Two-Higgs-doublet model matched to nonlinear effective theory

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We use functional methods to match the two-Higgs-doublet model with heavy scalars in the nondecoupling regime to the appropriate nonlinear effective field theory, which takes the form of an electroweak chiral Lagrangian (HEFT). The effective Lagrangian is derived to leading order in the chiral counting. This includes the loop induced $h \to \gamma\gamma$ and $h \to Z\gamma$ local terms, which enter at the same chiral order as their counterparts in the Standard Model. An algorithm is presented that allows us to compute the coefficient functions to all orders in h. Some of the all-orders results are given in closed form. The parameter regimes for decoupling, nondecoupling, and alignment scenarios in the effective field theory context and some phenomenological implications are briefly discussed.

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I. INTRODUCTION

Indirect effects of new physics (NP) at colliders can be consistently described with effective field theories (EFTs), where the new heavy particles are integrated out. Applying this approach to electroweak symmetry breaking and Higgs-boson properties, the nonlinear EFT in the form of an electroweak chiral Lagrangian (EWChL, also referred to as nonlinear Higgs-sector EFT, or HEFT) [1–19] provides us with the most natural framework [20]. It is economic and general and properly accounts for nondecoupling effects in the scalar sector. While the EFT is model independent, matching its parameters to a specific scenario connects the EFT coefficients to a given UV theory. Recently, there has been renewed interest in the two-Higgs doublet model (2HDM) [21] and the treatment of its properties at the electroweak scale in an EFT approximation [22–25] (for earlier work see, e.g., [26]). Our motivation for addressing this topic is essentially twofold. First, we would like to investigate the description of the 2HDM in the nondecoupling regime, which corresponds to interesting regions of parameter space. Second, our analysis exemplifies the structure of the Higgs-EWChL in the context of the 2HDM as a prototypical extension of the Higgs sector. In addition, we use functional methods throughout, which make the calculations rather efficient and transparent. Exploiting the advantages of the functional approach, we go beyond the existing literature in computing higher terms in the Higgs functions, including some all-orders results in powers of the Higgs field h, and an algorithmic prescription for their general derivation. The method used in the present study has been developed in detail in [27], where it was applied to the matching of a singlet extension of the SM to the nonlinear EFT.

The paper is organized as follows. In Sec. II we introduce polar coordinates for the scalar sector of the 2HDM, which are especially convenient for the matching to the nonlinear EFT. In Sec. III we perform the matching of the 2HDM in the nondecoupling regime to the leading-order (LO) chiral Lagrangian at tree level, integrating out the heavy scalars by functional methods. The matching calculation is extended to the one-loop induced $h \to \gamma\gamma$ and $h \to Z\gamma$ local EFT operators in Sec. IV. Section V summarizes important aspects of the 2HDM parameter space with heavy-scalar masses (of order TeV), including the decoupling, nondecoupling, and alignment regimes. Some phenomenological implications are discussed in Sec. VI, before we conclude in Sec. VII. Appendix A contains the solution $H_0(h)$ of the LO equations of motion (EOM) for the heavy-scalar field H_0 to all orders in h, Appendix B shows the one-loop matching for $h \to \gamma\gamma$ and $h \to Z\gamma$ to all orders in h, and Appendix C provides explicit expressions for the parameters of the 2HDM scalar potential.

II. 2HDM SCALAR SECTOR IN POLAR COORDINATES

The scalar sector of the 2HDM consists of two complex doublets H_1 and H_2, both in the fundamental representation of the weak gauge group $SU(2)$ and with weak hypercharge

$\mathcal{Y} = 1/2$. It is convenient to define the conjugate doublets $\tilde{H}_n \equiv i\sigma_2 H_n^*$, with $n = 1, 2$, and the matrix fields

$$S_n \equiv (\tilde{H}_n, H_n). \tag{1}$$

The Lagrangian of the scalar sector can then be expressed as

$$\mathcal{L}_S = \frac{1}{2}\langle D_\mu S_n^\dagger D^\mu S_n \rangle - V, \tag{2}$$

where $\langle \cdots \rangle$ denotes the trace, a sum over n is understood, and V is the potential to be discussed below. Following [28], the matrix fields S_n can be written in polar coordinates as

$$S_n \equiv U R_n, \qquad R_n = \frac{1}{\sqrt{2}}[(v_n + h_n)\mathbf{1} + iC_n \sigma_a \rho_a]. \tag{3}$$

Here $\sigma_a \equiv 2T_a$, $a = 1, 2, 3$, are the Pauli matrices, and $U \equiv \exp(2i\varphi_a T_a/v)$ is the matrix of the electroweak Goldstone bosons, where $v = 246$ GeV is the electroweak vacuum expectation value (VEV). The VEVs of the two Higgs doublet fields are v_1 and v_2, respectively, with $v_1^2 + v_2^2 = v^2$, and

$$C_1 = -\frac{v_2}{v} \equiv -\sin\beta, \qquad C_2 = \frac{v_1}{v} \equiv \cos\beta. \tag{4}$$

Using the decomposition in (3), the 8 real degrees of freedom in the complex doublets H_n are expressed through the eight real fields φ_a, ρ_a, and h_n. The electroweak quantum numbers of S_n and U imply that the covariant derivative reads

$$D_\mu \Phi = \partial_\mu \Phi + igW_\mu \Phi - ig' B_\mu \Phi T_3 \quad \text{for } \Phi = S_n, U, \tag{5}$$

where $W^\mu = W_a^\mu T_a$ and B^μ are the gauge fields of $SU(2)_L$ and $U(1)_Y$. It follows from (3) that

$$D_\mu R_n = \partial_\mu R_n + ig' B_\mu [T_3, R_n]. \tag{6}$$

Consequently, $h_{1,2}$ and ρ_3 are electroweak singlets, whereas $\rho_{1,2}$ are singlets of $SU(2)_L$, but charged under $U(1)_Y$. Hence,

$$D_\mu h_n = \partial_\mu h_n \quad \text{and} \quad D_\mu \rho_a = \partial_\mu \rho_a + g' B_\mu \varepsilon_{ab3} \rho_b. \tag{7}$$

For $a = 1, 2$, this can also be written in terms of the eigenstates $\rho^\pm$ of charge and hypercharge (with $Q = \mathcal{Y} = \pm 1$) as

$$D_\mu \rho^\pm = \partial_\mu \rho^\pm \pm ig' B_\mu \rho^\pm, \qquad \rho^\pm = \frac{1}{\sqrt{2}}(\rho_1 \mp i\rho_2). \tag{8}$$

Inserting (3) into (2), the kinetic term becomes

$$\begin{aligned}\mathcal{L}_{S,\text{kin}} &= \frac{1}{2}\langle D_\mu S_n^\dagger D^\mu S_n \rangle \\ &= \frac{1}{4}\langle D_\mu U^\dagger D^\mu U \rangle [(v_n + h_n)^2 + \rho_a \rho_a] \\ &\quad + \frac{1}{2}\partial_\mu h_n \partial^\mu h_n + \frac{1}{2} D_\mu \rho_a D^\mu \rho_a \\ &\quad + \langle iU^\dagger D_\mu U T_a \rangle [\varepsilon_{abc} \rho_b D^\mu \rho_c \\ &\quad + C_n(\rho_a \partial^\mu h_n - D^\mu \rho_a h_n)]. \end{aligned} \tag{9}$$

The potential in (2) can be written as [28]

$$\begin{aligned} V = &\frac{m_{11}^2}{2}\langle S_1^\dagger S_1 \rangle + \frac{m_{22}^2}{2}\langle S_2^\dagger S_2 \rangle - m_{12}^2 \langle S_1^\dagger S_2 \rangle \\ &+ \frac{\lambda_1}{8}\langle S_1^\dagger S_1 \rangle^2 + \frac{\lambda_2}{8}\langle S_2^\dagger S_2 \rangle^2 + \frac{\lambda_3}{4}\langle S_1^\dagger S_1 \rangle \langle S_2^\dagger S_2 \rangle \\ &+ \lambda_4 \langle S_1^\dagger S_2 P_+ \rangle \langle S_1^\dagger S_2 P_- \rangle + \frac{\lambda_5}{2}(\langle S_1^\dagger S_2 P_+ \rangle^2 + \langle S_1^\dagger S_2 P_- \rangle^2) \end{aligned} \tag{10}$$

in terms of the matrix fields S_n from (1). $P_\pm = (1 \pm \sigma_3)/2$ are projection operators. Here we assume invariance under $S_1 \to -S_1$, $S_2 \to S_2$, softly broken by m_{12}^2, and CP invariance, so that all parameters in (10) are real.

