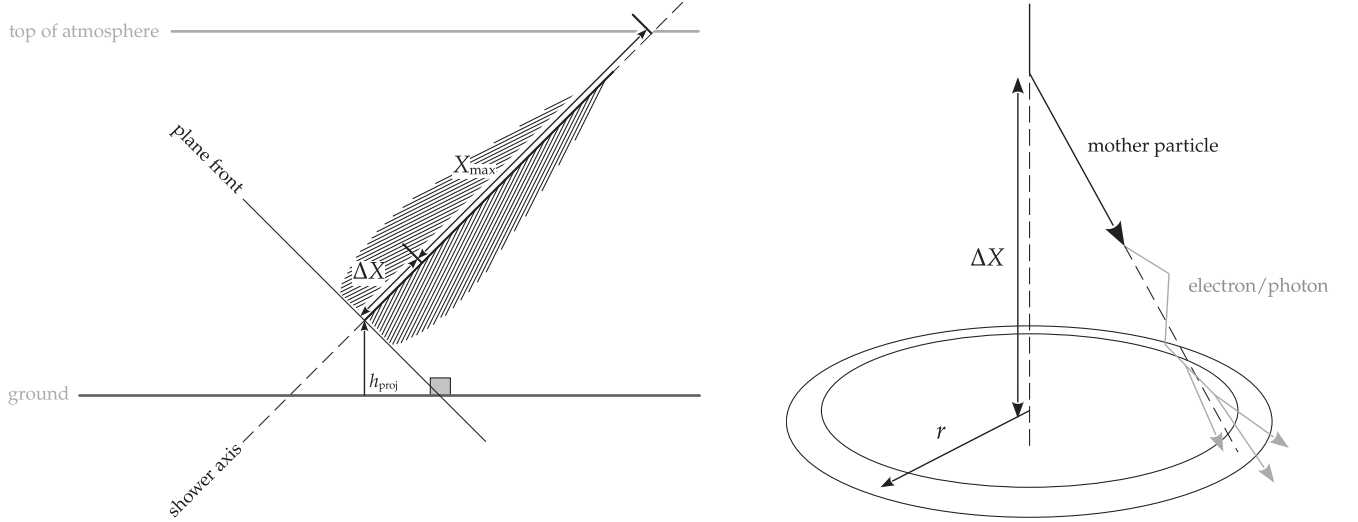


FIG. 3. Left: schematic of an event geometry. The axis of the shower is shown as a dashed line, and the path of the shower core is shown as a bold line. The slant atmospheric depth $X_{\max}$ of the shower maximum is measured from the top of the atmosphere along the shower axis. The depth parameter ΔX of a detector (gray box at the ground) is measured along the shower axis between shower maximum and the perpendicular (plane-front) projection of the detector onto the shower. The (physical) height of the shower core at depth ΔX is given by h_{proj} . Right: path of a mother particle (black arrow), produced close to the shower core and the shower maximum, which decays into electrons and photons (gray arrows). The resulting $e\gamma(\mu)$ and/or $e\gamma(\pi)$ components are therefore originating from depth ΔX and will hit the ground at a distance r from the shower axis (rings); image reproduced from Ref. [13].

The reference density ρ_{ref} at the depth ΔX_{ref} will be discussed in the next section. For the model described in Sec. IV, ΔX_{ref} is set to 200 g/cm^2 for all components, which is approximately the depth at which the surface detector is expected for most showers (the median zenith angle is $\theta \simeq 38^\circ$). Parameters λ , ΔX_1 , and $\Delta X_{\max}$ are individually parametrized for each component using simulations. As in the original Gaisser-Hillas profile, λ is related but not equal to the radiation length of the individual particles.⁶ The parameter ΔX_1 marks the effective start of the cascade for a component and is fixed to large negative constant values of $\approx -500 \text{ g/cm}^2$. Since Universality is only observed at and after the shower maximum, the values of ΔX_1 have no physical meaning and are not directly related to the first interaction. $\Delta X_{\max}$ parametrizes the radial retardation of the shower maximum, which could be equivalently described by a distance-dependent shower age [26,30]. Especially for the components μ and $e\gamma(\mu)$, the effective shower maximum is shifted towards $\Delta X > 0$ and increases with r . Both λ and $\Delta X_{\max}$ are parametrized as constant or linear functions in r .

C. The lateral reference profile

The lateral distribution of particles in air showers is well described by the functions that were derived in Refs. [24,31], and later approximated in Ref. [27]. For a shower with primary energy E_0 , the reference density ρ_{ref} ,

which describes the shower at a fixed depth ΔX_{ref} with respect to the maximum of the shower, can be parametrized using the NKG function as $f(r)$ and Eq. (5), resulting in

$$\rho_{\text{ref}}(r) = (E_0/E_{\text{ref}})^s f(r), \quad (8)$$

with $E_{\text{ref}} = 10^{19} \text{ eV}$ and

$$f(r) = f_0 \left(\frac{r}{r_G} \right)^{s-2} \left(1 + \frac{r}{r_G} \right)^{s-\frac{9}{2}}, \quad (9)$$

where

$$f_0 = N_{19} \frac{\Gamma(4.5 - s)}{2\pi r_G^2 \Gamma(s) \Gamma(4.5 - 2s)} \quad (10)$$

is used for the normalization. Since ρ_{ref} is always evaluated at $\Delta X = \Delta X_{\text{ref}}$, the parameter s as well as the reference distance r_G are fixed for each component.⁷ The dependence of the lateral distribution of particles with respect to ΔX is well described by the parametrization of λ and $\Delta X_{\max}$ as a function of the shower-plane radius r (see Sec. III B).

Due to changes in the opening angle of the particles relative to the shower axis and due to the attenuation and effects from a nonhomogeneous atmosphere, an additional

⁶For the $e\gamma$ component it is expected to be $\lambda \simeq 3X_0/2$, where X_0 is the electromagnetic radiation length in air.

⁷Because of the increasing thickness of the slanted atmosphere, this description is not valid for very inclined showers with $\theta \gtrsim 60^\circ$, for which r_G would have to be adjusted as a function of $\sec \theta$ [26].

correction factor $u(\psi)$ has to be applied to Eq. (9). A possible parametrization of $u(\psi)$ is given in Eq. (D1).

D. The areal density of particles

Combining the scaling of the size of the individual components according to Eq. (3) with the longitudinal and lateral profile functions g and f , discussed in Secs. III B and III C, as well as with a correction factor u for azimuthal asymmetry, a model of the particle density for each component according to Eq. (2) can be given. The particle density of each component as a function of ΔX and R_μ [14] reads as

$$\rho = (a(R_\mu - 1) + 1) \left(\frac{E_0}{E_{\text{ref}}} \right)^\gamma g(\Delta X) f(r) u(\psi), \quad (11)$$

where, again, indices i have been dropped for legibility. The total density of the particles is given by

$$\rho_{\text{tot}} = \sum_i \rho_i, \quad (12)$$

with the sum over the four particle components i . To describe the detector responses in an SD array, Eq. (11) is parametrized directly in terms of the signal for each component, since the Greisen factorization given in Eq. (2) also holds in terms of signals. The parametrization of the signal components according to Eq. (11) is discussed in Sec. IV.

