Stau pairs from natural SUSY at high luminosity LHC

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Natural supersymmetry (SUSY) with light Higgsinos is perhaps the most plausible of all weak scale SUSY models while a variety of motivations point to (right) tau sleptons as the lightest of all the sleptons. We examine a SUSY model line with rather light right staus embedded within natural SUSY. For light $\tilde{\tau}_1$ of a few hundred GeV, the decays $\tilde{\tau}_1 \to \tau\tilde{\chi}^0_{1,2}$ and $\nu_\tau\tilde{\chi}^-_1$ occur at comparable rates where the (Higgsino-like) $\tilde{\chi}^\pm_1$ and $\tilde{\chi}^0_2$ release only small visible energy: in this case, the expected $\tau^+\tau^- + \not{E}_T$ signature is diminished from the usual expectations due to the presence of the nearly invisible decay mode $\tilde{\tau}_1 \to \nu_\tau\tilde{\chi}^-_1$. However, once $m_{\tilde{\tau}_1} \gtrsim m(bino)$, decays to binos such as $\tilde{\tau}_1 \to \tau\tilde{\chi}^0_3$ open up where $\tilde{\chi}^0_3$ decays to Higgsinos plus $W^\pm$, Z^0, and h at comparable rates. For these heavier staus, the stau pair production gives rise to diboson $+ \not{E}_T$ events, which may contain 0, 1, or 2 additional hard τ leptons. From these considerations, we examine the potential for future discovery of tau-slepton pair production at a high-luminosity LHC. While we do not find a 5σ HL-LHC discovery reach for 3000 fb^{-1}, we do find a 95% CL exclusion reach, ranging between $m_{\tilde{\tau}_1}$: 100–450 GeV for $m_{\tilde{\chi}^0_1} \sim 100$ GeV. This latter reach disappears for $m_{\tilde{\chi}^0_1} \gtrsim 200$ GeV.

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I. INTRODUCTION

The search for weak scale supersymmetry (SUSY) [1] at hadron collider experiments often focusses on strongly interacting sparticles—the gluinos and squarks—since these have the largest production cross section for a given sparticle mass value. However, in many models, the gluinos and squarks also have the largest mass values, while sleptons and electroweakinos (EWinos) are much lighter. Thus, for a given point in model parameter space, sleptons and EWino pair production may dominate the production cross sections by virtue of their smaller mass values.

In a previous work [2], we examined prospects for EWino pair production at luminosity upgrades of the CERN Large Hadron Collider (LHC) in the context of natural SUSY, which is characterized by low values of an *electroweak* fine-tuning measure $\Delta_{\rm EW} \lesssim 30$ [3,4]. The value of $\Delta_{\rm EW}$ is a measure of practical naturalness [5]: that all independent contributions to an observable should be comparable to or less than its measured value. For the case of the minimal supersymmetric standard model, or MSSM, the weak scale (as typified by the Z-boson mass) is related to weak scale SUSY Lagrangian parameters as

$$\begin{aligned} m_Z^2/2 &= \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \\ &\simeq -m_{H_u}^2 - \mu^2 - \Sigma_u^u(\tilde{t}_{1,2}), \end{aligned} \tag{1}$$

where $m_{H_{u,d}}^2$ are soft breaking squared masses of the Higgs doublets, $\tan\beta = v_u/v_d$ is the ratio of Higgs field vevs, μ is the SUSY conserving μ parameter,[1] and the $\Sigma_{u,d}^{u,d}$ terms contain over 40 1-loop and some 2-loop corrections to the scalar potential (explicit expressions are given in Refs. [4,7]). The measure $\Delta_{\rm EW}$ is defined as

$$\Delta_{\rm EW} \equiv |\text{largest term on rhs of Eq.}(1)|/(m_Z^2/2), \tag{2}$$

so that no large unnatural fine-tunings are allowed in the derivation of m_Z. Computational evaluations of $\Delta_{\rm EW}$ are available in ISAJET [8] and DEW4SLHA [7] and include over 40 1-loop corrections in the $\Sigma_{u,d}^{u,d}$ terms [4,7] along with some 2-loop contributions from Dedes and Slavich [9]. A large negative value of A_t, which enters the expressions for

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[1]Twenty solutions to the SUSY μ problem are reviewed in Ref. [6].

$\Sigma_u^u(\tilde{t}_{1,2})$, leads to large cancellations (more naturalness) in both these terms [3] whilst lifting $m_h \to 125$ GeV [10].

Natural SUSY (natSUSY) models are considered more plausible than unnatural models in that they contain no implausible accidental tunings of unrelated terms in Eq. (1). Furthermore, it is now understood that natSUSY is the most likely expression of weak scale SUSY that ought to emerge from the string landscape [11–13]. This arises since low $\Delta_{\rm EW} \lesssim 30$ corresponds well with the anthropic Agrawal-Barr-Donoghue-Seckel (ABDS) [14,15] window of allowed weak scale values which give rise to complexity in the multiverse (atomic principle). For the case of fine-tuned models, then the available multiverse scan space shrinks to tiny volumes compared to natural models due to the fine-tuning, which is required.

In the present work, we examine prospects for slepton (specifically, the lightest τ-slepton $\tilde{\tau}_1$) pair production at the high-luminosity upgrade of LHC (HL-LHC) in a natural SUSY context. We focus on lightest right-tau-sleptons for several reasons.

In models with high scale slepton universality (such as mSUGRA [16–18]/CMSSM [19] or NUHM2, NUHM3 or NUHM4 [20,21] models), the stau soft mass RGEs are given by

$$\frac{dm_{L_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 - \frac{3}{10}g_1^2S + f_\tau^2X_\tau\right), \qquad (3)$$

$$\frac{dm_{E_3}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{12}{5}g_1^2M_1^2 + \frac{3}{5}g_1^2S + 2f_\tau^2X_\tau\right), \qquad (4)$$

where $m_{L_3}^2$ is the third generation doublet slepton soft mass squared (giving rise to left staus) and $m_{E_3}^2$ is the corresponding $SU(2)_L$ singlet slepton mass squared (giving rise to right staus). Also, $S = m_{H_u}^2 - m_{H_d}^2 + {\rm Tr}[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2]$ and $X_\tau = m_{L_3}^2 + m_{E_3}^2 + m_{H_d}^2 + A_\tau^2$ and $t = \log(Q)$. When running from high scales (e.g., $Q = m_{\rm GUT}$) to $Q = m_{\rm weak}$, the $SU(2)_L$ gauge term in Eq. (3) drives $m_{L_3}^2$ to larger values than $m_{E_3}^2$ at the weak scale while the rather large τ-Yukawa coupling term containing $2f_\tau^2$ in Eq. (4) drives the right-stau soft mass squared $m_{E_3}^2$ to smaller values than $m_{L_3}^2$. For the natSUSY models considered here, usually $S > 0$ so this term also drives right sleptons to smaller masses than left sleptons at the weak scale. Thus, in models with intra-generation universality of scalar masses [which are motivated by $SO(10)$ where all elements of each generation live in a single $\mathbf{16} - d$ spinor rep], we expect that right-stau masses are smaller than left-stau masses.[2]

[2] Light tau sleptons also arise in supersymmetric twin-Higgs models: see, e.g., Refs. [22–24].

