## Spectrum of molecular hexaquarks

Bo Wang<sup>(0)</sup>,<sup>1,2,3</sup> Kan Chen<sup>(0)</sup>,<sup>4,\*</sup> Lu Meng<sup>(0)</sup>,<sup>5,†</sup> and Shi-Lin Zhu<sup>(0)</sup>,<sup>4,‡</sup>

<sup>1</sup>College of Physics Science and Technology, Hebei University, Baoding, 071002, China <sup>2</sup>Hebei Key Laboratory of High-precision Computation and Application of Quantum Field Theory, Baoding, 071002, China

<sup>3</sup>Hebei Research Center of the Basic Discipline for Computational Physics, Baoding, 071002, China

<sup>4</sup>School of Physics, Northwest University, Xi'an, 710127, China

<sup>5</sup>Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany <sup>6</sup>School of Physics and Center of High Energy Physics, Peking University, Beijing, 100871, China

(Received 3 May 2024; revised 26 June 2024; accepted 28 June 2024; published 26 July 2024)

We investigate the mass spectra of molecular-type hexaquark states in the dibaryon systems. These systems are composed of the charmed baryons  $[\Sigma_c^{(*)}, \Xi_c^{(t,*)}]$ , doubly charmed baryons  $[\Xi_{cc}^{(*)}]$ , and hyperons  $[\Sigma^{(*)}, \Xi^{(*)}]$ . We consider all possible combinations of particle-particle and particle-antiparticle pairs, including the S-wave spin multiplets in each combination. We establish the underlying connections among the molecular tetraquarks, pentaquarks, and hexaquarks with the effective quark-level interactions. We find that the existence of molecular states in  $DD^*$ ,  $D\bar{D}^*$ , and  $\Sigma_c \bar{D}^{(*)}$  systems leads to the emergence of a large number of deuteronlike hexaquarks in the heavy flavor sectors. Currently, there have been several experimental candidates for molecular tetraquarks and pentaquarks. The experimental search for near-threshold hexaquarks will further advance the establishment of the underlying dynamical picture of hadronic molecules and deepen our understanding of the role of spin-flavor symmetry in near-threshold residual strong interactions.

DOI: 10.1103/PhysRevD.110.014038

## I. INTRODUCTION

In the past two decades, the field of hadron physics has witnessed a flourishing growth in both experimental investigations and theoretical studies [1]. One significant factor contributing to this progress is the abundant discovery of novel (exotic) hadronic states near threshold energies [2–12]. The peculiar hadrons such as the XYZ states [13–17], hidden-charm pentaquarks [18–20], doubly charmed tetraquarks [21,22], and charmed strange tetraquarks [23–26] have emerged as prominent examples. Theorists have extensively studied the mass spectra, decays, production mechanisms, and electromagnetic properties of these states [2–11]. However, due to limited experimental data and the reliance on specific models, the nature of these states still lacks a definitive consensus. Current theoretical interpretations include hadronic

Contact author: chenk10@nwu.edu.cn

molecular states, kinematic effects, and compact multiquark states, etc. [11]. The molecular picture is gaining increasing recognition of the communities, as the aforementioned exotic states are very close to the threshold of a pair of conventional hadrons. The weaker residual strong interaction ensures that the corresponding hadronic molecules naturally reside near the threshold, similar to the deuteron with its mass just about 2.2 MeV below the twonucleon threshold.

Inspired by the recent experimental observation of a large number of hadronic molecule candidates, a significant amount of research has been devoted to investigating possible molecular-type hexaquark states in heavy-heavy systems [27–58]. For example, Lee et al. investigated potential molecular states in systems composed of two charmed baryons using the one-boson exchange (OBE) model [27]. Their results indicated the absence of bound states in the  $\Lambda_c \Lambda_c$  system, while molecular states were found to exist in the  $\Xi_c \Xi_c$  and  $\Xi'_c \Xi'_c$  systems. This line of research has been further extended to the systems containing b quarks [28]. Other research works, such as those based on the OBE model [29–39], quark potential models [40–42], effective field theories [43–49], QCD sum rules [50-53], chromomagnetic interaction model [54,55], and lattice QCD calculations [56], have also extensively studied similar systems. In addition, Meng et al. and Yang et al.

<sup>&</sup>lt;sup>T</sup>Contact author: lu.meng@rub.de

<sup>&</sup>lt;sup>‡</sup>Contact author: zhusl@pku.edu.cn

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP<sup>3</sup>.

TABLE I. The molecular hexaquark systems under consideration and their quark compositions. The type-I and type-II systems' interactions are governed by the  $V_{qq}$  and  $V_{\bar{q}q}$ , respectively.

Type-I: V <sub>qq</sub>		Type-II: $V_{\bar{q}q}$							
Char	med baryon-(anti)	charmed baryo	n (III A)						
$\Sigma_c^{(*)}\Sigma_c^{(*)}$	[cqq][cqq]	$\Sigma_c^{(*)} ar{\Sigma}_c^{(*)}$	$[cqq][ar{c}ar{q}ar{q}]$						
$\Sigma_c^{(*)} \Xi_c^{(\prime,*)}$	[cqq][csq]	$\Sigma_c^{(*)} ar{\Xi}_c^{(\prime,*)}$	$[cqq][ar{c}ar{s}ar{q}]$						
$\Xi_c^{(\prime,*)}\Xi_c^{(\prime,*)}$	[csq][csq]	$\Xi_c^{(\prime,*)}\bar{\Xi}_c^{(\prime,*)}$	$[csq][c \bar{s} \bar{q}]$						
Doubly charmed baryon-(anti)doubly charmed baryon (III B)									
$\Xi_{cc}^{(*)}\Xi_{cc}^{(*)}$	[ccq][ccq]	$\Xi_{cc}^{(*)} \bar{\Xi}_{cc}^{(*)}$	$[ccq][ar{c}ar{c}ar{q}]$						
Hyperon-(anti)hyperon (III C)									
$\Sigma^{(*)}\Sigma^{(*)}$	[sqq][sqq]	$\Sigma^{(*)} ar{\Sigma}^{(*)}$	$[sqq][\bar{s}\bar{q}\bar{q}]$						
$\Sigma^{(*)} \Xi^{(*)}$	[sqq][ssq]	$\Sigma^{(*)} \bar{\Xi}^{(*)}$	$[sqq][\bar{s}\bar{s}\bar{q}]$						
$\Xi^{(*)}\Xi^{(*)}$	[ssq][ssq]	$\Xi^{(*)}\bar{\Xi}^{(*)}$	$[ssq][\bar{s}\bar{s}\bar{q}]$						
Charmed	baryon-(anti)dou	bly charmed ba	aryon (III D)						
$\Sigma_c^{(*)} \Xi_{cc}^{(*)}$	[cqq][ccq]	$\Sigma_c^{(*)} ar{\Xi}_{cc}^{(*)}$	$[cqq][ar{c}  ar{c}  ar{q}]$						
$\boldsymbol{\Xi}_{c}^{(\prime,*)}\boldsymbol{\Xi}_{cc}^{(*)}$	[csq][ccq]	$\Xi_c^{(\prime,*)} \bar{\Xi}_{cc}^{(*)}$	$[csq][c \ c \ q]$						
C	Charmed baryon-(	anti)hyperon (II	IIE)						
$\Sigma_c^{(*)}\Sigma^{(*)}$	[cqq][sqq]	$\Sigma_c^{(*)} ar{\Sigma}^{(*)}$	$[cqq][ar{s}ar{q}ar{q}]$						
$\Sigma_c^{(*)} \Xi^{(*)}$	[cqq][ssq]	$\Sigma_c^{(*)} \bar{\Xi}^{(*)}$	$[cqq][\bar{s}\ \bar{s}\ \bar{q}]$						
$\Xi_c^{(\prime,*)}\Sigma^{(*)}$	[csq][sqq]	$\Xi_c^{(\prime,*)} ar{\Sigma}^{(*)}$	$[csq][\bar{s}\bar{q}\bar{q}]$						
$\Xi_c^{(\prime,*)}\Xi^{(*)}$	[csq][ssq]	$\Xi_c^{(\prime,*)}\bar{\Xi}^{(*)}$	$[csq][\bar{s}\ \bar{s}\ \bar{q}]$						
Dout	oly charmed bary	on-(anti)hyperor	n (III F)						
$\Xi_{cc}^{(*)}\Sigma^{(*)}$	[ccq][sqq]	$\Xi^{(*)}_{cc}ar{\Sigma}^{(*)}$	$[ccq][ar{s}ar{q}ar{q}]$						
$\Xi_{cc}^{(*)}\Xi^{(*)}$	[ccq][ssq]	$\Xi_{cc}^{(*)}\bar{\Xi}^{(*)}$	$[ccq][\bar{s}\bar{s}\bar{q}]$						

have also studied the possible molecular states composed of two doubly charmed baryons [57,58].

As a natural extension of our previous two research works [59,60], in this paper, we will further systematically investigate possible molecular states in all systems formed by the combinations of hyperons, charmed baryons, and doubly charmed baryons (including both the particleparticle and particle-antiparticle combinations). See Table I for a list of the considered systems.

The structure of this article is arranged as follows: In Sec. II, we will present the extended heavy quark symmetry, the effective potentials at the quark level, the definition of parity for the neutral systems, and the systems considered in this paper. In Sec. III, we will provide the mass spectra of various possible molecular hexaquarks, and conduct a detailed comparison and discussion with existing results in the aforementioned works. In Sec. IV, we will summarize and provide an outlook for this work.

#### **II. FRAMEWORK**

#### A. Extended heavy quark symmetry

Generally, u, d, and s quarks are collectively referred to as light quarks because their masses are smaller compared to the nonperturbative scale of QCD,  $\Lambda_{\text{OCD}}$  (~200 MeV). However, in reality, inside hadrons, the masses of u and d quarks ( $m_u = 2.16^{+0.49}_{-0.26}$  MeV,  $m_d = 4.67^{+0.48}_{-0.17}$  MeV [1]) are much smaller than  $\Lambda_{\rm QCD}$ , and the mass of the s quark ( $m_s = 93.4^{+8.6}_{-3.4}$  MeV [1]) is less than  $\Lambda_{\rm QCD}$ . Therefore, in contrast to the isospin SU(2) symmetry, the flavor SU(3) symmetry is not as robust.

For the near-threshold residual strong interactions, nonperturbative dynamics between the color singlets (e.g.,  $D\bar{D}^*$  and  $DD^*$ ) also generates a nonperturbative scale similar to  $\Lambda_{\text{OCD}}$ , namely the binding momentum  $\gamma_b$ , which represents the scale of the typical momentum transfer between the interacting hadrons. In Ref. [59], we have analyzed that  $\gamma_b$  is typically less than 100 MeV. The mass of the constituent strange quark  $m_s^{\text{QM}}$  is about 500 MeV [61,62], indicating that  $\gamma_b$  is often much smaller than  $m_s^{\text{QM}}$ . This implies that the strength of near-threshold residual strong interactions is not sufficient to excite the strange quark inside hadrons. In this case, instead of light quarks, the strange quark behaves as an inert source and plays very similar role as a heavy quark. Therefore, we can further extend the range of heavy quarks to include strange quarks. In this way, the heavy quark spin-flavor symmetry [63] and heavy diquark-antiquark symmetry<sup>1</sup> [64] can be extended to encompass a wider range of systems. This allows us to clearly clarify the role of spin-flavor symmetry in hadronic molecular states, and describe the mass spectra of different molecular states using a few parameters. Based on the description that approximates the *s* quark as a heavy quark, we can naturally explain the emergence of the  $T_{cs0}(2900)$ and  $T^a_{c\bar{s}0}(2900)$  [59], and why the  $P^{\Lambda}_{ws}(4338)$  is so close to the  $\Xi_c \bar{D}$  threshold [60].

#### B. Effective potentials for the S-wave dihadron systems

From the perspective of the conventional mesonexchange model, a pair of heavy-light hadrons can exchange (quantum-number-allowed) pseudoscalar  $(\pi, \eta, ...)$ , scalar  $(a_0, f_0, ...)$ , vector  $(\rho, \omega, ...)$ , axial-vector  $(a_1, f_1, ...)$ , and tensor  $(a_2, f_2, ...)$  mesons. In Ref. [59], we utilized the basic ideas of the one-boson exchange model and effective field theory to construct the near-threshold interacting potentials for the S-wave dihadron systems. Compared to conventional approaches based on the hadronic degrees of freedom, starting from the quark level enables us to establish connections between different heavy-flavor dihadron systems.

Adopting the concepts from the nonrelativistic chiral quark model [65], we assume that the meson-exchange occurs between the light quarks  $q_1$  and  $q_2$  (where q = u, d, and  $q_1$  and  $q_2$  belong to different hadrons). Traditionally, we construct the effective Lagrangians for the couplings of light quarks and the exchanged mesons, from which the effective potentials are subsequently derived. However, the

<sup>&</sup>lt;sup>1</sup>For an introduction to the concept of heavy diquark-antiquark symmetry (HDAS), please refer to Sec. 2.2.2 of review [11].



FIG. 1. An illustrative diagram that transforms the nonlocal interaction into a local form in the case where  $q^2$  is much smaller than  $M_e^2$ . The bold (thin) and dashed lines represent the heavy (light) quarks, and the exchanged light mesons, respectively. The subfigures (a) and (b) represent the exchanged and local interactions, respectively.

current experimental data are insufficient to determine the coupling constants individually, thus we consider their collective contributions. For the near-threshold interactions, we can use a nonrelativistic framework. The contributions of these exchanged mesons to the nonrelativistic effective potentials can be parametrized in the following form:

$$V_{\text{eff}} \sim \sum_{e} \frac{\{1, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}), \ldots\}}{\boldsymbol{q}^2 + M_e^2}, \qquad (1)$$

in which the isospin coupling is omitted for simplicity. q and  $M_e$  denote the transferred momentum and the mass of the exchanged meson, respectively.  $\sigma_{1,2}$  represent the Pauli matrices in spin space. Thus, the first three terms in Eq. (1) signify the central potential, spin-spin coupling, and tensor potential, respectively, and their forms are determined by the specific quantum numbers of the exchanged particles.

Since we are concerned with the low-energy interactions near the threshold  $(q^2 \ll M_e^2)$ , we can transform the non-local potential [Fig. 1(a)] in Eq. (1) into a local form [Fig. 1(b)] using the following expansion:

$$\frac{1}{q^2 + M_e^2} = \frac{1}{M_e^2} \left( 1 - \frac{q^2}{M_e^2} + \dots \right).$$
(2)

In our calculations, considering the suppression of  $q^2/M_e^2$ , we will retain only the leading-order term of the expansion. Consequently, in Eq. (1), only the central potential and the spin-spin coupling term will remain.