E. The temporal distribution of particles

The temporal distributions of particles at the ground depend on the geometry of the event. Particles are considered to propagate with approximately the speed of light and to be mostly created in the shower core. If particles, for example, are created in a pointlike source above the ground, their arrival times would reflect an expanding sphere; if particles are created at an infinite distance, their arrival times would be in all detectors in coincidence with the arrival of the plane front. Assuming that particles reaching an SD arrive time ordered with respect to their creation in the shower core, the temporal distribution of particles (and thus the signal) in each detector can be related to the longitudinal profile of the shower [32] (for this reason we express all points in time relative to the time of the plane front t_{pf} reaching a detector station).

We assume that the distribution of the production depths of particles along the shower axis is similar to the longitudinal profile of the shower. This has been proven especially for the muonic component of the shower in Ref. [33]. Assuming a Gaisser-Hillas-like longitudinal profile, the 40% time quantile t_{40} of the signal deposited in a detector is directly related to X_{max} , since approximately 40% of the integrated longitudinal profile is contained

between $X = X_1$ and $X = X_{\text{max}}$ [34]. Furthermore, assuming that particles are produced in the shower core along the shower axis, the curvature of the shower front implicitly carries information about the depth of the shower maximum. We use a simplified ansatz, assuming that particles that reach an SD at the shower-plane distance r propagate rectilinearly over a distance of

$$\ell = \sqrt{D_{X'}^2 + r^2}. \quad (13)$$

Assuming an isothermal atmosphere, with vertical depth of the ground X_{vg} and scale height h_s , we try to identify the distance $D_{X'}$, which is the distance the shower plane travels from the point of origin of the particle at a depth X' until the shower plane passes through a detector. Depth X' can be expressed as

$$X' \simeq \frac{X_{\text{vg}}}{\cos \theta} \exp \left(-\frac{D_{X'} \cos \theta}{h_s} - \frac{h_{\text{proj}}}{h_s} \right), \quad (14)$$

$$= (\Delta X + X_{\text{max}}) \exp \left(-\frac{D_{X'} \cos \theta}{h_s} \right), \quad (15)$$

and therefore

$$D_{X'} \simeq \frac{h_s}{\cos \theta} \ln \left(\frac{\Delta X + X_{\text{max}}}{X'} \right). \quad (16)$$

The time quantile t_{40} , which is assumed to be related to the shower maximum, i.e., $D_{X_{\text{max}}}$, can thus be expressed as⁸

$$ct_{40} \simeq \sqrt{D_{X_{\text{max}}}^2 + r^2} - D_{X_{\text{max}}} \quad (17)$$

for particles propagating on straight trajectories with the speed of light c . We verified that t_{40} is directly dependent on X_{max} in the way implied by Eqs. (16) and (17).

In Ref. [14] it has been demonstrated that a shifted log-normal distribution function successfully describes the temporal distribution of the signal in an SD. Furthermore, the functional forms derived in Ref. [35], which relate the production of muons according to a longitudinal profile along the shower axis with the times of muons arriving at the ground, can also be numerically approximated by a shifted log-normal distribution as well. The shift is given by the start time of the signal with respect to the time of the plane front passing the detector. Therefore, the temporal distribution of n_{tot} particles in each component arriving at

⁸The arrival time of the first particles at a detector station at the distance r in the plane front at the depth ΔX is approximately related to the depth X_1 of the first interaction by the same relation evaluated at $X = X_1$.

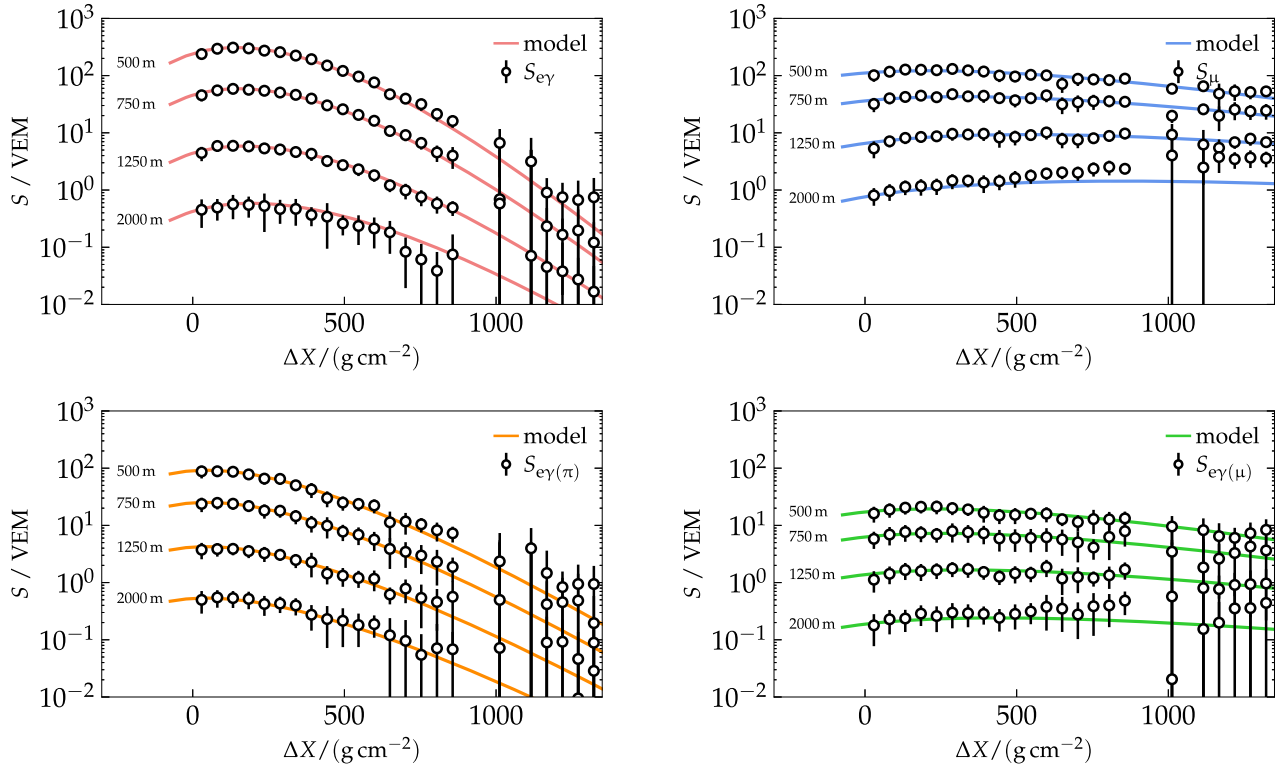


FIG. 4. Average simulated signals, $S_{e\gamma}$, S_{μ} , $S_{e\gamma(\pi)}$, and $S_{e\gamma(\mu)}$ (points), of WCDs to air showers as a function of the distance ΔX to the shower maximum along with the corresponding model for the four particle components (lines). The signal is shown as a function of the distance ΔX to the shower maximum for four different radii: 500, 750, 1250, and 2000 m.

