Exclusion bounds for neutral gauge bosons

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We study how the recent experimental results constrain the gauge sectors of $U(1)$ extensions of the standard model using a novel representation of the parameter space. We determine the bounds on the mixing angle between the massive gauge bosons or, equivalently, the new gauge coupling as a function of the mass $M_{Z'}$ of the new neutral gauge boson Z' in the approximate range $(10^{-2}, 10^4)$ GeV$/c^2$. We consider the most stringent bounds obtained from direct searches for the Z'. We also exhibit the allowed parameter space by comparing the predicted and measured values of the ρ parameter and those of the mass of the W boson. Finally, we discuss the prospects of Z' searches at future colliders.

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I. INTRODUCTION

The standard model of particle interactions (SM) has been tested to high precision both in low-energy experiments and at high-energy colliders [1]. Most recently, the large LHC experiments have found spectacular agreement between their experimental results and the SM predictions [2,3], which leaves very little room for new physics. Nevertheless, we do not have doubt that the SM cannot describe all observations in the microworld. Most notably, the masses of neutrinos, the baryon asymmetry and the origin of dark matter in the Universe are clear indications of the need for physics beyond the standard model (BSM). The nature of this new physics however, remains elusive.

There is also a 5σ tension between the measured value of the muon anomalous magnetic moment a_μ [4] and the SM prediction when the hadronic vacuum polarization to the photon is extracted from the measured total hadronic cross section in electron-positron annihilation at low energies [5]. A natural explanation for such a difference is the contribution of a new heavy neutral gauge boson Z' [6–8]. Presently, however, the size of the deviation between the SM prediction and the measurement is heavily debated. The Budapest-Marseille-Wuppertal Collaboration computed the hadronic vacuum polarization of the photon from first principles [9] and found a much less significant ($<2\sigma$) tension for a_μ between theory and experiment. Nevertheless, the exploration of the effects of a Z' on measurements is interesting because $U(1)_z$ extensions provide the simplest possible way to explain a potential fifth fundamental force.

Because of their simplicity, $U(1)_z$ extensions have a more than 40 year old history [10] and remain popular [1] at present. They have been investigated since the operation of the experiments of the Large Electron Positron collider in various forms, like gauged extra $U(1)$ symmetry [11], which can also be broken by a new scalar [12], giving rise to a new massive gauge boson, often called a dark photon [13], A'. Lately, more complete $U(1)$ extensions of the SM have been studied with the goal of explaining several beyond the standard model phenomena simultaneously, such as the superweak extension of the standard model (SWSM) [14]. Even the complete one-loop renormalization of the Dark Abelian Sector Model with identical gauge and scalar sectors as in the SWSM, but with somewhat different fermion content, has been carried out in Ref. [15].

The continuing theoretical interest is met with similarly ubiquitous experimental searches for dark photons or, more generally, new neutral gauge bosons. The experimental limits are typically presented in the parameter space of the dark photons, which provide serendipitous discovery potential for other types of vector particles [16]. Constraints have been

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placed on visible A' decays by beam-dump, collider, fixed-target, and rare-meson-decay experiments, as well as on invisible A' decays. New experiments have also been proposed to explore further the parameter space in the future [17,18].

In this work, we discuss the presently most constraining experimental limits in some part of the parameter space of general $U(1)$ extensions that contain right-handed neutrinos, not charged under the SM interactions, in the particle spectrum. We focus on two regions: the case of light and heavy Z', $\xi = M_{Z'}/M_Z \ll 1$ and $\xi \gg 1$, or quantitatively $M_{Z'} \in [0.02, 10]$ GeV and $M_{Z'} \in [0.2, 5]$ TeV [19]. The most stringent limits for a heavy Z' are provided by direct searches at the LHC in Drell-Yan pair production $pp \to Z' + X \to \ell^+\ell^- + X$ [20,21]. A complement study of constraining $SU(2)_\mathrm{L}$ singlet and triplet neutral gauge bosons in the context of standard model effective field theory operators was published recently [22]. We also present projections of the expected sensitivities to heavy Z' bosons at the planned future high-energy colliders.

For a light Z' in our mass range, the best limits have been obtained in direct searches for an invisibly decaying dark photon in the NA64 [23] and *BABAR* [24] experiments as well as for a dark photon decaying into an electron-positron pair in the FASER detector [25]. Similar studies have already been published. For instance, Refs. [26,27] focus on $U(1)_z$ extensions with a selection of benchmark z charges in the light $M_{Z'}$ region. In the present work, we use a parametrization of the z charges valid for any charge assignments that satisfy the conditions of anomaly cancellations and gauge invariance. The exclusion limits depend on a specific single combination of the free z charges such that we also take into account the uncertainty due to the choice of the renormalization scale where the z charges are set, neglected in Refs. [26,27]. A model with flavor-dependent z charges has also been investigated in Ref. [28], focusing on Z' mediated nonstandard interactions of neutrinos.

We also considered the Z' searches at the Belle II experiment presented in Refs. [29,30]. The limit set by the Belle experiment [29] is comparable to the constraint obtained from the measurement of the ρ parameter but not as severe as those obtained from the NA64 and *BABAR* experiments for any value of the z charges. Reference [30] presents results of a search for the production of a Z' boson in association with a dark scalar boson. In the models we discuss here, including the dark scalar in the analysis would introduce dependence on the free parameters of the extended scalar sector. As our aim is to present bounds purely on the Z' boson with as little reference to additional free parameters as possible, we postpone the investigation of the simultaneous bounds on a Z' boson with a dark scalar s to a future study.

It is worth mentioning that bounds on the parameter space of a new Z' can also be obtained from cosmological observations, leading to exclusion limits for significantly lower values of the new gauge coupling. Hence, these are useful in a complementary region of the parameter space [31,32]. In the case of cosmological bounds, the translation rules from one $U(1)$ model to another cannot be derived as simply as in the case of laboratory experiments as shown in Appendix D; hence, we do not consider those here.

As one might expect, in the regions far away from the mass of the Z boson, the mixing angle θ_Z between the massive neutral gauge bosons is small experimentally (around or below 10^{-3}), so one can use expansions around $\theta_Z = 0$. In this limit, the couplings of the Z' to chiral fermions are approximately vectorlike and universal in the sense that they all depend on a unique combination of the z charge of the right-handed neutrinos and Brout-Englert-Higgs (BEH) field. We also discuss how results of electroweak precision measurements constrain the parameter space.

II. MODEL DEFINITION

We consider the extensions of the standard model by a $U(1)_z$ gauge group with a complex scalar field χ and three generations of right-handed neutrinos. The new fields are neutral under the standard model gauge interactions. An example for such a model is the SWSM [14]. However, in the present work, we require only gauge and gravity anomaly cancellation and otherwise leave the z charges arbitrary. In this section, we collect the details of the model only to the extent used in the present analyses.

A. Scalar sector

In the scalar sector, in addition to the $SU(2)_\mathrm{L}$-doublet Brout-Englert-Higgs field

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + \mathrm{i}\phi_2 \\ \phi_3 + \mathrm{i}\phi_4 \end{pmatrix}, \tag{1}$$

the model contains a complex scalar SM singlet χ. The Lagrangian of the scalar fields contains the potential energy

$$V(\phi,\chi) = -\mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 + (|\phi|^2, |\chi|^2) \begin{pmatrix} \lambda_\phi & \frac{\lambda}{2} \\ \frac{\lambda}{2} & \lambda_\chi \end{pmatrix} \begin{pmatrix} |\phi|^2 \\ |\chi|^2 \end{pmatrix} \subset -\mathcal{L}, \tag{2}$$

where $|\phi|^2 = |\phi^+|^2 + |\phi^0|^2$. After spontaneous symmetry breaking, we parametrize the scalar fields as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -\mathrm{i}\sqrt{2}\sigma^+ \\ v + h' + \mathrm{i}\sigma_\phi \end{pmatrix}, \qquad \chi = \frac{1}{\sqrt{2}} (w + s' + \mathrm{i}\sigma_\chi), \tag{3}$$

where v and w are the vacuum expectation values (VEVs) of ϕ and χ. The fields h' and s' are real scalars, σ^+ is a charged, and σ_ϕ and σ_χ are neutral Goldstone bosons that are gauge eigenstates.

The gauge and mass eigenstates are related by the rotations

$$\begin{pmatrix} h \\ s \end{pmatrix} = \mathbf{Z}_S \begin{pmatrix} h' \\ s' \end{pmatrix}, \qquad \begin{pmatrix} \sigma_Z \\ \sigma_{Z'} \end{pmatrix} = \mathbf{Z}_G \begin{pmatrix} \sigma_\phi \\ \sigma_\chi \end{pmatrix}, \qquad (4)$$

with

$$\mathbf{Z}_X = \begin{pmatrix} \cos\theta_X & -\sin\theta_X \\ \sin\theta_X & \cos\theta_X \end{pmatrix} \qquad (5)$$

where we denoted the mass eigenstates with h, s and σ_Z, $\sigma_{Z'}$. The angles θ_S and θ_G are the scalar and Goldstone mixing angles that can be determined by the diagonalization of the mass matrix of the real scalars and that of the neutral Goldstone bosons. In the following, we are going to use the abbreviations $\mathrm{c}_X = \cos\theta_X$ and $\mathrm{s}_X = \sin\theta_X$ for mixing angles.

B. Gauge sector

The field strength tensors of the $U(1)$ gauge groups are gauge invariant, and kinetic mixing is allowed between the gauge fields belonging to the hypercharge $U(1)_Y$ and the new $U(1)_z$ gauge symmetries. Equivalently, one can choose a basis in which the gauge-field strengths do not mix [33], such that the covariant derivative corresponding to the $U(1)$ gauge groups can be parametrized as

$$D_\mu^{\mathrm{U}(1)} = -\mathrm{i}\begin{pmatrix} y & z \end{pmatrix} \begin{pmatrix} g_y & -g_{yz} \\ 0 & g_z \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix}, \qquad (6)$$

where B_μ and B'_μ are the $U(1)_y$ and $U(1)_z$ gauge fields, while y and z are the corresponding charges. The rotation angle α is not physical as it can be absorbed into the definition of the gauge fields [31]. The y charges are the eigenvalues of one-half times the hypercharge operator Y. The z charges are assigned such that Yukawa terms including the neutrinos and the scalar fields exist, and the gauge and gravity anomalies cancel in each family. Such a charge assignment can be paramterized with two numbers, usually chosen to be the charge of the left-handed quark doublet z_q and that of the right-handed u-type quarks z_u [34]. The z charge of the field χ can be fixed without the loss of generality as its normalization can be absorbed into the rescaling of g_z. In this work, we use $z_\chi = -1$.

In general, the z_i charges of the right-handed neutrinos have to satisfy

TABLE I. Field content and charge assignment of a generic $U(1)_z$ extension of the SM. The field ϕ is the Higgs doublet, and χ is a complex scalar field; the rest are Weyl fermions. We show the representations for $SU(3)_\mathrm{c} \otimes SU(2)_\mathrm{L}$ and the charges y and z for $U(1)_Y \otimes U(1)_z$.