Also, on the theory side, the string landscape pulls soft breaking terms as large as possible until they overcontribute beyond the ABDS window to the weak scale. This effect tends to pull first/second generation sfermion masses to the tens-of-TeV values whilst third generation sfermions, which contribute proportional to their Yukawa couplings squared, only get pulled up to values of several TeV [25] at the high scale.

Furthermore, in orbifold compactifications on the mini-landscape [26], first/second generation sfermions live near orbifold fixed points and "feel much less supersymmetry than third generation fields"[27], which instead live more in the bulk where they have large overlap with Higgs multiplets. Thus, third generation soft terms are more protected by SUSY and hence gain smaller soft masses than their first/second generation counterparts.

On the phenomenology side, light sleptons are preferred by the $(g-2)_\mu$ anomaly [28], and of all the sleptons, the right staus are expected to be lightest. Also, light tau sleptons with mass $m_{\tilde{\tau}_1} \sim m_{\tilde{\chi}_1^0}$ are required to *thermally* match the measured dark matter relic density in the so-called stau coannihilation region of SUSY model parameter space [29–31].

For these reasons, we examine prospects for detecting the lightest (right) tau sleptons, but within the context of natSUSY models. To this end, in Sec. II, we develop a natural SUSY model line with low $\Delta_{\rm EW}$ but with a variable right-stau soft mass. In Sec. III, we present stau pair production cross sections, which are expected at LHC14, and in Sec. IV, we compute expected $\tilde{\tau}_1$ branching fractions along our model line. Since we work within a natural SUSY context, Higgsinos are expected with mass $\sim\mu \sim 100$–350 GeV [32,33]. The presence of light Higgsinos is expected to diminish the LHC reach for light staus compared to usual simplified models in that in the natSUSY case, a substantial branching fraction $\tilde{\tau}_1 \to \nu_\tau \tilde{\chi}_1^-$ where the $\tilde{\chi}_1^\pm \to f\bar{f}'\tilde{\chi}_1^0$ and the small $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}$ mass gap leads to very low energy visible decay products. However, in the case where $m_{\tilde{\tau}_1} > m(bino) \gg m(Higgsino)$, then the decay to binos rapidly dominates the stau decay rate leading to possibly new discovery signatures: diboson $+ \tau\bar{\tau} + \not{E}_T$. In Sec. V, we evaluate the projected HL-LHC reach for stau pairs in natSUSY in the $m_{\tilde{\tau}_1}$ vs $m_{\tilde{\chi}_1^0}$ plane. While we do not find a 5σ discovery reach for HL-LHC, we do find a 95% CL exclusion reach that extends from $m_{\tilde{\tau}_1} \sim 200$–450 GeV for $m_{\tilde{\chi}_1^0} \sim 100$ GeV. We conclude in Sec. VI.

A. Brief review of some previous works

Many early works were focused on stau pair production in the stau coannihilation region of models like mSUGRA/CMSSM [18,19] with $\mu \gg m_Z$ and with a bino-like lightest supersymmetric particle [34–40].[3] Such models are nowadays regarded as unnatural under $\Delta_{\rm EW}$ and hence, rather implausible [43–45] as a realization of weak scale SUSY.

In contrast, natural SUSY models with $\mu \sim m_Z$ contain three light Higgsinos $\tilde{\chi}^0_{1,2}$ and $\tilde{\chi}^\pm_1$. Since Higgsinos annihilate and coannihilate at high rates in the early Universe [46], they have no dark matter overproduction problem and hence, no need for tuning the relic abundance into the stau coannihilation region, and there is no reason to expect a situation with long-lived light staus. Instead, any light staus are expected to decay promptly to the three light Higgsinos along with tau leptons or tau neutrinos (see upcoming Fig. 3 for branching fractions).

In Ref. [47], the ATLAS Collaboration reported on a search for stau pair production using run 2 data with 139 fb^{-1} at $\sqrt{s} = 13$ TeV. The final state signature searched for was $\tau_h\tau_h + \not{E}_T$, where τ_h is a hadronically decaying τ jet. No signal was seen above SM backgrounds, and limits were placed in the $m_{\tilde{\tau}}$ vs $m_{\tilde{\chi}^0_1}$ plane assuming

(1) degenerate left and right tau sleptons and
(2) just pair production of left tau sleptons.

In both cases, a simplified model with the decay $\tilde{\tau}_1 \to \tau\tilde{\chi}^0_1$ at 100% branching fraction was assumed. In the former case, with $m_{\tilde{\chi}^0_1} \sim 100$ GeV, then $m_{\tilde{\tau}_{L/R}}$: 230 GeV–350 GeV was excluded while for the second case no limit ensues for $m_{\tilde{\chi}^0_1} = 100$ GeV, although $m_{\tilde{\tau}_L}$: 150 GeV–320 GeV can be excluded for $m_{\tilde{\chi}^0_1} = 0$.

A similar search was reported on by CMS using run 2 data with 138 fb^{-1} in Ref. [48]. No excess was seen in the $\tau_h\tau_h + \not{E}_T$ signal channel above SM background leading CMS in case 1 to exclude $m_{\tilde{\tau}_{L/R}}$: 200 GeV–380 GeV while in case 2 $m_{\tilde{\tau}_L} \sim 280$ GeV could be excluded for $m_{\tilde{\chi}^0_1} = 100$ GeV while $m_{\tilde{\tau}_L}$: 120 GeV–350 GeV could be excluded for $m_{\tilde{\chi}^0_1} = 0$.

The ATLAS Collaboration also performed a HL-LHC reach study in 2016 [49] for stau pair production with $\sqrt{s} = 14$ TeV assuming 3000 fb^{-1} of integrated luminosity. This study examined the reach for case 1 and 2 as above but also included reach results for just $\tilde{\tau}_R\bar{\tilde{\tau}}_R$ pair production (case 3). For $m_{\tilde{\chi}^0_1} = 100$ GeV, they report a 95% CL exclusion reach in $m_{\tilde{\tau}}$ up to 540 GeV in case 3, 650 GeV in case 2, and 700 GeV in case 1. There was no 5σ discovery reach for any value of $m_{\tilde{\tau}}$ in case 3. An updated 2018 study by ATLAS was presented in Ref. [50]. A similar study was performed by CMS in 2019 [51] where a 95% CL exclusion reach for HL-LHC in $m_{\tilde{\tau}}$ up to 640 GeV was reported for case 1 with $m_{\tilde{\chi}^0_1} = 100$ GeV.

[3]Production of lighter stau pairs from heavy Higgs decay has been considered in Refs. [41] and [42].