an SD is modeled using a shifted log-normal distribution function $\mathcal{N}_{\mathcal{L}}$, which can be expressed in terms of t_{40} ,

$$\begin{aligned} \frac{d}{dt}n &= n_{\text{tot}}\mathcal{N}_{\mathcal{L}}(t - t_0, \sigma^2), \\ &= \frac{n_{\text{tot}}}{\sqrt{2\pi}\sigma(t - t_0)} \exp\left(-\frac{1}{2\sigma^2} \left[\ln\left(\frac{t - t_0}{t_{40} - t_0}\right) + k\right]^2\right), \end{aligned} \quad (18)$$

using⁹

$$k = \sqrt{2}\sigma \text{erf}^{-1}\left(2 \times \frac{40}{100} - 1\right) \approx -0.253\sigma. \quad (19)$$

The start time t_0 and the time quantile t_{40} are given relatively to the time t_{pf} of the plane front passing through the detector, which itself is defined by the reconstructed event geometry. The shape parameter σ and t_{40} are parametrized as a function of ΔX using detector simulations. σ is expressed as a function of the mean and standard deviation of the arrival times of the particles, which both can be approximated by a linear function in ΔX , and t_{40} is

⁹If the median time t_{50} is used, the argument of the inverse error function is 0 and $k = 0$.

given by a function of the form given in Eq. (17). Furthermore, for $t \leq t_0$, where Eq. (18) is not well defined, $dn/dt = 0$.

IV. PARAMETRIZATION OF THE EXPECTED SIGNAL IN A SURFACE-DETECTOR ARRAY

Using simulations of the response of the upgraded SD array of the Pierre Auger Observatory, AugerPrime [9], produced in the Off line software framework [19], a model of the particle-component densities can be parametrized. The model and parametrization described in this work is named Universality II and can be found as such within the Off line software. Parameterization is performed directly in terms of the signal S , so that no model of detector responses to individual particles is required. For each type of detector and particle components, it is individually assumed that Eq. (2) is valid directly in terms of the signal. This assumption breaks down for very small signals at large distances from the shower maximum or from the shower axis, when the particle density is in the region of the detection threshold.

Simulated responses of the Auger WCDs to proton showers with primary energies 10^{19} eV are shown in Fig. 4 for the four particle components individually, together with the fully parametrized model. The signal is

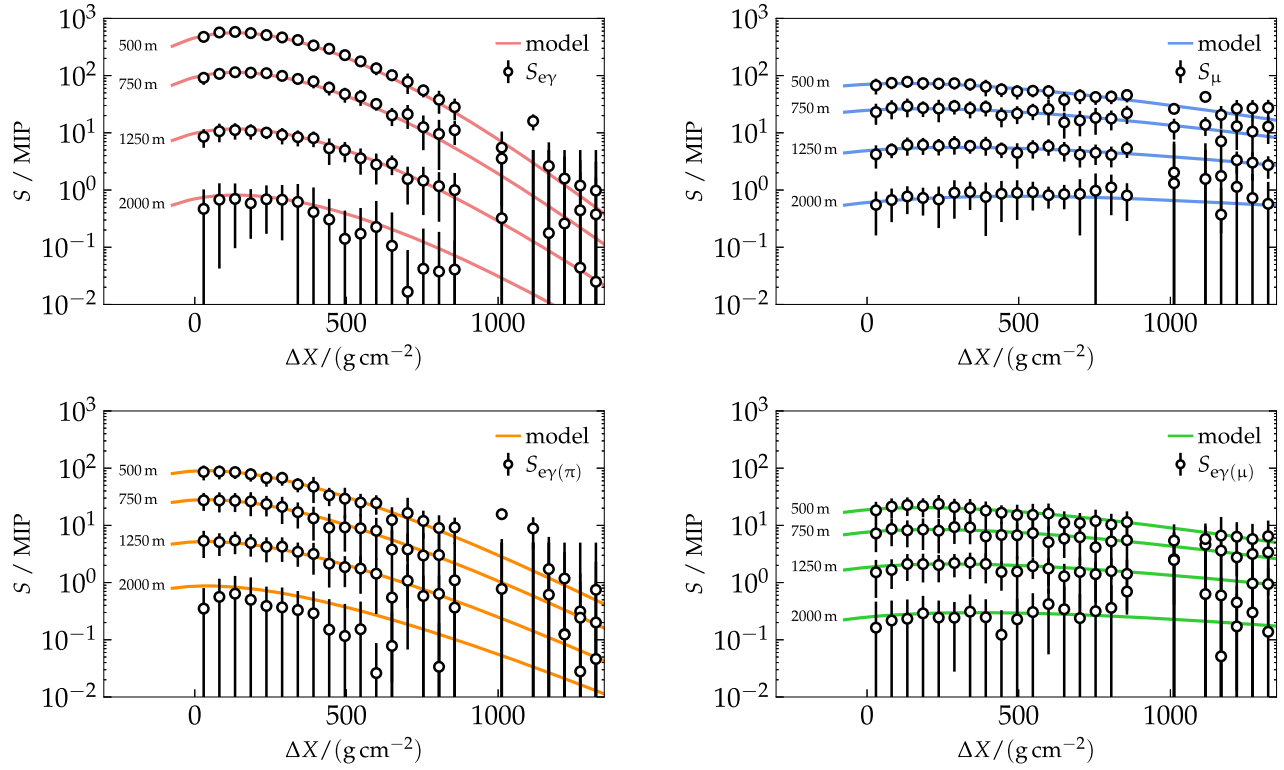


FIG. 5. Average simulated signals, $S_{e\gamma}$, S_{μ} , $S_{e\gamma(\pi)}$, and $S_{e\gamma(\mu)}$ (points), of SSDs to air showers as a function of the distance ΔX to the shower maximum along with the corresponding model for the four particle components (lines). The signal is shown as a function of the distance to the shower maximum for four different radii 500, 750, 1250, and 2000 m.

given in units of vertical equivalent muons [36]. Equivalent profiles for the model of the responses of SSDs are shown in Fig. 5, with the signal given in units of the most probable signal charge of a vertical minimum-ionizing particle [9]. The respective lateral profiles of the four components for both types of detectors are depicted in Fig. 6. The simulated air showers used to parametrize the model were produced using CORSIKA7 [37] employing the EPOS-LHC [8] model of hadronic interactions.