When S_n is expressed as in (3), the Goldstone field U disappears from the potential V in (10), which becomes a function of h_n and ρ_a. The VEVs $v_{1,2}$ are defined such that terms linear in $h_{1,2}$ vanish. The terms quadratic in the fields are diagonalized by $\rho^\pm$, ρ_3, and by h and H, which are related to $h_{1,2}$ by

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}. \tag{11}$$

Here and in the following, we define $\cos\phi \equiv c_\phi$ and $\sin\phi \equiv s_\phi$ for generic angles ϕ. The mass eigenstates of the scalar sector are then given by h, identified as the observed Higgs at $m_h = 125$ GeV, and the additional scalars $H \equiv H_0$, $H^\pm \equiv \pm i\rho^\pm$, and $A_0 \equiv -\rho_3$.

The eight parameters of V in (10), m_{11}^2, m_{22}^2, m_{12}^2, $\lambda_1, \ldots, \lambda_5$, can be traded for the VEVs, the particle masses, the Higgs mixing angle, and the soft breaking term:

$$\begin{aligned} &v_1, \qquad v_2, \qquad m_h, \qquad M_0 \equiv M_{H_0}, \\ &M_H \equiv M_{H^\pm}, \qquad M_A \equiv M_{A_0}, \qquad s_\alpha, \qquad m_{12}^2 \end{aligned} \tag{12}$$

or, equivalently,

$$\begin{aligned} &v, \qquad \tan\beta \equiv t_\beta = v_2/v_1, \qquad m_h, \qquad M_0, \qquad M_H, \\ &M_A, \qquad c_{\beta-\alpha}, \qquad \bar{m}^2 \equiv \frac{m_{12}^2}{s_\beta c_\beta}. \end{aligned} \tag{13}$$

Dropping an irrelevant additive constant, the potential finally takes the form

$$\begin{aligned} V = {} & \frac{1}{2}m_h^2h^2 + \frac{1}{2}M_0^2H^2 + M_H^2H^+H^- + \frac{1}{2}M_A^2A_0^2 - d_1h^3 - d_2h^2H - d_3hH^2 - d_4H^3 - d_5hH^+H^- - d_6hA_0^2 - d_7HH^+H^- \\ & - d_8HA_0^2 - z_1h^4 - z_2h^3H - z_3h^2H^2 - z_4hH^3 - z_5H^4 - z_6h^2H^+H^- - z_7hHH^+H^- - z_8H^2H^+H^- - z_9(H^+H^-)^2 \\ & - z_{10}h^2A_0^2 - z_{11}hHA_0^2 - z_{12}H^2A_0^2 - z_{13}H^+H^-A_0^2 - z_{14}A_0^4. \end{aligned} \tag{14}$$

The coefficients can be found in Appendix C.

The scalar sector couples to fermions through Yukawa interactions. We assume a type-II Yukawa sector given by the Lagrangian [21]

$$\mathcal{L}_Y = -\bar{q}_L H_1 Y_d d_R - \bar{q}_L \tilde{H}_2 Y_u u_R - \bar{\ell}_L H_1 Y_e e_R + \text{H.c.}, \tag{15}$$

where $q_L = (u_L, d_L)^T$ and $\ell_L = (\nu_L, e_L)^T$ are the left-handed doublets and u_R, d_R, e_R the right-handed singlets. The latter may be collected into $q_R = (u_R, d_R)^T$ and $\ell_R = (\nu_R, e_R)^T$. We suppress generation indices, which are understood for the fermion fields in (15). $\mathcal{L}_Y$ can be written in terms of the matrix fields S_n in (3) as

$$\mathcal{L}_Y = -\bar{q}_L Y_d S_1 P_- q_R - \bar{q}_L Y_u S_2 P_+ q_R - \bar{\ell}_L Y_e S_1 P_- \ell_R + \text{H.c.} \tag{16}$$

III. TREE-LEVEL MATCHING IN THE NONDECOUPLING REGIME

The full Lagrangian of the 2HDM can be written as

$$\mathcal{L}_{\text{2HDM}} = \mathcal{L}_0 + \mathcal{L}_{S,\text{kin}} - V + \mathcal{L}_Y, \tag{17}$$

where the scalar sector is represented by $\mathcal{L}_{S,\text{kin}}$, V, and $\mathcal{L}_Y$ from (9), (14), and (16), and $\mathcal{L}_0$ denotes the unbroken Standard Model (SM),

$$\begin{aligned} \mathcal{L}_0 = {} & -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ & + \bar{q}_L i\not{D} q_L + \bar{\ell}_L i\not{D}\ell_L + \bar{u}_R i\not{D} u_R \\ & + \bar{d}_R i\not{D} d_R + \bar{e}_R i\not{D} e_R. \end{aligned} \tag{18}$$

In terms of the model parameters in (12) and (13) the nondecoupling limit is defined by the hierarchy

$$v \sim m_h \sim \bar{m} \ll M_0, M_H, M_A \sim M_S \tag{19}$$

with t_β and $c_{\beta-\alpha}$ of order unity in general. To leading order, all terms in the effective Lagrangian that are unsuppressed by the heavy scale M_S [of order $(M_S)^0$] have to be retained.

The procedure of integrating out the heavy scalars at tree level in the nondecoupling scenario has been described in detail in [27]. It consists of the following steps:

(i) The EOM is solved to obtain the heavy field $H_0(h)$ to LO in the heavy-mass limit, $\mathcal{O}(M_S^0)$. This requires the LO terms in the full-theory Lagrangian of order M_S^2. A closed-form solution for $H_0(h)$ is derived in Appendix A. The $\mathcal{O}(M_S^2)$-Lagrangian contains the heavy fields A_0 and $H^\pm$ only at quadratic order or higher. Contributions with only internal lines from these fields, therefore, cannot arise at tree level. Integrating them out at tree level and to LO then implies $A_0 = H^\pm = 0$.

(ii) The EOM solutions $H_0 = H_0(h)$ and $A_0 = H^\pm = 0$ are inserted into the Lagrangian (17). The $\mathcal{O}(M_S^2)$-terms cancel and an expression of $\mathcal{O}(M_S^0)$ in the heavy-mass expansion is obtained.

(iii) The field redefinition

$$\tilde{h} = \int_0^h \sqrt{1 + \left(\frac{dH_0(s)}{ds}\right)^2} ds \tag{20}$$

is performed to achieve a canonically normalized kinetic term for the Higgs field $\tilde{h}$. For notational convenience we will drop the tilde in the end, taking $\tilde{h} \to h$.