II. A NATURAL SUSY MODEL LINE WITH LIGHT RIGHT STAUS

We would like to embed light tau sleptons within a natural SUSY model framework since it can be argued that SUSY models with low electroweak fine-tuning (with $\Delta_{\rm EW} \lesssim 30$) are the most plausible of SUSY models in that the weak scale $m_{\rm weak} \simeq m_{W,Z,h}$ is of order $\sim$100 GeV because all MSSM contributions (some positive, some negative) in Eq. (1) are comparable (within a factor of several) to the $m_{\rm weak}$ scale. Such models can be found, for instance, within the framework of nonuniversal Higgs models [20,21,52]. Here, we will work within the NUHM4 model[4] with parameters,

$$m_0(i), m_{1/2}, A_0, \tan\beta, m_{H_u}, m_{H_d} \quad (\text{NUHM4}') \tag{5}$$

[where $m_0(i)$ refers to separate soft masses $m_0(1,2,3)$ for each generation, as is expected from general supergravity models where no known symmetry enforces generational mass universality [53–55]] and where it is common to trade the high scale soft terms $m^2_{H_u}$ and $m^2_{H_d}$ for the more convenient weak scale parameters μ and m_A,

$$m_0(i), m_{1/2}, A_0, \tan\beta, \mu, m_A \quad (\text{NUHM4}), \tag{6}$$

and where $i = 1$–3 is a generation index. In NUHM models, the required $\mu \sim 100$–350 GeV parameter can be dialed to fulfill one of the requirements of low $\Delta_{\rm EW}$ in Eq. (1). Also, a large negative A_0 parameter lifts $m_h \to 125$ GeV [10,56] while reducing the top-squark loop corrections $\Sigma^u_u(\tilde{t}_{1,2})$ to Eq. (1) [3,4]. This latter effect reconciles natural SUSY with the rather large measured value of m_h and with $m_{\tilde{t}} \sim 1$–3 TeV (beyond present LHC top-squark mass bounds). For simplicity, we will take $m_0(1) = m_0(2) = m_0(3)$ since we are not concerned with the effects of the first two generations of sfermion masses. In the string landscape, then $m_0(1) \sim m_0(2) \gg m_0(3)$ leading to a decoupling/quasidegeneracy solution to the SUSY flavor and CP problems [57].

The benchmark point shown in Table I thus takes as parameter choices,

$$m_0(i) = 5\,\text{TeV}, \quad m_{1/2} = 1.2\,\text{TeV}, \quad A_0 = -1.6 m_0, \quad \tan\beta = 10 \ \text{with} \ \mu = 250\,\text{GeV} \ \text{and} \ m_A = 2\,\text{TeV}. \tag{7}$$

It yields $\Delta_{\rm EW} \sim 26$ with $m_h \simeq 125$ GeV whilst all sparticle masses are beyond present LHC bounds. The lightest neutralino $\tilde{\chi}^0_1$ is Higgsino-like with a thermally produced

[4]Four extra parameters nonuniversal Higgs model, where the four extra parameters beyond CMSSM include $m_0(2)$, $m_0(3)$, m_{H_u} and m_{H_d}.

TABLE I. Input parameters (TeV) and masses (GeV) for the light stau natural SUSY benchmark point from the NUHM2 + E3 model with $m_t = 173.2$ GeV using ISAJET7.91 [8].

Parameter	$\tilde{\tau}_1$BM point
m_0	5 TeV
$m_{1/2}$	1.2 TeV
A_0	−8 TeV
$\tan\beta$	10
m_{E_3}	1.11 TeV
μ	250 GeV
m_A	2 TeV
$m_{\tilde{g}}$	2826 GeV
$m_{\tilde{u}_L}$	5458 GeV
$m_{\tilde{u}_R}$	5484 GeV
$m_{\tilde{e}_R}$	4954 GeV
$m_{\tilde{t}_1}$	1517 GeV
$m_{\tilde{t}_2}$	3947 GeV
$m_{\tilde{b}_1}$	3987 GeV
$m_{\tilde{b}_2}$	5323 GeV
$m_{\tilde{\tau}_1}$	378 GeV
$m_{\tilde{\tau}_2}$	5054 GeV
$m_{\tilde{\nu}_\tau}$	5061 GeV
$m_{\tilde{\chi}_1^\pm}$	261.4 GeV
$m_{\tilde{\chi}_2^\pm}$	1019.0 GeV
$m_{\tilde{\chi}_1^0}$	248.0 GeV
$m_{\tilde{\chi}_2^0}$	259.1 GeV
$m_{\tilde{\chi}_3^0}$	539.3 GeV
$m_{\tilde{\chi}_4^0}$	1034.6 GeV
m_h	125.0 GeV
$\Omega_{\tilde{\chi}_1}^{std} h^2$	0.016
$\mathrm{BR}(b \to s\gamma) \times 10^4$	3.1
$\mathrm{BR}(B_s \to \mu^+\mu^-) \times 10^9$	3.8
$\sigma^{SI}(\tilde{\chi}_1^0, p)$ (pb)	2.2×10^{-9}
$\sigma^{SD}(\tilde{\chi}_1^0, p)$ (pb)	2.9×10^{-5}
$\langle \sigma v \rangle \vert_{v\to 0}$ (cm^3/sec)	1.3×10^{-25}
Δ_{EW}	26.4
θ_τ	89.9°

(TP) relic abundance of $\Omega_{\tilde{\chi}}^{TP} h^2 \sim 0.016$. Since we would also like to be natural in the context of the strong *CP* problem, we invoke SUSY axions in the DFSZ model and expect the bulk of dark matter to be axions along with a smattering of Higgsino-like weakly interacting massive particles [58–60].

To embed light sleptons within natSUSY, we create a model line with variable third generation MSSM soft mass m_{E_3}. Then, dialing m_{E_3} down in value, we can generate light right-slepton masses as shown in Table I, where we take $m_{E_3} = 1.11$ TeV which then generates a light tau slepton with mass $m_{\tilde{\tau}_1} = 378$ GeV. (We use ISAJETv7.91 [8] to generate the SUSY spectrum.) The lightest slepton eigenstate is given by [1] $\tilde{\tau}_1 = \cos\theta_\tau \tilde{\tau}_L - \sin\theta_\tau \tilde{\tau}_R$. The mixing angle $\theta_\tau = 89.9°$ listed in Table I shows that $\tilde{\tau}_1$ is dominantly $\tilde{\tau}_R$.