The model depicted in Figs. 4–6 accurately describes the expected detector responses, except for very large ΔX and r , where the particle density becomes very low and Poissonian fluctuations as well as detector threshold effects start to dominate the behavior of the signals. To better match the expected signal in the regions, where small signals are expected (at the order of magnitude of the detector threshold), a sigmoid-like function is multiplied to $f(r)$ [34]; its effect is visible in Fig. 6 (bottom), especially for the $e\gamma(\mu)$ component at $r \simeq 2000$ m. The model parameters for the four components of EPOS-LHC protons are given in Table I. Since the parametrization is performed in terms of signal rather than particle density, the interpretability of the individual parameters—especially the shower age parameter s —is limited (similarly as the parametrizations found in Ref. [30]).

The model of the signal components shown in Fig. 4 can be compared with data from simulated air-shower events using different hadronic interaction models and primary particles. In this way, the validity of Eqs. (3) and (4) can be tested. The result is depicted in Fig. 7. The particle densities are estimated by the signal deposited in a WCD, using the signal expected from a proton shower as a reference. The unambiguous dependence of the individual signal components on R_{μ} , illustrated in Fig. 7, shows that hadronic showers can be well described with a Universality-driven model by scaling the contribution of the individual components with R_{μ} . Figure 7 implicitly acts as a validation of the model, as the data is centered around 1 for the $e\gamma$ component, and around the identity for the other components. The slight deviation of the slope (given in the legend as a) from 0 for the $e\gamma$ component is due to the fact that particles from photopion production and subsequent decay are not correctly accounted for as part of the electromagnetic shower. These should be identified as a fifth shower component, $\mu(e\gamma)$, in future work.

The temporal distribution of particles is modelled using t_{40} and σ as obtained from simulated detector time traces, which were fit to a shifted log-normal distribution function. Equation (17) can be parametrized as a function of ΔX for each of the four particle components, using a correction

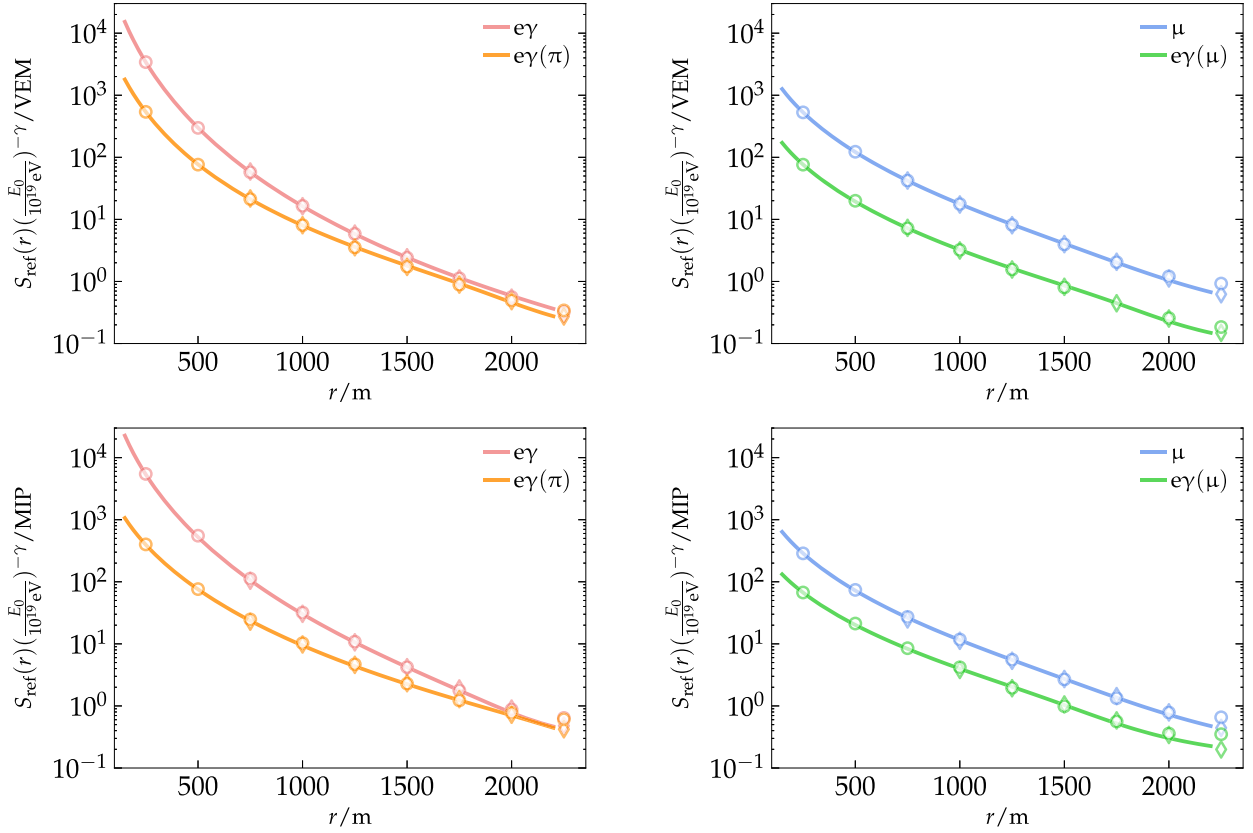


FIG. 6. Lateral distribution $S_{\text{ref}}(r)$ of the average signal deposited in the WCDs (top row) and SSDs (bottom row) by the four particle components at the reference distance $\Delta X_{\text{ref}} = 200 \text{ g/cm}^2$ from the shower maximum. The data are parametrized using Eq. (9) in terms of signal. Circular (diamond) markers show the lateral signal obtained from simulated proton showers with a primary energy of 10^{19} eV (10^{20} eV) along with the corresponding model as a solid line. The energy dependence is removed by normalizing according to Eq. (8).

shift of the form $\Delta X \rightarrow \Delta X + \delta X$ and an additional term to take into account the real shape¹⁰ of the shower front, $t_{40} \rightarrow t_{40} + t_{\text{corr}}$, where both δX and t_{corr} are parametrized as a function of r and θ [34]. The correction terms δX and t_{corr} also account for the nonrectilinear propagation of the particles and the finite detector response time, respectively, which are otherwise not part of the model described in Sec. III E. Using a global best fit, it was evaluated that the model describes the simulated data best when using¹¹ $c = 0.95c_0$, with the speed of light c_0 in vacuum. t_{40} as a function of ΔX is shown in Fig. 8 for four example radii. The magnitude of the slope of t_{40} as a function of ΔX shows the sensitivity of the Universality II model to the actual value of X_{max} as seen in the time information from individual stations. Only at large distances from the shower axis and/or the shower maximum (at $r \gtrsim 1800 \text{ m}$ and $\Delta X \gtrsim 600 \text{ g/cm}^2$) the model breaks down due to the decreasing multiplicity of particles. In these regions, the

traces are not smooth anymore and t_{40} shows no dependence on ΔX .

In addition to a minor variation, the average of the shape parameter σ is rather constant as a function of ΔX , although with a large uncertainty. The parametrization and the behavior of the mean σ as a function of the geometry is described in detail in Ref. [34].