Proceeding in this way, the effective theory takes the form of an electroweak chiral Lagrangian at chiral dimension two, $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{Uh,2}$, with

$$\begin{aligned} \mathcal{L}_{Uh,2} = {} & \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle(1 + F_U(h)) + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) \\ & - \left[\bar{q}_L\left(\mathcal{M}_u + \sum_{n=1}^\infty \mathcal{M}_u^{(n)}\left(\frac{h}{v}\right)^n\right)UP_+q_R \right. \\ & + \bar{q}_L\left(\mathcal{M}_d + \sum_{n=1}^\infty \mathcal{M}_d^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-q_R \\ & \left. + \bar{\ell}_L\left(\mathcal{M}_e + \sum_{n=1}^\infty \mathcal{M}_e^{(n)}\left(\frac{h}{v}\right)^n\right)UP_-\ell_R + \text{H.c.}\right], \end{aligned} \tag{21}$$

where

$$\begin{aligned} F_U(h) = {} & 2s_{\beta-\alpha}\frac{h}{v} + \left(1 - \frac{s_{2\alpha}}{s_{2\beta}}c_{\beta-\alpha}^2\right)\left(\frac{h}{v}\right)^2 \\ & - \frac{4}{3}\frac{s_{2\alpha}^2}{s_{2\beta}^2}c_{\beta-\alpha}^2 s_{\beta-\alpha}\left(\frac{h}{v}\right)^3 + \cdots, \end{aligned} \tag{22}$$

$$V(h) = \frac{v^2 m_h^2}{2}\left\{\left(\frac{h}{v}\right)^2 + \left[s_{\beta-\alpha} + \frac{2c_{\beta-\alpha}^2 c_{\beta+\alpha}}{s_{2\beta}}\left(1-\frac{\bar{m}^2}{m_h^2}\right)\right]\left(\frac{h}{v}\right)^3 + \left[\frac{1}{4} - \frac{c_{\beta-\alpha}^2}{4s_{2\beta}^2}\left(\frac{1}{6}(7-12c_{2(\beta+\alpha)}-19c_{4\alpha}) - (1-2c_{2\alpha}c_{2\beta}-3c_{4\alpha})\frac{\bar{m}^2}{m_h^2}\right)\right]\left(\frac{h}{v}\right)^4 - \frac{c_{\beta-\alpha}^2 s_{2\alpha}^2}{2s_{2\beta}^3}\left[c_{\beta+\alpha} + 3c_{\beta-3\alpha} - (2c_{\beta+\alpha} + 3c_{3\beta-\alpha} + 11c_{\beta-3\alpha})\frac{\bar{m}^2}{4m_h^2}\right]\left(\frac{h}{v}\right)^5 + \cdots\right\}, \tag{23}$$

$$\mathcal{M}_u + \sum_{n=1}^{\infty}\mathcal{M}_u^{(n)}\left(\frac{h}{v}\right)^n = \mathcal{M}_u\left[1 + \frac{c_\alpha}{s_\beta}\frac{h}{v} - \frac{c_{\beta-\alpha}}{2}\frac{s_\alpha^2 c_\alpha}{s_\beta^2 c_\beta}\left(\frac{h}{v}\right)^2 - \frac{c_{\beta-\alpha}}{6}\frac{s_{2\alpha}^2}{s_{2\beta}^2}(2s_{2\alpha} - (1-2c_{2\alpha})t_\beta^{-1})\left(\frac{h}{v}\right)^3 + \cdots\right], \tag{24}$$

$$\mathcal{M}_d + \sum_{n=1}^{\infty}\mathcal{M}_d^{(n)}\left(\frac{h}{v}\right)^n = \mathcal{M}_d\left[1 - \frac{s_\alpha}{c_\beta}\frac{h}{v} - \frac{c_{\beta-\alpha}}{2}\frac{s_\alpha c_\alpha^2}{s_\beta c_\beta^2}\left(\frac{h}{v}\right)^2 + \frac{c_{\beta-\alpha}}{6}\frac{s_{2\alpha}^2}{s_{2\beta}^2}(2s_{2\alpha} - (1+2c_{2\alpha})t_\beta)\left(\frac{h}{v}\right)^3 + \cdots\right]. \tag{25}$$

The mass matrices $\mathcal{M}_q$ are related to the Yukawa matrices in (16) through

$$\mathcal{M}_u = \frac{v}{\sqrt{2}}Y_u s_\beta, \qquad \mathcal{M}_d = \frac{v}{\sqrt{2}}Y_d c_\beta. \tag{26}$$

The expressions for the charged leptons, $\mathcal{M}_e$ and $\mathcal{M}_e^{(n)}$, are similar to those for the down-quark case.

Our method reproduces the results of [25] and gives several new expressions, the cubic coefficient of F_U, the coefficient of h^5 in $V(h)$, and the fermionic couplings. More generally, the procedure summarized at the beginning of Sec. III, together with the all-orders expression for $H_0(h)$ in Appendix A, defines an algorithm to extend the tree-level matching to all orders in h.

A. Other Yukawa interactions

Besides the type-II Yukawa interactions discussed above, there are three other possibilities without tree-level flavor changing neutral currents. Conventionally, these are given by

(i) Type-I

$$\mathcal{L} = -\bar{q}_L Y_d S_2 P_- q_R - \bar{q}_L Y_u S_2 P_+ q_R - \bar{\ell}_L Y_e S_2 P_- \ell_R. \tag{27}$$

(ii) Type-X (lepton-specific)

$$\mathcal{L} = -\bar{q}_L Y_d S_2 P_- q_R - \bar{q}_L Y_u S_2 P_+ q_R - \bar{\ell}_L Y_e S_1 P_- \ell_R. \tag{28}$$

(iii) Type-Y (flipped)

$$\mathcal{L} = -\bar{q}_L Y_d S_1 P_- q_R - \bar{q}_L Y_u S_2 P_+ q_R - \bar{\ell}_L Y_e S_2 P_- \ell_R. \tag{29}$$

Using the results of the matching for the type-II 2HDM, it is straightforward to find the matching for the other Yukawa structures. For example, in the type-I model, all terms depend only on S_2, so the matching will have the same form as the up-type terms of the type-II 2HDM. As a result, we find for the type-I 2HDM

$$\mathcal{M}_{u,d,e} + \sum_{n=1}^{\infty}\mathcal{M}_{u,d,e}^{(n)}\left(\frac{h}{v}\right)^n = \mathcal{M}_{u,d,e}\left[1 + \frac{c_\alpha}{s_\beta}\frac{h}{v} - \frac{c_{\beta-\alpha}}{2}\frac{s_\alpha^2 c_\alpha}{s_\beta^2 c_\beta}\left(\frac{h}{v}\right)^2 - \frac{c_{\beta-\alpha}}{6}\frac{s_{2\alpha}^2}{s_{2\beta}^2}(2s_{2\alpha} - (1-2c_{2\alpha})t_\beta^{-1})\left(\frac{h}{v}\right)^3 + \cdots\right] \tag{30}$$

with $\mathcal{M}_{u,d,e} = vY_{u,d,e}s_\beta/\sqrt{2}$. It is straightforward to obtain similar expressions for the type-X and type-Y models.

IV. NONDECOUPLING EFFECTS AT ONE LOOP

The procedure of integrating out the heavy scalars can be extended to one loop using functional methods [29]. The most important effects at this order are the local operators inducing $h \to \gamma\gamma$ and $h \to Z\gamma$ transitions, because they are loop suppressed in the SM. The EFT corrections are then at the same loop order as the leading contributions. In the 2HDM, the contributions to $h \to \gamma\gamma$ and $h \to Z\gamma$ due to the heavy sector come from charged scalars $H^\pm$ within the loop, or equivalently from the real fields $\rho_{1,2}$ in (3). To obtain the one-loop contributions with internal ρ_i from functional integration, the Lagrangian in (2) has to be expanded to quadratic order in these fields. The quadratic piece takes the form

$$\mathcal{L}_{\rho_{1,2}}^{(2)} = \frac{1}{2}\rho_i \Delta_{ij}\rho_j, \qquad \Delta = -D^2 - M_H^2 - \hat{Y}, \tag{31}$$

where

$$D^\mu_{ij} = \partial^\mu \delta_{ij} + X^\mu \varepsilon_{ij} \equiv (\partial^\mu + \hat{X}^\mu)_{ij}, \qquad \hat{Y}_{ij} = Y\delta_{ij}, \quad (32)$$

with $i, j \in \{1,2\}$, ε_{ij} the two-dimensional Levi-Civita symbol, and

$$X^\mu = eA^\mu + \frac{g}{2c_W}(1-2s_W^2)Z^\mu,$$
$$-Y = d_5 h + d_7 H + z_6 h^2 + z_7 hH + z_8 H^2. \quad (33)$$