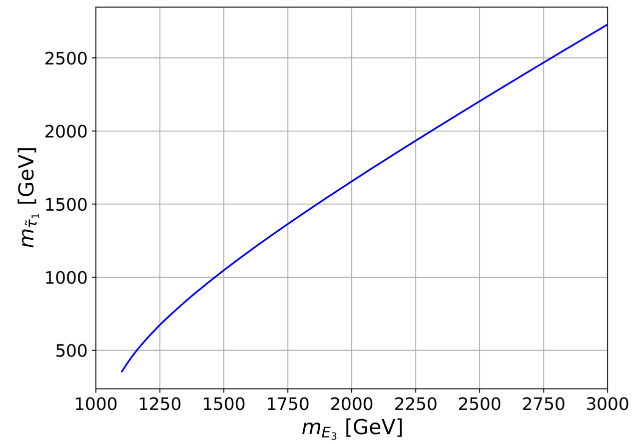


FIG. 1. Plot of $m_{\tilde{\tau}_1}$ vs m_{E_3} along the light stau natural SUSY model line.

A plot of $m_{\tilde{\tau}_1}$ vs. m_{E_3} is shown in Fig. 1 for the benchmark point but with variable m_{E_3}. The curve cuts off below $m_{E_3} \lesssim 1.1$ TeV in that $m_{E_3}^2$ is driven to tachyonic values at an intermediate iteration in the SUSY RGE solution in ISASUGRA [61]. The large S term [defined below Eq. (4)] becomes large positive for nonuniversal Higgs models with $m_{H_u} \gg m_{H_d}$, which then drives $m_{E_3}^2$ tachyonic for small enough GUT scale values of m_{E_3}. ($S = 0$ in models such as CMSSM with universal scalar masses.) We find this same behavior occurs also in SOFTSUSY [62]. Hence, to obtain smaller values of $m_{\tilde{\tau}_1}$, we implement the weak-scale SUSY parameters from the BM point into the pMSSM solution embedded in ISASUSY [63], which does not include RG running and so allows lighter tau sleptons as light as $m_{\tilde{\tau}_1} \simeq m_{\tilde{\chi}_1^0}$.

III. STAU PAIR PRODUCTION AT LHC14

Pair production of light right staus takes place via $q\bar{q} \to \gamma^*, Z^* \to \tilde{\tau}_1 \bar{\tilde{\tau}}_1 X$ at the LHC. (Light left staus can also be produced via $q\bar{q}' \to W^* \to \tilde{\tau}_1 \tilde{\nu}_\tau$.) Next-to-leading order QCD corrections were computed in Ref. [64] and are included in PROSPINO [65], which we use for the total cross section computation. The total cross section in fb for production of tau sleptons at LHC with $\sqrt{s} = 14$ TeV is shown vs $m_{\tilde{\tau}_1}$ in Fig. 2. From the plot, we see that $\tilde{\tau}_1 \bar{\tilde{\tau}}_1$ production occurs at $\sigma > 1$ fb for $m_{\tilde{\tau}_1} \lesssim 400$ GeV. For HL-LHC with an assumed integrated luminosity of 3000 fb^{-1}, we would drop below the 30 total event level for $m_{\tilde{\tau}_1} \gtrsim 850$ GeV level. Thus, we would expect any sensitivity of HL-LHC to tau-slepton pair production to lie in the few hundred GeV region, based solely on total production cross section.

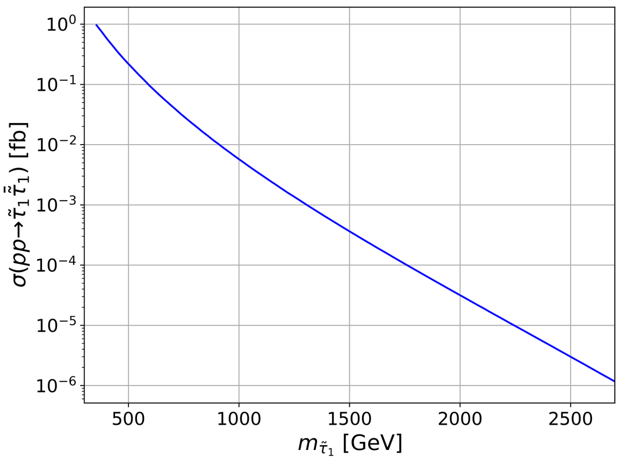


FIG. 2. NLO cross sections (in fb) for $pp \to \tilde{\tau}_1\tilde{\tau}_1^* X$ production at a pp collider with $\sqrt{s} = 14$ TeV versus $m_{\tilde{\tau}_1}$ for the light $\tilde{\tau}_1$ natural SUSY model line of the text.

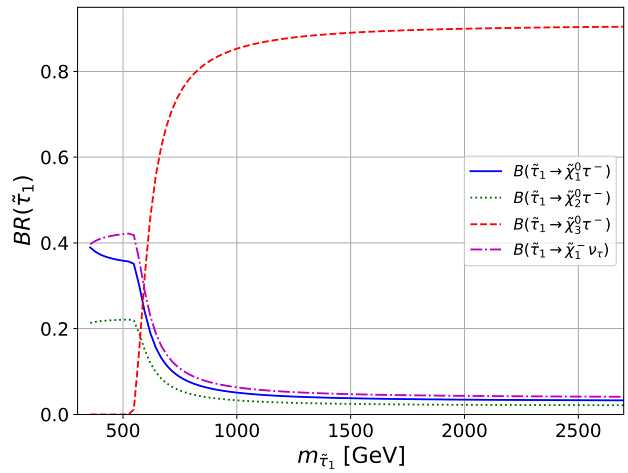


FIG. 3. Branching fractions of $\tilde{\tau}_1$ from natural SUSY versus $m_{\tilde{\tau}_1}$ for the light stau natural SUSY model line.

IV. RIGHT-STAU BRANCHING FRACTIONS IN NATURAL SUSY

In this section, we examine the expected light slepton branching fractions (BFs) within the context of natSUSY. The BFs of $\tilde{\tau}_1$ are computed using ISAJET7.91. In Fig. 3, we plot the dominant $BF(\tilde{\tau}_1)$ vs $m_{\tilde{\tau}_1}$ along our $\tilde{\tau}_1$ natSUSY model line with rather light Higgsinos. From the plot, we see that for $m_{\tilde{\tau}_1} \lesssim 550$ GeV, then the decay $\tilde{\tau}_1 \to \tilde{\chi}_1^- \nu_\tau$ is actually dominant at $\sim 40\%$. Since this decay would be followed by $\tilde{\chi}_1^- \to f\bar{f}'\tilde{\chi}_1^0$, with $m_{\tilde{\chi}_1^+}$ just a few GeV heavier than $m_{\tilde{\chi}_1^0}$, very soft visible energy will ensue, and the decay mode is likely to be hardly visible in the LHC detector environment.