The light meson exchange between two heavy-light hadrons includes both isospin triplet and isospin singlet. Taking the scalar field S as an example, we assume it can be written in the following form:

$$S = S \cdot \tau + \frac{1}{\sqrt{2}}S_1, \qquad (3)$$

where  $S_1$  and S respectively represent the isospin singlet and isospin triplet, with  $\tau$  being the Pauli matrices in isospin space. After integrating out various exchanged meson fields similar to those in Eq. (3) using the expansion in Eq. (2), the nonrelativistic effective potentials describing the  $q_1q_2$ and  $\bar{q}_1q_2$  correlations are respectively given as follows [59]:

$$V_{qq} = \left(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \frac{1}{2}\boldsymbol{\tau}_{0,1} \cdot \boldsymbol{\tau}_{0,2}\right) (c_s + c_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \quad (4)$$

$$V_{\bar{q}q} = \left(-\boldsymbol{\tau}_1^* \cdot \boldsymbol{\tau}_2 + \frac{1}{2}\boldsymbol{\tau}_{0,1} \cdot \boldsymbol{\tau}_{0,2}\right) (\tilde{c}_s + \tilde{c}_a \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2), \quad (5)$$

where  $\tau(\sigma)$  are the Pauli matrices denoting the isospin (spin) operators of light quarks, while  $\tau_0$  is the 2 × 2 identity matrix. The four terms in Eqs. (4) and (5) incorporate the isospin unrelated and related central potentials and spin-spin couplings. The low-energy constants (LECs)  $c_s$  ( $\tilde{c}_s$ ) and  $c_a$  ( $\tilde{c}_a$ ) are determined with the experimentally well-established states in the molecular scenario. For example, using the masses of  $P_{\psi}^N$  and  $T_{cc}$  states as the input, we obtained the values of  $c_s$  and  $c_a$  as

$$c_s = 146.4 \pm 10.8 \text{ GeV}^{-2},$$
  
 $c_a = -7.3 \pm 10.5 \text{ GeV}^{-2},$  (6)

while utilizing the bound state and virtual state pictures respectively for X(3872) and  $Z_c(3900)$  gives the ranges of  $\tilde{c}_s$  and  $\tilde{c}_a$  as

184.3 GeV<sup>-2</sup> < 
$$\tilde{c}_a + \tilde{c}_s < 187.5$$
 GeV<sup>-2</sup>,  
78.1 GeV<sup>-2</sup> <  $\tilde{c}_a - \tilde{c}_s < 180.3$  GeV<sup>-2</sup>. (7)

We ultimately need to translate the quark-level effective potentials to the hadron level. Here, we take  $V_{qq}$  as an example, and the translation of  $V_{\bar{q}q}$  can be done using the same form.

$$V_{\mathcal{H}_1\mathcal{H}_2}^{I,J} = \langle [\mathcal{H}_1\mathcal{H}_2]_J^I | V_{qq} | [\mathcal{H}_1\mathcal{H}_2]_J^I \rangle, \tag{8}$$

where  $[\mathcal{H}_1\mathcal{H}_2]_J^I$  represents the spin-flavor wave function of the dihadron system  $\mathcal{H}_1\mathcal{H}_2$  with isospin *I* and spin *J*. These wave functions are expanded in terms of the spin-flavor wave functions of individual hadrons  $\mathcal{H}_1$  and  $\mathcal{H}_2$ . For more specific details, we refer to Refs. [48,49].

With the hadron-level effective potentials in hand, we can insert them into the following Lippmann-Schwinger (LS) equation to solve for the existence of bound or virtual states in dihadron systems with specific quantum numbers,

$$t = v + vGt, \tag{9}$$

where v is the  $V_{\mathcal{H}_1\mathcal{H}_2}^{I,J}$  obtained in Eq. (8). The scattering information is encoded in the T-matrix t. The nonrelativistic form of the two-body propagator G is adopted,

$$G(E+i\epsilon) = \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3} \frac{2\mu}{p^2 - k^2 + i\epsilon}$$
$$= \frac{2\mu}{(2\pi)^3} \left[ p \tanh^{-1} \left(\frac{p}{\Lambda}\right) - \Lambda - \frac{i\pi}{2} p \right], \quad (10)$$

where we use a sharp cutoff  $\Lambda$  to regularize the divergent integral. The momentum *p* is related to the center of mass energy *E* via  $p = [2\mu(E - m_{\rm th})]^{1/2}$ , in which  $\mu$  and  $m_{\rm th}$ denote the reduced mass and threshold of the dihadron  $\mathcal{H}_1\mathcal{H}_2$ . To maintain consistency with Refs. [59,60], in this calculation, the cutoff value for  $\Lambda$  will also be taken as 0.4 GeV.

For the constant potential v, the integral equation (9) can be further simplified into the following algebraic equation,

$$t^{-1} = v^{-1} - G. \tag{11}$$

For single-channel scattering, the branch cut in Eq. (10) gives rise to two Riemann sheets, known as the physical sheet and the unphysical sheet. Bound and virtual states correspond to the below-threshold poles of the T-matrix locating at the real axis of the physical and unphysical Riemann sheets, respectively. These poles in *t* correspond to the zeros of  $t^{-1}$ . The physical and unphysical Riemann sheets can be reached by the following substitution of the *G* function,

Physical: 
$$G(E + i\epsilon)$$
,  
Unphysical:  $G(E + i\epsilon) + i \frac{\mu}{4\pi^2}p$ , (12)

where the  $G(E + i\epsilon)$  is given by Eq. (10).

## C. Parities of the neutral systems

We will encounter three types of neutral systems when studying the particle-antiparticle combinations, for which we can define their C parity.

(i) The neutral systems composed of a particle and its own antiparticle, such as the  $\Sigma_c \bar{\Sigma}_c$  and  $\Sigma_c^* \bar{\Sigma}_c^*$ , etc. Their *P*, *C*, and *G* parities are conventionally defined as

$$P = (-1)^{L+1}, (13)$$

$$C = (-1)^{L+S},$$
 (14)

$$G = (-1)^{L+S+I},$$
 (15)

where L, S, and I represent the orbital angular momentum, spin and isospin of the dihadron systems  $\mathcal{H}_1\mathcal{H}_2$ . For the S-wave case (L = 0), the total angular momentum J equals to S.

(ii) The neutral systems composed of two particles with different spins, such as the  $\sum_c \bar{\Sigma}_c^*, \Xi_c^{(\prime)} \bar{\Xi}_c^*$ , etc. Their *P* 

parity is still given by Eq. (13), while the *C* parity depends on the relative phase factor  $\eta$  in the wave function, as well as the convention for the transformation of the spin- $\frac{3}{2}$  field under charge conjugation. We take the  $\Sigma_c \overline{\Sigma}_c^*$  as an example, and its wave function is given by

$$|\Sigma_c \bar{\Sigma}_c^*\rangle = \frac{1}{\sqrt{2}} \left[ \Sigma_c \bar{\Sigma}_c^* + \eta \bar{\Sigma}_c \Sigma_c^* \right].$$
(16)

We adopt the following convention

$$\hat{C}\Sigma_c^* = \bar{\Sigma}_c^*,\tag{17}$$

which is similar to the one used in Ref. [66], but differs from the convention in Ref. [44] by a negative sign. Then the C and G parities for such systems will be defined as

$$C = \eta(-1)^{S}(-1)(-1)^{L} = \eta(-1)^{L+S+1}, \quad (18)$$

$$G = \eta(-1)^{L+S+I+1}.$$
 (19)

(iii) The neutral system composed of  $\Xi_c$  and  $\Xi'_c$ . Its *P* parity is also given by Eq. (13), while the *C* parity will depend on the relative phase factor  $\eta$  in its wave function,

$$|\Xi_c \bar{\Xi}'_c\rangle = \frac{1}{\sqrt{2}} \left[ \Xi_c \bar{\Xi}'_c + \eta \bar{\Xi}_c \Xi'_c \right], \qquad (20)$$

and its C and G parities are defined as

$$C = \eta(-1)^{S+1}(-1)(-1)^L = \eta(-1)^{L+S}, \quad (21)$$

$$G = \eta(-1)^{L+S+I}.$$
 (22)

For the dihadron systems composed of generalized identical particles, e.g., the  $\Sigma_c \Sigma_c$  and  $\Sigma_c^* \Sigma_c^*$ , where  $\Sigma_c^{(*)} = (\Sigma_c^{(*)++}, \Sigma_c^{(*)+}, \Sigma_c^{(*)0})$ , their quantum numbers must satisfy the following selection rule:

$$L + S + I + 2i = \text{Even number}, \qquad (23)$$

where *i* represents the isospin of the component particle, such as i = 1 for the  $\Sigma_c \Sigma_c$  and  $\Sigma_c^* \Sigma_c^*$  systems.

The various systems considered in this study are shown in Table I. We do not consider the systems that include the isospin singlet  $\Lambda_c$  and  $\Lambda$  due to the absence of isospin related interactions. As a result, the interactions in  $\Lambda_{(c)}$ -containing systems are often very weak, making it difficult to form molecular states in a single-channel scenario, such as the  $\Lambda_c \bar{D}^{(*)}$  [67–69] and  $\Lambda_c \Lambda_c$  systems [27].

## III. SPECTRUM OF THE MOLECULAR HEXAQUARKS

#### A. Charmed baryon-(anti)charmed baryon systems

Our results for the  $\Sigma_c^{(*)}\Sigma_c^{(*)}$  and  $\Sigma_c^{(*)}\overline{\Sigma}_c^{(*)}$  systems are shown in Table II. We obtain three (two) molecular states in the  $\Sigma_c \overline{\Sigma}_c$  ( $\Sigma_c \Sigma_c$ ) system, six (four) molecular states in the  $\Sigma_c \overline{\Sigma}_c^*$  ( $\Sigma_c \Sigma_c^*$ ) system, and five (four) molecular states in the  $\Sigma_c^* \overline{\Sigma}_c^*$  ( $\Sigma_c^* \Sigma_c^*$ ) system. It can be seen that the molecular states of the  $\Sigma_c^{(*)} \overline{\Sigma}_c^{(*)}$  systems are often found in the highest isospin or/and highest spin channels, while the molecular states in the  $\Sigma_c^{(*)} \Sigma_c^{(*)}$  systems are found in the low isospin channels, i.e., the channels with I = 0, 1, and there are no molecular states in the channel with I = 2. This is a typical feature of systems where molecular states are generated by the interaction between light quarks (via the  $V_{qq}$ ), such as the tetraquark state  $T_{cc}$ , the pentaquark states  $P_{\psi}^{N}$  and  $P_{\psi s}^{\Lambda}$ , etc., which all follow this pattern. The systems we considered in the following sections will mostly follow this pattern.

Meanwhile, we notice that the attractive interaction of most isotensor channels in  $\Sigma_c^{(*)} \bar{\Sigma}_c^{(*)}$  systems is stronger than that of other channels, such as the  $2^+(0^{-+})\Sigma_c \bar{\Sigma}_c$ ,  $2^{\mp}(1^{-\mp})\Sigma_c \bar{\Sigma}_c^*$ , and  $2^+(0^{-+})$ ,  $2^-(1^{--})\Sigma_c \bar{\Sigma}_c^*$ . The results from the OBE model in Ref. [27] are consistent with our results, for example, the attractive potential of the isotensor  $\Sigma_c \bar{\Sigma}_c$  is significantly stronger than that of other channels. The calculations of the molecular states of the  $\Sigma_c^{(*)} \Sigma_c^{(*)}$  systems in Ref. [33] are also qualitatively consistent with ours.

TABLE II. The  $I^{(G)}(J^{P(C)})$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma_c^{(*)}\Sigma_c^{(*)}$  and  $\Sigma_c^{(*)}\overline{\Sigma}_c^{(*)}$  systems. The masses/poles are denoted as  $E_B/E_V$  for the bound/virtual states, respectively. The thresholds of corresponding systems are given in the brackets in the first column with the form like " $\Sigma_c \overline{\Sigma}_c$  [4906.9]." The superscript  $\dagger$  represents that the state can also be a near-threshold virtual/bound state if it is labeled as a bound/virtual state. The masses (poles) are given in units of MeV.

Systems [m <sub>th</sub> ]	$I^G(J^{PC})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Sigma_c \bar{\Sigma}_c$ [4906.9]	$0^+(0^{-+})$	$8\tilde{c}_a - 6\tilde{c}_s$		$\Sigma_c \Sigma_c$	$0(0^+)$	$8c_a - 6c_s$	$[4861.6^{+12.0}_{-12.2}]_B$
	$0^{-}(1^{})$	$-\frac{8}{3}\tilde{c}_a-6\tilde{c}_s$	$[4882.0, 4895.2]_B$				121210
	$1^{-}(0^{-+})$	$\frac{8}{3}\tilde{c}_a - 2\tilde{c}_s$			$1(1^{+})$	$-\frac{8}{9}c_a - 2c_s$	$[4906.5^{+0.4}_{-0.9}]_B$
	$1^+(1^{})$	$-\frac{8}{9}\tilde{c}_a-2\tilde{c}_s$	$[4901.8, 4906.6]_V$				
	$2^+(0^{-+})$	$-8\tilde{c}_a + 6\tilde{c}_s$	$[4819.0, 4878.0]_B$		$2(0^{+})$	$-8c_{a}+6c_{s}$	
	$2^{-}(1^{})$	$\frac{8}{3}\tilde{c}_a + 6\tilde{c}_s$					
$\Sigma_c \bar{\Sigma}_c^*$ [4971.6]	$0^{-}(1^{})$	$\frac{22}{3}\tilde{c}_a - 6\tilde{c}_s$		$\Sigma_c \Sigma_c^*$	$0(1^{+})$	$6c_a - 6c_s$	$[4927.0^{+10.3}_{-10.5}]_B$
	$0^+(1^{-+})$	$6\tilde{c}_a - 6\tilde{c}_s$					
	$0^+(2^{-+})$	$-2\tilde{c}_a - 6\tilde{c}_s$	$[4953.0, 4967.2]_B$		$0(2^{+})$	$-2c_{a}-6c_{s}$	$[4932.0^{+7.0}_{-7.0}]_B$
	$0^{-}(2^{})$	$-6\tilde{c}_a - 6\tilde{c}_s$	$[4910.5, 4911.7]_B$		1(1+)	22	
	$1^+(1^{})$	$\frac{22}{9}\tilde{c}_a - 2\tilde{c}_s$	•••		$I(1^{+})$	$\frac{22}{9}c_a - 2c_s$	$[4970.3^{+1.1}_{-2.0}]_B$
	$1^{-}(1^{-+})$ $1^{-}(2^{-+})$	$2c_a - 2c_s$			$1(2^{+})$	2	[40 <b>5</b> 4 <b>2</b> ±0 31 <sup>†</sup>
	$1 (2^{-1})$ $1 + (2^{})$	$-\frac{2}{3}c_a - 2c_s$	$[4930.0, 4970.1]_V$		$I(2^{+})$	$-2c_a - 2c_s$	$[49/1.3^{+0.5}_{-1.3}]_B$
	$\frac{1}{2^{-}(1^{})}$	$-2c_a - 2c_s$	$[4907.3, 4907.7]_B$ [4803 5 4040 1]		$2(1^{+})$	$-6c \pm 6c$	
	$2^{+}(1^{-+})$	$-\frac{1}{3}c_a + 6c_s$	$[4014 \ 0 \ 4961 \ 7]$		2(1)	$-0\mathbf{c}_a + 0\mathbf{c}_s$	
	$2^{+}(2^{-+})$	$2\tilde{c}_a + 6\tilde{c}_s$	[+)1+.0, +)01.7]B		$2(2^{+})$	$2c_{2} + 6c_{2}$	
	$2^{-}(2^{})$	$6\tilde{c}_a + 6\tilde{c}_s$			-(- )	<i>u</i> + <i>s</i>	
$\Sigma_{c}^{*}\bar{\Sigma}_{c}^{*}$ [5036.2]	$0^+(0^{-+})$	$10\tilde{c}_a - 6\tilde{c}_s$		$\sum_{c}^{*} \sum_{c}^{*}$	$0(0^{+})$	$10c_a - 6c_s$	$[4989.0^{+13.8}]_{P}$
	$0^{-}(1^{})$	$\frac{22}{3}\tilde{c}_a - 6\tilde{c}_s$			. ,		L -14.01B
	$0^+(2^{-+})$	$2\tilde{c}_a - 6\tilde{c}_s$			$0(2^{+})$	$2c_a - 6c_s$	$[4994.0^{+7.0}_{-7.0}]_{B}$
	$0^{-}(3^{})$	$-6\tilde{c}_a - 6\tilde{c}_s$	$[4974.7, 4975.9]_B$				
	$1^{-}(0^{-+})$	$\frac{10}{3}\tilde{c}_a - 2\tilde{c}_s$	•••		$1(1^{+})$	$\frac{22}{9}c_a - 2c_s$	$[5034.9^{+1.2}_{-2.1}]_B$
	$1^+(1^{})$	$\frac{22}{9}\tilde{c}_a - 2\tilde{c}_s$					
	$1^{-}(2^{-+})$	$\frac{2}{3}\tilde{c}_a - 2\tilde{c}_s$			$1(3^{+})$	$-2c_{a}-2c_{s}$	$[5035.8^{+0.3}_{-1.4}]^{\dagger}_{B}$
	$1^+(3^{})$	$-2\tilde{c}_a - 2\tilde{c}_s$	$[5031.9, 5032.2]_B$				
	$2^{+}(0^{-+})$	$-10\tilde{c}_a + 6\tilde{c}_s$	$[4916.3, 4985.2]_B$		$2(0^{+})$	$-10c_a + 6c_s$	
	$2^{-}(1^{})$	$-\frac{22}{3}\tilde{c}_a+6\tilde{c}_s$	$[4957.7, 5013.3]_B$			<b>2</b>	
	$2^+(2^{-+})$	$-2\tilde{c}_a + 6\tilde{c}_s$	$[5033.3, 5036.1]_B^{\dagger}$		$2(2^{+})$	$-2c_a+6c_s$	
	2-(3)	$6\tilde{c}_a + 6\tilde{c}_s$					