Here e is the electromagnetic coupling, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$ with the Weinberg angle θ_W, A the photon, and Z the Z-boson field. Performing the Gaussian integration of $\exp(i\int d^4x \mathcal{L}^{(2)}_{\rho_{12}})$ over ρ_i gives the effective Lagrangian [29]

$$\mathcal{L}_{\rm eff} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int \frac{d^4p}{(2\pi)^4}\left\langle\left(\frac{2ip\cdot D + D^2 + \hat{Y}}{p^2 - M_H^2}\right)^n\right\rangle. \quad (34)$$

In a weakly coupled model of the heavy sector, a generic matrix $\hat{Y}$ scales at most with the first power of M_H. The series in (34) will then converge and only a finite number of terms will contribute to any given order in the $1/M_H$ expansion. By contrast, in the present nondecoupling scenario we have $\hat{Y} \sim M_H^2$, which is of the same order as the denominator $p^2 - M_H^2$. Therefore, an infinite number of terms in the sum over n contributes at a given order in the $1/M_H$ expansion. However, higher powers of $\hat{Y}$ come with higher powers of h, since $\hat{Y}^n = \mathcal{O}(h^n)$ in the field expansion. As a consequence, the infinite series generates a Higgs-function $F_O(h)$ that accompanies an EFT operator O, as it is characteristic for the Higgs-electroweak chiral Lagrangian. At any given order in h^n, the operator coefficient is well-defined and calculable.

Following this reasoning, we can extract the terms of interest here from (34). These contain two factors of the field strength $\hat{X}_{\mu\nu}$,

$$\hat{X}^{\mu\nu}_{ij} \equiv [D^\mu, D^\nu]_{ij} = X^{\mu\nu}\varepsilon_{ij} \quad (35)$$

corresponding to four covariant derivatives D, along with powers of $\hat{Y}$. Neglecting contributions with three or more powers of h, we need to include terms of order $\hat{Y}$ and $\hat{Y}^2$. The result is given by

$$32\pi^2\mathcal{L}_{\rm eff} = -\frac{1}{12M_H^2}\langle\hat{Y}\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}\rangle + \frac{1}{40M_H^4}\langle\hat{Y}^2\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}\rangle + \frac{1}{60M_H^4}\langle(\hat{Y}\hat{X}_{\mu\nu})^2\rangle, \quad (36)$$

which simplifies to

$$\mathcal{L}_{\rm eff} \equiv \mathcal{L}_{X,4} = \frac{X_{\mu\nu}X^{\mu\nu}}{192\pi^2}\left[\frac{Y}{M_H^2} - \frac{Y^2}{2M_H^4} + \mathcal{O}(h^3)\right]. \quad (37)$$

Using (33) and eliminating H in favor of h, we obtain

$$\mathcal{L}_{X,4} = \frac{e^2}{16\pi^2}\left(A_{\mu\nu}A^{\mu\nu} + \frac{1-2s_W^2}{s_W c_W}A_{\mu\nu}Z^{\mu\nu}\right)F_X(h), \quad (38)$$

$$F_X(h) = \frac{s_{\beta-\alpha}}{6}\frac{h}{v} - \frac{1}{12}\left(s^2_{\beta-\alpha} + \frac{s_{2\alpha}}{s_{2\beta}}c^2_{\beta-\alpha}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3) \quad (39)$$

with $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. We note that the field redefinition of h, needed to make its kinetic term canonically normalized, plays no role for F_X through order h^2.

The first term $\sim h$ in $F_X(h)$ agrees with the result of [25], the term $\sim h^2$ is new. Employing the procedure described above, it is straightforward to extend the calculation of F_X to higher orders in h. As discussed in Appendix B, in the alignment limit $c_{\beta-\alpha} = 0$ the function F_X, to all orders in h, takes the simple form

$$F_X(h) = \frac{1}{6}\ln\left(1 + \frac{h}{v}\right) \quad (40)$$

corresponding to the well-known low-energy theorems [30].

A. Custodial symmetry breaking

The scalar potential in (10) contains the custodial-symmetry violating term [21]

$$\Delta V_{\rm CSB} = (\lambda_5 - \lambda_4)\langle S_1^\dagger S_2 T_3\rangle^2.$$

When integrating out the heavy scalars, this term generates the two-derivative operator

$$\mathcal{L}_{\beta_1} = \beta_1 v^2 \langle U^\dagger D_\mu U T_3\rangle^2$$

but only at the one-loop level. The coefficient is directly related to the parameter T of oblique electroweak corrections, $\beta_1 = \alpha T/2$, with α the fine structure constant. One finds, up to a factor of order unity, that [21]

$$\beta_1 \sim \frac{\lambda_4 - \lambda_5}{16\pi^2} \sim \frac{M_A^2 - M_H^2}{16\pi^2 v^2}.$$

In accordance with the phenomenological requirement of approximate custodial symmetry, $\mathcal{L}_{\beta_1}$ cannot be a leading-order effect. Therefore, the difference $\lambda_4 - \lambda_5$ must be a weak coupling of $\mathcal{O}(1)$ and carries chiral dimension two. $\mathcal{L}_{\beta_1}$ is then counted as a next-to-leading-order (NLO) term of chiral dimension four, consistent with β_1 being small as a loop factor $1/16\pi^2$ [12]. The general analysis of such NLO effects is beyond the scope of the present paper.

V. PARAMETER SPACE AND THE DECOUPLING LIMIT

For the construction of a low-energy EFT, we consider the phenomenologically viable scenario where the masses of the new scalar degrees of freedom in the 2HDM are taken to be much larger than the electroweak scale, i.e.,

$$M_S \sim M_0, M_H, M_A \gg m_h \sim v. \tag{41}$$

Depending on the numerical values of the parameters, we can discern two basic scenarios, corresponding to weak and strong coupling, respectively. They are given by

(i) *Nondecoupling regime*[1] (strong coupling, nonlinear EFT)

$$\begin{aligned} &1 \ll |\lambda_i| \lesssim 16\pi^2, \\ &m_h \sim v \sim \bar{m} \ll M_S \Rightarrow c_{\beta-\alpha} = \mathcal{O}(1). \end{aligned} \tag{42}$$

While $c_{\beta-\alpha}$ is *a priori* unconstrained in this regime, we will also consider the case $c_{\beta-\alpha} \ll 1$, referred to as the *nondecoupling regime with (quasi)alignment*. We also note that the model with $\bar{m} = 0$, the $\mathbb{Z}_2$ symmetric 2HDM without soft breaking, has no decoupling limit [31–34].

(ii) *Decoupling regime*[2] (weak coupling, linear EFT)

$$\lambda_i = \mathcal{O}(1), \quad m_h \sim v \ll \bar{m} \sim M_S \Rightarrow c_{\beta-\alpha} \ll 1. \tag{43}$$

In the strong-coupling case, we require the λ_i to be somewhat below the nominal strong-coupling limit $M_S \approx 4\pi v$ corresponding to $|\lambda_i| \approx 16\pi^2$. Otherwise, a description of the heavy-scalar dynamics in terms of resonances would no longer be valid. To be more precise, the magnitude of the couplings is constrained by perturbative unitarity [35–41]. For loop corrections to the constraints, see Refs. [42–44]. Generally speaking, these give much stronger bounds, namely $|\lambda_i| \lesssim 4\pi$. Furthermore, the couplings are constrained such that the potential is bounded from below and that the symmetry breaking vacuum is the global minimum of the potential. For the 2HDM with (softly broken) $\mathbb{Z}_2$ symmetry, the necessary and sufficient conditions on the couplings read [45–49]

$$\begin{aligned} &\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq -\sqrt{\lambda_1 \lambda_2}, \\ &\lambda_3 + \lambda_4 - |\lambda_5| \geq -\sqrt{\lambda_1 \lambda_2}. \end{aligned} \tag{44}$$

To satisfy these bounds, the absolute values of the couplings have to be taken large uniformly, which limits the possible mass splitting between the heavy scalars. Especially the perturbative unitarity constraints severely restrict the possible parameter space of the nondecoupling regime. Nevertheless, masses of $M_S \lesssim 1$ TeV are still possible for $\bar{m} \sim v$, which clearly fulfills the power counting of the nondecoupling scenario.