The next largest BF comes from $\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau$ (blue curve), which occurs typically at the $\sim 35\%$ level for $m_{\tilde{\tau}_1} \lesssim 550$ GeV. For large enough $\tilde{\tau}_1 - \tilde{\chi}_1^0$ mass gap, this mode can give rise to visible isolated 1- and 3-prong τ jets. The green curve shows the decay $\tilde{\tau}_1 \to \tilde{\chi}_2^0 \tau$, where the $\tilde{\chi}_2^0$ is also mainly Higgsino-like but now can decay as $\tilde{\chi}_2^0 \to f\bar{f}\tilde{\chi}_1^0$. Again, for small $\tilde{\chi}_2^0 - \tilde{\chi}_1^0$ mass gap, this decay will typically yield only soft visible energy unless the $\tilde{\chi}_2^0$ is somewhat boosted. The τ lepton may again be visible as a distinctive τ jet. Thus, along the model line, and for $m_{\tilde{\tau}_1} \lesssim 550$ GeV, we expect stau pair production to yield either one or two hard τ jets plus missing energy, along with possibly soft visible debris from the quasidegenerate heavier Higgsino decays. This is at odds with simplified model analyses, which usually assume 100% stau decay to hard visible τ jets.

Of further note in Fig. 3 is that as $m_{\tilde{\tau}_1}$ exceeds the bino mass, where $\tilde{\chi}_3^0$ is dominantly bino-like, then new lucrative decay modes open up and rapidly dominate the $\tilde{\tau}_1$ branching fractions. For $m_{\tilde{\tau}_1} \gtrsim 550$ GeV, $\tilde{\tau}_1 \to \tilde{\chi}_3^0 \tau$, but the bino $\tilde{\chi}_3^0 \to \tilde{\chi}_1^\pm W^\mp$ at about 25% each, and also $\tilde{\chi}_3^0 \to \tilde{\chi}_{1,2}^0 Z$ and $\tilde{\chi}_1^0 h$ at $\sim$22%–25%. For this case, then stau pair production will yield final state events with two hard τ jets along with WW, WZ, Wh, ZZ, Zh, and hh. Such signatures would be very distinctive, but since the total production cross section tends to be rather low for such large values of $m_{\tilde{\tau}_1}$, it is unlikely that these would be easily visible at HL-LHC with $\sqrt{s} = 14$ TeV.

V. REACH OF HL-LHC FOR STAU PAIRS IN NATURAL SUSY

In this section, we examine the potential of HL-LHC (pp collisions at $\sqrt{s} = 14$ TeV with 3000 fb^{-1}) for probing $\tilde{\tau}_R^+\tilde{\tau}_R^-$ pair production in the context of natural SUSY.

A. Event generation for signal and background

We use ISAJET to first construct a SUSY Les Houches Accord (SLHA) file [66] for any natural SUSY parameter-space point and feed this into PYTHIA [67] to generate signal events. We also use PYTHIA to generate the various $2 \to 2$ background (BG) processes. For $2 \to 3$ background processes, we use MADGRAPH [68] coupled with PYTHIA. For our computation of SM backgrounds to the stau-pair signal, we include parton level production of $t\bar{t}$, $t\bar{t}V$, $V+$jets and VV production (here, V stands for $W^\pm$ or Z). Specifically, we normalize the stau pair production cross sections to their NLO values obtained from PROSPINO. For the most important SM backgrounds, we normalize the cross sections to their values at the NLO level or better when available. The NNLO/NNLL $t\bar{t}$ cross section is normalized to 985.7 pb,[5] the cross sections for $t\bar{t}V$ production are from Ref. [69], $V+j$ cross sections are calculated using the K-factor from the ratio of NLO and LO cross sections from MADGRAPH

[5]This is taken from https://twiki.cern.ch/twiki/bin/view/LHCPhysics/TtbarNNLO, where references to the literature for the calculation may also be found.

with parton jets defined using the anti-k_T algorithm with $p_{Tj} > 25$ GeV and $\Delta R = 0.4$, and finally, VV cross sections are normalized using the results in Ref. [70]. We use the DELPHES code [71] for detector simulation in our analysis.

Since our discovery channel contains backgrounds with high transverse momentum W and Z bosons decaying leptonically or hadronically, we focus on hard leptons and jets in the central part of the detector. With this in mind, we require isolated electrons and muons to satisfy

(i) $p_T(e) > 20$ GeV, $|\eta_e| < 2.47$, with $P_{TRatio} < 0.1$, and
(ii) $p_T(\mu) > 25$ GeV, $|\eta_\mu| < 2.5$ with $p_{TRatio} < 0.2$,

where P_{TRatio} is defined as the ratio of the transverse momentum (p_T^ℓ) of the lepton to the scalar sum of the transverse momenta of all other particles in a $\Delta R = 0.3$ cone around the lepton: $P_{TRatio} \equiv \frac{p_T^\ell}{\sum_{i\in \mathrm{cone}} p_T^i}$.

We construct jets using an anti-k_T jet algorithm and require

(i) $p_T(j) > 20$ GeV with a cone size $R \leq 0.4$ and $|\eta(j)| < 4.5$.

A jet is labeled as a b jet if, in addition, it is tagged as a b jet by DELPHES.

For our signal search, we require additional triggers to select candidates events. A hadronic τ jet τ_h satisfies

(1) the requirement of a baseline jet,
(2) $|\eta_j| < 2.4$, and
(3) be tagged as a τ-jet by DELPHES.[6]

B. $\tau_h\tau_h + \not{E}_T$ signal cuts

For this (dominant) signal channel and after scrutinizing various signal and BG distributions, we require

(i) At least two OS τ_h which satisfy the small radius τ_h jet candidate requirement for signal search, $p_T(\tau_1) > 115$ GeV, and $p_T(\tau_2) > 60$ GeV for the two τ_h selected as candidates.

Then we require the following cuts:

(i) $n(b) = 0$,
(ii) $\not{E}_T > 100$ GeV,
(iii) $\not{E}_{T,rel} \coloneqq \not{E}_T \cdot \sin(\min(\Delta\phi, \frac{\pi}{2})) > 100$ GeV, where $\Delta\phi$ is the azimuthal angle between the $\vec{\not{E}}_T$ and the closest lepton or jet with $p_T > 25$ GeV,
(iv) $\not{E}_T/\sqrt{H_T} > 5.5$,
(v) $|\eta(\tau_1) - \eta(\not{E}_T)| < 4.3$,
(vi) $m_T(\tau_1, \not{E}_T) + m_T(\tau_2, \not{E}_T) > 350$ GeV,
(vii) $\min(m_T(\tau_1), \not{E}_T), m_T(\tau_2, \not{E}_T)) > 105$ GeV,
(viii) $\Delta\phi(\tau_1, \vec{\not{E}}_T) > 55°$,
(ix) $\Delta\phi(\tau_1, \tau_2) > 50°$, and
(x) $R(\tau_1, \tau_2) < 3.3$.

[6]Efficiency and mistag rate taken from Ref. [72] (loose working point). For 1-prong, the efficiency is set to 85%. For 3-prong, it is 75%.

