Qualitative explanation of the data on the decays $D^0 \to a_0(980)^{\pm}\pi^{\mp}$ and $D^+ \to a_0(980)^{+(0)}\pi^{0(+)}$ in the four-quark model of the $a_0(980)$ resonance

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It is shown that the values of the ratios $\mathcal{B}(D^0 \to a_0(980)^+\pi^-)/\mathcal{B}(D^0 \to a_0(980)^-\pi^+)$ and $\mathcal{B}(D^+ \to a_0(980)^+\pi^0)/\mathcal{B}(D^+ \to a_0(980)^0\pi^+)$, recently measured by the BESIII Collaboration, are naturally explained in the four-quark model of the $a_0(980)$ resonance.

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I. INTRODUCTION

Phenomenological studies of two-particle and quasi-two-particle hadronic decays of D-mesons using the quark-diagram scheme have a long and seminal history, see, for example, Refs. [1–7]. Recently, the BESIII Collaboration [7] presented the first amplitude analysis of the Cabibbo-suppressed decays $D^0 \to \pi^+\pi^-\eta$ and $D^+ \to \pi^+\pi^0\eta$, as a result of which, for the first time, information about the contributions from the $a_0(980)\pi$ intermediate states into these decays was obtained with high statistical significances. In particular, for the ratios $\mathcal{B}(D^0 \to a_0(980)^+\pi^-)/\mathcal{B}(D^0 \to a_0(980)^-\pi^+)$ and $\mathcal{B}(D^+ \to a_0(980)^+\pi^0)/\mathcal{B}(D^+ \to a_0(980)^0\pi^+)$ were obtained the following data [7]:

$$r_{+/-} = \frac{\mathcal{B}(D^0 \to a_0(980)^+\pi^-)}{\mathcal{B}(D^0 \to a_0(980)^-\pi^+)} = 7.5^{+2.5}_{-0.8} \pm 1.7 \quad \text{and}$$
$$r_{+/0} = \frac{\mathcal{B}(D^+ \to a_0(980)^+\pi^0)}{\mathcal{B}(D^+ \to a_0(980)^0\pi^+)} = 2.6 \pm 0.6 \pm 0.3. \qquad (1)$$

In the quark-diagram scheme, it was expected that the value of the ratio $r_{+/-}$ would be less than 0.05 [6–8]. In this note, we propose a simple qualitative explanation of the values of the ratios $r_{+/-}$ and $r_{+/0}$ in the four-quark model for the $a_0(980)$-resonance [9–13], see also Ref. [14]. In this model, $a_0(980)^{(\pm,0)}$ mesons are states with symbolic quark structures of the form: $a_0(980)^+ = u\bar{d}s\bar{s}$, $a_0(980)^0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$, $a_0(980)^- = d\bar{u}s\bar{s}$ [9].

II. THE QUARK-DIAGRAM SCHEME

The formation of the four-quark $a_0(980)$ meson in a pair with the π meson in D decays can occur as a result of various quark fluctuations. The simplest quark diagrams giving examples of such fluctuations are shown in Figs. 1–4. The first diagram in Fig. 1 corresponds to the situation when after the c quark decay in the $D^0(c\bar{u})$ meson, $c\bar{u} \to (u\bar{s}s)\bar{u}$, the direct formation of the four-quark $a_0(980)^+$ meson and the π^- meson can occur owing to the vacuum $\bar{d}d$ fluctuation: $D^0 \to [c\bar{u} \to (u\bar{s}s)\bar{u} \to (u\bar{s}s\bar{d})(d\bar{u})] \to a_0(980)^+\pi^-$. This diagram is the main element in our scheme (it also appears in Figs. 3 and 4). The decay of $D^0 \to a_0(980)^+\pi^-$ with the formation of the four-quark $a_0(980)$ meson is also possible due to the second diagram in Fig. 1. Here the production of the $a_0(980)^+\pi^-$ system occurs via the $d\bar{d}$ intermediate state as a result of the double vacuum fluctuation $(\bar{u}u)(\bar{s}s)$. The decay of $D^0 \to a_0(980)^-\pi^+$ is described by the diagrams in Fig. 2. The diagram in Fig. 3 and diagrams in Fig. 4 correspond to the decays $D^+ \to a_0(980)^+\pi^0$ and $D^+ \to a_0(980)^0\pi^+$, respectively. Naturally, there are no diagrams with W^+ exchange between the valence quarks in the D^+ meson, and contributions from the annihilation diagrams $D^+ \to W^+ \to a_0(980)^{+(0)}\pi^{0(+)}$ are suppressed since G-parity is violated in such transitions [15]. Let us write the ratios $r_{+/-}$ and $r_{+/0}$ in terms of three amplitudes a, b and e corresponding to the diagrams in Figs. 1–4 (see figure captions):