In the decoupling regime, all NP effects are suppressed by powers of the heavy-mass scale M_S as formalised by the Appelquist-Carazzone decoupling theorem [50]. Several EFT matching calculations have been performed in the decoupling limit (see, e.g., [24,51,52]). A decoupling regime automatically implies the alignment limit $c_{\beta-\alpha} = 0$, where the h-couplings approach their SM values [31]. An explicit calculation gives

$$\begin{aligned} c_{\beta-\alpha}^2 = {}& \frac{v^4}{16\bar{m}^4} s_{2\beta}^2 [\lambda_1 - \lambda_2 + c_{2\beta}(\lambda_1 + \lambda_2 - 2\lambda_{345})]^2 \\ &+ \mathcal{O}(v^6/\bar{m}^6) \end{aligned} \tag{45}$$

with $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$. When $\bar{m} \gg v$, this indeed approaches zero. As mentioned above, there is no similar relation in the nondecoupling regime, and thus, $c_{\beta-\alpha}$ is unconstrained *a priori*.

To illustrate the two regimes discussed above, we take the hH_0^2-coupling d_3, given in Appendix C, as an example. In the nondecoupling regime, $M_0 \sim M_S \gg m_h, \bar{m}$, so $d_3 = \mathcal{O}(M_S^2)$, whereas in the decoupling regime, the masses and parameters of the model scale as

$$\begin{aligned} &M_0^2, M_H^2, M_A^2 = \bar{m}^2 + \mathcal{O}(v^2), \\ &m_h^2 = \mathcal{O}(v^2), \quad c_{\beta-\alpha} = \mathcal{O}(v^2/\bar{m}^2), \end{aligned} \tag{46}$$

leading to

$$d_3 = -\frac{m_h^2}{2v} - v s_\beta^2 c_\beta^2 (\lambda_1 + \lambda_2 - 2\lambda_{345}) + \mathcal{O}(v^3/\bar{m}^2). \tag{47}$$

Evidently, all heavy-mass dependence has canceled. Similar calculations show that this cancellation works for all d_i and z_i. It is now easy to see that all nondecoupling effects vanish in the decoupling regime. Obviously, all tree-level nondecoupling effects vanish in the decoupling limit, since they are all proportional to $c_{\beta-\alpha}$. Also the anomalous $h\gamma\gamma$ and $hZ\gamma$ couplings disappear as the ratios d_i/M_S^2 and z_i/M_S^2 go to zero in the limit $M_S \to \infty$.

[1] Although not stated explicitly, this limit was used to derive nondecoupling effects in [25].

[2] This limit has been studied extensively in [31]. The model we consider in this work is simpler because of an additional, softly broken, $\mathbb{Z}_2$ symmetry $S_1 \to -S_1$.

TABLE I. LO matching results for the 2HDM. c_u is the same for all up-type quarks (u, c, t), and c_d is the same for all down-type quarks (d, s, b) and charged leptons (e, μ, τ).

Tree level		Loop level	
c_V	$s_{\beta-\alpha}$	c_γ	$\frac{s_{\beta-\alpha}}{6}$
c_u	$s_{\beta-\alpha}+c_{\beta-\alpha}t_\beta^{-1}$	$c_{\gamma Z}$	$\frac{1-2s_W^2}{s_W c_W}\frac{s_{\beta-\alpha}}{6}$
c_d	$s_{\beta-\alpha}-c_{\beta-\alpha}t_\beta$	c_g	0

VI. PHENOMENOLOGICAL CONSIDERATIONS

The simplest way to confront the nondecoupling effects of the 2HDM with experiment is by using a global HEFT fit. Such a fit has been performed using LHC runs 1 and 2 data [53], where the authors fit the couplings of the HEFT Lagrangian in the form

$$\begin{aligned}\mathcal{L}_{\rm fit} &= 2c_V\left(m_W W_\mu^+ W^{-\mu}+\frac{1}{2}m_Z^2 Z_\mu Z^\mu\right)\frac{h}{v}-\sum_\psi c_\psi m_\psi \bar\psi\psi\frac{h}{v}\\ &\quad+\frac{e^2}{16\pi^2}c_\gamma A_{\mu\nu}A^{\mu\nu}\frac{h}{v}+\frac{e^2}{16\pi^2}c_{\gamma Z}A_{\mu\nu}Z^{\mu\nu}\frac{h}{v}\\ &\quad+\frac{g_s^2}{16\pi^2}c_g\langle G_{\mu\nu}G^{\mu\nu}\rangle\frac{h}{v}\end{aligned}\tag{48}$$

with $\psi\in\{t,b,c,\tau,\mu\}$.[3] Our matching results are given in Table I. The strongest constraint is derived from the Higgs–vector boson coupling

$$c_V = 1.01\pm 0.06 \Rightarrow s_{\beta-\alpha}\gtrsim 0.95, \tag{49}$$

where the given error corresponds to the 68% probability interval. This motivates the (quasi)alignment limit as it constrains $c_{\beta-\alpha}\ll 1$.

Applying the above bound to the anomalous Higgs-photon coupling, we find

$$c_\gamma\in[0.16,0.17]. \tag{50}$$

This coupling is particularly important, as it is bounded from below in the alignment limit. We see here that this is a direct consequence of the bound on the Higgs-vector boson coupling. From the global HEFT fit, the bound on c_γ is given by

$$c_\gamma = 0.05\pm 0.20, \tag{51}$$

which is consistent with the matching prediction. Nevertheless, with more data from the LHC, it is plausible that the limits on c_γ could be sufficiently improved to exclude the nondecoupling regime experimentally. Local couplings with more than one Higgs in (38) and (39), such as $h^2 A_{\mu\nu}A^{\mu\nu}$, could in principle be probed at a photon collider [54] in a process like $\gamma\gamma\to hh$.

Aside from using an EFT approach, there is a large amount of literature using global fits for the 2HDM directly. Depending on the structure of the Yukawa interactions, these fits can give much stronger bounds on $s_{\beta-\alpha}$ than the global HEFT fit (see, e.g., [55,56]). However, a detailed analysis lies beyond the scope of this work.

[3]The couplings to the lighter fermions are so small that they are not included in the fit.

VII. CONCLUSIONS

We presented a systematic derivation of the EFT at the electroweak scale for the 2HDM in the nondecoupling regime. In this regime, the EFT takes the form of an electroweak chiral Lagrangian (nonlinear EFT). Our discussion follows closely the detailed discussion given in [27] for the nondecoupling regime of the SM extension with a heavy-scalar singlet. The scalar sector of the 2HDM is written in polar coordinates, with a nonlinear representation of the Goldstone fields, which facilitates the use of functional methods that we employ throughout. An algorithmic procedure is given, by which the LO EFT Lagrangian can be worked out to arbitrary order in the Higgs field h. We confirm previous results for the EFT Higgs couplings and extend the derivation to additional terms. The main results are displayed in (21)–(25), (38), and (39). Some all-orders expressions are given in closed form (Appendixes A and B). We derive the LO EFT Lagrangian, including the fermionic Yukawa interactions and the loop-induced local terms for $h\to\gamma\gamma$ and $h\to Z\gamma$. As already pointed out in [25,57], the latter terms have interesting nondecoupling effects that survive in the alignment limit. Those are still compatible with present data. They could be discovered or ruled out in future measurements of anomalous Higgs-boson couplings.

Note added. Recently, the article [58] appeared on arXiv. It also addresses the HEFT matching of models with extended scalar sectors and partially overlaps with our results on the 2HDM.