After these cuts, we next plot the m_{T2} distribution [73] for the $\tau_h\tau_h + \not{E}_T$ events for signal and SM BGs. The results are shown in Fig. 4. The colored solid histograms show various SM backgrounds of which WW is dominant for $m_{T2} \gtrsim 100$ GeV. Two signal BM points with various assumed $m_{\tilde{\tau}_1}$ values are shown as dot-dashed histograms. From the plot, we see that the signal histogram for $m_{\tilde{\tau}_1} = 400$ GeV is comparable to the summed background for $m_{T2} \sim 100$–300 GeV while the $m_{\tilde{\tau}_1} = 580$ GeV signal distribution lies well below BG. For the $m_{\tilde{\tau}_1} = 400$ GeV BM point, we expect of order tens of signal events around $m_{T2} \sim 100$–200 GeV for 3000 fb^{-1} of integrated luminosity. With the run 2 integrated luminosity of ~139 fb^{-1}, the event levels would instead be of order 0–1, so no current limits are expected in this case.

C. $\tau_h\ell + \not{E}_T$ signal cuts

For this (subdominant) signal channel, after scrutinizing the signal and BG distributions, we require the following cuts:

(i) At least one pair of OS lepton and τ_h, $p_T(\tau_h) > 165$ GeV.

Then we require

(i) $n(b) = 0$,
(ii) $\not{E}_T > 100$ GeV,
(iii) $\not{E}_{T,rel} \coloneqq \not{E}_T \cdot \sin(\min(\Delta\phi, \frac{\pi}{2})) > 100$ GeV, where $\Delta\phi$ is the azimuthal angle between the $\vec{\not{E}}_T$ and the closest lepton or jet with $p_T > 25$ GeV,
(iv) $|\eta(\ell)| < 2$,
(v) $m_T(\tau, \not{E}_T) + m_T(l, \not{E}_T) > 425$ GeV,
(vi) $\Delta\phi([\tau_h + \ell], \not{E}_T) > 150°$,
(vii) $m_T(\tau, \not{E}_T) > 145$ GeV, and
(viii) $R(\tau_h, \ell) < 3.1$.

The resulting distributions in m_{T2} are shown in Fig. 5 with color coding as in Fig. 4. In this case, we find the signal histograms to be well below BG by at least an order of magnitude even in the most propitious bins.

D. Reach of HL-LHC for stau pair production

For each of the two signal channels from Secs. V B and V C, we examine the binned m_{T2} distributions shown in Figs. 4–5. For exclusion of the stau-pair signal, we assume that the true distribution we would observe in an experiment would correspond to a background only distribution. Upper limits on $m_{\tilde{\tau}_1}$ are then evaluated using a modified frequentist CL_S method [74] with the profile likelihood ratio as the test statistic. The likelihood is built as a product of Poissonian terms for each of the bins in the distributions. A background systematic uncertainty is accounted for by introducing an independent nuisance parameter for each bin of each channel, and the likelihood is modified by log-normal terms to account for these nuisance parameters, with uncertainty that we take to be 25%. Then, the largest value of $m_{\tilde{\tau}_1}$ that can be excluded at

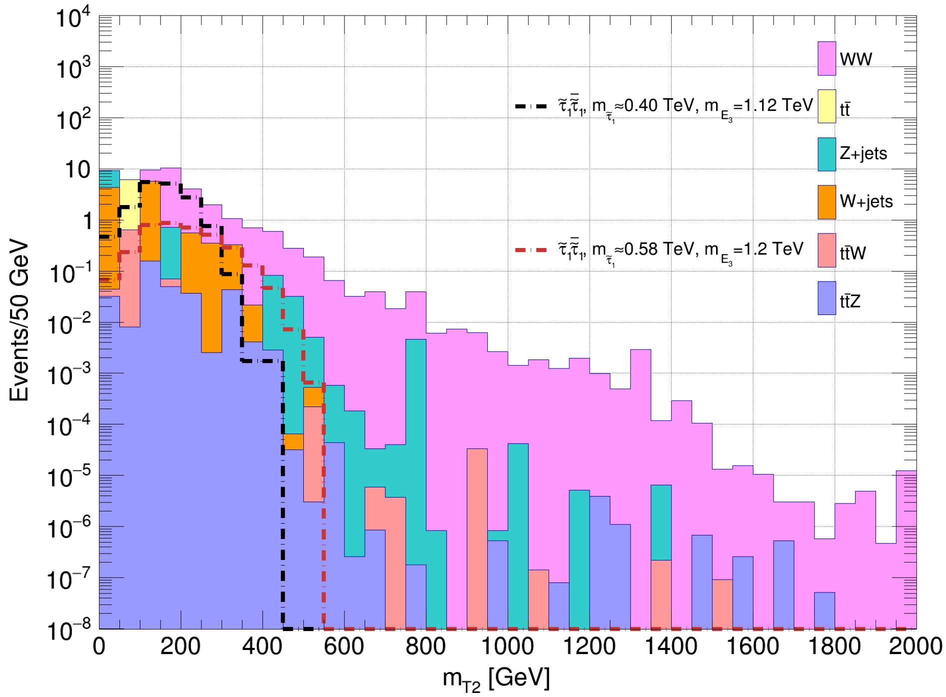


FIG. 4. Distributions in m_{T2} for $\tau_h\tau_h + \not{E}_T$ events from several right-stau pair production models and SM backgrounds at HL-LHC where we assume 3000 fb^{-1} of integrated luminosity.