The full prediction of the Universality II model for the response of three detectors to a simulated example event is shown in Fig. 9. The model accurately describes the temporal distribution of the signal for the four components and its total, except for spikes of a single particles, as can be seen in the last panel of Fig. 9.

V. APPLICATION OF THE MODEL

The Universality II model discussed in Sec. IV can be used to reconstruct observables, most prominently X_{max} and R_μ , from SD data collected by the Pierre Auger Observatory. Reconstruction of both X_{max} and R_μ using a Universality-based model and only SD data has been a topic of research for many years [13,14]. The main challenge of the estimation at event level of R_μ from SD data is the correlation of the number of muons N_μ with the

¹⁰The ideal shower front is a perfect expanding sphere, the real shape is, however, slightly parabolic.

¹¹Using $c < c_0$ captures possible kinematic delays and scattering.

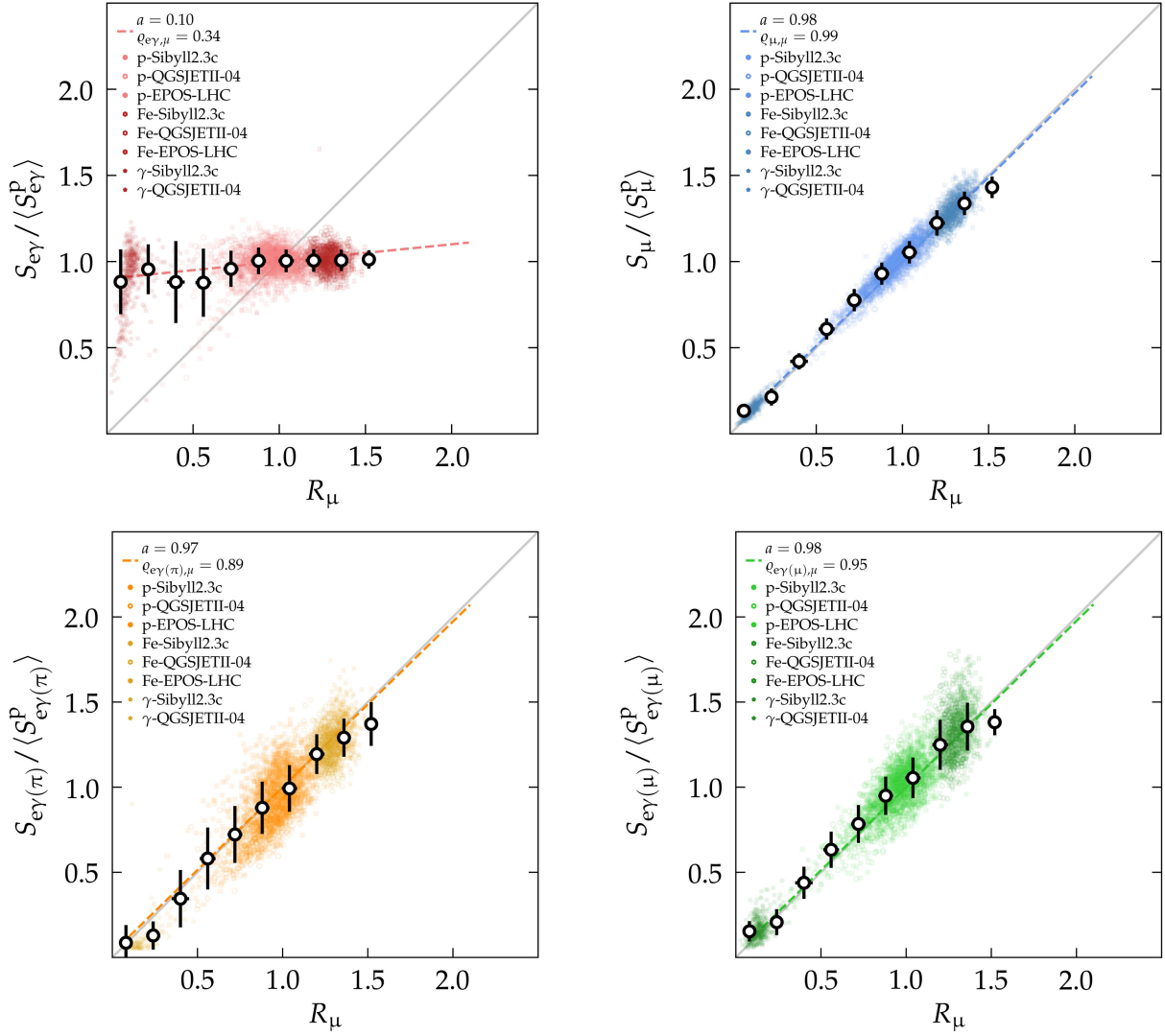


FIG. 7. Normalized signal $S/\langle S^p \rangle$ of the particle components as a function of R_μ . Data are obtained from simulated WCD signals at 750, 1000, and 1250 m using different hadronic interaction models and primary particles. Showers were simulated with a primary energy $E_0 = 10^{19}$ eV with zenith angles in a range from 0° to 60° . For each component, Eq. (3) is expressed in terms of signal. The component scaling factor a and the Pearson correlation coefficient ρ of the data are placed in the upper-left corner of each panel.

primary energy E_0 of the shower, the latter governing the total amount of particles produced. An independent energy estimator, such as given by the Auger fluorescence detector or the SSDs, can help to disentangle this ambiguity. Furthermore, as outlined in Sec. IV, only little information about $X_{\max}$ is contained in SD data from a single detector station. At high energies, however, where many stations are triggered by a single event, $X_{\max}$ can be reconstructed with reasonable accuracy and precision using the combined information of the station time traces (see Fig. 9). The ability of the Universality-based reconstruction to estimate $X_{\max}$ and R_μ is evaluated using simulations of the AugerPrime SD responses to the EAS produced by CORSIKA. Reconstructed observables of showers are depicted in Fig. 10. The results are obtained by minimizing

an event-level likelihood function in terms of $X_{\max}$ and R_μ . Stations that are expected to be located at a distance greater than $r = 1800$ m or $\Delta X = 1000$ g/cm² from the shower axis or the maximum shower, respectively, are not considered in the fit for the reasons described in Sec. IV. The reconstruction of these Monte Carlo events was performed under the same conditions as later expected for measured data. A global bias correction is performed for both the reconstructed $X_{\max}$ and R_μ to accurately match the expectation values of showers simulated with the EPOS-LHC model. Since the mean values for $X_{\max}$ (and R_μ) for showers of different primary particles are subject to systematic uncertainties, arising from the differences in hadronic interaction models, it is foreseen to calibrate the reconstruction using the data from direct fluorescence