The results for  $\Sigma_c^{(*)}\Xi_c^{(\prime,*)}$ ,  $\Sigma_c^{(*)}\overline{\Xi}_c^{(\prime,*)}$ , and  $\Xi_c^{(\prime,*)}\Xi_c^{(\prime,*)}$ ,  $\Xi_c^{(\prime,*)}\overline{\Xi}_c^{(\prime,*)}$  systems are presented in Tables III and IV, respectively. The systems such as  $\Xi_c\Xi_c$ ,  $\Xi_c'\Xi_c'$ ,  $\Xi_c^*\Xi_c^*$  as well as the corresponding particle-antiparticle pairs were also considered in Refs. [27,31]. Their results in most channels are in line with ours. It is worth noting that in our framework, the effective potentials of channels  $0^{-}(1^{--})\Xi_c\overline{\Xi}_c$ ,  $0^{\pm}(0^{-\pm})\Xi_c\overline{\Xi}_c'$ ,  $0^{\mp}(2^{-\mp})\Xi_c\overline{\Xi}_c^*$ ,  $0^{+}(1^{-+})\Xi_c'\overline{\Xi}_c^*$ , and  $0^{-}(3^{--})\Xi_c'\overline{\Xi}_c^*$  are exactly the same as that of  $0^{+}(1^{++})D\overline{D}^*$  (see Table IV of Ref. [59]), which means that if *X*(3872) is a weakly bound state of  $D\overline{D}^*$ , there must be corresponding molecular states in these systems as well.

# B. Doubly charmed baryon-(anti)doubly charmed baryon systems

Our results for the  $\Xi_{cc}^{(*)}\Xi_{cc}^{(*)}$  and  $\Xi_{cc}^{(*)}\overline{\Xi}_{cc}^{(*)}$  systems are shown in Table V, where  $\Xi_{cc}^{*}$  has not been observed. Therefore, we refer to both the predictions based on heavy diquark-antiquark symmetry and the calculations of the quark model to obtain a mass difference of  $m_{\Xi_{cc}^{*}} - m_{\Xi_{cc}}$  of  $85 \pm 15$  MeV [60]. As can be seen from Table V, we use the notation  $\pm 15$  MeV to represent the mass uncertainty of  $\Xi_{cc}^{*}$ .

Our calculation gives a binding energy of about 10 MeV in the  $0(1^+)\Xi_{cc}\Xi_{cc}$  channel, which is consistent with the results in Ref. [57]. As for the isovector  $\Xi_{cc}\bar{\Xi}_{cc}$  system,

TABLE III. The  $I(J^P)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma_c^{(*)} \Xi_c^{(*)}$  and  $\Sigma_c^{(*)} \overline{\Xi}_c^{(*)}$  systems. The superscript  $\sharp$  represents that this state might be nonexistent in the range of parameters. Other notations are the same as those in Table II.

Systems [m <sub>th</sub> ]	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Sigma_c \bar{\Xi}_c$ [4922.5]	$\frac{1}{2}(0^{-})$	$6\tilde{c}_a - 3\tilde{c}_s$		$\Sigma_c \Xi_c$	$\frac{1}{2}(0^+)$	$6c_a - 3c_s$	$[4912.0^{+6.0}_{-6.7}]_B$
	$\frac{1}{2}(1^{-})$	$-2\tilde{c}_a - 3\tilde{c}_s$	$[4915.5, 4918.7]_B$		$\frac{1}{2}(1^+)$	$-2c_a - 3c_s$	$[4915.7^{+3.0}_{-3.4}]_B$
	$\frac{3}{2}(0^{-})$	$-6\tilde{c}_a + 3\tilde{c}_s$	$[4864.4, 4901.4]_B$		$\frac{3}{2}(0^+)$	$-6c_a + 3c_s$	
	$\frac{3}{2}(1^{-})$	$2\tilde{c}_a + 3\tilde{c}_s$			$\frac{3}{2}(1^+)$	$2c_a + 3c_s$	
$\Sigma_c \bar{\Xi}'_c$ [5031.7]	$\frac{1}{2}(0^{-})$	$-2\tilde{c}_a - 3\tilde{c}_s$	$[5024.2, 5027.5]_B$	$\Sigma_c \Xi_c'$	$\frac{1}{2}(0^+)$	$-2c_{a} - 3c_{s}$	$[5024.5^{+3.1}_{-3.5}]_B$
	$\frac{1}{2}(1^{-})$	$\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(1^+)$	$\frac{2}{3}c_a - 3c_s$	$[5023.2^{+2.4}_{-2.6}]_B$
	$\frac{3}{2}(0^{-})$	$2\tilde{c}_a + 3\tilde{c}_s$			$\frac{3}{2}(0^+)$	$2c_{a} + 3c_{s}$	
	$\frac{3}{2}(1^{-})$	$-\frac{2}{3}\tilde{c}_a+3\tilde{c}_s$	$[\sim 5010.6]_V^{\sharp}$		$\frac{3}{2}(1^+)$	$-\frac{2}{3}c_a + 3c_s$	
$\Sigma_c \bar{\Xi}_c^*$ [5098.6]	$\frac{1}{2}(1^{-})$	$\frac{10}{3}\tilde{c}_a - 3\tilde{c}_s$		$\Sigma_c \Xi_c^*$	$\frac{1}{2}(1^+)$	$\frac{10}{3}c_a - 3c_s$	$[5088.6^{+4.3}_{-4.7}]_{B}$
	$\frac{1}{2}(2^{-})$	$-2\tilde{c}_a - 3\tilde{c}_s$	$[5090.9, 5094.2]_B$		$\frac{1}{2}(2^+)$	$-2c_a - 3c_s$	$[5091.1^{+3.1}_{-3.5}]_{B}$
	$\frac{3}{2}(1^{-})$	$-\frac{10}{3}\tilde{c}_a+3\tilde{c}_s$	$[5078.7, 5098.1]_B$		$\frac{3}{2}(1^+)$	$-\frac{10}{3}c_a + 3c_s$	
	$\frac{2}{3}(2^{-})$	$2\tilde{c}_a + 3\tilde{c}_s$			$\frac{2}{3}(2^+)$	$2c_a + 3c_s$	
$\Sigma_{c}^{*}\bar{\Xi}_{c}$ [4987.2]	$\frac{1}{2}(1^{-})$	$5\tilde{c}_a - 3\tilde{c}_s$		$\Sigma_c^* \Xi_c$	$\frac{1}{2}(1^+)$	$5c_a - 3c_s$	$[4976.8^{+5.3}_{-6.0}]_{B}$
	$\frac{1}{2}(2^{-})$	$-3\tilde{c}_a - 3\tilde{c}_s$	$[4970.8, 4971.4]_B$		$\frac{1}{2}(2^+)$	$-3c_{a} - 3c_{s}$	$[4980.6^{+3.5}_{-4.0}]_{B}$
	$\frac{3}{2}(1^{-})$	$-5\tilde{c}_a + 3\tilde{c}_s$	$[4943.7, 4975.3]_B$		$\frac{2}{3}(1^+)$	$-5c_{a}+3c_{s}$	
	$\frac{3}{2}(2^{-})$	$3\tilde{c}_a + 3\tilde{c}_s$			$\frac{3}{2}(2^+)$	$3c_{a} + 3c_{s}$	
$\Sigma_{c}^{*}\bar{\Xi}_{c}'$ [5096.3]	$\frac{1}{2}(1^{-})$	$-\frac{5}{3}\tilde{c}_a-3\tilde{c}_s$	$[5091.2, 5094.8]_B$	$\Sigma_c^* \Xi_c'$	$\frac{1}{2}(1^+)$	$-\frac{5}{3}c_a - 3c_s$	$[5088.7^{+3.0}_{-3.3}]_{B}$
	$\frac{1}{2}(2^{-})$	$\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(2^+)$	$c_a - 3c_s$	$[5087.4^{+2.7}_{-2.9}]_B$
	$\frac{3}{2}(1^{-})$	$\frac{5}{3}\tilde{c}_a + 3\tilde{c}_s$			$\frac{3}{2}(1^+)$	$\frac{5}{3}c_a + 3c_s$	
	$\frac{3}{2}(2^{-})$	$-\tilde{c}_a + 3\tilde{c}_s$	$[\sim 5093.1]_V^{\sharp}$		$\frac{3}{2}(2^+)$	$-c_a + 3c_s$	
$\Sigma_{c}^{*}\bar{\Xi}_{c}^{*}$ [5163.2]	$\frac{1}{2}(0^{-})$	$5\tilde{c}_a - 3\tilde{c}_s$		$\Sigma_c^* \Xi_c^*$	$\frac{1}{2}(0^+)$	$5c_a - 3c_s$	$[5152.1^{+5.4}_{-6.1}]_{B}$
	$\frac{1}{2}(1^{-})$	$\frac{11}{3}\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(1^+)$	$\frac{11}{3}c_a - 3c_s$	$[5153.0^{+4.5}_{-5.0}]_B$
	$\frac{1}{2}(2^{-})$	$\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(2^+)$	$c_a - 3c_s$	$[5154.1^{+2.7}_{-2.9}]_B$
	$\frac{1}{2}(3^{-})$	$-3\tilde{c}_a - 3\tilde{c}_s$	$[5146.0, 5146.6]_B$		$\frac{1}{2}(3^+)$	$-3c_a - 3c_s$	$[5156.0^{+3.7}_{-4.0}]_B$
	$\frac{3}{2}(0^{-})$	$-5\tilde{c}_a + 3\tilde{c}_s$	$[5118.7, 5150.6]_B$		$\frac{3}{2}(0^+)$	$-5c_{a}+3c_{s}$	
	$\frac{3}{2}(1^{-})$	$-\frac{11}{3}\tilde{c}_a+3\tilde{c}_s$	$[5138.3, 5161.1]_B$		$\frac{3}{2}(1^+)$	$-\frac{11}{3}c_a + 3c_s$	
	$\frac{3}{2}(2^{-})$	$-\tilde{c}_a + 3\tilde{c}_s$	$[\sim 5160.3]_V^{\sharp}$		$\frac{3}{2}(2^+)$	$-c_a + 3c_s$	
	$\frac{3}{2}(3^{-})$	$3\tilde{c}_a + 3\tilde{c}_s$	•••		$\frac{3}{2}(3^+)$	$3c_a + 3c_s$	

Systems [m <sub>th</sub> ]	$I^G(J^{PC})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_c \bar{\Xi}_c$ [4938.2]	$0^+(0^{-+})$	$\frac{15}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_c \Xi_c$	$0(1^+)$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[4935.5^{+2.0}_{-2.8}]_B$
	$0^{-}(1^{})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[4928.5, 4929.0]_B$		1(0)	0 2	
	$1^{-}(0^{-+})$ $1^{+}(1^{})$	$-\frac{9}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[4902.0, 4925.5]_B$		$1(0^{+})$	$-\frac{9}{2}c_a + \frac{3}{2}c_s$	•••
 - - - - - - - - - - - - - - - - - -	$(1)^{+}(0^{-+})$	$2c_a + 2c_s$	[5027.2, 5027.6]		$0(0^{+})$	5 5	[5044 2+2]]
$\Xi_c \Xi_c' [5047.3]$	$0^{-}(0^{-})$ $0^{\mp}(1^{-\mp})$	$-\frac{1}{2}c_a - \frac{1}{2}c_s$ $\frac{5}{2}\tilde{c} - \frac{5}{2}\tilde{c}$	$[5057.2, 5057.0]_B$	$\underline{\neg}_{c}\underline{\neg}_{c}$	$0(0^{+})$ $0(1^{+})$	$-\frac{1}{2}c_a - \frac{1}{2}c_s$ $\frac{5}{2}c_a - \frac{5}{2}c_s$	$[5044.3_{-2.9}]_B$ $[5043.1^{+1.8}]$
	$1^{\mp}(0^{-\pm})$	$\frac{6}{3}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(0^+)$	$\frac{6}{3}c_{a} + \frac{3}{2}c_{s}$	[5015.1_2.0] <i>B</i>
	$1^{\pm}(1^{-\mp})$	$-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 5005.1]_V^{\sharp}$		$1(1^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	•••
$\Xi_c \bar{\Xi}_c^*$ [5114.2]	$0^{\pm}(1^{-\pm})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_c \Xi_c^*$	$0(1^{+})$	$\frac{25}{6}c_a - \frac{5}{2}c_s$	$[5108.4^{+3.7}_{-4.5}]_{B}$
	$0^{\mp}(2^{-\mp})$	$-\frac{5}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	$[5103.8, 5104.3]_B$		$0(2^+)$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[5111.0^{+2.2}_{-2.9}]_B$
	$1^{\mp}(1^{-\pm})$	$-\frac{5}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[5104.8, 5114.1]_B$		$1(1^+)$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1^{\pm}(2^{-+})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	•••		$1(2^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
$\Xi_c' \bar{\Xi}_c'$ [5156.4]	$0^+(0^{-+})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_c' \Xi_c'$	$0(1^{+})$	$-\frac{5}{18}c_a - \frac{5}{2}c_s$	$[5152.3^{+1.5}_{-1.7}]_V$
	$0^{-}(1^{})$ $1^{-}(0^{-+})$	$-\frac{5}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	$[5051.6, 5153.5]_V$		$1(0^{+})$	1 3 .	
	$1^{+}(1^{})$	$-\frac{1}{2}c_a + \frac{1}{2}c_s$ $\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 5117.4]_V^*$		$I(0^{+})$	$-\frac{1}{2}c_a+\frac{1}{2}c_s$	
₽/ ₽* [5222.2]	$0^{-}(1^{})$	$5 \approx 5 \approx$	[5122 7 5220 6]	<b>□</b> / <b>□</b> *	0(1+)	5 . 5 .	[5210.0+1.5]
$\Xi_c \Xi_c [3223.3]$	$0^{+}(1^{-+})$	$-\frac{1}{18}C_a - \frac{1}{2}C_s$ $-\frac{5}{2}\tilde{C}_a - \frac{5}{2}\tilde{C}_a$	$[5122.7, 5220.0]_V$ $[5212.5, 5212.9]_P$		0(1*)	$-\frac{1}{18}c_a - \frac{1}{2}c_s$	$[5219.0^{+1.7}_{-1.7}]_B$
	$0^+(2^{-+})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	····		$0(2^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[5219.9^{+2.3}_{-3.0}]_{B}$
	$0^{-}(2^{})$	$-\frac{5}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	$[5212.5, 5212.9]_B$			2 2	5.015
	$1^{+}(1^{})$	$\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	••••		$1(1^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
	$1^{-}(1^{-+})$ $1^{-}(2^{-+})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	 [5212 5 5222 2]		$1(2^{+})$	5 3 .	
	$1^{+}(2^{})$	$-\frac{1}{2}c_a + \frac{1}{2}c_s$ $\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[5215.5, 5225.5]_B$		$I(2^{+})$	$-\frac{1}{2}c_a+\frac{1}{2}c_s$	
⊐*⊡* [5200.2]	$0^+(0^{-+})$	$2^{\circ}a + 2^{\circ}s$		<b>*</b> * <b>*</b>	$0(1^{+})$	55 0 5 0	[5204 2+3.2]
$\Xi_c \Xi_c [3290.2]$	$0^{-}(1^{})$	$\frac{\overline{6}}{\overline{6}} c_a - \frac{\overline{2}}{\overline{2}} c_s$ $\frac{55}{\overline{6}} \tilde{c}_a - \frac{5}{\overline{6}} \tilde{c}_s$		$\underline{\neg c} \underline{\neg c}$	0(1)	$\frac{1}{18}c_a - \frac{1}{2}c_s$	$[3284.3_{-3.7}]_B$
	$0^+(2^{-+})$	$\frac{18}{5}\tilde{c}_{a}^{2}-\frac{5}{2}\tilde{c}_{s}^{2}$			$0(3^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[5286.6^{+2.4}_{-3.1}]_{B}$
	0-(3)	$-\frac{5}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	$[5279.1, 5279.6]_B$			2 2	. 5.175
	$1^{-}(0^{-+})$	$-\frac{5}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[5280.1, 5290.1]_B$		$1(0^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1^+(1^{})$ $1^-(2^{-+})$	$-\frac{11}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[5286.4, 5290.2]_V$		$1(2^{+})$	1 3	
	1 (2') $1^+(3^{})$	$-\frac{1}{2}c_a + \frac{1}{2}\dot{c}_s$	$[\sim 5254.6]_V^*$		$I(2^{+})$	$-\frac{1}{2}c_a+\frac{3}{2}c_s$	
	1 (5 )	$\overline{2}c_a + \overline{2}c_s$					

TABLE IV. The  $I^{(G)}(J^{P(C)})$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Xi_c^{(\prime,*)}\Xi_c^{(\prime,*)}$  and  $\Xi_c^{(\prime,*)}\bar{\Xi}_c^{(\prime,*)}$  systems. The notations are the same as those in Tables II and III.