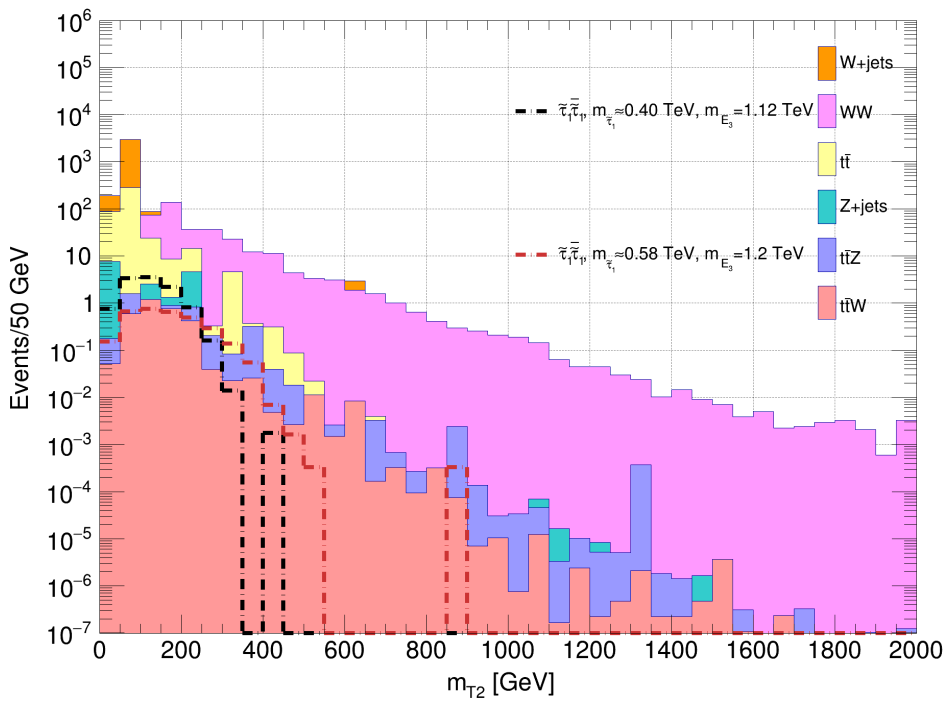


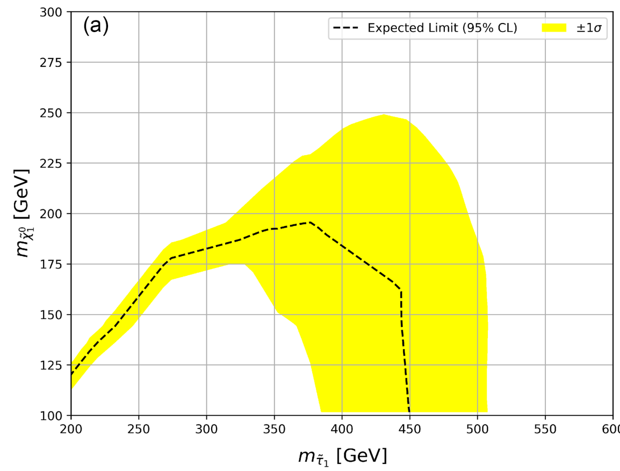
FIG. 5. Distributions in m_{T2} for $\tau_h\ell^\pm + \not{E}_T$ events from several right-stau pair production models and SM backgrounds at HL-LHC.

95% CL for a given assumed value of $m_{\tilde{\chi}_1^0}$ is the exclusion limit. For discovery, we assume that the distribution one would observe in an experiment corresponds to signal-plus-background. We then test this against the background only distribution for each value of $m_{\tilde{\tau}_1}$. If the background only hypothesis can be rejected at at least the 5σ level, we deem that the HL-LHC would discover staus with a mass corresponding to that choice of $m_{\tilde{\tau}_1}$. For both the exclusion and discovery limits, we use the asymptotic expansion for obtaining the median significance [75].[7]

Our HL-LHC reach results are shown in Fig. 6(a) in the $m_{\tilde{\tau}_1}$ vs $m_{\tilde{\chi}_1^0}$ plane assuming the natSUSY benchmark scenario for the cse of no assumed systematic error. We vary μ in order to vary $m_{\tilde{\chi}_1^0}$. In our case of $\tilde{\tau}_1\bar{\tilde{\tau}}_1$ production within natSUSY, we do not find any discovery reach. However, the 95% CL exclusion curve is shown as the black dashed curve along with 1σ fluctuation limits shown as the yellow band.[8] Unlike the ATLAS and CMS results, our $m_{\tilde{\chi}_1^0}$ values only extend down to ∼100 GeV since LEP2 is expected to exclude Higgsino-like charginos with mass $m_{\tilde{\chi}_1^\pm} \lesssim 100$ GeV. For $m_{\tilde{\chi}_1^0} \sim 100$ GeV, then we expect LHC experiments to be able to exclude $m_{\tilde{\tau}_1}$: 200–450 (400) GeV, assuming 0% (25%) systematic uncertainty. For lower values of $m_{\tilde{\tau}_1} \lesssim 200$ GeV, then the final state τs become too soft for our cuts, while for $m_{\tilde{\tau}_1} \gtrsim 450$ GeV, then the expected signal rates become too tiny for exclusion. We see that we do expect some exclusion for $m_{\tilde{\chi}_1^0}$ values as high as ∼200 GeV; for higher $m_{\tilde{\chi}_1^0} \sim \mu \gtrsim$ 200 GeV values, then most of the final-state energy goes into making to $\tilde{\chi}_1^0$ rest mass, and too little visible energy is left to distinguish a signal. For our exclusion plot, the above cuts were optimized for $\mu \sim 200$ GeV, so some small extension of this region may be gained if a lighter value of μ is assumed (but then one may begin to conflict with ATLAS/CMS bounds on μ from soft isolated dilepton plus jets plus $\not{E}_T$ search results [76,77]). In frame Fig. 6(b), we show how the reach is diminished if instead we include an assumed 25% systematic error.

Comparing our results to ATLAS and CMS, we find in the ATLAS (2018) Ref. [50] HL-LHC reach study that there also is no 5σ discovery reach for $\tilde{\tau}_R\bar{\tilde{\tau}}_R$ pairs, although there is a 95% CL exclusion region for $m_{\tilde{\chi}_1^0} \lesssim 100$ GeV. This study includes some systematic errors plus an assumed pileup that we have not included. This helps us to gain a larger exclusion region than ATLAS even though some of our staus decay invisibly.

Comparing with the CMS HL-LHC reach study, they do obtain a 5σ discovery reach even for $m_{\tilde{\chi}_1^0} > 100$ GeV. There reach should be better than our since they include both $\tilde{\tau}_L\tilde{\tau}_L^*$ and $\tilde{\tau}_R\tilde{\tau}_R^*$ production (but see the discussion below on this dangerous assumption). Meanwhile, their 95% CL exclusion limit ranges from $m_{\tilde{\tau}_1} \sim 100$–650 GeV for $m_{\tilde{\chi}_1^0} \sim 100$ GeV, which is broader than our result, although we include the invisible stau decay modes, which are generic for natural SUSY, while they assume $\tilde{\tau}_1 \to \tau\tilde{\chi}_1^0$ at 100% branching fraction.



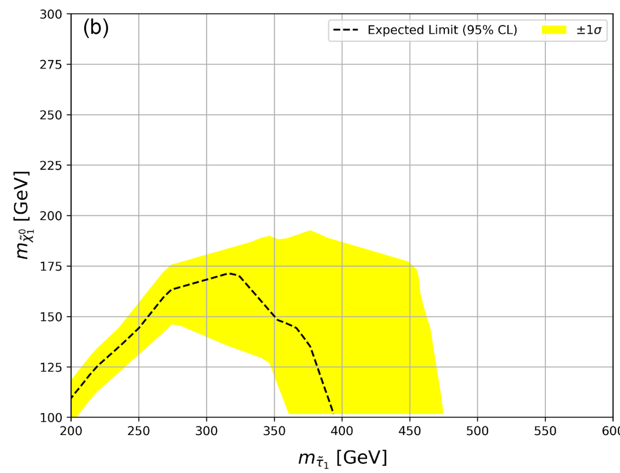


FIG. 6. The 95% CL exclusion reach of HL-LHC for right-stau pair production events with a natSUSY setup. In frame (a), we assume no systematic error whilst in frame (b) we assume 25% systematic error.

VI. SUMMARY AND DISCUSSION

We have examined right-stau pair production at HL-LHC in the context of a natural SUSY model line wherein all independent contributions to the weak scale are comparable to $m_{\rm weak}$ (thus, no weak scale fine-tuning is needed). This class of models can be considered as much more plausible

[7]We have checked that for every channel that we study there are at least ten (frequently significantly more) background events in the "sensitive regions" of the histograms in Figs. 4–5. This is large enough to justify the use of asymptotic formulae since for discovery (exclusion) we are concerned with fluctuations of the background (signal plus background).