Field	$SU(3)_\mathrm{c}$	$SU(2)_\mathrm{L}$	y	z
Q_L	3	2	$\frac{1}{6}$	$z_q = \frac{1}{3}(z_\phi - z_N)$
U_R	3	1	$\frac{2}{3}$	$z_u = \frac{1}{3}(4z_\phi - z_N)$
D_R	3	1	$-\frac{1}{3}$	$z_d = -\frac{1}{3}(2z_\phi + z_N)$
ℓ_L	1	2	$-\frac{1}{2}$	$z_\ell = z_N - z_\phi$
N_R	1	1	0	z_N
e_R	1	1	-1	$z_e = z_N - 2z_\phi$
ϕ	1	2	$\frac{1}{2}$	z_ϕ
χ	1	1	0	$z_\chi = -1$

$$\frac{1}{3}\sum_{i=1}^{n} z_i = z_u - 4z_q \equiv z_N, \quad \text{and} \quad \left(\sum_{i=1}^{n} z_i\right)^3 = 9\sum_{i=1}^{n} z_i^3. \qquad (7)$$

A simple and natural choice is to have $n = 3$ generations of sterile neutrinos and generation independent z charges, i.e., $z_i = z_N$ for any $i = 1, 2, 3$. We find that for phenomenology it is more convenient to choose z_N and the z charge of the SM scalar field z_ϕ as independent charges. We exhibit the corresponding z charge assignment in Table I.

A $D = 4$ operator corresponding to a Majorana mass term for the sterile neutrinos is allowed only for $z_\chi + 2z_N = 0$, which implies $z_N = 1/2$ with our normalization $z_\chi = -1$. For example, in the $B - L$ $U(1)$ extension, $z_N = 1/2$ and $z_\phi = 0$, while in the SWSM, $z_N = 1/2$ and $z_\phi = 1$.

The neutral gauge fields are related to their mass eigenstates A_μ, Z_μ, and Z'_μ via two rotations [35],

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \\ B'_\mu \end{pmatrix} = \begin{pmatrix} \mathrm{c}_W & -\mathrm{s}_W & 0 \\ \mathrm{s}_W & \mathrm{c}_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{c}_Z & -\mathrm{s}_Z \\ 0 & \mathrm{s}_Z & \mathrm{c}_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}. \qquad (8)$$

The two mixing angles are (i) the weak mixing angle θ_W and (ii) the $Z - Z'$ mixing angle $\theta_Z \in [-\pi/4, \pi/4]$. The former is defined as $\mathrm{s}_W = \frac{g_y}{g_{Z^0}}$, with ${g_{Z^0}}^2 = g_y^2 + {g_\mathrm{L}}^2$, so $e = g_\mathrm{L}\mathrm{s}_W$ where g_L is the SU(2) gauge coupling and e is the elementary charge. The new mixing angle is defined as

$$\tan(2\theta_Z) = -\frac{2\kappa}{1 - \kappa^2 - \tau^2} \qquad (9)$$

in terms of effective couplings

$$\kappa = 2\frac{g_z}{g_{Z^0}} z_\phi(\mu) \quad \text{and} \quad \tau = 2\frac{g_z}{g_{Z^0}} \tan\beta, \qquad (10)$$

where $\tan\beta = \frac{w}{v}$, and we introduced the effective charge

$$z_\phi(\mu) = z_\phi - \frac{g_{yz}}{2g_z} \tag{11}$$

as the charge z_ϕ appears always together with this ratio of the new couplings throughout our computations. In Eq. (11), we indicated the dependence of the couplings on the renormalization scale μ to emphasize the scale dependence of the effective charge defined as abbreviation. It is possible to choose a basis of the fundamental gauge fields such that $g_{yz}(\mu_0) = 0$ at a fixed, but arbitrary renormalization scale μ_0. Clearly, $z_\phi(\mu_0) = z_\phi$ at this scale; i.e., μ_0 is the scale where all z charges are set. The scale μ_0 can be chosen at will, but the running of $z_\phi(\mu)$ introduces some theoretical uncertainty to our predictions, whose size depends on the actual choice of μ_0. To assess this uncertainty, we discuss the one-loop running of the ratio $\eta = g_{yz}/g_z$ in Appendix A.

In terms of the mixing angles and effective couplings, the masses of the gauge bosons are $M_W = \frac{1}{2}g_{\rm L}v$,

$$M_Z = \frac{M_W}{c_W}\sqrt{R(c_Z, s_Z)}, \qquad M_{Z'} = \frac{M_W}{c_W}\sqrt{R(s_Z, -c_Z)}, \tag{12}$$

with $R(x,y) = (x-\kappa y)^2 + (\tau y)^2$. The coupling parameters κ and τ can be expressed in terms of the experimentally more accessible parameters $M_{Z'}$ and θ_Z as

$$\kappa = -c_Z s_Z \frac{M_Z^2 - M_{Z'}^2}{c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2} \quad \text{and} \quad \tau = \frac{M_Z M_{Z'}}{c_Z^2 M_Z^2 + s_Z^2 M_{Z'}^2}. \tag{13}$$

Taking the ratio κ/τ, Eqs. (10) and (13) imply

$$-s_Z c_Z (1-\xi^2) = \frac{\xi z_\phi(\mu)}{\tan\beta}, \tag{14}$$

where we remind the reader that $\xi = M_{Z'}/M_Z$.

C. Modified ρ parameter

The well-known SM tree-level relationship between the masses of the W and Z bosons is usually expressed as $\rho = 1$, where

$$\rho = \frac{M_W^2}{c_W^2 M_Z^2}. \tag{15}$$

In the extended model, it is no longer equal to 1 at the tree level as it is modified [36] to

$$\rho = 1 - {s_Z}^2(1-\xi^2), \tag{16}$$

also used for a Z' exclusion study in Ref. [37]. Experimentally, from global fits [1], one has

$$\rho = 1.00038 \pm 0.00020, \tag{17}$$

which implies that ${s_Z}^2 \ll 1$ for either a light or a heavy Z' boson. Utilizing the smallness of s_Z, we can also express the ρ parameter in terms of the effective couplings, $\rho = 1 - \kappa^2/(1-\tau^2) + \mathcal{O}({s_Z}^4)$, or using the Lagrangian couplings and $M_{Z'}$ as

$$\rho = 1 - \frac{v^2}{M_Z^2 - M_{Z'}^2}(z_\phi(\mu) g_z)^2 + \mathcal{O}({s_Z}^4), \tag{18}$$

which we use below. Equivalently, we can express ρ using $\tan\beta$. In the limit of a heavy Z', we have $\rho \simeq 1 + (z_\phi(\mu)/\tan\beta)^2$, whereas a light Z' implies $\rho \simeq 1 - (z_\phi(\mu)\xi/\tan\beta)^2$.

D. Vector-axial vector couplings of the Z-prime boson

Direct Z' searches at colliders are most often based on the Drell-Yan process, hence on decays of the Z' into fermion pairs. The relevant theoretical predictions, discussed in detail in the following sections, rely on the interaction of the Z' boson and the fermions, which in the Dirac basis reads as (neglecting the mixing among the neutrinos)

$$\mathcal{L}_{\rm NC}^{(Z')} = -\frac{e}{2 s_W c_W} Z'_\mu \sum_f \bar{f}\gamma^\mu (v_{Z',f} - a_{Z',f}\gamma_5) f. \tag{19}$$

We recall the vector and axial (V-A) vector couplings $v_{Z',f}$ and $a_{Z',f}$ using a parametrization convenient to our analysis in Table II, obtained using the chiral couplings presented in Appendix B. Expanding these couplings in terms of the small parameter s_Z, one obtains the following expressions (recall that $\xi = M_{Z'}/M_Z$):

TABLE II. The vector and axial-vector couplings of the Z' boson to fermions in $U(1)_z$ extensions of the SM. The corresponding couplings of the Z boson $v_{Z,f}$ and $a_{Z,f}$ are obtained by the replacement $(c_Z, s_Z) \to (s_Z, -c_Z)$ in $v_{Z',f}$ and $a_{Z',f}$.

f	$v_{Z',f}$	$a_{Z',f}$
ν	$-\frac{1}{2}s_Z + \frac{1}{2}(-\kappa + 2\frac{\tau}{\tan\beta}z_N)c_Z$	$-\frac{1}{2}(s_Z + \kappa c_Z)$
ℓ	$-(-\frac{1}{2} + 2{s_W}^2)s_Z + \frac{1}{2}(-3\kappa + 2\frac{\tau}{\tan\beta}z_N)c_Z$	$\frac{1}{2}(s_Z + \kappa c_Z)$
u	$-(\frac{1}{2} - \frac{4}{3}{s_W}^2)s_Z + \frac{1}{6}(5\kappa - 2\frac{\tau}{\tan\beta}z_N)c_Z$	$-\frac{1}{2}(s_Z + \kappa c_Z)$
d	$-(-\frac{1}{2} + \frac{2}{3}{s_W}^2)s_Z - \frac{1}{6}(\kappa + 2\frac{\tau}{\tan\beta}z_N)c_Z$	$\frac{1}{2}(s_Z + \kappa c_Z)$

$$v_{Z',\nu} \simeq \frac{z_N \xi}{\tan\beta} - \frac{1}{2} s_Z \xi^2, \qquad a_{Z',\nu} \simeq -\frac{1}{2} s_Z \xi^2,$$
$$v_{Z',\ell} \simeq \frac{z_N \xi}{\tan\beta} + \frac{1}{2} s_Z (4 c_W{}^2 - 3\xi^2), \qquad a_{Z',\ell} \simeq \frac{1}{2} s_Z \xi^2,$$
$$v_{Z',u} \simeq -\frac{z_N \xi}{3\tan\beta} + \frac{1}{6} s_Z (-8 c_W{}^2 + 5\xi^2), \qquad a_{Z',u} \simeq -\frac{1}{2} s_Z \xi^2,$$
$$v_{Z',d} \simeq -\frac{z_N \xi}{3\tan\beta} + \frac{1}{6} s_Z (4 c_W{}^2 - \xi^2), \qquad a_{Z',d} \simeq \frac{1}{2} s_Z \xi^2. \tag{20}$$

It is useful to distinguish the cases of $\xi \to \infty$ (heavy Z') and $\xi \to 0$ (light Z'). In the case of a heavy Z' boson, Eq. (14) implies that

$$\frac{\xi}{\tan\beta} \simeq \frac{s_Z \xi^2}{z_N \mathscr{Z}} \tag{21}$$

for small values of $|s_Z|$. In Eq. (21), we introduced the effective charge ratio

$$\mathscr{Z}(\mu) = \frac{z_\phi(\mu)}{z_N} \tag{22}$$

that contains all dependence on the specific $U(1)$ extension. Of course, this $\mathscr{Z}(\mu)$ cannot be defined for models with vanishing sterile neutrino z charge, in which case the phenomenology would be quite different from ours, and we do not consider it further. As the effective charge of the BEH field depends on the renormalization scale, so does $\mathscr{Z}$, which we suppress in the following but take into account as theoretical uncertainty of our predictions as discussed in Appendix A.