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APPENDIX A: EXACT SOLUTION FOR $H_0(h)$

The LO term for H is calculated from the equations of motion at $\mathcal{O}(M_S^2)$. In particular, we can set $H^\pm = A = 0$ in this approximation. As a result, retaining only $\mathcal{O}(M_S^2)$ terms the Lagrangian simplifies to

$$\mathcal{L}_M = -m_{11}^2\phi_1^2 - m_{22}^2\phi_2^2 - \frac{\lambda_1}{2}\phi_1^4 - \frac{\lambda_2}{2}\phi_2^4 - \lambda_{345}\phi_1^2\phi_2^2, \tag{A1}$$

where $\phi_n^2 \equiv (v_n + h_n)^2/2$ and

$$\lambda_1 = \frac{M_0^2}{v^2}\frac{c_\alpha^2}{c_\beta^2}, \quad \lambda_2 = \frac{M_0^2}{v^2}\frac{s_\alpha^2}{s_\beta^2}, \quad \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5 = \frac{M_0^2}{v^2}\frac{s_\alpha c_\alpha}{s_\beta c_\beta},$$
$$m_{11}^2 = -\frac{M_0^2}{2}\left(c_\alpha^2 + \frac{s_\alpha c_\alpha s_\beta}{c_\beta}\right), \quad m_{22}^2 = -\frac{M_0^2}{2}\left(s_\alpha^2 + \frac{s_\alpha c_\alpha c_\beta}{s_\beta}\right). \tag{A2}$$

After expressing h_1 and h_2 through h and H, the two fields take the form

$$\phi_1 = \frac{1}{\sqrt{2}}(c_\beta v + c_\alpha H - s_\alpha h),$$
$$\phi_2 = \frac{1}{\sqrt{2}}(s_\beta v + s_\alpha H + c_\alpha h). \tag{A3}$$

By defining the combination

$$R^2 = \frac{c_\alpha}{c_\beta}\phi_1^2 + \frac{s_\alpha}{s_\beta}\phi_2^2, \tag{A4}$$

the Lagrangian (A1) can be rewritten as

$$\mathcal{L}_M = \frac{M_0^2}{2}(s_\alpha s_\beta + c_\alpha c_\beta)R^2 - \frac{M_0^2}{2v^2}R^4. \tag{A5}$$

At this point we can solve the EOM by analogy to the heavy singlet model studied in [27]. By direct comparison to the results in the appendix of [27], we can identify

$$\phi_1 \to S, \qquad \phi_2 \to \phi, \qquad v \to f,$$
$$s_\beta \to \sqrt{\xi}, \qquad s_\alpha \to \sqrt{\omega}, \qquad M_0 \to M, \tag{A6}$$

which exactly reproduces the corresponding terms of the heavy singlet model. The solution to the EOM is then given by

$$H_0(h) = \frac{v + \left(\frac{s_\alpha^2 c_\alpha}{s_\beta} - \frac{s_\alpha c_\alpha^2}{c_\beta}\right)h}{\frac{s_\alpha^3}{s_\beta} + \frac{c_\alpha^3}{c_\beta}} \times \left[\sqrt{1 - \frac{\left(\frac{s_\alpha^3}{s_\beta} + \frac{c_\alpha^3}{c_\beta}\right)\left(\frac{s_\alpha c_\alpha^2}{s_\beta} + \frac{s_\alpha^2 c_\alpha}{c_\beta}\right)h^2}{\left(v + \left(\frac{s_\alpha^2 c_\alpha}{s_\beta} - \frac{s_\alpha c_\alpha^2}{c_\beta}\right)h\right)^2}} - 1\right]. \tag{A7}$$

This expression fulfills

$$R^2 = \frac{v^2}{2}(s_\alpha s_\beta + c_\alpha c_\beta) = \frac{v^2}{2}c_{\beta-\alpha}, \tag{A8}$$

which, when inserted back into the Lagrangian (A5), shows that the $\mathcal{O}(M_S^2)$-terms cancel up to a constant. Furthermore, the solution starts at $\mathcal{O}(h^2)$ with coefficients that are functions of s_α, c_α, s_β, and c_β. Note that the combination

$$\frac{s_\alpha c_\alpha^2}{s_\beta} + \frac{s_\alpha^2 c_\alpha}{c_\beta} = -c_{\beta-\alpha}[1 - 2c_{\beta-\alpha}^2 + 2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)] \tag{A9}$$

vanishes in the alignment limit. Then, the square root in (A7) reduces to 1, which gives $H_0(h) = 0$. As a result, all tree-level nondecoupling effects vanish in the alignment limit.

APPENDIX B: ONE-LOOP MATCHING OF $h^n X_{\mu\nu} X^{\mu\nu}$ TO ALL ORDERS IN n

When calculating the one-loop EFT contributions of the form $h^n X_{\mu\nu} X^{\mu\nu}$, we noted that, since $\hat{Y} = \mathcal{O}(M_H^2)$, the series does not converge. Therefore, to calculate the full Higgs function associated with the operator $X_{\mu\nu} X^{\mu\nu}$, we need all coefficients C_n of the expression

$$\mathcal{L}_{\rm eff} \supset \sum_{n=1}^{\infty} C_n \langle \hat{Y}^n \hat{X}_{\mu\nu} \hat{X}^{\mu\nu} \rangle. \tag{B1}$$

In writing the above, we made essential use of the fact that $\hat{Y} \propto \mathbf{1}$ and thus commutes with $\hat{X}^{\mu\nu}$. This is, however, a special case. In general, $\hat{Y}$ does not commute, giving more possible operator structures for each n.

To derive an all-orders result for the C_n, we start from the general expression for the one-loop effective Lagrangian given in (34). We now use a slightly adapted form of a trick explained in the appendix of [59]: We evaluate expression (34) in the special configuration $\partial_\mu \hat{X}_\nu = \partial_\mu \hat{Y} = 0$, allowing us to drop all derivatives. In this case

$$D_\mu \hat{G} \to [\hat{X}_\mu, \hat{G}], \qquad \hat{X}_{\mu\nu} \to [\hat{X}_\mu, \hat{X}_\nu], \tag{B2}$$

where $\hat{G}$ is any matrix valued function of $\hat{Y}$ and $\hat{X}_\mu$. In the final expressions, we can express everything through D_μ and $\hat{X}_{\mu\nu}$, regaining the general result.

In our special case, we are only interested in the terms of the form $\hat{Y}^n \hat{X}_{\mu\nu} \hat{X}^{\mu\nu}$, which contribute to the $h^n\gamma\gamma$ nondecoupling effects. Setting $[\hat{X}_\mu, \hat{Y}] = 0$ automatically removes all terms of the form $D_\mu \hat{Y}$.

In this way, it is easy to evaluate all terms from (34) with four derivatives (four factors of X_μ) and n factors of Y, which reduce to the terms of interest due to the formal gauge invariance of the functional integral. Finally, we obtain

$$\mathcal{L}_{\rm eff} = \frac{1}{32\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{12nM_H^{2n}}\langle \hat{Y}^n \hat{X}_{\mu\nu} \hat{X}^{\mu\nu} \rangle = \frac{X_{\mu\nu}X^{\mu\nu}}{192\pi^2}\ln\left(1 + \frac{Y(h)}{M_H^2}\right). \tag{B3}$$

The first two terms in the sum over n agree with those given in [29], and the third and fourth terms agree with those given in [60], in the special case that $\hat{Y}$ and $\hat{X}_{\mu\nu}$ commute.

The second expression in (B3) represents the full Higgs function after taking

$$Y(h) = -d_5^{(0)} h - d_7^{(0)} H_0(h) - z_6^{(0)} h^2 - z_7^{(0)} h H_0(h) - z_8^{(0)} H_0(h)^2 \tag{B4}$$

and expressing h in terms of the canonically normalized Higgs field $\tilde{h}$, $h = h(\tilde{h})$, through the inverse of the function $\tilde{h}(h)$ defined in (20). Here $d_5^{(0)}$ is the $\mathcal{O}(M_S^2)$ part of d_5, and similarly for the other coefficients, and $H_0(h)$ is the function derived in Appendix A.