[8]For Fig. 6(a), the yellow band is purely statistical uncertainty. For Fig. 6(b), the band includes the combined effects of statistical and systematic uncertainty. The reasons leading to larger (statistical) uncertainties for heavier mass scales of staus or neutralinos region are 1) signal yield is small when $m_{\tilde{\tau}_1}$ is large and 2) when the $m_{\tilde{\chi}_1^0}$ is heavier, the mass difference between the stau and the $\tilde{\chi}_1^0$ becomes smaller such that the visible decay products become softer and thus the signal signatures are less distinguishable from the backgrounds. In both cases, the statistical uncertainty tends to be large which widens the uncertainty band. Such features are consistent with the reach contour from the current ATLAS search on direct stau production via $\tau_h\tau_h + \not{E}_T$. See, e.g., Fig. 7 of [47].

than fine-tuned models, which require accidentally large cancellations to obtain $m_{W,Z,h} \sim 100$ GeV. Furthermore, the right staus are usually expected to be the lightest of the tau-sleptons. Thus, we embed light right staus within a natSUSY model line.

In such models, the four Higgsino-like EWinos are the lightest of sparticles, so $\tilde{\tau}_1 \to \tilde{\chi}_1^- \nu_\tau$ (nearly invisible) at rates comparable to $\tilde{\tau}_1 \to \tilde{\chi}_{1,2}^0 \tau$. The latter decays lead to ditau $+\not{E}_T$ events at a reduced rate compared to the usual simplified models. The hadronic ditau $+ \not{E}_T$ ends up being a more lucrative search channel than $\tau_h \ell + \not{E}_T$. By computing signal and SM BG in the $m_{\tilde{\tau}_1}$ vs $m_{\tilde{\chi}_1^0}$ plane, we do not find any 5σ discovery regions at HL-LHC, but we do obtain a 95% CL exclusion reach. This region extends from $m_{\tilde{\tau}_1}$: 200–450 (400) GeV, assuming 0% (25%) systematic uncertainty, for $m_{\tilde{\chi}_1^0} \sim 100$ GeV, but disappears entirely for $m_{\tilde{\chi}_1^0} \gtrsim 200$ GeV. The net reach is of course reduced by including an overall systematic error. These results illustrate the difficulty of finding light tau sleptons at HL-LHC in a natSUSY context.

A. Comparison of stau pair searches in natural and unnatural SUSY

Most experimental search projections occur within simplified models which assume stau pair production ($\tilde{\tau}_L \tilde{\tau}_L^*$ and/or $\tilde{\tau}_R \tilde{\tau}_R^*$) along with decay to a single light neutralino $\tilde{\chi}_1^0$ with $BF(\tilde{\tau}_i \to \tau \tilde{\chi}_1^0)$ at 100%. Can one distinguish these presumably unnatural models (with decoupled Higgsinos so that μ must be large) from our case of natSUSY, which includes light Higgsinos since $|\mu| \lesssim 350$ GeV? Most likely, the answer is yes. The natural case with light Higgsinos will be accompanied by Higgsino pair production signals [33] such as $pp \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm \tilde{\chi}_2^0$ with $\tilde{\chi}_2^0 \to \ell \bar{\ell} \tilde{\chi}_1^0$. Higgsino pair production thus gives rise to soft opposite-sign dileptons $+\not{E}_T$, which may be visible if the soft leptons recoil against a hard initial state jet radiation [78–80]. In fact, there are some small excesses in both ATLAS [76] and CMS [77] run 2 data in this channel. There is also a distinctive same-sign diboson signature in natSUSY which can occur from wino pair production followed by decay to lighter Higgsinos [81,82]. These Higgsino related signatures would not occur for unnatural SUSY models. For a review of collider signals from natural SUSY, see, e.g., the review [83].

B. The case of left vs right staus

The projected reach of ATLAS [50] and CMS [51] has been computed for stau pair production followed by 100% branching fraction $\tilde{\tau}_i \to \tau \tilde{\chi}_1^0$ where $i = L$ and/or R. Another question arises: can one tell whether one is producing $\tilde{\tau}_L \tilde{\tau}_L^*$ from $\tilde{\tau}_R \tilde{\tau}_R^*$ production, or indeed a mixture of both as is assumed in some experimental simplified model scenarios. One way may be to use the different decay energy distributions from left- versus right-tau lepton decays that arise from their parent $\tilde{\tau}_L$ or $\tilde{\tau}_R$ particles.

So far, we have argued that the lighter right stau $m_{\tilde{\tau}_R} \ll m_{\tilde{\tau}_L}$ is more theoretically motivated, and so we have focused on his case. In fact, the ATLAS and CMS studies assume certain simplified models which violate major theoretical constraints. Aside from assuming 100% stau branching fractions into a single mode $\tau \tilde{\chi}_1^0$, if one assumes a light left stau $\tilde{\tau}_L$, then necessarily it comes with a light tau sneutrino. The weak scale mass relations are [13] (neglecting small mixing effects)

$$m_{\tilde{\tau}_L}^2 \simeq m_{L_3}^2 + m_\tau^2 + m_Z^2 \cos 2\beta(-1/2 + \sin \theta_W) \quad (8)$$

$$m_{\tilde{\nu}_{\tau L}}^2 \simeq m_{L_3}^2 + m_Z^2 \cos 2\beta(+1/2) \quad \text{and} \quad (9)$$

$$m_{\tilde{\tau}_R}^2 \simeq m_{E_3}^2 + m_\tau^2 + m_Z^2 \cos 2\beta(-\sin \theta_W), \quad (10)$$

so that most of the mass of $\tilde{\tau}_L$ and $\tilde{\nu}_{\tau L}$ comes from $m_{L_3}^2$. This actually means that if you assume light left staus, one must also include $pp \to W^* \to \tilde{\tau}_L \tilde{\nu}_{\tau L}$ and $\tilde{\nu}_{\tau L} \tilde{\nu}_{\tau L}^*$ production where now the $\tilde{\nu}_{\tau L}$ also usually decays visibly. (Even if one assumes an invisible $\tilde{\nu}_{\tau L} \to \nu_\tau \tilde{\chi}_1^0$ decay, the W^* production channel will give rise to a large rate for mono-tau-jet $+ \not{E}_T$ events, which should be included in any search strategy.) The W^* mediated production cross section dominates slepton pair production [84] and sneutrino pair production is comparable to stau-left pair production cross section. Thus, the total cross sections for left-slepton pair production will be much higher than typically assumed in simplified models, and the decay signatures will be more complex, for a given value of $m_{\tilde{\tau}_L}$. We expect this much more complex, but more realistic, case of left-slepton pair production to be readily distinguishable from right stau pair production. At present, realistic analyses are lacking.

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