Ref. [57] also obtained a bound state, while in our calculation, the  $1^{-}(0^{-+})\Xi_{cc}\bar{\Xi}_{cc}$  may be a virtual state or may not exist within the parameter range.

We also obtain bound states in the isoscalar  $\Xi_{cc}^{(*)}\Xi_{cc}^*$ systems, with binding energies around 10 MeV. The results in Ref. [58] are consistent with our calculations. Furthermore, most of the results of  $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^*$  systems in Ref. [58] agree with ours.

The  $0^+(1^{-+})$  and  $0^-(2^{--})\Xi_{cc}\bar{\Xi}_{cc}^*$  as well as the  $0^-(3^{--})\Xi_{cc}^*\bar{\Xi}_{cc}^*$  can be regarded as the quadruply heavy counterparts of X(3872), since they share the same

effective potentials under the heavy diquark-antiquark symmetry [see Table IV of Ref. [59] for the effective potentials of X(3872)].

### C. Hyperon-(anti)hyperon systems

The results of the hyperon-(anti)hyperon systems are shown in Tables VI and VII. Under the extended heavy quark symmetry, this type of system is very similar to the charmed baryon-(anti)charmed baryon ones. In our model, the  $\Lambda\Lambda$  system lacks significant isospin-isospin coupling interactions, resulting in weak interactions that make it

Systems $[m_{\rm th}]$	$I^G(J^{PC})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_{cc}\bar{\Xi}_{cc}$ [7243.1]	$0^+(0^{-+})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_{cc}\Xi_{cc}$	$0(1^+)$	$-\frac{5}{18}c_a - \frac{5}{2}c_s$	$[7233.5^{+2.1}_{-2.2}]_B$
	$0^{-}(1^{})$ $1^{-}(0^{-+})$ $1^{+}(1^{})$	$-\frac{5}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[7209.0, 7243.1]_V$ $[\sim 7232.8]_V^{\sharp}$ 		$1(0^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}\bar{\Xi}^*_{cc}$ [7328.1 ± 15]	$0^{-}(1^{})$ $0^{+}(1^{-+})$	$-\frac{5}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{5}{2}\tilde{c}_s - \frac{5}{2}\tilde{c}_s$	$[7311.7 \pm 16.4 \pm 15]_V$ $[7310.7 \pm 0.3 \pm 15]_V$	$\Xi_{cc}\Xi_{cc}^{*}$	$0(1^{+})$	$-\frac{5}{18}c_a - \frac{5}{2}c_s$	$[7318.3^{+2.1}_{-2.2} \pm 15]_B$
	$0^{+}(2^{-+})$ $0^{-}(2^{})$	$\frac{-\frac{1}{2}c_a - \frac{1}{2}c_s}{\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s}$ $-\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$	$[7310.7 \pm 0.3 \pm 15]_B$ 		$0(2^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[7319.4^{+3.5}_{-3.8}\pm15]_B$
	$1^+(1^{})$ $1^-(1^{-+})$	$\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $\frac{3}{2}\tilde{c}_s + \frac{3}{2}\tilde{c}_s$	···· []R		$1(1^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
	$1^{-}(2^{-+})$ $1^{+}(2^{})$	$-\frac{5}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $-\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[7318.6 \pm 7.0 \pm 15]_B$		$1(2^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}^* \bar{\Xi}_{cc}^*$ [7413.1 ± 30]	$0^+(0^{-+})$ $0^-(1^{})$	$\frac{\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s}{\frac{55}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s}$		$\Xi_{cc}^*\Xi_{cc}^*$	$0(1^{+})$	$\frac{55}{18}c_a - \frac{5}{2}c_s$	$[7401.3^{+4.2}_{-4.5}\pm30]_B$
	$0^+(2^{-+})$ $0^-(3^{})$	$\frac{\frac{5}{6}\tilde{c}_{a}}{-\frac{5}{2}\tilde{c}_{s}} - \frac{\frac{5}{2}\tilde{c}_{s}}{-\frac{5}{2}\tilde{c}_{s}}$	$[7395.5 \pm 0.3 \pm 30]_{B}$		$0(3^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[7404.2^{+3.6}_{-3.9}\pm30]_B$
	$1^{-}(0^{-+})$ $1^{+}(1^{})$	$-\frac{5}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[7403.4 \pm 7.0 \pm 30]_B$		$1(0^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1^{-}(2^{-+})$	$-\frac{1}{6}c_a + \frac{1}{2}c_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[7409.2 \pm 3.9 \pm 30]_B^{\sharp}$ $[\sim 7403.8 \pm 30]_V^{\sharp}$		$1(2^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
	$1^+(3^{})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	•••				

TABLE V. The  $I^{(G)}(J^{P(C)})$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Xi_{cc}^{(*)}\Xi_{cc}^{(*)}$  and  $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$  systems. The notations are the same as those in Tables II and III.

TABLE VI. The  $I^{(G)}(J^{P(C)})$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma^{(*)}\Sigma^{(*)}$  and  $\Sigma^{(*)}\overline{\Sigma}^{(*)}$  systems. The notations are the same as those in Tables II and III.

Systems [m <sub>th</sub> ]	$I^G(J^{PC})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
ΣΣ [2386.3]	$0^+(0^{-+})$	$8\tilde{c}_a - 6\tilde{c}_s$		ΣΣ	$0(0^+)$	$8c_a - 6c_s$	$[2368.5^{+8.9}_{-10.1}]_{B}$
	$0^{-}(1^{})$	$-\frac{8}{3}\tilde{c}_a-6\tilde{c}_s$	$[2382.5, 2386.3]_B^{\dagger}$				-10.13D
	$1^{-}(0^{-+})$	$\frac{8}{3}\tilde{c}_a - 2\tilde{c}_s$			$1(1^{+})$	$-\frac{8}{9}c_a - 2c_s$	$[2363.2^{+7.9}_{-11.4}]_V$
	$1^+(1^{})$	$-\frac{8}{9}\tilde{c}_a - 2\tilde{c}_s$	$[2274.2, 2336.6]_V$			,	
	$2^+(0^{-+})$	$-8\tilde{c}_a + 6\tilde{c}_s$	$[2331.3, 2380.2]_{B}$		$2(0^{+})$	$-8c_{a}+6c_{s}$	
	$2^{-}(1^{})$	$\frac{8}{3}\tilde{c}_a + 6\tilde{c}_s$					
$\Sigma \bar{\Sigma}^{*}$ [2577.7]	0-(1)	$\frac{22}{3}\tilde{c}_a - 6\tilde{c}_s$		$\Sigma\Sigma^*$	$0(1^+)$	$6c_a - 6c_s$	$[2557.8^{+8.1}_{-8.9}]_{B}$
	$0^+(1^{-+})$	$6\tilde{c}_a - 6\tilde{c}_s$					-0.715
	$0^{+}(2^{-+})$	$-2\tilde{c}_a - 6\tilde{c}_s$	$[2574.6, 2577.7]^{\dagger}_{B}$		$0(2^{+})$	$-2c_a - 6c_s$	$[2561.7^{+5.2}_{-5.7}]_{B}$
	$0^{-}(2^{})$	$-6\tilde{c}_a - 6\tilde{c}_s$	$[2543.6, 2544.7]_B$				
	$1^+(1^{})$	$\frac{22}{9}\tilde{c}_a - 2\tilde{c}_s$			$1(1^{+})$	$\frac{22}{9}c_a - 2c_s$	$[2566.5^{+6.2}_{-11.3}]_V$
	$1^{-}(1^{-+})$	$2\tilde{c}_a - 2\tilde{c}_s$				ŕ	
	$1^{-}(2^{-+})$	$-\frac{2}{3}\tilde{c}_a-2\tilde{c}_s$	$[2409.1, 2521.6]_V$		$1(2^{+})$	$-2c_{a}-2c_{s}$	$[2560.0^{+8.7}_{-14.9}]_V$
	$1^+(2^{})$	$-2\tilde{c}_a - 2\tilde{c}_s$	$[2573.8, 2574.2]_V$				
	$2^{-}(1^{})$	$-\frac{22}{3}\tilde{c}_a+6\tilde{c}_s$	$[2528.3, 2573.7]_B$		$2(1^{+})$	$-6c_{a}+6c_{s}$	
	$2^+(1^{-+})$	$-6\tilde{c}_a + 6\tilde{c}_s$	$[2546.7, 2577.7]_{B}^{\dagger}$				
	$2^+(2^{-+})$	$2\tilde{c}_a + 6\tilde{c}_s$			$2(2^{+})$	$2c_{a} + 6c_{s}$	
	$2^{-}(2^{})$	$6\tilde{c}_a + 6\tilde{c}_s$	•••		. ,		

(Table continued)

Systems [m <sub>th</sub> ]	$I^G(J^{PC})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
Σ*Σ̄* [2769.2]	$0^+(0^{-+})$	$10\tilde{c}_a - 6\tilde{c}_s$	••••	$\Sigma^*\Sigma^*$	$0(0^+)$	$10c_a - 6c_s$	$[2744.0^{+11.4}]_{P}$
	$0^{-}(1^{})$	$\frac{22}{3}\tilde{c}_a - 6\tilde{c}_s$			. ,		-12.5 JB
	$0^+(2^{-+})$	$2\tilde{c}_a - 6\tilde{c}_s$			$0(2^{+})$	$2c_a - 6c_s$	$[2748.0^{+5.7}_{-6.0}]_{B}$
	0-(3)	$-6\tilde{c}_a - 6\tilde{c}_s$	$[2731.4, 2732.5]_B$				-0.015
	$1^{-}(0^{-+})$	$\frac{10}{3}\tilde{c}_a - 2\tilde{c}_s$			$1(1^{+})$	$\frac{22}{9}c_a - 2c_s$	$[2762.3^{+4.4}_{-8.5}]_V$
	$1^+(1^{})$	$\frac{22}{9}\tilde{c}_a - 2\tilde{c}_s$					
	$1^{-}(2^{-+})$	$\frac{2}{3}\tilde{c}_a - 2\tilde{c}_s$			$1(3^{+})$	$-2c_a - 2c_s$	$[2757.0^{+6.4}_{-11.5}]_V$
	$1^+(3^{})$	$-2\tilde{c}_a - 2\tilde{c}_s$	$[2767.3, 2767.6]_V$				-11.537
	$2^+(0^{-+})$	$-10\tilde{c}_a + 6\tilde{c}_s$	$[2676.1, 2740.8]_B$		$2(0^{+})$	$-10c_{a}+6c_{s}$	
	$2^{-}(1^{})$	$-\frac{22}{3}\tilde{c}_{a}+6\tilde{c}_{s}$	$[2715.7, 2763.2]_B$				
	$2^+(2^{-+})$	$-2\tilde{c}_a+6\tilde{c}_s$	$[\sim 2765.2]_V^{\sharp}$		$2(2^{+})$	$-2c_{a}+6c_{s}$	
	2-(3)	$6\tilde{c}_a + 6\tilde{c}_s$	••••				

TABLE VI. (Continued)

TABLE VII. The  $I^{(G)}(J^{P(C)})$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma^{(*)}\Xi^{(*)}$ ,  $\Sigma^{(*)}\bar{\Xi}^{(*)}$ ,  $\Xi^{(*)}\Xi^{(*)}$ ,  $\Xi^{(*)}\bar{\Xi}^{(*)}$ , and  $\Xi^{(*)}\bar{\Xi}^{(*)}$  systems. The notations are the same as those in Tables II and III.