In the alignment limit $c_{\beta-\alpha} = 0$, the Lagrangian in (B3) can be given in closed form to all orders in h. In this case, $H_0(h)$ vanishes and Y in (33) reduces to $Y/M_H^2 = 2h/v + (h/v)^2$. Then (B3) takes the form of (38) with the function $F_X(h)$ given by (40).

APPENDIX C: PARAMETERS OF THE 2HDM POTENTIAL

The full scalar potential is given in (14). In this section we give all coefficients of the potential in terms of the input parameters

$$v, \quad m_h, \quad M_0, \quad M_H, \quad M_A, \quad \bar{m}, \quad t_\beta, \quad c_{\beta-\alpha}. \tag{C1}$$

The cubic couplings read

$$vd_1 = c_{\beta-\alpha}^2 (\bar{m}^2 - m_h^2)(s_{\beta-\alpha} + c_{\beta-\alpha} \cot(2\beta)) - \frac{m_h^2}{2} s_{\beta-\alpha}, \tag{C2}$$

$$vd_2 = \frac{c_{\beta-\alpha}}{2} [(2m_h^2 + M_0^2 - 3\bar{m}^2)(1 - 2c_{\beta-\alpha}^2 + 2s_{\beta-\alpha}c_{\beta-\alpha} \cot(2\beta)) - \bar{m}^2], \tag{C3}$$

$$vd_3 = -\frac{s_{\beta-\alpha}}{2} [(2M_0^2 + m_h^2 - 3\bar{m}^2)(1 - 2c_{\beta-\alpha}^2 + 2s_{\beta-\alpha}c_{\beta-\alpha} \cot(2\beta)) + \bar{m}^2], \tag{C4}$$

$$vd_4 = s_{\beta-\alpha}^2 (\bar{m}^2 - M_0^2)(c_{\beta-\alpha} - s_{\beta-\alpha} \cot(2\beta)) - \frac{M_0^2}{2} c_{\beta-\alpha}, \tag{C5}$$

$$vd_5 = 2s_{\beta-\alpha}\left(\bar{m}^2 - M_H^2 - \frac{m_h^2}{2}\right) + 2c_{\beta-\alpha} \cot(2\beta)(\bar{m}^2 - m_h^2), \tag{C6}$$

$$vd_6 = s_{\beta-\alpha}\left(\bar{m}^2 - M_A^2 - \frac{m_h^2}{2}\right) + c_{\beta-\alpha} \cot(2\beta)(\bar{m}^2 - m_h^2), \tag{C7}$$

$$vd_7 = 2c_{\beta-\alpha}\left(\bar{m}^2 - M_H^2 - \frac{M_0^2}{2}\right) + 2s_{\beta-\alpha} \cot(2\beta)(M_0^2 - \bar{m}^2), \tag{C8}$$

$$vd_8 = c_{\beta-\alpha}\left(\bar{m}^2 - M_A^2 - \frac{M_0^2}{2}\right) + s_{\beta-\alpha} \cot(2\beta)(M_0^2 - \bar{m}^2), \tag{C9}$$

and the quartic couplings are given by

$$\begin{aligned} v^2 z_1 = &-\frac{m_h^2}{8} + \frac{c_{\beta-\alpha}^2}{8} [4s_{\beta-\alpha}^2 \bar{m}^2 + (-3 + 4c_{\beta-\alpha}^4) m_h^2 - (1 - 2c_{\beta-\alpha}^2)^2 M_0^2 \\ &+ 4c_{\beta-\alpha}s_{\beta-\alpha} \cot(2\beta)(2\bar{m}^2 - (1 + 2c_{\beta-\alpha}^2) m_h^2 - (1 - 2c_{\beta-\alpha}^2) M_0^2) \\ &+ 4c_{\beta-\alpha}^2 \cot^2(2\beta)(\bar{m}^2 - c_{\beta-\alpha}^2 m_h^2 - s_{\beta-\alpha}^2 M_0^2)], \end{aligned} \tag{C10}$$

$$\begin{aligned} v^2 z_2 = &\frac{s_{\beta-\alpha}c_{\beta-\alpha}}{2} (1 - 2c_{\beta-\alpha}^2)[m_h^2 + M_0^2 - 2c_{\beta-\alpha}^2 (M_0^2 - m_h^2) - 2\bar{m}^2] \\ &+ c_{\beta-\alpha}^2 \cot(2\beta)[m_h^2 (1 + 2c_{\beta-\alpha}^2 - 4c_{\beta-\alpha}^4) + 2M_0^2 (1 - 3c_{\beta-\alpha}^2 + 2c_{\beta-\alpha}^4) \\ &+ \bar{m}^2 (-3 + 4c_{\beta-\alpha}^2)] + 2c_{\beta-\alpha}^3 s_{\beta-\alpha} \cot^2(2\beta)[M_0^2 - \bar{m}^2 - c_{\beta-\alpha}^2 (M_0^2 - m_h^2)], \end{aligned} \tag{C11}$$

$$v^2 z_3 = \frac{1}{4}[(2-12c_{\beta-\alpha}^2+12c_{\beta-\alpha}^4)\bar{m}^2+(1-2c_{\beta-\alpha}^2)((-1-3c_{\beta-\alpha}^2+6c_{\beta-\alpha}^4)m_h^2+(-2+9c_{\beta-\alpha}^2-6c_{\beta-\alpha}^4)M_0^2) \\ +2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)((6-12c_{\beta-\alpha}^2)\bar{m}^2+(-1-6c_{\beta-\alpha}^2+12c_{\beta-\alpha}^4)m_h^2+(-5+18c_{\beta-\alpha}^2-12c_{\beta-\alpha}^4)M_0^2) \\ +12c_{\beta-\alpha}^2 s_{\beta-\alpha}^2\cot^2(2\beta)(\bar{m}^2-s_{\beta-\alpha}^2M_0^2-c_{\beta-\alpha}^2m_h^2)], \tag{C12}$$

$$v^2 z_4 = \frac{s_{\beta-\alpha}c_{\beta-\alpha}}{2}(1-2c_{\beta-\alpha}^2)[m_h^2-3M_0^2+2c_{\beta-\alpha}^2(M_0^2-m_h^2)+2\bar{m}^2] \\ +s_{\beta-\alpha}^2\cot(2\beta)[m_h^2(2c_{\beta-\alpha}^2-4c_{\beta-\alpha}^4)+M_0^2(1-6c_{\beta-\alpha}^2+4c_{\beta-\alpha}^4)+\bar{m}^2(-1+4c_{\beta-\alpha}^2)] \\ +2c_{\beta-\alpha}s_{\beta-\alpha}^3\cot^2(2\beta)[M_0^2-\bar{m}^2-c_{\beta-\alpha}^2(M_0^2-m_h^2)], \tag{C13}$$

$$v^2 z_5 = \frac{1}{8}[4s_{\beta-\alpha}^2c_{\beta-\alpha}^2\bar{m}^2-s_{\beta-\alpha}^2(1-2c_{\beta-\alpha}^2)^2m_h^2-c_{\beta-\alpha}^2(3-2c_{\beta-\alpha}^2)^2M_0^2 \\ +4c_{\beta-\alpha}s_{\beta-\alpha}^3\cot(2\beta)(-2\bar{m}^2+(-1+2c_{\beta-\alpha}^2)m_h^2+(3-2c_{\beta-\alpha}^2)M_0^2) \\ +4s_{\beta-\alpha}^4\cot^2(2\beta)(\bar{m}^2-M_0^2+c_{\beta-\alpha}^2(M_0^2-m_h^2))], \tag{C14}$$