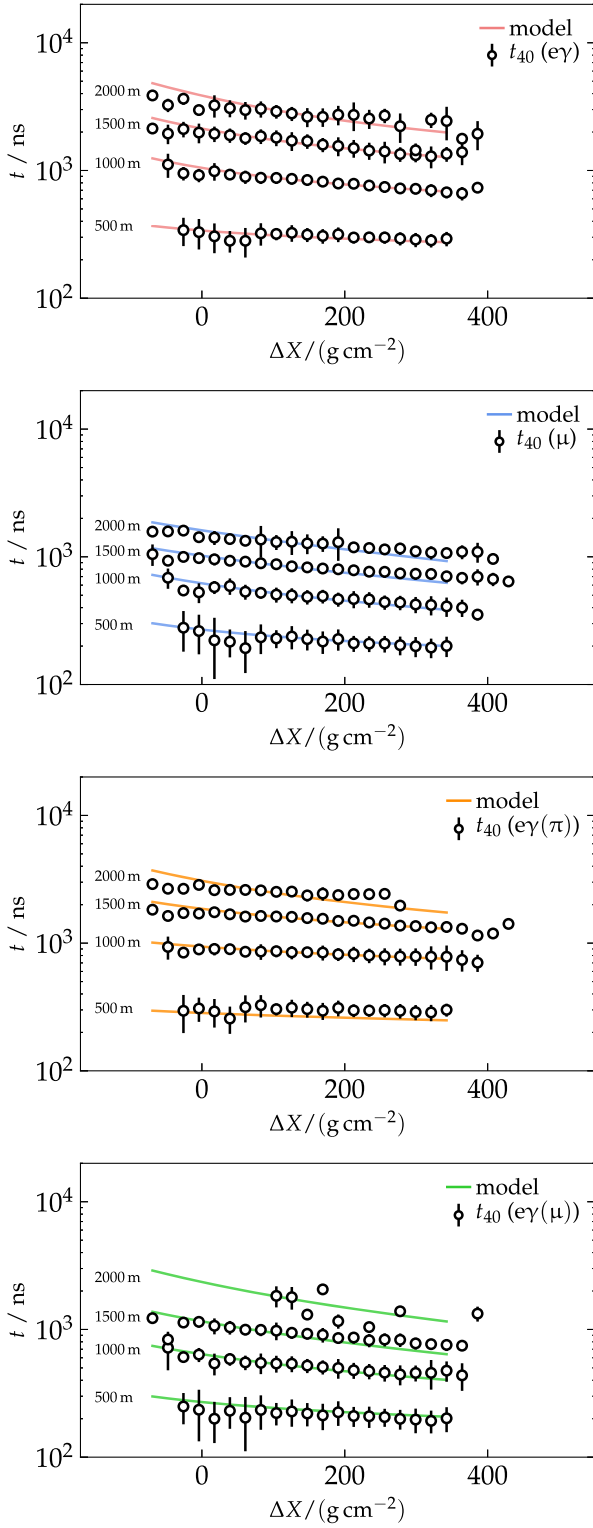


FIG. 8. The average 40% time quantile t_{40} of simulated signals of individual detectors (points) as a function of the distance ΔX to the shower maximum, as well as the corresponding model for the four components (lines). The data are depicted for four sample radii. The data for stations at 2000 m distance, where the model is not applicable anymore, is shown just for reference.

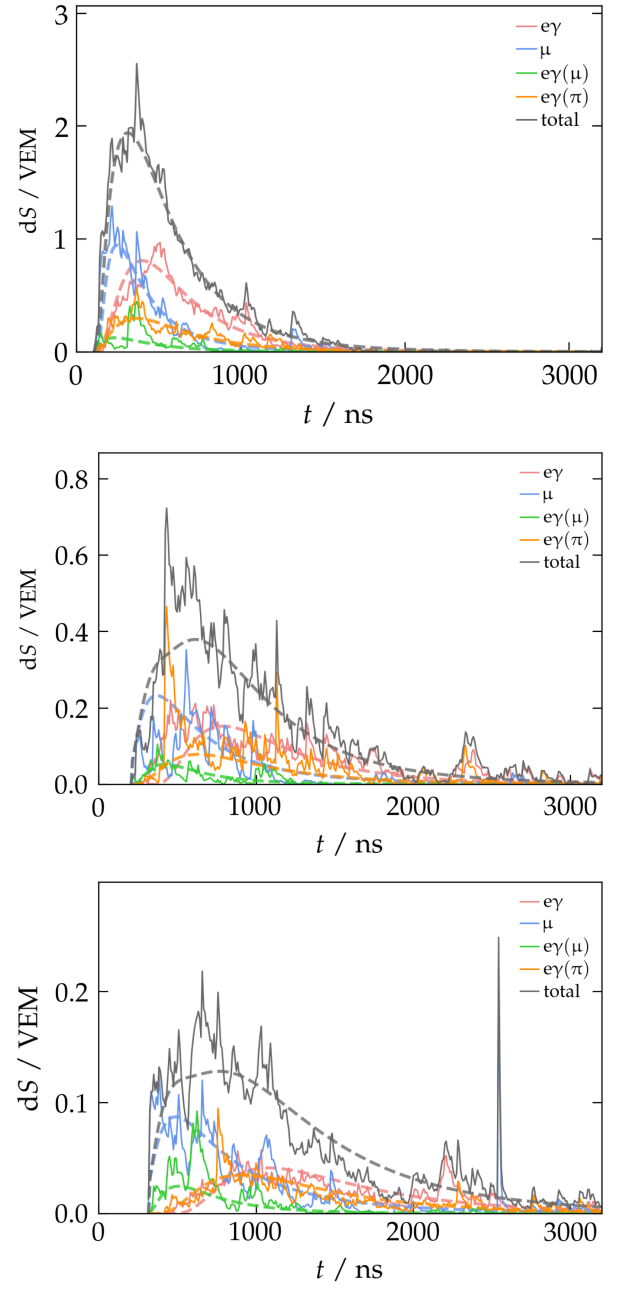


FIG. 9. Visualization of the model prediction for the time-dependent signal from a proton shower with primary energy $E_0 = 10^{19}$ eV and zenith angle $\theta = 22^\circ$ at three different distances $r = 750$ (top), 1000 (middle), and 1250 m (bottom). The model prediction (dashed lines) for three example WCDs is shown along with simulated detector responses (thin solid lines) for the four components as well as for the total signal. $t = 0$ marks the time of the plane front arriving at the position of the detector.

detector measurements. Note that the reconstruction of $X_{\max}$ depicted here is based on only the WCD signal time traces, which are more smooth than the SSD signal time traces. The reported performance is thus also expected from Auger SD data recorded before the upgrade of the SD.