Systems $[m_{\rm th}]$	$I^{(G)}(J^{P(C)})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
ΣΞ̈́ [2511.4]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$-2\tilde{c}_a - 3\tilde{c}_s$ $\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$ $-\frac{2}{3}\tilde{c}_a + 3\tilde{c}_s$	$[2506.9, 2510.3]_V$	ΣΞ	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{3}{2}(0^{+})$ $\frac{3}{2}(1^{+})$	$-2c_a - 3c_s$ $\frac{2}{3}c_a - 3c_s$ $2c_a + 3c_s$ $-\frac{2}{3}c_a + 3c_s$	$ [2510.2^{+1.2}_{-3.3}]_V  [2510.8^{+0.6}_{-1.6}]_V  \dots \\ \dots \\ \dots \\ \dots $
ΣĒ* [2726.6]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$ \begin{array}{l} \frac{10}{3} \tilde{c}_a - 3 \tilde{c}_s \\ -2 \tilde{c}_a - 3 \tilde{c}_s \\ -\frac{10}{3} \tilde{c}_a + 3 \tilde{c}_s \\ 2 \tilde{c}_a + 3 \tilde{c}_s \end{array} $	$[2724.2, 2726.3]_V$ $[2712.3, 2726.6]_V$ $\dots$	∑ <b>Ξ</b> *	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$\frac{10}{3}c_a - 3c_s$ $-2c_a - 3c_s$ $-\frac{10}{3}c_a + 3c_s$ $2c_a + 3c_s$	$ \begin{bmatrix} 2726.5^{+0.1}_{-1.2} \end{bmatrix}_B^{\dagger} \\ \begin{bmatrix} 2726.2^{+0.3}_{-2.1} \end{bmatrix}_V^{\dagger} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$
Σ*Ξ [2702.9]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$-\frac{5}{3}\tilde{c}_a - 3\tilde{c}_s$ $\tilde{c}_a - 3\tilde{c}_s$ $\frac{5}{3}\tilde{c}_a + 3\tilde{c}_s$ $-\tilde{c}_a + 3\tilde{c}_s$	$[2694.8, 2701.3]_V$   $[\sim 2636.9]_V^{\sharp}$	∑*Ξ	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$-\frac{5}{3}c_a - 3c_s$ $c_a - 3c_s$ $\frac{5}{3}c_a + 3c_s$ $-c_a + 3c_s$	$ [2702.6^{+0.3}_{-1.7}]^{\dagger}_{V} \\ [2702.8^{+0.1}_{-0.8}]^{\dagger}_{V} \\ \cdots \\ \cdots \\ \cdots$
Σ*Ξ <sup>*</sup> [2918.0]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{1}{2}(2^{-})}{\frac{1}{2}(3^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$ $\frac{3}{2}(2^{-})$ $\frac{3}{2}(2^{-})$ $\frac{3}{2}(3^{-})$	$5\tilde{c}_{a} - 3\tilde{c}_{s}$ $\frac{11}{3}\tilde{c}_{a} - 3\tilde{c}_{s}$ $\tilde{c}_{a} - 3\tilde{c}_{s}$ $-3\tilde{c}_{a} - 3\tilde{c}_{s}$ $-5\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\frac{11}{3}\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\tilde{c}_{a} + 3\tilde{c}_{s}$ $3\tilde{c}_{a} + 3\tilde{c}_{s}$	$[2914.7, 2915.0]_B$ $[2893.8, 2916.9]_B$ $[2909.7, 2918.0]_B^{\dagger}$ $\dots$	Σ*Ξ*	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{1}{2}(2^{+})}{\frac{1}{2}(3^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$ $\frac{3}{2}(2^{+})$ $\frac{3}{2}(3^{+})$	$5c_a - 3c_s$ $\frac{11}{3}c_a - 3c_s$ $c_a - 3c_s$ $-3c_a - 3c_s$ $-5c_a + 3c_s$ $-\frac{11}{3}c_a + 3c_s$ $-c_a + 3c_s$ $3c_a + 3c_s$	$ \begin{array}{c} [2917.4^{+0.5}_{-2.7}]^{+}_{B} \\ [2917.6^{+0.4}_{-1.9}]^{+}_{B} \\ [2917.9^{+0.1}_{-0.7}]^{+}_{B} \\ [2917.9^{+0.1}_{-0.6}]^{+}_{V} \\ \dots \end{array} $
EĒ [2636.6]	$\begin{array}{c} 0^+(0^{-+})\\ 0^-(1^{})\\ 1^-(0^{-+})\\ 1^+(1^{}) \end{array}$	$\frac{5}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s} \\ -\frac{5}{18}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s} \\ -\frac{1}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s} \\ \frac{1}{6}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s} $	$[\sim 2559.9]_V^{\sharp}$	ΞΞ	$0(1^+)$ $1(0^+)$	$-\frac{5}{18}c_{a} - \frac{5}{2}c_{s}$ $-\frac{1}{2}c_{a} + \frac{3}{2}c_{s}$	$[2633.1^{+1.8}_{-2.8}]_V$

(Table continued)

Systems $[m_{\rm th}]$	$I^{(G)}(J^{P(C)})$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
ΞĒ* [2851.7]	0-(1)	$-\frac{5}{18}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[\sim 2796.4]_V^{\sharp}$	$\Xi\Xi^*$	$0(1^+)$	$-\frac{5}{18}c_a - \frac{5}{2}c_s$	$[2850.1^{+1.1}_{-1.9}]_V$
	$0^+(1^{-+})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[2851.4, 2851.6]_B$				
	$0^+(2^{-+})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(2^+)$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[2849.1^{+2.2}_{-5.4}]_V$
	$0^{-}(2^{})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[2851.4, 2851.6]_B$				
	$1^+(1^{})$	$\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(1^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
	$1^{-}(1^{-+})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$					
	$1^{-}(2^{-+})$	$-\frac{5}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[2834.6, 2851.7]_V^\dagger$		$1(2^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1^+(2^{})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$					
Ξ*Ξ̃* [3066.8]	$0^+(0^{-+})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi^*\Xi^*$	$0(1^{+})$	$\frac{55}{18}c_a - \frac{5}{2}c_s$	$[3066.7^{+0.1}_{-1.8}]^{\dagger}_{V}$
	$0^{-}(1^{})$	$\frac{55}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$				10 2	- 1017
	$0^+(2^{-+})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(3^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[3065.8^{+1.0}_{-3.6}]_V$
	$0^{-}(3^{})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[3065.9, 3066.1]_B$				
	$1^{-}(0^{-+})$	$-\frac{5}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[3055.4, 3066.8]_V^\dagger$		$1(0^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1^+(1^{})$	$-\frac{11}{6}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[3016.7, 3065.1]_V$				
	$1^{-}(2^{-+})$	$-\frac{1}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$			$1(2^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
	$1^+(3^{})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	•••				

#### TABLE VII. (Continued)

difficult to form bound states. Hence, we have focused our attention on the two-body systems containing  $\Sigma^{(*)}$  and  $\Xi^{(*)}$  hyperons.

In Ref. [70], Polinder *et al.* investigated the leading-order interaction of the  $\Sigma\Sigma$  system using the chiral effective field theory. They obtained large scattering length in the  $\Sigma\Sigma$ system when the effective potential is iterated into the LS equation. Additionally, they observed that the scattering cross-section in the  $0(0^+)$  channel of the  $\Sigma\Sigma$  system can become significantly large near the threshold. This implies that the  $\Sigma\Sigma$  system exhibits strong attractive interactions in the  $0(0^+)$  partial wave, which could potentially lead to the formation of a bound state close to the threshold. This implication is consistent with our calculations, for example, we obtained a bound state with binding energy about 20 MeV in the  $0(0^+)$  channel (see Table VI).

The mass spectra of other systems can be directly obtained from Tables VI and VII.

# D. Charmed baryon-(anti)doubly charmed baryon systems

The spectra of charmed baryon-(anti)doubly charmed baryon systems are given in Tables VIII and IX.

TABLE VIII.	The $I(J^P)$ quantum numbers, effective potentials, and bound/virtual state solutions of the $\Xi_{cc}^{(*)}\Sigma_c^{(*)}, \Xi_{cc}^{(*)}\overline{\Sigma}_c^{(*)}, \Xi_{cc}\Xi_c^{(/,*)},$	
and $\Xi_{cc}\bar{\Xi}_{c}^{(\prime,*)}$ s	systems. The notations are the same as those in Tables II and III.	

Systems [m <sub>th</sub> ]	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\overline{\Xi_{cc}}\overline{\Sigma}_{c}$ [6075.0]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$-2\tilde{c}_a - 3\tilde{c}_s$ $\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$ $-\frac{2}{3}\tilde{c}_a + 3\tilde{c}_s$	$[6064.6, 6068.4]_B$   $[\sim 6064.8]_V^{\sharp}$	$\Xi_{cc}\Sigma_{c}$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$	$-2c_a - 3c_s$ $\frac{2}{3}c_a - 3c_s$ $2c_a + 3c_s$ $-\frac{2}{3}c_a + 3c_s$	$ \begin{array}{c} [6064.9^{+3.5}_{-3.8}]_B \\ [6063.5^{+2.7}_{-2.8}]_B \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$
$\Xi_{cc}\bar{\Sigma}_{c}^{*}$ [6139.7]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$\begin{aligned} &-\frac{5}{3}\tilde{c}_a-3\tilde{c}_s\\ &\tilde{c}_a-3\tilde{c}_s\\ &\frac{5}{3}\tilde{c}_a+3\tilde{c}_s\\ &-\tilde{c}_a+3\tilde{c}_s\end{aligned}$	$[6131.9, 6136.3]_B$   $[\sim 6138.9]_V^{\sharp}$	$\Xi_{cc}\Sigma_{c}^{*}$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$-\frac{5}{3}c_a - 3c_s$ $c_a - 3c_s$ $\frac{5}{3}c_a + 3c_s$ $-c_a + 3c_s$	$ \begin{bmatrix} 6129.1^{+3.4}_{-3.6} \end{bmatrix}_B \\ \begin{bmatrix} 6127.7^{+3.0}_{-3.1} \end{bmatrix}_B \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$

(Table continued)

Systems $[m_{\rm th}]$	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_{cc}\bar{\Xi}_{c}$ [6090.6]	0(0-)	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[6077.4, 6077.8]_B$	$\Xi_{cc}\Xi_{c}$	$0(0^+)$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[6085.4^{+2.9}_{-3.4}]_B$
	$0(1^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(1^{+})$	$\frac{5}{6}c_a - \frac{5}{2}c_s$	$[6083.9^{+2.2}_{-2.3}]_B$
	$1(0^{-})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(0^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
	$1(1^{-})$	$-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 6067.2]_V^{\sharp}$		$1(1^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}\bar{\Xi}_{c}'$ [6199.8]	$0(0^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_{cc}\Xi_{c}'$	$0(0^+)$	$\frac{5}{6}c_a - \frac{5}{2}c_s$	$[6192.6^{+2.2}_{-2.4}]_B$
	$0(1^{-})$	$-\frac{5}{18}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[6136.4, 6199.1]_V$		$0(1^{+})$	$-\frac{5}{18}c_a - \frac{5}{2}c_s$	$[6193.1^{+1.8}_{-2.0}]_B$
	$1(0^{-})$	$-\frac{1}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[\sim 6178.5]_V^{\sharp}$		$1(0^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
	$1(1^{-})$	$\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(1^{+})$	$\frac{1}{6}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}\bar{\Xi}_{c}^{*}$ [6266.7]	$0(1^{-})$	$-\frac{25}{18}\tilde{c}_{a}-\frac{5}{2}\tilde{c}_{s}$	$[6262.4, 6265.4]_B$	$\Xi_{cc}\Xi_{c}^{*}$	$0(1^{+})$	$-\frac{25}{18}c_a - \frac{5}{2}c_s$	$[6260.3^{+2.5}_{-2.7}]_{B}$
	$0(2^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(2^{+})$	$\frac{5}{6}c_a - \frac{5}{2}c_s$	$[6259.3^{+2.2}_{-2.4}]_B$
	$1(1^{-})$	$\frac{5}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(1^{+})$	$\frac{5}{6}c_a + \frac{3}{2}c_s$	
	$1(2^{-})$	$-\frac{1}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[\sim 6245.9]_V^{\sharp}$		$1(2^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}^* \bar{\Sigma}_c  [6160.0 \pm 15]$	$\frac{1}{2}(1^{-})$	$\frac{10}{3}\tilde{c}_a - 3\tilde{c}_s$		$\Xi_{cc}^*\Sigma_c$	$\frac{1}{2}(1^+)$	$\frac{10}{3}c_a - 3c_s$	$[6147.0^{+4.7}_{-5.0} \pm 15]_{B}$
	$\frac{1}{2}(2^{-})$	$-2\tilde{c}_a - 3\tilde{c}_s$	$[6151.3 \pm 1.9 \pm 15]_B$		$\frac{1}{2}(2^+)$	$-2c_a - 3c_s$	$[6149.7^{+3.6}_{-3.8} \pm 15]_B$
	$\frac{3}{2}(1^{-})$	$-\frac{10}{3}\tilde{c}_a+3\tilde{c}_s$	$[6147.4 \pm 10.7 \pm 15]_B$		$\frac{3}{2}(1^+)$	$-\frac{10}{3}c_a + 3c_s$	
	$\frac{3}{2}(2^{-})$	$2\tilde{c}_a + 3\tilde{c}_s$			$\frac{3}{2}(2^+)$	$2c_{a} + 3c_{s}$	
$\Xi_{cc}^* \bar{\Sigma}_c^*  \left[ 6224.7 \pm 15  ight]$	$\frac{1}{2}(0^{-})$	$5\tilde{c}_a - 3\tilde{c}_s$		$\Xi_{cc}^*\Sigma_c^*$	$\frac{1}{2}(0^{+})$	$5c_a - 3c_s$	$[6210.4^{+6.0}_{-6.4}\pm15]_B$
	$\frac{1}{2}(1^{-})$	$\frac{11}{3}\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(1^+)$	$\frac{11}{3}c_a - 3c_s$	$[6211.1^{+5.0}_{-5.3}\pm15]_B$
	$\frac{1}{2}(2^{-})$	$\tilde{c}_a - 3\tilde{c}_s$			$\frac{1}{2}(2^+)$	$c_a - 3c_s$	$[6212.5^{+3.0}_{-3.1} \pm 15]_B$
	$\frac{1}{2}(3^{-})$	$-3\tilde{c}_a - 3\tilde{c}_s$	$[6204.2 \pm 0.3 \pm 15]_B$		$\frac{1}{2}(3^+)$	$-3c_a - 3c_s$	$[6214.6^{+4.2}_{-4.6}\pm15]_B$
	$\frac{3}{2}(0^{-})$	$-5\tilde{c}_a + 3\tilde{c}_s$	$[6192.3 \pm 16.4 \pm 15]_B$		$\frac{3}{2}(0^+)$	$-5c_{a}+3c_{s}$	• • •
	$\frac{3}{2}(1^{-})$	$-\frac{11}{3}\tilde{c}_a+3\tilde{c}_s$	$[6208.3 \pm 12.3 \pm 15]_B$		$\frac{3}{2}(1^+)$	$-\frac{11}{3}c_a + 3c_s$	
	$\frac{3}{2}(2^{-})$	$-\tilde{c}_a + 3\tilde{c}_s$	$[\sim\!6224.0\pm15]_V^\dagger$		$\frac{3}{2}(2^+)$	$-c_a + 3c_s$	
	$\frac{3}{2}(3^{-})$	$3\tilde{c}_a + 3\tilde{c}_s$	•••		$\frac{3}{2}(3^+)$	$3c_{a} + 3c_{s}$	•••

TABLE VIII. (Continued)

TABLE IX. The  $I(J^p)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Xi_{cc}^* \Xi_c^{(\prime,*)}$  and  $\Xi_{cc}^* \bar{\Xi}_c^{(\prime,*)}$  systems. The notations are the same as those in Tables II and III.