$$r_{+/-} = \frac{|a+e|^2}{|b+e|^2} \quad \text{and} \quad r_{+/0} = \frac{|a|^2}{|a-b|^2}. \qquad (2)$$

Let us restrict ourselves to a simplest possible solution of these relations. We neglect the amplitude e due to

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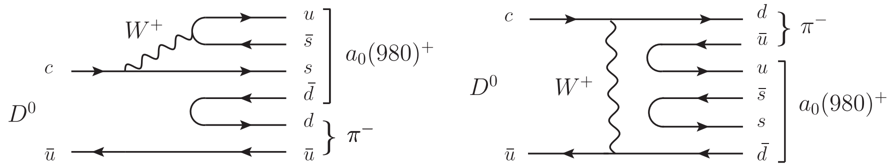

FIG. 1. Diagrams for $D^0 \to a_0(980)^+\pi^-$. The amplitudes of the first and second diagrams are labeled as a and e, respectively.

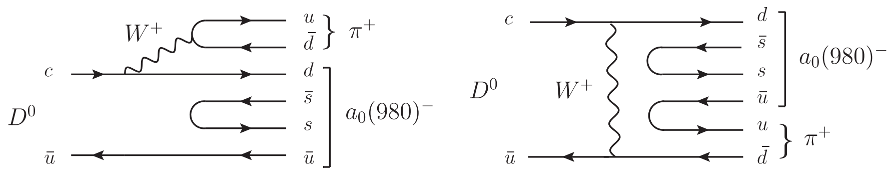

FIG. 2. Diagrams for $D^0 \to a_0(980)^-\pi^+$. The amplitude of the first diagram is labeled as b. The amplitude of the second diagram is equal to e (see Fig. 1 caption).

the double vacuum fluctuation and assume that the amplitudes a and b are real and identical in sign. Then Eq. (2) leads to the following relation between $r_{+/-}$ and $r_{+/0}$:

$$r_{+/0} = \frac{r_{+/-}}{|\sqrt{r_{+/-}} - 1|^2}. \tag{3}$$

We put $r_{+/-} = 7.5$ [see. Eq. (1)]. Then from Eq. (3) we find $r_{+/0} \approx 2.48$. And vice versa, assuming $r_{+/0} = 2.6$ [see Eq. (1)], from Eq. (3) we get $r_{+/-} \approx 6.93$. How $r_{+/0}$ changes depending on $r_{+/-}$ is shown in Fig. 5. Thus, with the assumptions made, the structure of the mechanism of the four-quark $a_0(980)$ meson production in the $D \to a_0(980)\pi$ decays [see Figs. 1–4 and Eq. (2)] allows us to qualitatively understand the BESIII data [7] for $r_{+/-}$ and $r_{+/0}$ and even reasonably describe them by

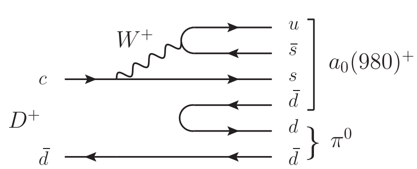

FIG. 3. The diagram for $D^+ \to a_0(980)^+\pi^0$. The amplitude of this diagram is equal to $-a/\sqrt{2}$ (see Fig. 1 caption).

using Eq. (3). Note that the previously obtained evidences in favor of the four-quark nature of the $a_0(980)$ state [9–13] are based on the analyzes of experimental results and confirmed by experiments.