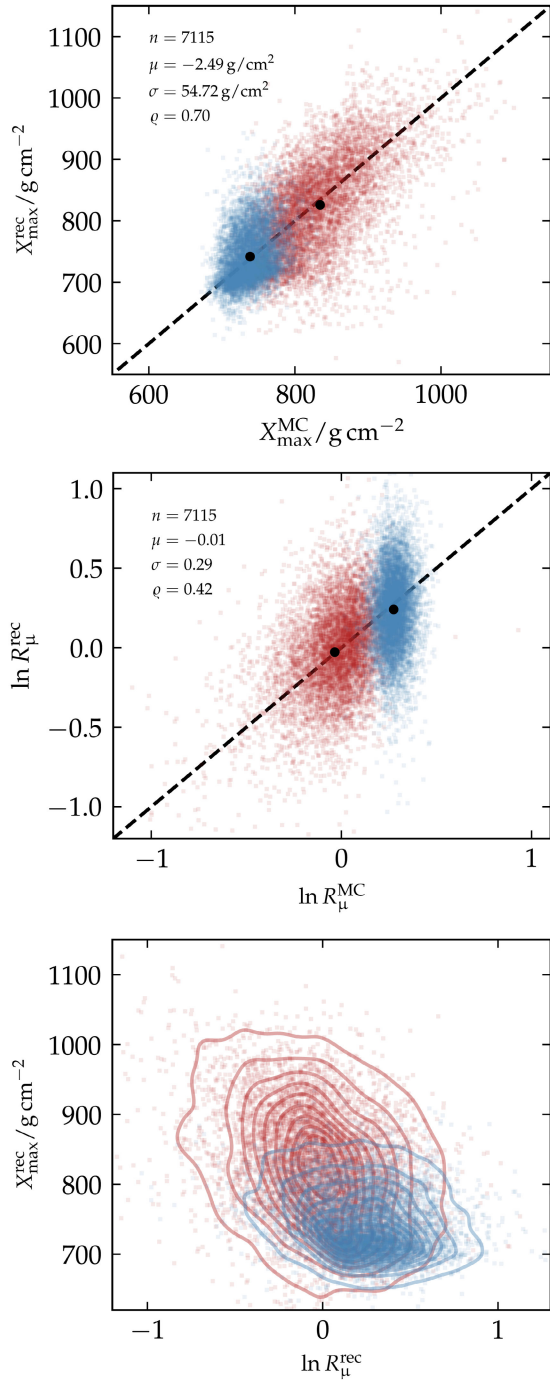


FIG. 10. Simulated and reconstructed values of (top) the depth of the shower maximum $X_{\max}^{\text{MC}}$ and $X_{\max}^{\text{rec}}$, and (middle) scaling $\ln R_{\mu}^{\text{MC}}$ and $\ln R_{\mu}^{\text{rec}}$ for simulated air showers from primary iron particles (blue) and primary proton (red) with energies between $10^{18.5}$ and 10^{20} eV with zenith angles in range from 0° to 60° . The inset legend indicates the number of events n , accuracy μ , and precision σ of the reconstructed values, and the Pearson correlation ρ of the bivariate data. Mean values for $X_{\max}$ and $\ln R_{\mu}$ are indicated by black circular markers. The identity function is given as a dashed line in the top two plots. Bottom: two-dimensional distributions in reconstructed $X_{\max}$ and $\ln R_{\mu}$ shown as individual data points alongside contour lines of estimated equal density.

At the energies considered, the average precision to reconstruct values of $X_{\max}$, as depicted in Fig. 10, is approximately 55 g/cm^2 (slightly improving with increasing primary energy). This is clearly no competition to direct shower profile observations provided by fluorescence detector measurements, which yield a precision of up to $\approx 15 \text{ g/cm}^2$ [38]. This disadvantage is, however, countered by the largely increased statistics of SD measurements. Furthermore, while a higher precision can be achieved using machine learning techniques [39], the Universality II model provides a classical alternative to neural networks and its performance is well enough to distinguish a heavy and light component, given a mixed composition dataset. The same holds true for the relative number of muons, which can be on average reconstruction with a precision of about 25%. This number could improve, for example, if novel methods to estimate the primary energy of UHECRs are successfully employed [40]. We consider the achieved precision reasonable, as it is smaller than the difference in expectation values of the respective observables for proton and iron primary particles (which in simulations is approximately 80 g/cm^2 in terms of $X_{\max}$ and approximately 30% in muons, depending on the underlying hadronic interaction model). Moreover, because of shower-to-shower fluctuations, individual showers—especially from light primary particles—are expected to produce values in $X_{\max}$ (and $\ln R_{\mu}$) that are by far larger (smaller) than the average of a given distribution of a mixed composition, as can be seen by the outlier data points shown in Fig. 10, top two panels. These can be easily recovered with the Universality II model and its given precision, if they are present in the reconstructed data.

VI. DISCUSSION AND SUMMARY

In this article we presented a new model of SD responses to EAS from UHECRs. The Universality II model is derived from the principle that the particle densities of air showers can be uniquely described based on the depth of the shower maximum, the number of muons in the shower, and the primary energy, independently of the nuclear mass of the primary cosmic ray. Both the signal model and the time model are based on earlier work, but are thoroughly revisited and improved, especially in terms of the employed functional forms. The time model is now based on considerations concerning the creation and propagation of the individual shower particles, and the signal model is formulated using common shower-profile functions, with a minimal set of free (fitted) parameters. The model provides a complete and physically motivated picture of the expected particle densities in air-shower experiments. Furthermore, the parametrization is performed for both the WCD and SSD detectors of the Pierre Auger Observatory using full detector simulations, while previous parametrizations were

relying on tabulated detector responses. Using the model realization for the SSDs, it is thus possible to describe also the data from the updated SD. We show that the expected signals in simulated showers from different primary particles and different hadronic interaction models are well described. Lastly, we demonstrate qualitatively that the model can be applied to air-shower data to estimate the number of muons and the depth of the shower maximum from the spatial and temporal distribution of particles at the ground with a reasonable precision for showers with a primary energy above 3 EeV. The application of the method will be performed in a forthcoming article.

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APPENDIX A: RELATIVE RATE OF CHANGE OF THE LONGITUDINAL PROFILE

The relative rate of change λ_1 of the longitudinal profile g of a shower as a function of depth t in radiation-length units, $t = X/X_0$, is given by

$$\lambda_1 := \frac{1}{g(t)} \frac{dg(t)}{dt}. \quad (\text{A1})$$

Reference [26] introduces the approximation

$$\lambda_1 \simeq \frac{1}{2}(s - 1 - 3 \ln s), \quad (\text{A2})$$

which, when inserted into Eq. (A1) and solved for t , results in the Greisen profile (interesting derivations of the Greisen profile are given in Refs. [11,34]). In case of the Gaisser-Hillas profile, the function λ_1 simply evaluates into

$$\lambda_1 = -\frac{2(t - t_{\max})}{3(t - t_1)} \quad (\text{A3})$$

with $t_{\max} = X_{\max}/X_0$ and $t_1 = X_1/X_0$. An exact calculation of λ_1 under approximation B of the cascade equations is given in [11].