Systems $[m_{\rm th}]$	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_{cc}^*\bar{\Xi}_c \ [6175.6 \pm 15]$	$0(1^{-})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_{cc}^*\Xi_c$	$0(1^+)$	$\frac{25}{6}c_a - \frac{5}{2}c_s$	$[6167.2^{+4.4}_{-4.9} \pm 15]_B$
	$0(2^{-})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[6162.4 \pm 0.2 \pm 15]_B$		$0(2^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[6170.3^{+2.9}_{-3.4} \pm 15]_B$
	$1(1^{-})$	$-\frac{\tilde{5}}{2}\tilde{c}_a+\frac{\tilde{3}}{2}\tilde{c}_s$	$[6169.1 \pm 5.8 \pm 15]_B$		$1(1^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1(2^{-})$	$\frac{3}{2}\tilde{\tilde{c}}_a + \frac{3}{2}\tilde{\tilde{c}}_s$			$1(2^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}^* \bar{\Xi}_c'  [6284.8 \pm 15]$	$0(1^{-})$	$-\frac{25}{18}\tilde{c}_{a}-\frac{5}{2}\tilde{c}_{s}$	$[6282.1 \pm 1.5 \pm 15]_B$	$\Xi_{cc}^*\Xi_c'$	$0(1^{+})$	$-\frac{25}{18}c_a - \frac{5}{2}c_s$	$[6278.5^{+2.5}_{-2.7}\pm15]_B$
	$0(2^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(2^{+})$	$\frac{5}{6}c_a - \frac{5}{2}c_s$	$[6277.5^{+2.2}_{-2.4}\pm15]_B$
	$1(1^{-})$	$\frac{5}{6}\tilde{c}_{a}+\frac{3}{2}\tilde{c}_{s}$			$1(1^{+})$	$\frac{5}{6}c_a + \frac{3}{2}c_s$	
	$1(2^{-})$	$-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 6264.4 \pm 15]_V^{\sharp}$		$1(2^{+})$	$-\frac{1}{2}c_a + \frac{3}{2}c_s$	
$\Xi_{cc}^* \bar{\Xi}_c^*  [6351.7 \pm 15]$	$0(0^{-})$	$\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$		$\Xi_{cc}^*\Xi_c^*$	$0(0^+)$	$\frac{25}{6}c_a - \frac{5}{2}c_s$	$[6342.5^{+4.5}_{-5.0} \pm 15]_B$
	$0(1^{-})$	$\frac{55}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(1^{+})$	$\frac{55}{18}c_a - \frac{5}{2}c_s$	$[6343.0^{+3.8}_{-4.1} \pm 15]_B$
	$0(2^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$			$0(2^{+})$	$\frac{5}{6}c_a - \frac{5}{2}c_s$	$[6344.1^{+2.3}_{-2.4} \pm 15]_B$
	$0(3^{-})$	$-\frac{5}{2}\tilde{c}_a-\frac{5}{2}\tilde{c}_s$	$[6337.6 \pm 0.2 \pm 15]_B$		$0(3^{+})$	$-\frac{5}{2}c_a - \frac{5}{2}c_s$	$[6345.7^{+3.1}_{-3.5} \pm 15]_{B}$
	$1(0^{-})$	$-\frac{5}{2}\tilde{c}_a+\frac{3}{2}\tilde{c}_s$	$[6344.6 \pm 6.2 \pm 15]_B$		$1(0^{+})$	$-\frac{5}{2}c_a + \frac{3}{2}c_s$	
	$1(1^{-})$	$-\frac{\tilde{11}}{6}\tilde{c}_a+\frac{\tilde{3}}{2}\tilde{c}_s$	$[6349.2 \pm 2.5 \pm 15]^{\dagger}_{B}$		$1(1^{+})$	$-\frac{11}{6}c_a + \frac{3}{2}c_s$	
	$1(2^{-})$	$-\frac{\tilde{1}}{2}\tilde{c}_a+\frac{\tilde{3}}{2}\tilde{c}_s$	$[\sim 6332.3 \pm 15]_V^{\dagger}$		$1(2^{+})$	$-\frac{1}{2}c_a+\frac{3}{2}c_s$	
	$1(3^{-})$	$\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$			$1(3^{+})$	$\frac{3}{2}c_a + \frac{3}{2}c_s$	

The doubly charmed baryon  $\Xi_{cc}^{(*)}$  can be associated with the anticharm meson  $\bar{D}^{(*)}$  with the heavy diquark-antiquark symmetry. Consequently, the  $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$  and  $\Xi_{cc}^{(*)}\Xi_c^{(*)}$  systems correspond respectively to the  $\bar{D}^{(*)}\Sigma_c^{(*)}$  and  $\bar{D}^{(*)}\Xi_c^{(*)}$  systems. If the hidden-charm pentaquarks  $P_{\psi}^N$  and  $P_{\psi s}^\Lambda$  are indeed molecular states of  $\bar{D}^{(*)}\Sigma_c$  and  $\bar{D}^{(*)}\Xi_c$ , it implies the presence of corresponding molecular states in the  $\Xi_{cc}^{(*)}\Sigma_c$ and  $\Xi_{cc}^{(*)}\Xi_c$  systems as well. A comprehensive analysis of the spin-flavor symmetry between these systems can be found in Ref. [49].

In Ref. [71], Chen *et al.* investigated the  $\Xi_{cc}\Sigma_{c}^{(*)}$  and  $\Xi_{cc}\Xi_{c}^{(*)}$  systems using the OBE model, taking into account the coupled-channel effects. They obtained molecular states in both the highest isospin channels and the lowest isospin channels. The results in the lowest isospin channels align with our calculations. However, our calculations indicate the absence of bound states in the highest isospin channel in such systems. Junnarkar et al. obtained a molecular state with a binding energy of  $-8 \pm 17$  MeV in the  $\Xi_{cc}\Sigma_c$  system with  $J^P = 1^+$  through the lattice QCD calculations [72]. This result is in good agreement with our findings in the  $\Xi_{cc}\Sigma_c$  system with  $O(1^+)$ , where we obtained a binding energy of  $-6.7^{+2.2}_{-2.3}$  MeV. The results presented in Ref. [72] have also been used by Pan et al. to determine the spin of the  $P_{\psi}^{N}(4440)$  and  $P_{\psi}^{N}(4457)$ states [73]. The mass hierarchy of the  $0^+$  and  $1^+$  states in the  $\Xi_{cc}\Sigma_c$  system exactly opposes that of the  $\bar{D}^*\Sigma_c$  system, which consists of  $\frac{1}{2}^{-}$  and  $\frac{3}{2}^{-}$  states. Therefore, if the mass splitting of the  $0^+$  and  $1^+$  states in the  $\Xi_{cc}\Sigma_c$  system can be accurately calculated on the lattice, it can be used to infer the spin of the  $P_{\psi}^{N}(4440)$  and  $P_{\psi}^{N}(4457)$  based on their masses.

Furthermore, we also calculated the mass spectra of the  $\Xi_{cc}^{(*)}\bar{\Sigma}_{c}^{(*)}$  and  $\Xi_{cc}^{(*)}\bar{\Xi}_{c}^{(\prime,*)}$  systems. These systems will correspond to the  $\bar{D}^{(*)}\bar{\Sigma}_{c}^{(*)}$  and  $\bar{D}^{(*)}\bar{\Xi}_{c}^{(\prime,*)}$  systems, respectively, within the heavy diquark-antiquark symmetry. Therefore, if the  $\bar{D}^{(*)}\bar{\Sigma}_{c}^{(*)}$  and  $\bar{D}^{(*)}\bar{\Xi}_{c}^{(\prime,*)}$  systems both contain double-charm pentaquarks [60,74,75], it naturally follows that the  $\Xi_{cc}^{(*)}\bar{\Sigma}_{c}^{(*)}$  and  $\Xi_{cc}^{(*)}\bar{\Xi}_{c}^{(\prime,*)}$  systems also possess triple-charm hexaquarks.

#### E. Charmed baryon-(anti)hyperon systems

The results for charmed baryon-(anti)hyperon systems are listed in Tables X–XII.

This type of systems such as  $\Sigma_c^{(*)}\Sigma^{(*)}, \Sigma_c^{(*)}\bar{\Sigma}^{(*)}, \Xi_c^{(\prime,*)}\Xi^{(*)}$ , and  $\Xi_c^{(\prime,*)}\bar{\Xi}^{(*)}$  were investigated in literature [36,38,39]. In Ref. [36], Kong *et al.* studied the possible molecular states in  $\Sigma_c^{(*)}\Sigma^{(*)}$  and  $\Sigma_c^{(*)}\bar{\Sigma}^{(*)}$  systems within the OBE model, and they obtained a series of bound states in the isoscalar, isovector, and isotensor channels. Their results in the

isoscalar  $\Sigma_c^{(*)} \Sigma^{(*)}$  systems are consistent with ours, while in our work, the isovector states become virtual states due to the weaker attractions, and no bound/virtual states can be obtained in the isotensor channels. In Ref. [39], Wu *et al.* noticed that the  $\Sigma_c^* \overline{\Sigma}$  system in  $0(2^-)$  and  $1(2^-)$  channels are likely to form bound states. We obtained the bound and virtual state solutions in these two channels, respectively, and we also found that the more attractive interaction in  $2(1^-)$  channel gives rise to deeper bound state.

#### F. Doubly charmed baryon-(anti)hyperon systems

The results for doubly charmed baryon-(anti)hyperon systems are given in Table XIII. We obtained several spin multiplets in the lowest isospin  $\Xi_{cc}^{(*)}\Sigma^{(*)}$  and  $\Xi_{cc}^{(*)}\Xi^{(*)}$  systems, and they can be related to the anticharmed strange pentaquarks predicted in Ref. [60] within the heavy diquark-antiquark symmetry. A series of bound/virtual states were also obtained in  $\Xi_{cc}^{(*)}\overline{\Sigma}^{(*)}$  and  $\Xi_{cc}^{(*)}\overline{\Xi}^{(*)}$  systems.

### G. General discussions

We noticed that most results from the OBE model are qualitatively consistent with our calculations. Here, we briefly discuss the possible reasons behind this consistency.

Although our calculations also adopt the idea of the OBE model, we do not explicitly determine the coupling constants between baryon fields and the exchanged mesons. Instead, we absorb the contributions of all possible exchanged mesons into the LECs. These LECs are determined using known experimental data (i.e., the mass spectra of molecular tetraquarks and pentaquarks), and are then used to calculate the corresponding mass spectrum of molecular hexaquarks.

In contrast, the OBE model calculations explicitly determine the coupling constants of various exchange interactions using experimental data and phenomenological approaches, and then derive the effective potential to solve for the mass spectrum. Because the interactions in heavy-flavor dihadron systems are dominated by the correlations of  $q_1q_2$  ( $\bar{q}_1q_2$ ), these two approaches may yield similar results. Therefore, as long as the effective interaction of  $q_1q_2$  ( $\bar{q}_1q_2$ ) is described appropriately, consistent results will be obtained.

However, in specific channels, such as the systems dominated by  $V_{qq}$  interactions, the two approaches may yield inconsistent results in high isospin channels. This discrepancy could be attributed, on one hand, to the uncertainties in the coupling constants determined using phenomenological approaches, and on the other hand, to the calculations in OBE model to vary the cutoff over a large range to search for binding solutions. When the cutoff becomes excessively large, the OBE potential may be extended into an energy region where it is not applicable, thus leading to spurious bound state solutions.

Systems [m <sub>th</sub> ]	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Sigma_c \bar{\Sigma}$ [3646.6]	$\begin{array}{c} 0(0^{-}) \\ 0(1^{-}) \\ 1(0^{-}) \\ 1(1^{-}) \\ 2(0^{-}) \\ 2(1^{-}) \end{array}$	$8\tilde{c}_{a} - 6\tilde{c}_{s}$ $-\frac{8}{3}\tilde{c}_{a} - 6\tilde{c}_{s}$ $\frac{8}{3}\tilde{c}_{a} - 2\tilde{c}_{s}$ $-\frac{8}{9}\tilde{c}_{a} - 2\tilde{c}_{s}$ $-8\tilde{c}_{a} + 6\tilde{c}_{s}$ $\frac{8}{3}\tilde{c}_{a} + 6\tilde{c}_{s}$	$[3634.4, 3644.0]_B$  $[3608.5, 3633.3]_V$ $[3576.0, 3631.0]_B$ 	$\Sigma_c \Sigma$	$\begin{array}{c} 0(0^+) \\ 0(1^+) \\ 1(0^+) \\ 1(1^+) \\ 2(0^+) \\ 2(1^+) \end{array}$	$8c_a - 6c_s$ $-\frac{8}{3}c_a - 6c_s$ $\frac{8}{3}c_a - 2c_s$ $-\frac{8}{9}c_a - 2c_s$ $-8c_a + 6c_s$ $\frac{8}{3}c_a + 6c_s$	$ \begin{bmatrix} 3616.4^{+10.8}_{-11.4} \end{bmatrix}_B \\ \begin{bmatrix} 3622.2^{+6.6}_{-7.0} \end{bmatrix}_B \\ \begin{bmatrix} 3644.7^{+1.7}_{-4.7} \end{bmatrix}_V \\ \begin{bmatrix} 3644.7^{+2.3}_{-3.8} \end{bmatrix}_V \\ & \ddots \\ & \ddots \end{bmatrix} $
$\Sigma_c \bar{\Sigma}^*$ [3838.0]	$0(1^{-}) \\ 0(2^{-}) \\ 1(1^{-}) \\ 1(2^{-}) \\ 2(1^{-}) \\ 2(2^{-})$	$\frac{20}{3}\tilde{c}_a - 6\tilde{c}_s$ $-4\tilde{c}_a - 6\tilde{c}_s$ $\frac{20}{9}\tilde{c}_a - 2\tilde{c}_s$ $-\frac{4}{3}\tilde{c}_a - 2\tilde{c}_s$ $-\frac{20}{3}\tilde{c}_a + 6\tilde{c}_s$ $4\tilde{c}_a + 6\tilde{c}_s$	$[3810.0, 3818.4]_B$ $[3833.3, 3836.5]_V$ $[3782.9, 3830.6]_B$	$\Sigma_c \Sigma^*$	$0(1^+)$ $0(2^+)$ $1(1^+)$ $1(2^+)$ $2(1^+)$ $2(2^+)$	$\frac{20}{3}c_{a} - 6c_{s} \\ -4c_{a} - 6c_{s} \\ \frac{20}{9}c_{a} - 2c_{s} \\ -\frac{4}{3}c_{a} - 2c_{s} \\ -\frac{20}{3}c_{a} + 6c_{s} \\ 4c_{a} + 6c_{s} \end{cases}$	$ \begin{bmatrix} 3804.8^{+10.1}_{-10.5} \end{bmatrix}_{B} \\ \begin{bmatrix} 3810.7^{+7.8}_{-8.1} \end{bmatrix}_{B} \\ \begin{bmatrix} 3837.4^{+0.6}_{-2.5} \end{bmatrix}_{V} \\ \begin{bmatrix} 3836.3^{+1.4}_{-3.1} \end{bmatrix}_{V} \\ & \ddots \\ & \ddots \end{bmatrix} $
$\Sigma_c \bar{\Xi}$ [3771.7]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$-2\tilde{c}_a - 3\tilde{c}_s$ $\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$ $-\frac{2}{3}\tilde{c}_a + 3\tilde{c}_s$	$[3770.7, 3771.7]_B$  $[\sim 3688.7]_V^{\sharp}$	$\Sigma_c \Xi$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$	$-2c_a - 3c_s$ $\frac{2}{3}c_a - 3c_s$ $2c_a + 3c_s$ $-\frac{2}{3}c_a + 3c_s$	$[3770.8^{+0.9}_{-1.9}]_B$ $[3770.2^{+1.1}_{-1.5}]_B$ $\dots$
$\Sigma_c \bar{\Xi}^*$ [3986.9]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$\frac{10}{3}\tilde{c}_a - 3\tilde{c}_s$ $-2\tilde{c}_a - 3\tilde{c}_s$ $-\frac{10}{3}\tilde{c}_a + 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$	$[3984.5, 3986.3]_B$ $[3975.0, 3986.9]_B^{\dagger}$ 	$\Sigma_c \Xi^*$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$\frac{10}{3}c_a - 3c_s$ $-2c_a - 3c_s$ $-\frac{10}{3}c_a + 3c_s$ $2c_a + 3c_s$	$ [3983.0^{+2.7}_{-3.6}]_B  [3984.7^{+1.7}_{-2.4}]_B  \dots \\ \dots $
$\Sigma_c^* \bar{\Sigma}$ [3711.3]	$0(1^{-}) \\ 0(2^{-}) \\ 1(1^{-}) \\ 1(2^{-}) \\ 2(1^{-}) \\ 2(2^{-})$	$\frac{20}{3}\tilde{c}_a - 6\tilde{c}_s$ $-4\tilde{c}_a - 6\tilde{c}_s$ $\frac{20}{9}\tilde{c}_a - 2\tilde{c}_s$ $-\frac{4}{3}\tilde{c}_a - 2\tilde{c}_s$ $-\frac{20}{3}\tilde{c}_a + 6\tilde{c}_s$ $4\tilde{c}_a + 6\tilde{c}_s$	$[3686.6, 3694.6]_B$ $[3702.7, 3707.6]_V$ $[3660.0, 3705.9]_B$	$\Sigma_c^*\Sigma$	$0(1^+)$ $0(2^+)$ $1(1^+)$ $1(2^+)$ $2(1^+)$ $2(2^+)$	$\frac{20}{3}c_{a} - 6c_{s} \\ -4c_{a} - 6c_{s} \\ \frac{20}{9}c_{a} - 2c_{s} \\ -\frac{4}{3}c_{a} - 2c_{s} \\ -\frac{20}{3}c_{a} + 6c_{s} \\ 4c_{a} + 6c_{s} \end{cases}$	$ \begin{bmatrix} 3681.4^{+9.8}_{-10.3} \end{bmatrix}_B \\ \begin{bmatrix} 3687.2^{+7.5}_{-8.0} \end{bmatrix}_B \\ \begin{bmatrix} 3709.3^{+1.7}_{-4.2} \end{bmatrix}_V \\ \begin{bmatrix} 3707.3^{+2.5}_{-4.7} \end{bmatrix}_V \\ & \dots \\ & \dots \\ & \dots \\ \end{bmatrix} $
$\Sigma_{c}^{*}\bar{\Sigma}^{*}$ [3902.7]	$\begin{array}{c} 0(0^{-}) \\ 0(1^{-}) \\ 0(2^{-}) \\ 0(3^{-}) \\ 1(0^{-}) \\ 1(1^{-}) \\ 1(2^{-}) \\ 1(3^{-}) \\ 2(0^{-}) \\ 2(1^{-}) \\ 2(2^{-}) \\ 2(3^{-}) \end{array}$	$10\tilde{c}_{a} - 6\tilde{c}_{s}$ $\frac{22}{3}\tilde{c}_{a} - 6\tilde{c}_{s}$ $2\tilde{c}_{a} - 6\tilde{c}_{s}$ $-6\tilde{c}_{a} - 6\tilde{c}_{s}$ $\frac{10}{3}\tilde{c}_{a} - 2\tilde{c}_{s}$ $\frac{22}{9}\tilde{c}_{a} - 2\tilde{c}_{s}$ $\frac{2}{3}\tilde{c}_{a} - 2\tilde{c}_{s}$ $-2\tilde{c}_{a} - 2\tilde{c}_{s}$ $-10\tilde{c}_{a} + 6\tilde{c}_{s}$ $-2\tilde{c}_{a} + 6\tilde{c}_{s}$ $-2\tilde{c}_{a} + 6\tilde{c}_{s}$ $-2\tilde{c}_{a} + 6\tilde{c}_{s}$	$[3853.6, 3854.8]_B$ $[3853.6, 3854.8]_B$ $[3853.6, 3854.8]_B$ $[3902.4, 3902.5]_B$ $[3902.4, 3902.5]_B$ $[3796.5, 3863.7]_B$ $[3837.2, 3889.5]_B$ $[\sim 3902.6]_V^{\sharp}$	$\Sigma_c^* \Sigma^*$	$\begin{array}{c} 0(0^+)\\ 0(1^+)\\ 0(2^+)\\ 0(3^+)\\ 1(0^+)\\ 1(1^+)\\ 1(2^+)\\ 1(3^+)\\ 2(0^+)\\ 2(1^+)\\ 2(2^+)\\ 2(2^+)\\ 2(3^+)\end{array}$	$10c_{a} - 6c_{s}$ $\frac{22}{3}c_{a} - 6c_{s}$ $2c_{a} - 6c_{s}$ $-6c_{a} - 6c_{s}$ $\frac{10}{3}c_{a} - 2c_{s}$ $\frac{22}{9}c_{a} - 2c_{s}$ $\frac{2}{3}c_{a} - 2c_{s}$ $-2c_{a} - 2c_{s}$ $-10c_{a} + 6c_{s}$ $-\frac{22}{3}c_{a} + 6c_{s}$ $-2c_{a} + 6c_{s}$ $6c_{s} - 6c_{s}$	$ \begin{bmatrix} 3867.2^{+12.8}_{-13.4} \end{bmatrix}_B \\ \begin{bmatrix} 3868.7^{+10.7}_{-11.1} \end{bmatrix}_B \\ \begin{bmatrix} 3871.7^{+6.4}_{-6.6} \end{bmatrix}_B \\ \begin{bmatrix} 3876.1^{+9.2}_{-9.7} \end{bmatrix}_B \\ \begin{bmatrix} 3902.4^{+0.3}_{-2.9} \end{bmatrix}_V^+ \\ \begin{bmatrix} 3902.2^{+0.4}_{-2.5} \end{bmatrix}_V^+ \\ \begin{bmatrix} 3902.2^{+0.4}_{-2.5} \end{bmatrix}_V^+ \\ \begin{bmatrix} 3901.8^{+0.7}_{-1.6} \end{bmatrix}_V \\ \begin{bmatrix} 3900.8^{+1.6}_{-4.2} \end{bmatrix}_V \\ & \ddots \\ & & \ddots \\ & & \ddots \\ \end{pmatrix} $