Then, in the limit of small neutral gauge mixing and heavy Z', the V-A couplings simplify to

$$v_{Z',\nu} \simeq s_Z \xi^2 \left(\frac{1}{\mathscr{Z}} - \frac{1}{2}\right), \qquad a_{Z',\nu} \simeq -\frac{1}{2} s_Z \xi^2,$$
$$v_{Z',\ell} \simeq s_Z \xi^2 \left(\frac{1}{\mathscr{Z}} - \frac{3}{2}\right), \qquad a_{Z',\ell} \simeq \frac{1}{2} s_Z \xi^2,$$
$$v_{Z',u} \simeq -\frac{s_Z \xi^2}{3} \left(\frac{1}{\mathscr{Z}} - \frac{5}{2}\right), \qquad a_{Z',u} \simeq -\frac{1}{2} s_Z \xi^2,$$
$$v_{Z',d} \simeq -\frac{s_Z \xi^2}{3} \left(\frac{1}{\mathscr{Z}} + \frac{1}{2}\right), \qquad a_{Z',d} \simeq \frac{1}{2} s_Z \xi^2. \tag{23}$$

As for a light Z', Eq. (14) implies

$$\frac{\xi}{\tan\beta} \simeq -\frac{s_Z}{z_N \mathscr{Z}}, \tag{24}$$

and for the V-A couplings, one has negligible $a_{Z',f}$, and

$$v_{Z',\nu} \simeq -\frac{s_Z}{\mathscr{Z}}, \qquad v_{Z',\ell} \simeq s_Z \left(-\frac{1}{\mathscr{Z}} + 2 c_W{}^2\right),$$
$$v_{Z',u} \simeq \frac{s_Z}{3} \left(\frac{1}{\mathscr{Z}} - 4 c_W{}^2\right), \qquad v_{Z',d} \simeq \frac{s_Z}{3} \left(\frac{1}{\mathscr{Z}} + 2 c_W{}^2\right). \tag{25}$$

We may also use the new gauge couplings as input parameters. To write the V-A couplings as functions of g_z, we first observe that Eq. (13) implies

$$\kappa \simeq -s_Z \quad \text{for} \quad \xi \to 0,$$
$$\kappa \simeq s_Z \xi^2 \quad \text{for} \quad \xi \to \infty. \tag{26}$$

Then, for a heavy Z', the axial couplings are $a_{Z',f} \simeq z_N \frac{2 g_z}{g_{Z^0}} a_f^{(h)}$ and $v_{Z',f} \simeq z_N \frac{2 g_z}{g_{Z^0}} v_f^{(h)}$. Using the definition of κ in Eq. (10), we obtain

$$a_\nu^{(h)} = a_u^{(h)} = -\frac{1}{4}, \qquad a_\ell^{(h)} = a_d^{(h)} = +\frac{1}{4}, \tag{27}$$

while from Eq. (23)

$$v_\nu^{(h)} = 1 - \frac{1}{2}\mathscr{Z}, \qquad v_\ell^{(h)} = 1 - \frac{3}{2}\mathscr{Z},$$
$$v_u^{(h)} = -\frac{1}{3} + \frac{5}{6}\mathscr{Z}, \qquad v_d^{(h)} = -\frac{1}{3} - \frac{1}{6}\mathscr{Z}. \tag{28}$$

A light Z' boson implies $a_{Z',f} \simeq 0$ and $v_{Z',f} \simeq z_N \frac{2 g_z}{g_{Z^0}} v_f^{(\ell)}$, where

$$v_\nu^{(\ell)} = 1, \qquad v_\ell^{(\ell)} = 1 - 2 c_W{}^2 \mathscr{Z},$$
$$v_u^{(\ell)} = -\frac{1}{3} + \frac{4}{3} c_W{}^2 \mathscr{Z}, \qquad v_d^{(\ell)} = -\frac{1}{3} - \frac{2}{3} c_W{}^2 \mathscr{Z}. \tag{29}$$

III. DIRECT Z' BOSON SEARCHES

Collider experiments such as LEP, Tevatron, and the LHC performed direct searches for a Z' boson. The nonobservation of such a particle can be and was translated into exclusion bands for the parameters of certain models predicting a Z' boson.

According to Ref. [34], the results of the LEPII experiment show that Z' is either heavier than the largest center-of-mass energy (209 GeV) or $|z_\ell g_z| \lesssim 10^{-3}$. Tevatron searched for a Z' in the range 200 GeV $< M_{Z'} <$ 800 GeV [38]. Finally, ATLAS [20] and CMS [21] at the LHC performed the most recent searches up to $M_{Z'} <$ 5500 GeV. Below M_Z, the NA64 and *BABAR* experiments together with FASER below the mass of the pion provide strong bounds for a light Z' boson [23,24]. In this study, we focus on the exclusion bounds obtained from ATLAS and CMS for a heavy Z' and NA64, *BABAR*, and FASER for a light Z'.

To compare model predictions to experimental results at proton-proton (pp) colliders, one has to compute the cross section for the process $pp \to Z' + X \to \ell^+\ell^- + X$, which is usually performed in the narrow width approximation

$$\sigma(pp \to Z' + X \to \ell^+\ell^- + X) = \sigma(pp \to Z'X)\mathrm{Br}(Z' \to \ell^+\ell^-), \qquad (30)$$

assuming that the total width of the Z' boson $\Gamma_{Z'}$ is much smaller than its mass, $\gamma_{Z'} = \Gamma_{Z'}/M_{Z'} \ll 1$. Equation (30) is usually presented as

$$\sigma(pp \to Z' + X \to \ell^+\ell^- + X) = \frac{\pi}{6s}(c_U w_U(s, M_{Z'}) + c_D w_D(s, M_{Z'})), \qquad (31)$$

where and $U \in \{u, c, t\}$, $D \in \{d, s, b\}$ and the coefficients c_q collect model-dependent contributions to the cross section

$$c_q = (2\sqrt{2}G_F M_Z^2 \rho)\mathrm{Br}(Z' \to \ell^+\ell^-)(a_{Z',q}^2 + v_{Z',q}^2). \qquad (32)$$

The hadronic structure functions $w_{U/D}$ (cf. Ref. [38]) collect the QCD corrections. For the production of a heavy neutral gauge boson, they depend only on the M of the gauge boson and the center-of-mass energy squared s,

$$w_{U/D} = \sum_{q\in U/D} \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}z \delta\left(\frac{M^2}{s} - z x_1 x_2\right) \times [f_g(x_1, M)(f_q(x_2, M) + f_{\bar{q}}(x_2, M))\Delta_{gq}(z, M^2) + (f_q(x_1, M) f_{\bar{q}}(x_2, M))\Delta_{qq}(z, M^2) + (x_1 \leftrightarrow x_2)], \qquad (33)$$

where the functions $f_i(x, \mu_F)$ are the parton distribution functions inside the proton for parton i at factorization scale μ_F. We use the NNPDF3.0 next-to-learding-order (NLO) PDF set in our numerical computations. At the NLO accuracy, the coefficient functions Δ_{ab} for vector boson production are known [39]. One also needs to compute the total decay width of the Z' to obtain the cross section (30). We collect the coefficient functions and the decay width formulas of the Z' boson, needed to compute the cross section in Eq. (31) in Appendix C.

IV. NUMERICAL ANALYSIS

A. Parameter scanning

The model predictions can be expressed as functions of the free parameters of the theory. At the most fundamental level, these are the free z charges, new couplings, and the VEV ratio,

$$z_\phi, z_N, g_z, g_{yz}, \tan\beta, \qquad (34)$$

which are not independent, and a certain combination of them appears in the model predictions. For instance, the tree-level ρ parameter estimates the constraints from the electroweak precision observables, and it depends only on

$$(\mathrm{s}_Z, M_{Z'}) \quad \text{or} \quad (z_N g_z, M_{Z'}, \mathscr{L}). \qquad (35)$$

The NA64 experiment presents exclusion bounds for a dark photon in the $(\epsilon, M_{A'})$ plane. Those constraints can be translated to our model parameters using the relation derived in Appendix D. This shows one that exclusion bounds depend on either

$$(\mathrm{s}_Z, M_{Z'}, \mathscr{L}) \quad \text{or} \quad (z_N g_z, M_{Z'}, \mathscr{L}). \qquad (36)$$

Presently, the most stringent bounds on the parameter space for heavy Z' bosons can be obtained from direct searches using the Drell-Yan pair production process

$$p + p \to Z' + X \to \ell^+ + \ell^- + X, \qquad (37)$$

described in Sec. III. The corresponding cross section (31) can be rewritten as

$$\sigma = \frac{4\pi^2}{3s}\frac{\Gamma_{Z'}}{M_{Z'}}\mathrm{Br}(Z' \to \ell^+\ell^-)(\mathrm{Br}(Z' \to U\bar{U})w_U(s, M_{Z'}) + \mathrm{Br}(Z' \to D\bar{D})w_D(s, M_{Z'})), \qquad (38)$$

where the branching fractions are listed in Appendix C, while $w_{U/D}$ are given by Eq. (33). In this case as well, the predictions depend on the parameter set (36) or equivalently on

$$(\gamma_{Z'}, M_{Z'}, \mathscr{L}), \qquad (39)$$

where $\gamma_{Z'} = \Gamma_{Z'}/M_{Z'}$.

B. Constraints on a light neutral gauge boson

Light vector-type particles, usually called dark photons (A'), are often considered as a portal to a secluded sector in particle physics or downright as dark matter candidates. Presently, the most stringent, 90% C.L. exclusion bound in the dark photon mass range $M_{A'} \in (1\ \text{MeV}, 8\ \text{GeV})$ comes from the combined results of the NA64 [23], *BABAR* [24], and more recently the FASER [25] experiments. The dark photon model probed in these experiments has a single vector type coupling ϵe to the electromagnetic current. The parameters $(\epsilon, M_{A'})$ can be matched to a generic $U(1)_z$ as detailed in Appendix D. We scanned the parameter planes $(\mathrm{s}_Z, M_{Z'})$ and $(|z_N g_z|, M_{Z'})$ for several benchmark values of $\mathscr{L}$. Using the value for the ρ parameter given in Eq. (17), we set upper bounds

$$|s_Z| \lesssim 4.5 \times 10^{-3} \quad \text{and} \quad |z_N g_z| \lesssim \frac{1.7 \times 10^{-3}}{|\mathscr{X}|} \tag{40}$$

for $M_{Z'} \ll M_Z$.

The experimental bounds obtained from NA64, *BABAR*, and FASER all depend on v_ℓ^ℓ given in Eq. (29). The former two experiments searched for invisible decay products of dark photons, whereas the latter one searches for decays $A' \to e^+e^-$, which introduces further dependence on the corresponding branching fractions. The mapping of ϵ onto s_Z or $|z_N g_z|$ and $M_{Z'}$ thus leads to a dependence on $\mathscr{X}$ in the exclusion bands. For instance, as $\mathscr{X}$ approaches $1/(2c_W^2)$, the reduced vector coupling tends to zero, $v_\ell^\ell \to 0$, which renders $|s_Z|$ and $|z_N g_z|$ unconstrained. The exclusion band obtained from the FASER experiment is even more sensitive as the branching fraction $\mathrm{Br}(Z' \to e^+e^-)$ also depends on v_ℓ^ℓ.