APPENDIX B: UNIVERSALITY OF THE ELECTROMAGNETIC COMPONENT

According to the Heitler-Matthews model of hadronic air showers [21], a shower induced by a primary particle with atomic mass A and energy E_0 produces

$$N_{\mu}^{(A)} = A^{1-\beta} \left(\frac{E_0}{\varepsilon_c^{\pi}} \right)^{\beta} \quad (\text{B1})$$

muons, where $\varepsilon_c^{\pi} \simeq 20$ GeV is the critical energy, below which pions decay into muons, and $\beta \simeq 0.95$ is the relative muon growth rate. In this simplistic picture, at the decay stage of the shower development all the energy, available at the time in charged pions, is converted into muons. Thus, in total the energy

$$E_{\mu/\pi}^{(A)} = \varepsilon_c^{\pi} N_{\mu}^{(A)} = \varepsilon_c^{\pi} A^{1-\beta} \left(\frac{E_0}{\varepsilon_c^{\pi}} \right)^{\beta} \quad (\text{B2})$$

is deposited into the muonic and hadronic component of the shower. Suppose an extreme scenario, where none of the muons produced decays into the electromagnetic component. We are left with $E_0 = \varepsilon_c^{\gamma} N_{e\gamma}^{(A)} + E_{\mu/\pi}^{(A)}$ and thus

$$N_{e\gamma}^{(A)} = (E_0 - A^{1-\beta} E_{\mu/\pi}^{(1)}) \frac{1}{\varepsilon_c^{\gamma}} \quad (\text{B3})$$

particles are being produced in the maximum of the $e\gamma$ component. In this scenario, the relative change of $N_{e\gamma}^{(A)}$ as a function of A is thus given as

$$\frac{1}{N_{e\gamma}^{(1)}} \frac{\partial N_{e\gamma}^{(A)}}{\partial A} = A^{-\beta} (\beta - 1) \frac{E_{\mu/\pi}^{(1)}}{E_0 - E_{\mu/\pi}^{(1)}} \approx -\frac{0.0125}{A^{\beta}}, \quad (\text{B4})$$

where the last expression is evaluated for $E_0 \approx 10^{19}$ eV. It is thus safe to assume that the number of particles in the $e\gamma$ component does not change significantly with the atomic mass A of the primary particle for hadronic UHECRs. The size of the pure electromagnetic cascade is thus universal.

If, however, a fraction w of the muons and hadrons created in a shower decays into electromagnetic particles, one expects an proportional increase of the electromagnetic particles,

$$\Delta N_{e\gamma(\mu/\pi)} = \frac{w E_{\mu/\pi}^{(A)}}{\varepsilon_c^{\gamma}}. \quad (\text{B5})$$

In relative terms, for showers induced by proton primaries this makes up a contribution of

$$\frac{\Delta N_{e\gamma(\mu/\pi)}}{N_{e\gamma}^{(1)} + \Delta N_{e\gamma(\mu/\pi)}} = \frac{w E_{\mu/\pi}^{(A)}}{E_0 + (w - 1) E_{\mu/\pi}^{(A)}} \approx \frac{w}{w + 1.7}. \quad (\text{B6})$$

where, again, the last expression is evaluated for $E_0 \approx 10^{19}$ eV. A realistic estimate is $w \simeq 0.15$, so that in an ultra-high-energy proton shower $\approx 10\%$ (and more so for heavier primary particles) of the electromagnetic particles are not part of the pure and universal electromagnetic component. These particles follow different longitudinal and lateral distributions and therefore have to be treated individually. The number of these particles is expected to scale linearly with the number of muons produced in the shower.

APPENDIX C: THE DISTANCE TO THE SHOWER MAXIMUM

The distance ΔX of a SD station to the shower maximum is defined by the depth of the shower core, relative to $X_{\max}$, when the plane-front shower is passing through the detector. Using an isothermal approximation for the atmospheric density profile with a scale height h_s and the total vertical depth of the ground X_{vg} , the distance ΔX is given by

$$\Delta X = \frac{X_{\text{vg}}}{\cos \theta} \exp\left(-\frac{h_{\text{proj}}}{h_s}\right) - X_{\max}, \quad (\text{C1})$$

using the projected height of the shower core

$$h_{\text{proj}} = r \cos \psi \sin \theta \quad (\text{C2})$$

for a detector with the shower-plane coordinates r and ψ . To take into account the effects of a nonisothermal atmosphere, h_s can be expanded in the first order as a function of h_{proj} ,

$$h_s = h_s^{(0)} + h_{\text{proj}} h_s^{(1)}, \quad (\text{C3})$$

with seasonally dependent parameters X_{vg} , $h_s^{(0)}$, and $h_s^{(1)}$.

APPENDIX D: CORRECTION FOR AZIMUTHAL ASYMMETRY

We can model the azimuthal asymmetry for Eq. (9) as

$$u(\psi) = \exp\left[\zeta\left(\frac{\Delta X - \Delta X_1}{\lambda}\right)\left(\frac{r}{r_{\text{ref}}}\right)\cos \psi \sin \theta\right], \quad (\text{D1})$$

using an arbitrary but fixed reference distance $r_{\text{ref}} = 1000$ m and a constant $\zeta \simeq \mathcal{O}(10^{-2})$ for each component.

APPENDIX E: TABLE OF PARAMETERS

TABLE I. Parameters for the un2 signal model based on shower simulations using proton primary particles and the EPOS-LHC model.

WCD parameters	$e\gamma$	μ	$e\gamma(\mu)$	$e\gamma(\pi)$
a	0.0	1.0	1.0	1.0
γ	0.982	0.963	0.949	0.947
N_{19}	1.602×10^{10}	7.211×10^8	1.042×10^8	8.147×10^8
r_G/m	410	660	725	174
s	0.4	1.1	1.14	1.33
$\Delta X_1/(\text{g}/\text{cm}^2)$	-600	-400	-400	-500
$\Delta X_{\max}/(\text{g}/\text{cm}^2)$	140	$0.426(r/\text{m})$	$154 + 0.141(r/\text{m})$	45
$\lambda/(\text{g}/\text{cm}^2)$	$58 + 0.014(r/\text{m})$	$280 + 0.201(r/\text{m})$	$278 + 0.162(r/\text{m})$	$98 + 0.011(r/\text{m})$
ζ	0.03	0.01	0.03	0.1
SSD parameters	$e\gamma$	μ	$e\gamma(\mu)$	$e\gamma(\pi)$
a	0.0	1.0	1.0	1.1
γ	0.982	0.963	0.949	0.947
N_{19}	4.077×10^{10}	4.028×10^8	1.078×10^8	5.392×10^8
r_G/m	649	579	613	658
s	0.2	1.28	1.43	0.89
$\Delta X_1/(\text{g}/\text{cm}^2)$	-600	-400	-400	-500
$\Delta X_{\max}/(\text{g}/\text{cm}^2)$	135	$5 + 0.236(r/\text{m})$	$121 + 0.122(r/\text{m})$	40
$\lambda/(\text{g}/\text{cm}^2)$	$61 + 0.015(r/\text{m})$	$250 + 0.187(r/\text{m})$	$259 + 0.174(r/\text{m})$	$111 + 0.019(r/\text{m})$
ζ	0.03	0.01	0.03	0.1

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