$$v^2 z_6 = \frac{1}{2}[2s_{\beta-\alpha}^2(\bar{m}^2-M_H^2)-m_h^2+c_{\beta-\alpha}^2(1-2c_{\beta-\alpha}^2)(M_0^2-m_h^2)+2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)(2\bar{m}^2-m_h^2-M_0^2+3c_{\beta-\alpha}^2(M_0^2-m_h^2)) \\ +4c_{\beta-\alpha}^2\cot^2(2\beta)(\bar{m}^2-s_{\beta-\alpha}^2M_0^2-c_{\beta-\alpha}^2m_h^2)], \tag{C15}$$

$$v^2 z_7 = c_{\beta-\alpha}s_{\beta-\alpha}(2\bar{m}^2-2M_H^2-(1-2c_{\beta-\alpha}^2)(M_0^2-m_h^2))+2\cot(2\beta)(M_0^2-\bar{m}^2+2c_{\beta-\alpha}^2\bar{m}^2+c_{\beta-\alpha}^2M_0^2(-4+3c_{\beta-\alpha}^2) \\ +c_{\beta-\alpha}^2m_h^2(2-3c_{\beta-\alpha}^2))+4c_{\beta-\alpha}s_{\beta-\alpha}\cot^2(2\beta)(M_0^2-\bar{m}^2-c_{\beta-\alpha}^2(M_0^2-m_h^2)), \tag{C16}$$

$$v^2 z_8 = \frac{1}{2}[-m_h^2+c_{\beta-\alpha}^2(2(\bar{m}^2-M_H^2)+(M_0^2-m_h^2)(-3+2c_{\beta-\alpha}^2)) \\ +2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)(-2\bar{m}^2-2m_h^2+4M_0^2-3c_{\beta-\alpha}^2(M_0^2-m_h^2))+4s_{\beta-\alpha}^2\cot^2(2\beta)(\bar{m}^2-M_0^2+c_{\beta-\alpha}^2(M_0^2-m_h^2))], \tag{C17}$$

$$v^2 z_9 = \frac{1}{2}[-s_{\beta-\alpha}^2m_h^2-c_{\beta-\alpha}^2M_0^2+4\cot(2\beta)c_{\beta-\alpha}s_{\beta-\alpha}(M_0^2-m_h^2)+4\cot^2(2\beta)(\bar{m}^2-M_0^2+c_{\beta-\alpha}^2(M_0^2-m_h^2))], \tag{C18}$$

$$v^2 z_{10} = \frac{1}{4}[2s_{\beta-\alpha}^2(\bar{m}^2-M_A^2)-m_h^2(1+c_{\beta-\alpha}^2-2c_{\beta-\alpha}^4)+c_{\beta-\alpha}^2M_0^2(1-2c_{\beta-\alpha}^2) \\ +2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)(2\bar{m}^2-m_h^2-M_0^2+3c_{\beta-\alpha}^2(M_0^2-m_h^2))+4c_{\beta-\alpha}^2\cot^2(2\beta)(\bar{m}^2-s_{\beta-\alpha}^2M_0^2-c_{\beta-\alpha}^2m_h^2)], \tag{C19}$$

$$v^2 z_{11} = \frac{1}{2}[c_{\beta-\alpha}s_{\beta-\alpha}(2\bar{m}^2-2M_A^2+(2c_{\beta-\alpha}^2-1)(M_0^2-m_h^2)) \\ +2\cot(2\beta)(M_0^2-\bar{m}^2+2c_{\beta-\alpha}^2\bar{m}^2+c_{\beta-\alpha}^2M_0^2(-4+3c_{\beta-\alpha}^2)+c_{\beta-\alpha}^2m_h^2(2-3c_{\beta-\alpha}^2)) \\ +4c_{\beta-\alpha}s_{\beta-\alpha}\cot^2(2\beta)(M_0^2-\bar{m}^2-c_{\beta-\alpha}^2(M_0^2-m_h^2))], \tag{C20}$$

$$v^2 z_{12} = \frac{1}{4}[-m_h^2+c_{\beta-\alpha}^2(2\bar{m}^2+3m_h^2-2M_A^2-3M_0^2)+2c_{\beta-\alpha}^4(M_0^2-m_h^2) \\ +2c_{\beta-\alpha}s_{\beta-\alpha}\cot(2\beta)(4M_0^2-2m_h^2-2\bar{m}^2-3c_{\beta-\alpha}^2(M_0^2-m_h^2)) \\ +4s_{\beta-\alpha}^2\cot^2(2\beta)(\bar{m}^2-M_0^2+c_{\beta-\alpha}^2(M_0^2-m_h^2))], \tag{C21}$$

$$v^2 z_{13} = 4v^2 z_{14} = \frac{1}{2}[-s_{\beta-\alpha}^2m_h^2-c_{\beta-\alpha}^2M_0^2+4\cot(2\beta)c_{\beta-\alpha}s_{\beta-\alpha}(M_0^2-m_h^2)+4\cot^2(2\beta)(\bar{m}^2-M_0^2+c_{\beta-\alpha}^2(M_0^2-m_h^2))]. \tag{C22}$$

In the alignment limit ($c_{\beta-\alpha}\to 0$) the coupling constants simplify to

$$vd_1=-\frac{m_h^2}{2},\qquad vd_2=0,\qquad vd_3=\bar m^2-M_0^2-\frac{m_h^2}{2},$$
$$vd_5=2\bar m^2-2M_H^2-m_h^2,\qquad vd_6=\bar m^2-M_A^2-\frac{m_h^2}{2},$$
$$d_4=\frac{1}{2}d_7=d_8=\cot(2\beta)\frac{(M_0^2-\bar m^2)}{v},\tag{C23}$$

$$v^2z_1=-\frac{m_h^2}{8},\qquad v^2z_2=0,\qquad v^2z_3=\frac{1}{2}\left(\bar m^2-M_0^2-\frac{m_h^2}{2}\right),$$
$$v^2z_6=\bar m^2-M_H^2-\frac{m_h^2}{2},$$
$$z_4=\frac{z_7}{2}=z_{11}=\cot(2\beta)\frac{(M_0^2-\bar m^2)}{v^2},$$
$$v^2z_{10}=-\frac{1}{2}(M_A^2-\bar m^2)-\frac{m_h^2}{4},$$
$$4z_5=z_8=z_9=2z_{12}=z_{13}=4z_{14}$$
$$=-\frac{m_h^2}{2v^2}-2\cot^2(2\beta)\frac{(M_0^2-\bar m^2)}{v^2}.\tag{C24}$$

It is also useful to express the parameters of the potential in the original form of (10) in terms of the physical parameters in (C1). The relations read

$$m_{11}^2=s_\beta^2\bar m^2-\frac{1}{2}(c_\alpha^2M_0^2+s_\alpha^2m_h^2)-\frac{s_\beta^2}{2}\frac{s_{2\alpha}}{s_{2\beta}}(M_0^2-m_h^2),\tag{C25}$$

$$m_{22}^2=c_\beta^2\bar m^2-\frac{1}{2}(s_\alpha^2M_0^2+c_\alpha^2m_h^2)-\frac{c_\beta^2}{2}\frac{s_{2\alpha}}{s_{2\beta}}(M_0^2-m_h^2),\tag{C26}$$

$$\lambda_1=\frac{1}{c_\beta^2v^2}(c_\alpha^2M_0^2+s_\alpha^2m_h^2-s_\beta^2\bar m^2),\tag{C27}$$

$$\lambda_2=\frac{1}{s_\beta^2v^2}(s_\alpha^2M_0^2+c_\alpha^2m_h^2-c_\beta^2\bar m^2),\tag{C28}$$

$$\lambda_3=\frac{1}{v^2}\left(2M_H^2-\bar m^2+\frac{s_{2\alpha}}{s_{2\beta}}(M_0^2-m_h^2)\right),\tag{C29}$$

$$\lambda_4=\frac{1}{v^2}(M_A^2-2M_H^2+\bar m^2),\tag{C30}$$

$$\lambda_5=\frac{1}{v^2}(\bar m^2-M_A^2).\tag{C31}$$

The absence of a decoupling limit for $\bar m=0$ is obvious from these formulas.

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