The Z' search at the Belle II experiment presented in Ref. [29] sets stringent bounds on the $L_\mu - L_\tau$ model [11] via the process

$$e^+ + e^- \to \mu^+ + \mu^- + Z' \to \mu^+ + \mu^- + \text{invisible}. \tag{41}$$

We computed the corresponding cross section in the $U(1)_z$ models considered here and found that Belle II bounds on s_Z and $|z_N g_z|$ are comparable to the limits obtained from the ρ parameter, but not as severe as the ones from NA64 or *BABAR*.

Our findings for selected benchmark values of $\mathscr{X}$ are summarized in Fig. 1. The regions in the parameter planes above the dashed line and gray bands are excluded at 90% C.L. The dashed lines correspond to the experimental value of the ρ parameter in Eq. (17), whereas the regions above the gray bands correspond to the exclusions by direct searches at fixed values of the effective charge ratio $\mathscr{X}$. The width of the gray bands is the uncertainty due to the number of right-handed neutrinos lighter than $M_{Z'}/2$. A light Z' boson may always decay into the three families of active neutrinos, but decays into right-handed neutrinos may be kinematically forbidden depending on the specific values of $M_{Z'}$.

C. Constraints on a heavy neutral gauge boson

Direct searches for heavy Z' bosons were performed at the LEPII, Tevatron, and LHC as well and are of continued interest for future colliders [17,40]. We perform the scan in the parameter sets given in Eq. (36) using the 95% C.L. exclusion bands presented by the ATLAS [20] and CMS [21] experiments. Our findings are summarized in Fig. 2. The exclusion limit by the ρ parameter is represented again with a dashed line: the region above it is excluded. Analytically, this corresponds to

$$|s_Z| \lesssim 0.0025 \left[\frac{1\ \text{TeV}}{M_{Z'}}\right] \quad \text{and} \quad |g_z z_N| \lesssim \frac{0.11}{\mathscr{X}} \left[\frac{M_{Z'}}{1\ \text{TeV}}\right]. \tag{42}$$

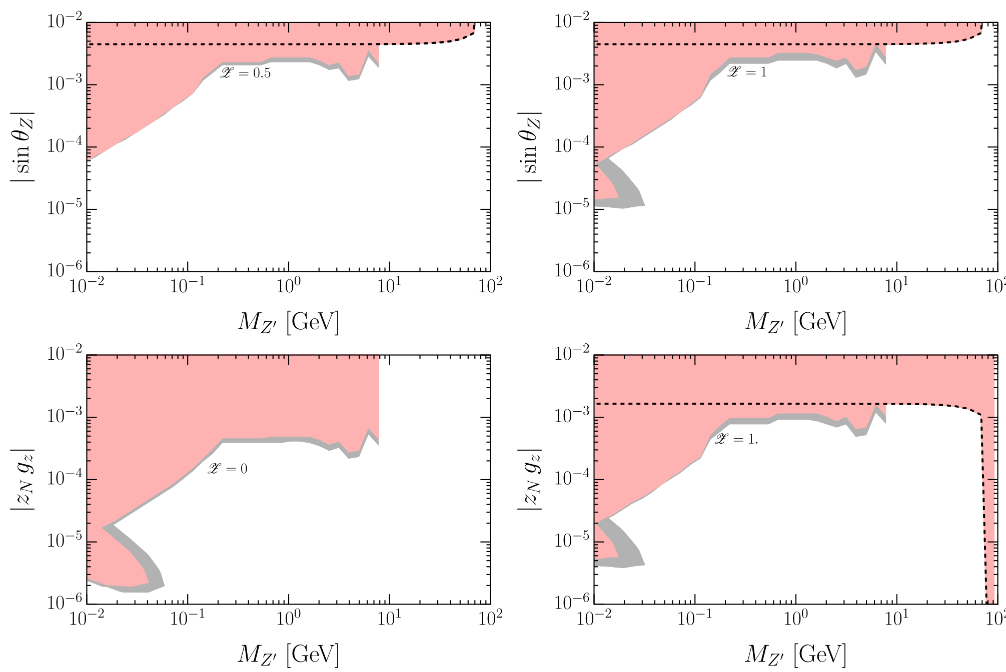

FIG. 1. 90% C.L. exclusion bounds for light Z' bosons obtained from the NA64, *BABAR*, and FASER experiments. The width of the band corresponds to the uncertainty in the number of sterile neutrino families where the decay $Z \to N + N$ is kinematically allowed. The region above the dashed line is excluded due to the ρ parameter, and the area above the gray bands is also excluded for a selected value of $\mathscr{X}$.

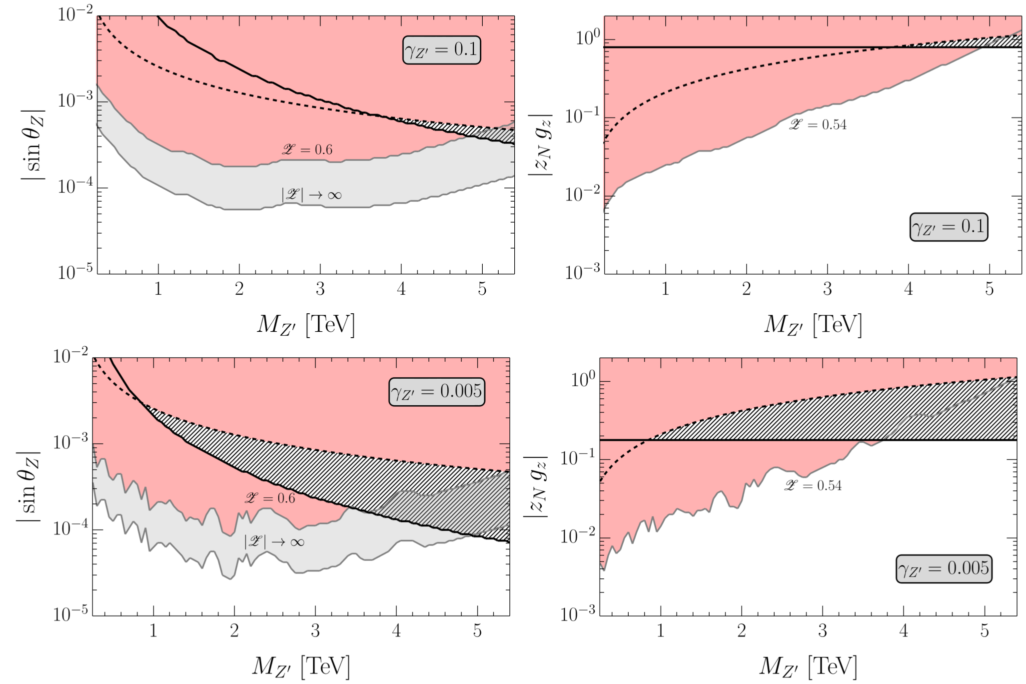

FIG. 2. 95% C.L. exclusion bounds for heavy Z' bosons obtained from the CMS and ATLAS experiments at the LHC for fixed ratios $\gamma_{Z'}$. The region above the dashed line is excluded due to the ρ parameter, and the area above the gray line is also excluded for a select value of $\mathscr{Z}$.

The collider searches by ATLAS and CMS are performed for fixed values of the ratio $\gamma_{Z'}$. We chose the datasets corresponding to the largest ($\gamma_{Z'} = 10\%$) and smallest ($\gamma_{Z'} = 0.5\%$) presented values. It is possible that the cross section (38) is large enough that the process is excluded experimentally for given values of the input parameters (36), but the corresponding ratio $\gamma_{Z'}$ is larger than that searched for in the experiment in a region whose lower boundary is denoted by a solid curve in Fig. 2. The hatched region corresponds to this exact case, i.e., where no strict exclusion applies. The region shaded in red in the parameter plane presented in Fig. 2 is excluded at 95% C.L. The branching fractions and the ratio $\gamma_{Z'}$ in Eq. (38) depend on the charge ratio $\mathscr{Z}$, and hence so do the exclusion bounds. We find that there exist a value of $\mathscr{Z}$ both for s_Z and $z_N g_z$ which corresponds to a loosest, i.e., the most conservative, bound on these parameters. For the mixing s_Z, this value is $\mathscr{Z} \simeq 0.6$. Any other fixed $\mathscr{Z}$ value presents a more severe bound than that shown in Fig. 2. It is interesting that the cross section in Eq. (38) diverges as $|\mathscr{Z}| \to 0$ (vanishing z_N charge), which means that only zero mixing $s_Z = 0$ is allowed for small $\mathscr{Z}$. Conversely, Eq. (38) saturates at a finite value for $|\mathscr{Z}| \to \infty$; the corresponding exclusion bands are shown on the left-hand side plots of Fig. 2.

As for $z_N g_z$, the most conservative bound corresponds to $\mathscr{Z} \simeq 0.54$ as can be seen in the plots on the right-hand side of Fig. 2. As opposed to the exclusion bound on s_Z, in this case, the cross section is finite at $\mathscr{Z} = 0$, and hence one has a well-defined exclusion bound on $z_N g_z$ for $\mathscr{Z} = 0$.

D. Constraints on the parameter space of specific $U(1)$ extensions

We showcase the exclusion bounds on two specific $U(1)_z$ extensions, one with a light Z' boson and one with a heavy Z' boson. We consider here the uncertainty due to the renormalization group equation (RGE) running of η, and we also use the mass of the W boson as a constraint.

The tree-level ρ parameter discussed in Sec. II C is a useful quantity to gauge the exclusion from electroweak precision observables in a model-independent manner. The effect of one-loop BSM corrections might become important for a given region in the parameter space, and thus the use of a precise prediction is warranted. The drawback is that the radiative BSM corrections are in general complicated functions of the free parameters and the z charges. Once the z charges are set and η is considered as an uncertainty, there are two free parameters from the gauge sector ($M_{Z'}$ and either s_Z or g_z) and two from the scalar sector (M_S and s_S). Using these four parameters, we compute the complete one-loop corrections to M_W in $U(1)_z$ extensions presented in Ref. [36] based on the computational method of Ref. [41]; consult also Ref. [42] for the renormalization of s_Z. Our input parameters are

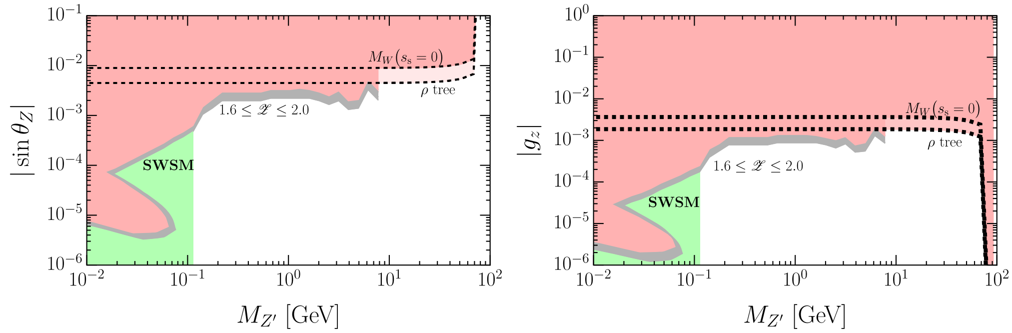

FIG. 3. Exclusion bounds for models with a light $M_{Z'}$ and $z_N = 1/2$ and $z_\phi = 1$. The red region is excluded at 90% C.L. The green region is the preferred parameter space of the SWSM. The widths of the lines take into account the uncertainty in the η parameter. The gray line corresponds to the NA64, *BABAR*, and FASER experiments whereas the dashed ones correspond to the bounds from M_W and ρ.

$$M_W^{\rm SM} = 80.353\ \text{GeV}, \qquad M_W^{\rm exp} = 80.377\ \text{GeV}, \qquad (43)$$

with a combined experimental and theoretical uncertainty of $\sigma = 15$ MeV.