Let us now provide a few additional comments. Certainly, the BESIII data [7] demonstrate the importance of taking into account the amplitudes of seed multiquark fluctuations along with the simplest seed quark amplitudes T, C, E and A [2,3]. Of course, our assumption about the seed amplitudes a and b is rather rough. These amplitudes must be dressed by $a_0(980)\pi$ interactions in the final state and acquire phases in accordance with their decomposition into components with a defined isospin. It is remarkable that more than 10 years ago an example of the S-wave interaction amplitude between the π and $a_0(980)$ mesons was constructed, taking into account the $a_0(980)$ coupling to the $K\bar{K}$-channels [16]. The problem of taking into account the $a_0(980)\pi$ final state interactions requires further careful study.

Of course, the quark processes in Figs. 1–4 can be taken as an initial start out of which to build the $a_0(980)$ resonance due to coupled-channel interactions involving the $K\bar{K}$ and $\pi\eta$ channels. For instance, this was similarly done in Ref. [17] for $J/\psi \to \phi\pi\pi(K\bar{K})$ decays, where final state interactions are taken into account as rescattering effects in the system of the two pseudoscalar mesons.

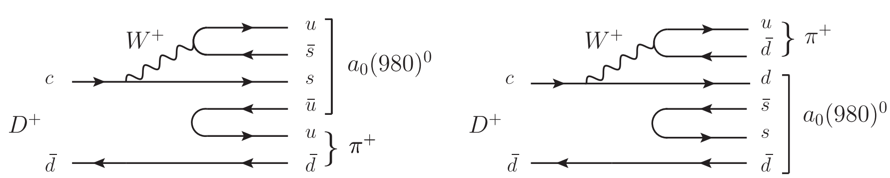

FIG. 4. Diagrams for $D^+ \to a_0(980)^0\pi^+$. The amplitudes of the first and second diagrams are equal to $a/\sqrt{2}$ and $-b/\sqrt{2}$, respectively (see captions for Figs. 1 and 2).

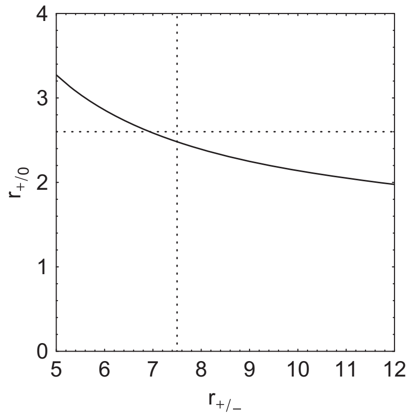

FIG. 5. The solid curve shows $r_{+/0}$ as a function of $r_{+/-}$ [see Eq. (3)] within the uncertainties of $r_{+/-}$ indicated in Eq. (1). The the dotted vertical and horizontal lines mark the central values of $r_{+/-}$ and $r_{+/0}$ in Eq. (1), respectively.

Here, it would also be appropriate to note the molecular $K\bar{K}$ model for the $a_0(980)$ state (see for review Refs. [12,13]. In these reviews, it was noted that in a number of cases the four-quark and molecular models for the $a_0(980)$- and $f_0(980)$-mesons lead to similar predictions, but there are also a number of significant differences, for example, in predictions for their two-photon widths, and these differences do not favor the molecular model [12,13]. As for the considered case of the $D \to a_0(980)\pi$ decays, it is difficult to indicate any differences between these models in the language of the symbolic quark amplitudes.

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