TABLE X. The  $I(J^P)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma_c^{(*)}\Sigma^{(*)}, \Sigma_c^{(*)}\overline{\Sigma}^{(*)}, \Sigma_c^{(\pm)}\overline{\Sigma}^{(*)}, \Sigma_c^{(\pm)}\overline{\Sigma}^{(*)}, \Sigma_c^{(\pm)}\overline{\Sigma}^{(\pm)}, \Sigma_c^{(\pm)}\overline{\Sigma}$ 

Systems [m <sub>th</sub> ]	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\overline{\Sigma_c^* \bar{\Xi}}$ [3836.4]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$-\frac{5}{3}\tilde{c}_a - 3\tilde{c}_s$ $\tilde{c}_a - 3\tilde{c}_s$ $\frac{5}{3}\tilde{c}_a + 3\tilde{c}_s$ $-\tilde{c}_a + 3\tilde{c}_s$	$[3835.6, 3836.4]_V^{\dagger}$ $[\sim 3812.2]_V^{\sharp}$	$\Sigma_c^*\Xi$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{3}{2}(1^{+})$ $\frac{3}{2}(2^{+})$	$-\frac{5}{3}c_a - 3c_s$ $c_a - 3c_s$ $\frac{5}{3}c_a + 3c_s$ $-c_a + 3c_s$	$[3835.3^{+1.0}_{-1.8}]_B$ $[3834.6^{+1.2}_{-1.7}]_B$ $\cdots$
$\Sigma_c^* \bar{\Xi}^*$ [4051.5]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{1}{2}(2^{-})}{\frac{1}{2}(3^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$ $\frac{\frac{3}{2}(2^{-})}{\frac{3}{2}(3^{-})}$	$5\tilde{c}_{a} - 3\tilde{c}_{s}$ $\frac{11}{3}\tilde{c}_{a} - 3\tilde{c}_{s}$ $\tilde{c}_{a} - 3\tilde{c}_{s}$ $-3\tilde{c}_{a} - 3\tilde{c}_{s}$ $-5\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\frac{11}{3}\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\tilde{c}_{a} + 3\tilde{c}_{s}$ $3\tilde{c}_{a} + 3\tilde{c}_{s}$	$ \begin{array}{c} \dots\\ [4041.9, 4042.4]_B\\ [4017.0, 4045.6]_B\\ [4035.2, 4051.5]_B^{\dagger}\\ [\sim 4035.1]_V^{\dagger}\\ \dots\\ \end{array} $	$\Sigma_c^* \Xi^*$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{1}{2}(2^{+})}{\frac{1}{2}(3^{+})}$ $\frac{3}{2}(0^{+})$ $\frac{3}{2}(1^{+})$ $\frac{3}{2}(2^{+})$ $\frac{3}{2}(3^{+})$	$5c_a - 3c_s$ $\frac{11}{3}c_a - 3c_s$ $c_a - 3c_s$ $-3c_a - 3c_s$ $-5c_a + 3c_s$ $-\frac{11}{3}c_a + 3c_s$ $-c_a + 3c_s$ $3c_a + 3c_s$	
$\Xi_c \bar{\Sigma}$ [3662.2]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$6\tilde{c}_a - 3\tilde{c}_s$ $-2\tilde{c}_a - 3\tilde{c}_s$ $-6\tilde{c}_a + 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$	$[3661.8, 3662.2]_B^{\dagger}$ $[3620.1, 3653.0]_B$	$\Xi_c \Sigma$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$	$6c_a - 3c_s$ $-2c_a - 3c_s$ $-6c_a + 3c_s$ $2c_a + 3c_s$	$ [3660.2^{+2.0}_{-4.3}]_B  [3661.9^{+0.3}_{-1.4}]^{\dagger}_B \\ \dots \\ \dots \\ \dots$
$\Xi_c \bar{\Sigma}^*$ [3853.7]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$5\tilde{c}_a - 3\tilde{c}_s$ $-3\tilde{c}_a - 3\tilde{c}_s$ $-5\tilde{c}_a + 3\tilde{c}_s$ $3\tilde{c}_a + 3\tilde{c}_s$	$[3845.9, 3846.3]_B$ $[3821.8, 3849.3]_B$ 	$\Xi_c \Sigma^*$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$5c_a - 3c_s$ $-3c_a - 3c_s$ $-5c_a + 3c_s$ $3c_a + 3c_s$	$ [3850.3^{+2.9}_{-4.4}]_B  [3852.5^{+1.1}_{-2.5}]_B  \dots \\ \dots $
Ξ <sub>c</sub> Ξ [3787.4]	$0(0^{-})$ $0(1^{-})$ $1(0^{-})$ $1(1^{-})$	$-\frac{5}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[3784.9, 3785.2]_B$   $[\sim 3635.3]_V^{\sharp}$	$\Xi_c \Xi$	$egin{array}{l} 0(0^+) \ 0(1^+) \ 1(0^+) \ 1(1^+) \end{array}$	$-\frac{5}{2}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} - \frac{5}{2}c_{s}$ $\frac{3}{2}c_{a} + \frac{3}{2}c_{s}$ $-\frac{1}{2}c_{a} + \frac{3}{2}c_{s}$	$[3787.3^{+0.1}_{-1.6}]^{\dagger}_{V}$ $[3787.3^{+0.1}_{-0.5}]^{\dagger}_{B}$ $\dots$
$\Xi_c \bar{\Xi}^*$ [4002.5]	$0(1^{-})$ $0(2^{-})$ $1(1^{-})$ $1(2^{-})$	$\frac{\frac{25}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s}{-\frac{5}{2}\tilde{c}_a - \frac{5}{2}\tilde{c}_s}$ $-\frac{5}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $\frac{3}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[3998.3, 3998.6]_B$ $[3999.0, 4002.5]_B^{\dagger}$ 	$\Xi_c \Xi^*$	$0(1^+) \\ 0(2^+) \\ 1(1^+) \\ 1(2^+)$	$\frac{\frac{25}{6}c_{a} - \frac{5}{2}c_{s}}{-\frac{5}{2}c_{a} - \frac{5}{2}c_{s}}$ $-\frac{5}{2}c_{a} + \frac{3}{2}c_{s}$ $\frac{3}{2}c_{a} + \frac{3}{2}c_{s}$	$ \begin{array}{c} [4001.3^{+1.2}_{-2.9}]_B \\ [4002.4^{+0.1}_{-1.2}]_B^{\dagger} \\ & \ddots \\ & \ddots \end{array} $
$\Xi_c' \bar{\Sigma}$ [3771.4]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$-2\tilde{c}_a - 3\tilde{c}_s$ $\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$ $-\frac{2}{3}\tilde{c}_a + 3\tilde{c}_s$	$egin{array}{c} [3770.8, 3771.4]^{\dagger}_{B} & \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	$\Xi_c'\Sigma$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$	$-2c_a - 3c_s$ $\frac{2}{3}c_a - 3c_s$ $2c_a + 3c_s$ $-\frac{2}{3}c_a + 3c_s$	$[3770.9^{+0.5}_{-1.5}]_B$ $[3770.4^{+0.8}_{-1.3}]_B$ $\dots$

TABLE XI. The  $I(J^P)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Sigma_c^* \Xi^{(*)}, \Sigma_c^* \bar{\Xi}^{(*)}, \Xi_c \bar{\Sigma}^{(*)}, \Xi_c \bar{\Xi}^{(*)}, \Xi_c$ 

Systems [m <sub>th</sub> ]	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_c' \bar{\Sigma}^*$ [3962.8]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$-\frac{5}{3}\tilde{c}_a - 3\tilde{c}_s$ $\tilde{c}_a - 3\tilde{c}_s$ $\frac{5}{3}\tilde{c}_a + 3\tilde{c}_s$ $-\tilde{c}_a + 3\tilde{c}_s$	$[3962.3, 3962.8]_V^{\dagger}$  $[\sim 3941.7]_V^{\sharp}$	$\Xi_c' \Sigma^*$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$-\frac{5}{3}c_a - 3c_s$ $c_a - 3c_s$ $\frac{5}{3}c_a + 3c_s$ $-c_a + 3c_s$	$[3961.2^{+1.3}_{-2.0}]_B$ $[3960.4^{+1.5}_{-1.9}]_B$ $\dots$
<i>Ξ</i> ′ <sub>c</sub> Ξ [3896.5]	$0(0^{-})$ $0(1^{-})$ $1(0^{-})$ $1(1^{-})$	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{5}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 3872.2]_V^{\sharp}$ $[\sim 3754.3]_V^{\dagger}$ 	$\Xi_c'\Xi$	$egin{array}{l} 0(0^+) \ 0(1^+) \ 1(0^+) \ 1(1^+) \end{array}$	$\frac{\frac{5}{6}c_{a} - \frac{5}{2}c_{s}}{-\frac{5}{18}c_{a} - \frac{5}{2}c_{s}} \\ -\frac{\frac{1}{2}c_{a} + \frac{3}{2}c_{s}}{\frac{1}{6}c_{a} + \frac{3}{2}c_{s}}$	$ [3896.4^{+0.1}_{-0.6}]^{\dagger}_{B}  [3896.4^{+0.1}_{-0.4}]^{\dagger}_{B} \\ \dots \\ \dots \\ \dots$
$\Xi_c' \bar{\Xi}^*$ [4111.6]	$0(1^{-})$ $0(2^{-})$ $1(1^{-})$ $1(2^{-})$	$-\frac{25}{18}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{5}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{5}{6}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $-\frac{1}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$	$[4109.9, 4111.6]_V$   $[\sim 4008.6]_V^{\sharp}$	$\Xi_c' \Xi^*$	$egin{array}{l} 0(1^+) \ 0(2^+) \ 1(1^+) \ 1(2^+) \end{array}$	$-\frac{25}{18}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} + \frac{3}{2}c_{s}$ $-\frac{1}{2}c_{a} + \frac{3}{2}c_{s}$	$ \begin{array}{c} [4111.3^{+0.3}_{-1.1}]^{\dagger}_{B} \\ [4110.9^{+0.6}_{-1.1}]_{B} \\ & \dots \\ & \dots \end{array} $
$\Xi_c^* \bar{\Sigma}$ [3838.3]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$\frac{10}{3}\tilde{c}_a - 3\tilde{c}_s$ $-2\tilde{c}_a - 3\tilde{c}_s$ $-\frac{10}{3}\tilde{c}_a + 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$	$[3837.6, 3838.3]_B^{\dagger}$ $[3830.1, 3838.3]_B^{\dagger}$ $\dots$	$\Xi_c^*\Sigma$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$\frac{10}{3}c_{a} - 3c_{s} \\ -2c_{a} - 3c_{s} \\ -\frac{10}{3}c_{a} + 3c_{s} \\ 2c_{a} + 3c_{s}$	$ [3836.6^{+1.5}_{-2.8}]_B  [3837.7^{+0.5}_{-1.6}]^{\dagger}_B \\ \dots \\ \dots \\ \dots$
Ξ <sup>*</sup> <sub>c</sub> Σ̄* [4029.7]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{1}{2}(2^{-})}{\frac{1}{2}(3^{-})}$ $\frac{3}{2}(0^{-})$ $\frac{3}{2}(1^{-})$ $\frac{3}{2}(2^{-})$ $\frac{3}{2}(3^{-})$	$5\tilde{c}_{a} - 3\tilde{c}_{s}$ $\frac{11}{3}\tilde{c}_{a} - 3\tilde{c}_{s}$ $\tilde{c}_{a} - 3\tilde{c}_{s}$ $-3\tilde{c}_{a} - 3\tilde{c}_{s}$ $-5\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\frac{11}{3}\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\tilde{c}_{a} + 3\tilde{c}_{s}$ $3\tilde{c}_{a} + 3\tilde{c}_{s}$	$[4021.3, 4021.7]_B$ $[3996.9, 4024.8]_B$ $[4014.8, 4029.7]_B^{\dagger}$ $[\sim 4010.0]_V^{\sharp}$	$\Xi_c^* \Sigma^*$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{1}{2}(2^{+})}{\frac{1}{2}(3^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$ $\frac{3}{2}(2^{+})$ $\frac{3}{2}(3^{+})$	$5c_a - 3c_s$ $\frac{11}{3}c_a - 3c_s$ $c_a - 3c_s$ $-3c_a - 3c_s$ $-5c_a + 3c_s$ $-\frac{11}{3}c_a + 3c_s$ $-c_a + 3c_s$ $3c_a + 3c_s$	$ \begin{bmatrix} 4025.9^{+3.1}_{-4.6} \end{bmatrix}_B \\ \begin{bmatrix} 4026.4^{+2.6}_{-3.6} \end{bmatrix}_B \\ \begin{bmatrix} 4027.2^{+1.5}_{-1.9} \end{bmatrix}_B \\ \begin{bmatrix} 4027.2^{+1.5}_{-1.9} \end{bmatrix}_B \\ \begin{bmatrix} 4028.3^{+1.4}_{-2.6} \end{bmatrix}_B \\ & \ddots \\ & & \ddots \\ & & \ddots \\ & & \ddots \\ & & & &$
Ξ <sub>c</sub> <sup>*</sup> Ē [3963.4]	$0(1^{-}) \\ 0(2^{-}) \\ 1(1^{-}) \\ 1(2^{-})$	$-\frac{25}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[3959.6, 3963.1]_V$  $[\sim 3825.9]_V^{\sharp}$	$\Xi_c^*\Xi$	$\begin{array}{c} 0(1^+) \\ 0(2^+) \\ 1(1^+) \\ 1(2^+) \end{array}$	$-\frac{25}{18}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} + \frac{3}{2}c_{s}$ $-\frac{1}{2}c_{a} + \frac{3}{2}c_{s}$	$[3963.3^{+0.1}_{-0.5}]^{\dagger}_{B}$ $[3963.3^{+0.1}_{-0.7}]^{\dagger}_{B}$ $\cdots$
Ξ <sub>c</sub> *Ξ* [4178.5]	$\begin{array}{c} 0(0^{-}) \\ 0(1^{-}) \\ 0(2^{-}) \\ 0(3^{-}) \\ 1(0^{-}) \\ 1(1^{-}) \\ 1(2^{-}) \\ 1(3^{-}) \end{array}$	$\frac{25}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{55}{18}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{5}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $-\frac{5}{2}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $-\frac{5}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $-\frac{11}{6}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $-\frac{1}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $\frac{3}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$	 [4173.9, 4174.2] <sub>B</sub> [4174.6, 4178.5] <sup>†</sup> <sub>B</sub> [4159.1, 4178.5] <sup>†</sup> <sub>V</sub> [ $\sim$ 4078.6] <sup>†</sup> <sub>V</sub> 	<u></u> Ξ <sub>c</sub> *Ξ*	$\begin{array}{c} 0(0^+) \\ 0(1^+) \\ 0(2^+) \\ 0(3^+) \\ 1(0^+) \\ 1(1^+) \\ 1(2^+) \\ 1(3^+) \end{array}$	$\frac{25}{6}c_a - \frac{5}{2}c_s$ $\frac{55}{18}c_a - \frac{5}{2}c_s$ $\frac{5}{6}c_a - \frac{5}{2}c_s$ $-\frac{5}{2}c_a - \frac{5}{2}c_s$ $-\frac{5}{2}c_a + \frac{3}{2}c_s$ $-\frac{11}{6}c_a + \frac{3}{2}c_s$ $-\frac{1}{2}c_a + \frac{3}{2}c_s$ $\frac{3}{2}c_a + \frac{3}{2}c_s$	$ \begin{array}{c} [4177.0^{+1.5}_{-3.1}]_B \\ [4177.3^{+1.2}_{-2.4}]_B \\ [4177.7^{+0.7}_{-1.2}]_B \\ [4177.7^{+0.7}_{-1.4}]_B^{\dagger} \\ & \ddots \end{array} $