The BSM corrections can amount to either a positive or negative contribution to M_W. A heavy Z' boson and a light S scalar ($M_S < M_H$) increase, while a light Z' boson and a heavy S scalar ($M_S > M_H$) decrease the predicted value of M_W. In this work, we focus on the effect of the gauge sector, and thus we present exclusion bounds obtained from M_W at $s_S = 0$, i.e., when the extended scalar sector does not affect the mass of the W boson.

Our case study for a $U(1)_z$ extension with a light Z' boson is the SWSM (recall that $z_N = 1/2$ and $z_\phi = 1$). This model can explain the observed dark matter abundance in the Universe with freeze-out scenario if $10\ \text{MeV} \lesssim M_{Z'} \lesssim m_\pi \ll M_Z$ and the dark matter candidate is the lightest sterile neutrino, which is considered to be lighter than $M_{Z'}/2$, while the other sterile neutrinos are much heavier [31].

Our findings are summarized in Fig. 3. The region in red is excluded at 90% C.L. The gray band is the lower boundary of the exclusion region from the NA64, *BABAR*, and FASER experiments, and the width of the gray band corresponds to the combined uncertainty from decays of the Z' boson and the running of η. Solving the RGEs of Appendix A, we find that the largest possible value of η is 0.4; hence, $\mathscr{Z}$ can take values in the range (1.6, 2.0). The dashed lines correspond to the bound from M_W computed with $s_S = 0$ and to the bound from the tree-level ρ parameter as a reference. The scalar sector has the potential to significantly affect the bound obtained from M_W. In fact, for a heavy scalar ($M_S \gg M_h$) and a light Z' boson, one may write the BSM correction $\delta M_W^{\rm BSM}$ to the mass of the W boson as

$$\delta M_W^{\rm BSM} \simeq -\left[5.6(100 s_Z)^2 + 1.5(10 s_S)^2 \times \left(1 + 0.57 \log\left(\frac{M_S}{1\ \text{TeV}}\right)\right)\right]\ \text{MeV}, \qquad (44)$$

which is independent of the z charges. For instance, a scalar with a mass $M_S \simeq 1$ TeV and a mixing of $s_S \simeq 0.2$ would increase the difference $|M_W - M_W^{\rm exp}|$ above 2σ, excluding any nonzero value of s_Z.

As for a $U(1)_z$ extension with a heavy Z' boson, $M_Z \ll M_{Z'}$, we choose to investigate the $B-L$ extension

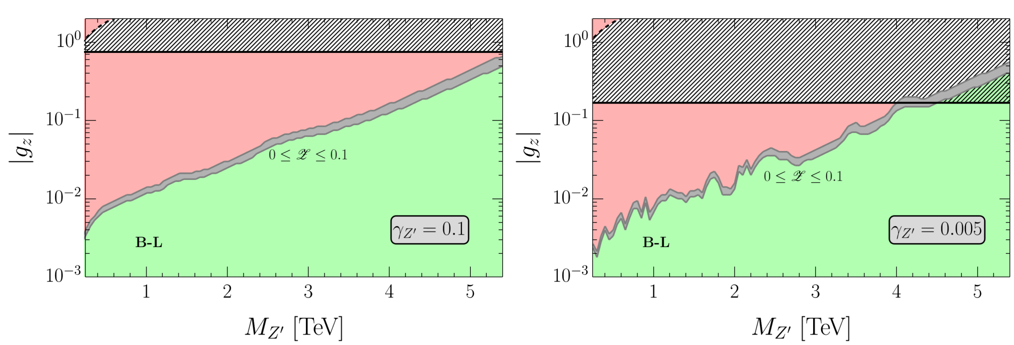

FIG. 4. Exclusion bounds for the $B-L$ extension of the SM where one has $z_N = 1/2$ and $z_\phi = 0$. The red region is excluded at 95% C.L. The green region is the preferred parameter space of the $B-L$ model. The widths of the lines take into account the uncertainty in the η parameter. The gray line corresponds to the CMS and ATLAS direct searches at $\gamma_{Z'} = 0.1$ and at 0.005. The dashed line in the top left corner corresponds to the exclusion from M_W at $s_S = 0$.

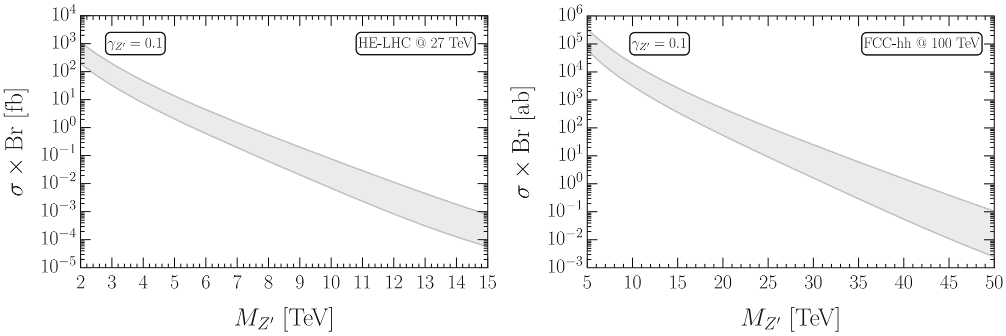

FIG. 5. Production cross sections $\sigma(pp \to Z')$ times the leptonic branching fraction $\mathrm{Br}(Z' \to \ell^+ + \ell^-)$ as the function of M_Z' for center-of-mass energies $\sqrt{s} = 27$ TeV (left, for the HE-LHC) and 100 TeV (right, for FCC-hh) and fixed ratio $\gamma_{Z'} = 0.1$. The cross section for any value of $\mathscr{X}$ is inside of the gray band.

of the SM, which has $z_N = 1/2$ and $z_\phi = 0$. It is interesting to note that in this case $\mathscr{X} = 0$ at the default scale μ_0, and the uncertainty from the RG running of η is also essentially negligible, at most about 0.1. Hence, there is effectively no mixing between the Z and Z' bosons, $\mathrm{s}_Z \simeq 0$, and consequently, there is no bound from the tree-level ρ parameter. Our findings are summarized in Fig. 4. The region in red is excluded at 95% C.L. The dashed line corresponds to the exclusion from M_W at $\mathrm{s}_S = 0$. Since the tree-level ρ parameter equals 1, this corresponds purely to one-loop BSM corrections to M_W. The hatched region is not excluded by M_W, and the width-to-mass ratio $\gamma_{Z'}$ of the Z' boson is larger than the one considered experimentally. The green region displays the presently allowed parameter space of the $B - L$ model.

E. Projections for future proton-proton collider experiments

The High-Energy LHC (HE-LHC) experiment is planned to operate at $\sqrt{s} = 27$ TeV center-of-mass energy, while the Future Circular Collider will collide hadrons (FCC-hh) at $\sqrt{s} = 100$ TeV. The experimental programs of both machines include direct searches for Z' bosons. The cross section (38) for the process $p + p \to Z' + X \to \ell^+ + \ell^- + X$, which is the main search channel for Z' bosons, is shown on Fig. 5 for relevant values of $\sqrt{s}$. The cross sections are inside of the gray band in Fig. 5 for any value of $\mathscr{X}$ for $\gamma_{Z'} = 0.1$. Note that the gray band for a different value of $\gamma_{Z'}$ can be obtained by the linear rescaling of those on Fig. 5.

Detector simulations are already available both for the HE-LHC and the FCC-hh. We compute projected 95% C.L. exclusion bands both for $|\mathrm{s}_Z|$ and $|g_z z_N|$ using the simulations for the HE-LHC at 15 ab^{-1} integrated luminosity [17] and for the FCC-hh at 30 ab^{-1} [43]. Our predictions are shown in Fig. 6. The widths of the gray bands correspond to the 2σ uncertainty of the simulation in the location of the exclusion band.

It is noteworthy that for large ($\gtrsim$10 TeV) masses of the Z' boson the cross section

$$\sigma(p + p \to Z' + X \to Z + W^+ + W^- + X) \quad (45)$$

may become comparable to or larger than the Drell-Yan pair production cross section in Eq. (38) as the ratio of the two cross sections is

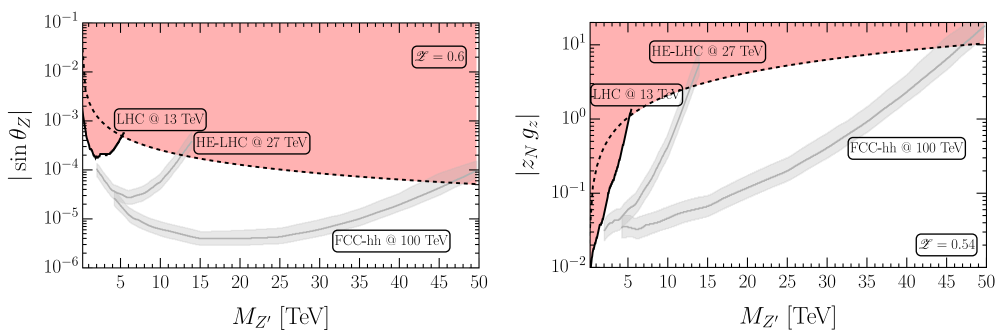

FIG. 6. Projected exclusion bounds on $|\mathrm{s}_Z|$ at $\mathscr{X} = 0.6$ and on $|z_N g_z|$ at $\mathscr{X} = 0.54$ for the HE-LHC and FCC-hh experiments using the simulated exclusion bands obtained in Refs. [17] and [43]. The dark gray line is the expected median exclusion limit and the width of the gray bands correspond to the 95% C.L. expected limit. The dashed line represents the exclusion by the ρ parameter, and the red shaded area is already excluded at 95% C.L.

$$\frac{\mathrm{Br}(Z' \to ZW^+W^-)}{\mathrm{Br}(Z' \to \ell^+\ell^-)} = \frac{\mathscr{Z}^2}{2-6\mathscr{Z}+5\mathscr{Z}^2}\left(C_{ff}\frac{7c_W^4}{160\pi}\right)\frac{M_{Z'}^2}{M_Z^2} \simeq 0.368\frac{\mathscr{Z}^2}{2-6\mathscr{Z}+5\mathscr{Z}^2}\left[\frac{M_{Z'}}{10\ \mathrm{TeV}}\right]^2. \tag{46}$$

V. CONCLUSIONS

In this paper, we studied the parameter space of $U(1)_z$ extensions of the standard model with an additional complex scalar field and three families of right-handed neutrinos with generation-independent z charges and no exotic fermions. Anomaly cancellation constrains the z charges such that two z charges remain arbitrary. The vector–axial vector couplings, which are critical in the analysis presented in this work, depend on a special combination $\mathscr{Z}$ of z_ϕ the z charge of the BEH field and z_N as given in Eq. (22). We presented our predictions using $\mathscr{Z}$, the mass $M_{Z'}$ of the Z' boson, and either the mixing s_Z between Z and Z' or the right-handed neutrino z charge times the new gauge coupling, $z_N g_z$. Our exclusion bounds on these parameters used the results of the NA64, *BABAR*, and FASER experiments for a light Z' boson and those of the ATLAS and CMS experiments for a heavy Z' boson as constraints from direct searches. We also studied the limits obtained from the measured values of the ρ parameter and the mass of the W boson as constraints from indirect sources.

For a light Z' boson, the ρ parameter provides a bound on $|s_Z|$, independent of $\mathscr{Z}$, and a bound on $|z_N g_z|$, proportional to $1/\mathscr{Z}$ as given in Eq. (40). The bound on $|s_Z|$ obtained from the NA64, *BABAR*, and FASER experiments is in general more severe, and it is proportional to $1/\mathscr{Z}$. The constraint on $|z_N g_z|$ from such direct searches depends weekly on $\mathscr{Z}$ unless $\mathscr{Z}$ is fine tuned to $\mathscr{Z} = 1/(2c_W{}^2)$ [see Eq. (D7)].

In the case of a heavy Z' boson, the value $\mathscr{Z} \simeq 0.6$ corresponds to the loosest bound on $|s_Z|$ (or $\simeq$0.54 on $|z_N g_z|$) obtained from the ATLAS and CMS experiments, which means that one has a model-independent way to constrain the parameter space of $U(1)_z$ extensions with different charge assignments. In this region, the ρ parameter excludes a decent portion of the parameter space, but it is never as severe as the exclusion bound obtained from direct searches.

We also used detector simulations for the HE-LHC and FCC experiments to provide projected exclusion bounds on the parameter space. We found that the minimum of the excluded $|s_Z|$ values will improve by 1 order of magnitude in HE-LHC compared to LHC and an additional 1 order of magnitude in FCC-hh compared to HE-LHC. As for $|g_z z_N|$, we find no such improvement. The exclusion bound obtained from the LHC and the projected ones are based on leptonic final states. We find that for very large values of $M_{Z'}$ (>10 TeV) the decay $Z' \to Z + W^+ + W^-$ might become dominant over $Z' \to \ell^+ + \ell^-$ depending on $\mathscr{Z}$. This means that we propose direct searches at FCC for the final state $Z + W^+ + W^-$ as it is well motivated experimentally.

ACKNOWLEDGMENTS

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APPENDIX A: RUNNING ETA PARAMETER

We defined the ratio of the mixing coupling g_{yz} and the new gauge coupling g_z as

$$\eta = \frac{g_{yz}}{g_z}. \tag{A1}$$

It always appears as a correction to the z charge of the BEH field as in Eq. (11). Like any other coupling, η depends on the renormalization scale μ, as described by the RGEs

$$\begin{aligned} \dot{g}_y &= \frac{g_y}{16\pi^2}\left(\frac{41}{6}g_y^2 + \frac{5}{3}g_z^2\eta b_\eta\right), \\ \dot{g}_z &= \frac{g_z^3}{16\pi^2}\left(\frac{5}{9} + \frac{4000}{369}z_N^2 + \frac{10}{41}b_\eta^2\right), \\ \dot{\eta} &= \frac{g_y^2}{16\pi^2}b_\eta, \end{aligned} \tag{A2}$$

where

$$b_\eta = \frac{16}{3}z_N - \frac{41}{3}\left(z_\phi - \frac{\eta}{2}\right) = \frac{z_N}{3}(16 - 41\mathscr{Z}) \tag{A3}$$

is a linear function of η. The derivative $\dot{f} = \partial f/\partial t$ is meant with respect to $t = \ln(\mu/\mu_0)$. We chose the z charge assignment according to Table I at an arbitrarily chosen fixed scale μ_0 where $\eta(\mu_0) = 0$. Then, one can investigate the uncertainty due to the choice of the unknown scale μ_0.

The scale μ_0 can be chosen arbitrarily. In our study, the most reasonable choices are either $\mu_0 = m_\mathrm{t}$ or $\mu_0 = M_{Z'}$. The values of $M_{Z'}$ considered here are at maximum a few tens of TeV; then, the running of η from $\mu_0 = M_{Z'}$ down to the electroweak scale does not affect the value of $\mathscr{Z}$ in any way relevant for the phenomenology considered in this work. One may set μ_0 as high as M_Pl, in which case $\eta(m_\mathrm{t}) = \mathrm{O}(1)$, with the exact value depending on z_N and z_ϕ. For instance, in the SWSM, choosing $\eta(M_\mathrm{Pl}) = 0$ at the Planck scale, the renormalization group running implies that $\eta \simeq 0.67$ at the electroweak scale [31]. Here, we choose

$\mu_0 = m_t$. Then, one initial condition, $g_y(m_t) \simeq 0.36$, is known, while the initial conditions for g_z, η as well as the z charges z_N and z_ϕ are free parameters. One can show that the coupling $g_z(\mu)$ has a Landau pole below the Planck scale $M_{\rm Pl}$ if $g_z(m_t)$ is larger than a critical value. Assuming a constant η, this value is

$$\alpha_z(m_t) = \frac{g_z(m_t)^2}{4\pi} \gtrsim \frac{11.95}{41 + 800 z_N^2 + 18 b_\eta^2}. \tag{A4}$$

Taking the running of η into account, this formula is not exact, and the actual upper bound on $g_z(m_t)$ to avoid the Landau pole below the Planck scale is about 15% lower. The loosest constraint obtained from avoiding the Landau pole corresponds to $z_N = z_\phi = 0$ with $\eta(m_t) = 0$. Then, one has the upper bound $g_z(m_t) \lesssim 1.91$. Any different z charge assignment results in a considerably more severe upper limit. For instance, in the SWSM, it is $g_z(m_t) \lesssim 0.22$. Then, the initial conditions $\eta(M_{\rm Pl}) = 0$ and $g_z(m_t) \in [0, 0.22]$ in the one-loop RGEs of Eq. (A2) yield η at the electroweak scale in the range $\eta(m_t) \in [0.4, 0.375]$ [the larger $g_z(m_t)$, the smaller $\eta(m_t)$].

APPENDIX B: CHRIAL COUPLINGS OF THE Z AND Z' BOSONS

We list here the chiral couplings of the Z' bosons to fermions in terms of the neutral mixing angle and effective couplings κ and τ in Table III. The chiral couplings to the Z boson can be obtained by the replacement $(c_Z, s_Z) \to (s_Z, -c_Z)$.

We recall here the chiral couplings of the neutrinos; for a detailed discussion, see Ref. [44]. As the neutral currents are written in terms of flavor eigenstates, the interactions between the neutral gauge bosons and the propagating mass eigenstate neutrinos include also the neutrino mixing matrices,

$$\Gamma^\mu_{V\nu_i\nu_j} = -\mathrm{i}e\gamma^\mu(\Gamma^L_{V\nu\nu}P_L + \Gamma^R_{V\nu\nu}P_R)_{ij}, \tag{B1}$$

where

$$\Gamma^L_{V\nu\nu} = C^L_{V\nu\nu}\mathbf{U}_L^\dagger\mathbf{U}_L - C^R_{V\nu\nu}\mathbf{U}_R^T\mathbf{U}_R^* \tag{B2}$$

and

$$\Gamma^R_{V\nu\nu} = -C^L_{V\nu\nu}\mathbf{U}_L^T\mathbf{U}_L^* + C^R_{V\nu\nu}\mathbf{U}_R^\dagger\mathbf{U}_R = -(\Gamma^L_{V\nu\nu})^* \tag{B3}$$

for both $V = Z$ and $V = Z'$. To recover the SM vector and axial vector couplings of the Z boson and the neutrinos, the right-handed mixing matrices have to vanish, and

$$\mathbf{U}_L^\dagger\mathbf{U}_L \to \mathbf{1}_3 \quad \text{and} \quad \mathbf{U}_L^T\mathbf{U}_L^* \to 0. \tag{B4}$$

If one estimates the chiral couplings of the Z and Z' bosons in the presence of sterile neutrinos but with the mixing neglected, then one needs to use the replacements

$$\begin{aligned} \mathbf{U}_L^\dagger\mathbf{U}_L \to \mathbf{1}_3 \quad &\text{and} \quad \mathbf{U}_L^T\mathbf{U}_L^* \to 0, \\ \mathbf{U}_R^\dagger\mathbf{U}_R \to \mathbf{1}_3 \quad &\text{and} \quad \mathbf{U}_R^T\mathbf{U}_R^* \to 0, \end{aligned} \tag{B5}$$

which we adopted throughout.

APPENDIX C: COEFFICIENT FUNCTIONS FOR HADROPRODUCTION AND DECAYS OF THE Z' BOSON

The theoretical input needed to compute the Drell-Yan pair production cross section in Eq. (30) is the coefficient functions and branching ratios, which we present here explicitly. The coefficient functions for the production of a neutral gauge boson at NLO accuracy in QCD read [39]

$$\begin{aligned} \Delta_{qq}(z,\mu_R^2) &= \delta(1-z) + \frac{\alpha_s(\mu_R^2)}{2\pi}C_F\left[\delta(1-z)\left(\frac{2\pi^2}{3} - 8\right) + 4(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln(z)\right], \\ \Delta_{gq}(z,\mu_R^2) &= \frac{\alpha_s(\mu_R^2)}{2\pi}T_R\left[(1-2z+2z^2)\ln\left(\frac{(1-z)^2}{z}\right) + \frac{1}{2} + 3z - \frac{7}{2}z^2\right], \end{aligned} \tag{C1}$$

with color factors $C_F = 4/3$ and $T_R = 1/2$.

TABLE III. The chiral couplings of the Z' boson to fermions in $U(1)_z$ extensions of the SM.

f	$C^R_{Z',f\bar{f}}$	$C^L_{Z',f\bar{f}}$
ν	$\frac{\tau}{\tan\beta}z_N c_Z$	$-s_Z + (-\kappa + \frac{\tau}{\tan\beta}z_N)c_Z$
ℓ	$-2{s_W}^2 s_Z + (-2\kappa + \frac{\tau}{\tan\beta}z_N)c_Z$	$(1 - 2{s_W}^2)s_Z + (-\kappa + \frac{\tau}{\tan\beta}z_N)c_Z$
u	$\frac{4}{3}{s_W}^2 s_Z + (\frac{4}{3}\kappa - \frac{1}{3}\frac{\tau}{\tan\beta}z_N)c_Z$	$(-1 + \frac{4}{3}{s_W}^2)s_Z + \frac{1}{3}(\kappa - \frac{\tau}{\tan\beta}z_N)c_Z$
d	$-\frac{2}{3}{s_W}^2 s_Z - (\frac{2}{3}\kappa + \frac{1}{3}\frac{\tau}{\tan\beta}z_N)c_Z$	$(1 - \frac{2}{3}{s_W}^2)s_Z + \frac{1}{3}(\kappa - \frac{\tau}{\tan\beta}z_N)c_Z$

The model considered here is defined in Sec. II. The SM particle spectrum is extended with the Z' boson, a new scalar s, and three right-handed neutrinos $N_i, i = 1, 2, 3$. The mass of the new scalar M_s has to be larger than half the mass of the Higgs boson M_h; otherwise, the decay width of the Higgs particle becomes too large as compared to the experimental upper limit $3.2^{+2.8}_{-2.2}$ MeV [45]. This means that a light Z' ($M_{Z'} < M_Z$) can only decay into fermion pairs,

$$\Gamma(Z' \to f + \bar{f}) = N_C \rho C_{ff} M_{Z'} (v^2_{Z',f} + a^2_{Z',f}), \tag{C2}$$

where ρ is defined in Eq. (16), $C_{ff} = \frac{G_F M_Z^2}{6\sqrt{2}\pi} \simeq 3.6383 \times 10^{-3}$, and the vector and axial-vector couplings are given in Sec. II D. Equation (C2) is valid for Dirac fermions. The formula for Majorana neutrinos can be obtained by the replacement $C_{ff} \to \frac{1}{2} C_{ff}$. The invisible branching fraction of a light Z' boson is

$$\mathrm{Br}(Z' \to \mathrm{inv}) = \frac{3n_N}{3n_N + 3(1 - 2c_W^2 \mathscr{Z})^2 n_\ell + (1 + 2c_W^2 \mathscr{Z})^2 n_d + (1 - 4c_W^2 \mathscr{Z})^2 n_u}, \tag{C3}$$

where n_f counts the kinematically allowed decays into n_f families of fermion type f. The number n_N counts the Dirac-type neutrinos. The same formula applies for Majorana neutrinos with the replacement $n_N \to n_N/2$. The parameter $\mathscr{Z}$ is defined in Eq. (22) and $c_W = \cos\theta_W$.

The larger $M_{Z'}$, the more decay channels are allowed. For the case of a heavy Z', $M_{Z'} \gg M_h$, we neglect the finite mass effects of the particles Z, W and h in the following decay formulas. However, we keep the full dependence on the unknown M_s, as it is a free parameter of the model. The decays into a pair of charged W bosons are [46]

$$\begin{aligned}
\Gamma(Z' \to W^+ + W^-) &= \varepsilon M_{Z'} \frac{\xi^2 C_{ff}}{4\rho}, \\
\Gamma(Z' \to Z + W^+ + W^-) &= \varepsilon M_{Z'} \frac{7\xi^4 C_{ff}^2}{320\pi} {c_W}^4, \\
\Gamma(Z' \to \gamma + W^+ + W^-) &= \varepsilon M_{Z'} \frac{301\xi^2 C_{ff}^2}{800\pi} {c_W}^2 {s_W}^2,
\end{aligned} \tag{C4}$$

where $\varepsilon = (\xi s_Z)^2$. The decays into scalar particles also include two- and three-body ones as

$$\begin{aligned}
\Gamma(Z' \to Z + S) &= \varepsilon M_{Z'} \frac{\xi^2 C_{ff}}{4\rho^2} |\Gamma_{Z,Z',S_i}|^2 (1 - \zeta_S^2)^3, \\
\Gamma(Z' \to Z + S + S) &= \varepsilon M_{Z'} \frac{3\xi^4 C_{ff}^2}{64\pi\rho^2} |\Gamma_{Z,Z',S_i,S_i}|^2 \Bigg[\sqrt{1 - 4\zeta_S^2}\left(1 + \frac{26}{3}\zeta_S^2 - \frac{62}{3}\zeta_S^4 + 20\zeta_S^6\right) - 4\zeta_S^2(1 - 3\zeta_S^2 + 6\zeta_S^4 - 5\zeta_S^6) \\
&\quad \times \ln\left(\frac{(1 - 2\zeta_S^2)(\sqrt{1 - 4\zeta_S^2} + 1 - 2\zeta_S^2)}{2\zeta_S^4} - 1\right)\Bigg], \\
\Gamma(Z' \to Z + s + h) &= \varepsilon M_{Z'} \frac{3\xi^4 C_{ff}^2}{32\pi\rho^2} \left(s_S c_S + \frac{s_S c_S}{\tan\beta^2}\right)^2 \left[1 + \frac{1}{3}\zeta_S^2 (10 + 12\ln(\zeta_S^2) - 18\zeta_S^2 + 6\zeta_S^4 - \zeta_S^6)\right],
\end{aligned} \tag{C5}$$

where $S = h, s$, $\zeta_S = M_S/M_{Z'}$. For a heavy Z', $M_{Z'} \gtrsim 1$ TeV, we neglected $\zeta_h \lesssim \mathcal{O}(10^{-1})$. The triple and quartic vector-scalar vertices are

$$\begin{aligned}
\Gamma_{Z,Z',h} &= c_S + s_S/\tan\beta, \qquad \Gamma_{Z,Z',s} = s_S - c_S/\tan\beta, \\
\Gamma_{Z,Z',h,h} &= {c_S}^2 - (s_S/\tan\beta)^2, \qquad \Gamma_{Z,Z',s,s} = {s_S}^2 - (c_S/\tan\beta)^2.
\end{aligned} \tag{C6}$$

The largest contribution from the scalar sector to the Z' decay width is obtained by setting $\zeta_S = 0$. In that case, we find the sum of partial decay widths with a scalar in the final state independent of the scalar mixing angle as

$$\sum \Gamma(Z' \to Z + \mathrm{scalar}) \lesssim \varepsilon M_{Z'} \frac{1}{4\rho^2} \left[\xi^2 C_{ff} (1 + \tan^{-2}\beta) + \frac{3}{16\pi} \xi^4 C_{ff}^2 (1 + \tan^{-4}\beta)\right]. \tag{C7}$$

We use this upper limit (C7) in our numerical calculation to take into account the effect of the scalar sector in the total decay width of Z'. Consulting Eq. (16), one can recognize that $\varepsilon \simeq \rho - 1$ is a small parameter for a heavy Z'.

We computed the fermionic branching fractions of a heavy Z' boson using the V-A couplings obtained in Sec. II D and obtained

$$\mathrm{Br}(Z'\to \ell^+\ell^-) = \frac{2-6\mathscr{Z}+5\mathscr{Z}^2}{16-32\mathscr{Z}+\mathscr{Z}^2(41+C_{w,s}\xi^2)},$$
$$\mathrm{Br}(Z'\to U\bar{U}) = \frac{2-10\mathscr{Z}+17\mathscr{Z}^2}{48-96\mathscr{Z}+3\mathscr{Z}^2(41+C_{w,s}\xi^2)},$$
$$\mathrm{Br}(Z'\to D\bar{D}) = \frac{2+2\mathscr{Z}+5\mathscr{Z}^2}{48-96\mathscr{Z}+3\mathscr{Z}^2(41+C_{w,s}\xi^2)}, \qquad \text{(C8)}$$

where the coupling constant is

$$C_{w,s} = C_{ff}\frac{15+7\mathrm{c}_W^4}{160\pi} \simeq 1.4\times 10^{-4}. \qquad \text{(C9)}$$

We note that for $M_{Z'} > 14$ TeV the decays into scalars and W bosons start to dominate and the fermionic branching fractions may decrease significantly depending on the charge assignment encoded in $\mathscr{Z}$.

APPENDIX D: KINETIC MIXING FOR $U(1)$ EXTENSIONS

Low-energy experiments, such as NA64, *BABAR*, and FASER place a stringent constraints on models which are extended by a $U(1)$ gauge group introducing a new gauge boson coupled to the SM fermions, which can be interpreted as a dark photon A' that has kinetic mixing with the known photon. In the dark photon model, the interaction term involving the A' coupled to the electromagnetic current J^μ_{EM} can be written as

$$\mathcal{L}_{\mathrm{int}} = -\epsilon e A'_\mu J^\mu_{\mathrm{EM}}, \qquad \text{(D1)}$$

where ϵ can be viewed as the kinetic mixing parameter. The experimental exclusion bounds are placed on the parameter plane $(\epsilon, M_{A'})$. To extend those constraints to the parameters of a more general $U(1)$ extension, one may set $M_{A'} \equiv M_{Z'}$, but relating ϵ to the free parameters discussed Sec. IV A. involves some subtlety, discussed below.

The NA64 and *BABAR* experiments search for dark photon brehmsstrahlung in the invisible decay channel of the A' [23]. Following Ref. [31] (see also Ref. [16]), we have

$$\frac{\sigma(e^-+Z\longrightarrow e^-+Z+A')}{\sigma(e^-+Z\longrightarrow e^-+Z+Z')} = \frac{\mathrm{Br}(Z'\to \mathrm{inv})}{\mathrm{Br}(A'\to \mathrm{inv})}, \qquad \text{(D2)}$$

where $\mathrm{Br}(Z'\to\mathrm{inv})$ is given in Eq. (C3) and $\mathrm{Br}(A'\to\mathrm{inv}) = 1$ in the NA64 experiment. Computing the cross sections of the A' and Z' bremsstrahlung processes yields

$$\frac{\sigma(e+Z\longrightarrow e+Z+A')}{\sigma(e+Z\longrightarrow e+Z+Z')} = \epsilon^2\frac{(4\mathrm{s}_W^2\mathrm{c}_W^2)}{v_{Z',\ell}^2 + a_{Z',\ell}^2(1+f(M_{Z'}))} + \mathcal{O}\left(\frac{m_e^2}{s}\right), \qquad \text{(D3)}$$

where $v_{Z',\ell}$ and $a_{Z',\ell}$ are given in Table II, $\sqrt{s} = 100$ GeV in the NA64 experiment, and

$$f(M) = 2\left(\frac{m_e^2}{M^2}\right)\left(1-\log^{-1}\left(\frac{M^2}{4s}\right)\right) + \mathcal{O}\left(\frac{m_e^4}{M^4}\right) \qquad \text{(D4)}$$

collects the finite mass effects of the electron and the Z' boson. Assembling the pieces for Eq. (D2) gives the matching relation

$$\epsilon = \frac{\sqrt{v_{Z',\ell}^2 + a_{Z',\ell}^2(1+f(M_{Z'}))}}{2\mathrm{s}_W\mathrm{c}_W}\sqrt{\mathrm{Br}(Z'\to\mathrm{inv})}. \qquad \text{(D5)}$$

The axial-vector couplings can be neglected for a light Z' boson, and hence the matching relation reduces to

$$\epsilon = \frac{|\mathrm{s}_Z|}{2\mathrm{s}_W\mathrm{c}_W}\left|2\mathrm{c}_W{}^2 - \frac{1}{\mathscr{Z}}\right|\sqrt{\mathrm{Br}(Z'\to\mathrm{inv})}, \qquad \text{(D6)}$$

or in terms of g_z, it is given as

$$\epsilon = \frac{|z_N g_z|}{e}|2\mathrm{c}_W{}^2\mathscr{Z} - 1|\sqrt{\mathrm{Br}(Z'\to\mathrm{inv})}, \qquad \text{(D7)}$$

where $\mathrm{Br}(Z'\to\mathrm{inv})$ is given in (C3). This formula reproduces the corresponding one given in Ref. [31] for $M_{Z'} < m_\pi = 130$ MeV and $\mathscr{Z} = 2$ (with $z_N = 1/2$ and $z_\phi = 1$), which is the superweak extension of the SM.

In the FASER experiment, predominantly a light neutral meson ($m^0 = \pi^0$, η^0, or ω^0) decays into a neutral gauge boson pair, which may include the $m^0 \to \gamma + A'$ production channel, where the dark photon is assumed to subsequently decay into an electron-positron pair. The partial rate of the neutral pion into a photon and dark photon is then given as

$$\Gamma(\pi^0\to A'+\gamma) = 2\left(1-\frac{M_{A'}^2}{m_\pi^2}\right)^3(2\,\mathrm{tr(gens)})^2\Gamma(\pi^0\to\gamma+\gamma), \qquad \text{(D8)}$$

where the first factor of 2 is due to the symmetry factor difference between the $A'+\gamma$ and $\gamma+\gamma$ final states and the factor containing $M_{A'}$ is due to the polarization sum of a massive vector boson times the phase-space factor. The trace of the generators is

$$2\mathrm{tr(gens)} = 2N_C\mathrm{tr}(\tau^a QQ)\epsilon = \epsilon, \qquad \text{(D9)}$$

with the matrices

$$\tau^a = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}, \quad \text{and} \quad Q = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}. \tag{D10}$$

The trace of the generators in $U(1)$ extensions considered in this paper is

$$2\mathrm{tr}(\mathrm{gens}) = 2N_C \mathrm{tr}(\tau^a Q \mathbf{v}_{Z',q}) = \left| \frac{v_{Z',\ell}}{2\mathrm{s}_W \mathrm{c}_W} \right|, \tag{D11}$$

where

$$\mathbf{v}_{Z',q} = \begin{pmatrix} v_{Z',u} & 0 \\ 0 & v_{Z',d} \end{pmatrix} = \frac{\mathrm{s}_Z}{3} \begin{pmatrix} \frac{1}{\mathscr{X}} - 4\mathrm{c}_W^2 & 0 \\ 0 & \frac{1}{\mathscr{X}} + 2\mathrm{c}_W^2 \end{pmatrix}. \tag{D12}$$

To match the exclusion bounds of FASER, one has to solve two equations for the two parameters $(M_{A'}, \epsilon)$. The first one expresses the equality of the signal events in the two models,

$$\Gamma(\pi^0 \to A' + \gamma)\mathrm{Br}(A' \to e^+e^-) = \Gamma(\pi^0 \to Z' + \gamma)\mathrm{Br}(Z' \to e^+e^-), \tag{D13}$$

where $\mathrm{Br}(A' \to e^+e^-) = 1$ in the FASER experiment. The second equation,

$$M_{A'}\Gamma_{A'} = M_{Z'}\Gamma_{Z'}, \tag{D14}$$

ensures that the decay length of the dark photon A' and that of the Z' boson are the same as those are required to decay in the detector itself. For $M_{A'} \ll m_\pi$, the matching relations yield

$$M_{Z'} = \mathrm{Br}(Z' \to e^+e^-)M_{A'}, \qquad |z_N g_z| = \frac{e\epsilon}{\sqrt{\mathrm{Br}(Z' \to e^+e^-)}} \frac{1}{|1 - 2\mathrm{c}_W^2 \mathscr{X}|}. \tag{D15}$$

In the $B - L$ extension considered in Ref. [25], one has $\mathscr{X} = 0$, $\mathrm{Br}(Z' \to e^+e^-) = 2/5$ and $z_N = 1$ as they use a different normalization for the z charges.

The Belle II Collaboration searched for Z' bosons in the process

$$e^+e^- \to \mu^+\mu^- + Z', \tag{D16}$$

where the Z' subsequently decays into invisible particles [29] at center-of-mass energies $\sqrt{s} = 10.58$ GeV. The collaboration investigates the $L_\mu - L_\tau$ model, where the Z' boson does not couple to the first generation of charged leptons and places exclusion limits on the cross section times branching fraction $\sigma_{\mathrm{exp}} = \sigma(e^+e^- \to \mu^+\mu^- + Z') \times \mathrm{Br}(Z' \to \mathrm{inv})$ as a function of $M_{Z'}$ for $M_{Z'} < 8.5$ GeV (cf. Fig. 2 of Ref. [29]). To incorporate the Belle2 results into our bounds on s_Z and g_z, we compute the cross section of the process (D16) in the $U(1)$ extensions considered here. For these extensions—as opposed to the $L_\mu - L_\tau$ model—the Z' can also be radiated off the initial-state particles. We take into account four Feynman graphs, which correspond to the process $e^+e^- \to \gamma \to \mu^+\mu^-$ combined with the initial- and final-state radiation of a Z' boson from each fermion leg, denoted here as

$$e^+e^- \to \gamma \to \mu^+\mu^- + Z'. \tag{D17}$$

We neglect the process $e^+e^- \to Z/Z' \to \mu^+\mu^- + Z'$ as these give negligibly small contribution to the total cross section compared to the process (D17). The Z' mediation is suppressed by an extra factor of $\mathrm{s}_Z{}^4$ (or $|z_N g_z|^4$), and since $\sqrt{s} = 10.58$ GeV with $M_{Z'} < 8.5$ GeV, it does not receive an enhancement from the resonance peak near $M_{Z'} \simeq \sqrt{s}$. Consulting Sec. II D, one concludes that $a_{Z',\ell}$ are negligible for $M_{Z'} < 8.5$ GeV, in which case we obtain

$$\sigma(e^+e^- \to \gamma \to \mu^+\mu^- + Z') = v_{Z',\ell}^2 \times \sigma_{\mathrm{theo}}, \tag{D18}$$

where σ_{theo}. depends only on M_Z'. Then, the matching relation is simply

$$|v_{Z',\ell}| = \sqrt{\frac{\sigma_{\mathrm{exp}}}{\sigma_{\mathrm{theo}}}} \frac{1}{\sqrt{\mathrm{Br}(Z' \to \mathrm{inv})}}. \tag{D19}$$

[1] R. L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
[2] G. Aad et al. (ATLAS Collaboration), Report No. ATL-PHYS-PUB-2022-009.
[3] https://cms-results.web.cern.ch/cms-results/public-results/publications/SMP
[4] D. P. Aguillard et al. (Muon $g-2$ Collaboration), Phys. Rev. Lett. **131**, 161802 (2023).
[5] T. Aoyama et al., Phys. Rep. **887**, 1 (2020).
[6] M. Pospelov, Phys. Rev. D **80**, 095002 (2009).
[7] H. Davoudiasl, H.-S. Lee, and W. J. Marciano, Phys. Rev. D **86**, 095009 (2012).

[8] M. Cadeddu, N. Cargioli, F. Dordei, C. Giunti, and E. Picciau, Phys. Rev. D **104**, 011701 (2021).
[9] S. Borsanyi *et al.*, Nature (London) **593**, 51 (2021).
[10] L. B. Okun, Sov. Phys. JETP **56**, 502 (1982), http://jetp.ras.ru/cgi-bin/e/index/e/56/3/p502?a=list.
[11] X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. D **43**, 22 (1991).
[12] S. Gopalakrishna, S. Jung, and J. D. Wells, Phys. Rev. D **78**, 055002 (2008).
[13] After diagonalization of the mass matrix of the neutral gauge bosons.
[14] Z. Trócsányi, Symmetry **12**, 107 (2020).
[15] S. Dittmaier, J. Rehberg, and H. Rzehak, J. High Energy Phys. 01 (2024) 037.
[16] P. Ilten, Y. Soreq, M. Williams, and W. Xue, J. High Energy Phys. 06 (2018) 004.
[17] A. Abada *et al.* (FCC Collaboration), Eur. Phys. J. Special Topics **228**, 1109 (2019).
[18] J. L. Feng, I. Galon, F. Kling, and S. Trojanowski, Phys. Rev. D **97**, 035001 (2018).
[19] The cases of an intermediate mass Z', $M_{Z'} \approx M_Z$, and a very light Z', $M_{Z'} \lesssim 20$ MeV have been discussed elsewhere.
[20] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. B **796**, 68 (2019).
[21] A. M. Sirunyan *et al.* (CMS Collaboration), J. High Energy Phys. 07 (2021) 208.
[22] A. Crivellin, M. Hoferichter, M. Kirk, C. A. Manzari, and L. Schnell, J. High Energy Phys. 10 (2021) 221.
[23] D. Banerjee *et al.* (NA64 Collaboration), Phys. Rev. Lett. **123**, 121801 (2019).
[24] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **119**, 131804 (2017).
[25] H. Abreu *et al.* (FASER Collaboration), Phys. Lett. B **848**, 138378 (2024).
[26] K. Asai, A. Das, J. Li, T. Nomura, and O. Seto, Phys. Rev. D **106**, 095033 (2022).
[27] K. Asai, A. Das, J. Li, T. Nomura, and O. Seto, Phys. Rev. D **109**, 075026 (2024).
[28] R. Enberg, Y. Hiçyılmaz, S. Moretti, C. Pérez de los Heros, and H. Waltari, arXiv:2404.04717.
[29] I. Adachi *et al.* (Belle-II Collaboration), Phys. Rev. Lett. **130**, 231801 (2023).
[30] F. Abudinén *et al.* (Belle-II Collaboration), Phys. Rev. Lett. **130**, 071804 (2023).
[31] S. Iwamoto, K. Seller, and Z. Trócsányi, J. Cosmol. Astropart. Phys. 01 (2022) 035.
[32] C. A. Manzari, J. Martin Camalich, J. Spinner, and R. Ziegler, Phys. Rev. D **108**, 103020 (2023).
[33] This is the convention used in SARAH for models with multiple gauged U(1) symmetries.
[34] T. Appelquist, B. A. Dobrescu, and A. R. Hopper, Phys. Rev. D **68**, 035012 (2003).
[35] Our sign convention for θ_Z agrees with the convention of Ref. [14], which differs by a factor of (-1) from that of Ref. [31].
[36] Z. Péli and Z. Trócsányi, Phys. Rev. D **108**, L031704 (2023).
[37] M. P. Bento, H. E. Haber, and J. P. Silva, Phys. Lett. B **850**, 138501 (2024).
[38] M. Carena, A. Daleo, B. A. Dobrescu, and T. M. P. Tait, Phys. Rev. D **70**, 093009 (2004).
[39] G. Altarelli, R. K. Ellis, and G. Martinelli, Nucl. Phys. **B157**, 461 (1979).
[40] M. Mangano, CERN Yellow Report 3/2017, 10.23731/CYRM-2017-003 (2017).
[41] P. Athron, M. Bach, D. H. J. Jacob, W. Kotlarski, D. Stöckinger, and A. Voigt, Phys. Rev. D **106**, 095023 (2022).
[42] Z. Péli, Acta Phys. Pol. B Proc. Suppl. **17**, 2-A16 (2024).
[43] C. Helsens, D. Jamin, and M. Selvaggi, Report No. CERN-ACC-2019-028, https://cds.cern.ch/record/2642473/files/.
[44] S. Iwamoto, T. J. Kärkkäinen, Z. Péli, and Z. Trócsányi, Phys. Rev. D **104**, 055042 (2021).
[45] A. M. Sirunyan *et al.* (CMS Collaboration), Phys. Rev. D **99**, 112003 (2019).
[46] P. Langacker, Rev. Mod. Phys. **81**, 1199 (2009).