TABLE XII. The  $I(J^P)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Xi'_c \Sigma^*$ ,  $\Xi'_c \bar{\Sigma}^*$ ,  $\Xi'_c \bar{\Xi}^{(*)}$ ,  $\Xi'_c \bar{\Xi}^{(*)}$ ,  $\Xi'_c \bar{\Sigma}^{(*)}$ ,  $\Xi'_c \bar{\Xi}^{(*)}$ ,  $\Xi'_c \bar{\Xi}^{(*$ 

Systems $[m_{\rm th}]$	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$	Systems	$I(J^P)$	$V^{I,J}_{\mathcal{H}_1\mathcal{H}_2}$	$E_B/E_V$
$\Xi_{cc}\bar{\Sigma}$ [4814.7]	$\frac{\frac{1}{2}(0^{-})}{\frac{1}{2}(1^{-})}$ $\frac{\frac{3}{2}(0^{-})}{\frac{3}{2}(1^{-})}$	$-2\tilde{c}_a - 3\tilde{c}_s$ $\frac{2}{3}\tilde{c}_a - 3\tilde{c}_s$ $2\tilde{c}_a + 3\tilde{c}_s$ $-\frac{2}{3}\tilde{c}_a + 3\tilde{c}_s$	$[4813.1, 4814.5]_B$ [~4745.4]_V^{\sharp}	$\Xi_{cc}\Sigma$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{3}{2}(0^{+})}{\frac{3}{2}(1^{+})}$	$-2c_a - 3c_s$ $\frac{2}{3}c_a - 3c_s$ $2c_a + 3c_s$ $-\frac{2}{3}c_a + 3c_s$	$ \begin{array}{c} [4813.2^{+1.3}_{-2.2}]_B \\ [4812.5^{+1.3}_{-1.7}]_B \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$
$\Xi_{cc} \bar{\Sigma}^*$ [5006.1]	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$-\frac{5}{3}\tilde{c}_a - 3\tilde{c}_s$ $\tilde{c}_a - 3\tilde{c}_s$ $\frac{5}{3}\tilde{c}_a + 3\tilde{c}_s$ $-\tilde{c}_a + 3\tilde{c}_s$	$[5004.7, 5006.0]_B$   $[\sim 4993.7]_V^{\dagger}$	$\Xi_{cc}\Sigma^*$	$\frac{\frac{1}{2}(1^{+})}{\frac{1}{2}(2^{+})}$ $\frac{\frac{3}{2}(1^{+})}{\frac{3}{2}(2^{+})}$	$-\frac{5}{3}c_a - 3c_s$ $c_a - 3c_s$ $\frac{5}{3}c_a + 3c_s$ $-c_a + 3c_s$	$ [5002.9^{+2.0}_{-2.5}]_B  [5002.0^{+1.9}_{-2.3}]_B  \dots \\ \dots $
Ξ <sub>cc</sub> Ξ [4939.8]	0(0 <sup>-</sup> ) 0(1 <sup>-</sup> ) 1(0 <sup>-</sup> ) 1(1 <sup>-</sup> )	$\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{5}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $\frac{1}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[\sim 4924.4]_V^{\sharp}$ $[\sim 4838.5]_V^{\sharp}$ $\dots$	$\Xi_{cc}\Xi$	$0(0^+) \\ 0(1^+) \\ 1(0^+) \\ 1(1^+)$	$\frac{\frac{5}{6}c_{a} - \frac{5}{2}c_{s}}{-\frac{5}{18}c_{a} - \frac{5}{2}c_{s}} \\ -\frac{1}{2}c_{a} + \frac{3}{2}c_{s} \\ \frac{1}{6}c_{a} + \frac{3}{2}c_{s}$	$ \begin{array}{c} [4939.1^{+0.6}_{-1.1}]_B \\ [4939.3^{+0.5}_{-0.8}]_B \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$
$\Xi_{cc}\bar{\Xi}^{*}$ [5155.0]	0(1 <sup>-</sup> ) 0(2 <sup>-</sup> ) 1(1 <sup>-</sup> ) 1(2 <sup>-</sup> )	$-\frac{25}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[5154.6, 5155.0]_B^{\dagger}$   $[\sim 5083.8]_V^{\sharp}$	$\Xi_{cc}\Xi^*$	$0(1^+) \\ 0(2^+) \\ 1(1^+) \\ 1(2^+)$	$-\frac{25}{18}c_a - \frac{5}{2}c_s$ $\frac{5}{6}c_a - \frac{5}{2}c_s$ $\frac{5}{6}c_a + \frac{3}{2}c_s$ $-\frac{1}{2}c_a + \frac{3}{2}c_s$	$ [5153.6^{+1.1}_{-1.7}]_B  [5153.0^{+1.2}_{-1.6}]_B  \dots \\ \dots $
$\Xi_{cc}^*\bar{\Sigma}  \left[4899.7 \pm 15\right]$	$\frac{\frac{1}{2}(1^{-})}{\frac{1}{2}(2^{-})}$ $\frac{\frac{3}{2}(1^{-})}{\frac{3}{2}(2^{-})}$	$ \begin{array}{l} \frac{10}{3} \tilde{c}_a - 3 \tilde{c}_s \\ -2 \tilde{c}_a - 3 \tilde{c}_s \\ -\frac{10}{3} \tilde{c}_a + 3 \tilde{c}_s \\ 2 \tilde{c}_a + 3 \tilde{c}_s \end{array} $	 [4898.7 ± 0.7 ± 15] <sub>B</sub> [4894.4 ± 4.7 ± 15] <sup>†</sup> <sub>B</sub> 	$\Xi_{cc}^*\Sigma$	$\frac{\frac{1}{2}(1^+)}{\frac{1}{2}(2^+)}$ $\frac{\frac{3}{2}(1^+)}{\frac{3}{2}(2^+)}$	$\frac{10}{3}c_a - 3c_s$ $-2c_a - 3c_s$ $-\frac{10}{3}c_a + 3c_s$ $2c_a + 3c_s$	$ \begin{split} & [4896.6^{+2.4}_{-3.4}\pm15]_B \\ & [4898.1^{+1.3}_{-2.2}\pm15]_B \\ & & \dots \\ & & \dots \\ & & \dots \end{split} $
$\Xi_{cc}^* \bar{\Sigma}^* $ [5091.1 ± 15]	$\begin{array}{c} \frac{1}{2}(0^{-})\\ \frac{1}{2}(1^{-})\\ \frac{1}{2}(2^{-})\\ \frac{1}{2}(3^{-})\\ \frac{3}{2}(0^{-})\\ \frac{3}{2}(1^{-})\\ \frac{3}{2}(2^{-})\\ \frac{3}{2}(3^{-}) \end{array}$	$5\tilde{c}_{a} - 3\tilde{c}_{s}$ $\frac{11}{3}\tilde{c}_{a} - 3\tilde{c}_{s}$ $\tilde{c}_{a} - 3\tilde{c}_{s}$ $-3\tilde{c}_{a} - 3\tilde{c}_{s}$ $-5\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\frac{11}{3}\tilde{c}_{a} + 3\tilde{c}_{s}$ $-\tilde{c}_{a} + 3\tilde{c}_{s}$ $3\tilde{c}_{a} + 3\tilde{c}_{s}$	 $[5080.4 \pm 0.3 \pm 15]_B$ $[5069.3 \pm 15.1 \pm 15]_B$ $[5082.2 \pm 9.0 \pm 15]_B$ $[\sim 5079.1 \pm 15]_V^{\sharp}$ 	$\Xi_{cc}^*\Sigma^*$	$\frac{\frac{1}{2}(0^{+})}{\frac{1}{2}(1^{+})}$ $\frac{\frac{1}{2}(2^{+})}{\frac{1}{2}(3^{+})}$ $\frac{3}{2}(0^{+})$ $\frac{3}{2}(1^{+})$ $\frac{3}{2}(2^{+})$ $\frac{3}{2}(3^{+})$	$5c_a - 3c_s$ $\frac{11}{3}c_a - 3c_s$ $c_a - 3c_s$ $-3c_a - 3c_s$ $-5c_a + 3c_s$ $-\frac{11}{3}c_a + 3c_s$ $-c_a + 3c_s$ $3c_a + 3c_s$	$ \begin{split} & [5085.3^{+4.0}_{-5.1} \pm 15]_B \\ & [5085.9^{+3.3}_{-4.2} \pm 15]_B \\ & [5086.9^{+2.0}_{-2.3} \pm 15]_B \\ & [5088.3^{+2.3}_{-3.2} \pm 15]_B \\ & & \cdots \\ & & & &$
$\Xi_{cc}^* \bar{\Xi} \; [5024.8 \pm 15]$	$0(1^{-})$ $0(2^{-})$ $1(1^{-})$ $1(2^{-})$	$-\frac{25}{18}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a - \frac{5}{2}\tilde{c}_s$ $\frac{5}{6}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$ $-\frac{1}{2}\tilde{c}_a + \frac{3}{2}\tilde{c}_s$	$[5024.1 \pm 0.7 \pm 15]_V$   $[\sim 4925.6 \pm 15]_V^{\sharp}$	$\Xi_{cc}^*\Xi$	$0(1^+) \\ 0(2^+) \\ 1(1^+) \\ 1(2^+)$	$-\frac{25}{18}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} - \frac{5}{2}c_{s}$ $\frac{5}{6}c_{a} + \frac{3}{2}c_{s}$ $-\frac{1}{2}c_{a} + \frac{3}{2}c_{s}$	$ \begin{split} [5024.4^{+0.4}_{-1.2} \pm 15]_B \\ [5024.1^{+0.7}_{-1.2} \pm 15]_B \\ & \dots \\ & \dots \\ & \dots \end{split} $
$\Xi_{cc}^* \bar{\Xi}^* \ [5240.0 \pm 15]$	$\begin{array}{c} 0(0^{-}) \\ 0(1^{-}) \\ 0(2^{-}) \\ 0(3^{-}) \\ 1(0^{-}) \\ 1(1^{-}) \\ 1(2^{-}) \\ 1(3^{-}) \end{array}$	$\frac{25}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{55}{18}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $\frac{5}{6}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $-\frac{5}{2}\tilde{c}_{a} - \frac{5}{2}\tilde{c}_{s}$ $-\frac{5}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $-\frac{11}{6}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $-\frac{1}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$ $\frac{3}{2}\tilde{c}_{a} + \frac{3}{2}\tilde{c}_{s}$	$$ $[5233.3 \pm 0.2 \pm 15]_B$ $[5236.8 \pm 3.0 \pm 15]_B^{\dagger}$ $[5234.3 \pm 5.7 \pm 15]_V^{\dagger}$ $[\sim 5171.0 \pm 15]_V^{\sharp}$	$\Xi_{cc}^{*}\Xi^{*}$	$\begin{array}{c} 0(0^+)\\ 0(1^+)\\ 0(2^+)\\ 0(3^+)\\ 1(0^+)\\ 1(1^+)\\ 1(2^+)\\ 1(3^+)\\ \end{array}$	$\frac{\frac{25}{6}c_{a}-\frac{5}{2}c_{s}}{\frac{55}{18}c_{a}-\frac{5}{2}c_{s}}$ $\frac{\frac{5}{5}c_{a}-\frac{5}{2}c_{s}}{-\frac{5}{2}c_{a}-\frac{5}{2}c_{s}}$ $-\frac{5}{2}c_{a}-\frac{5}{2}c_{s}$ $-\frac{5}{2}c_{a}+\frac{3}{2}c_{s}$ $-\frac{11}{6}c_{a}+\frac{3}{2}c_{s}$ $-\frac{1}{2}c_{a}+\frac{3}{2}c_{s}$ $\frac{3}{2}c_{a}+\frac{3}{2}c_{s}$	$ \begin{split} & [5236.9^{+2.6}_{-3.8}\pm15]_B \\ & [5237.3^{+2.1}_{-3.0}\pm15]_B \\ & [5238.0^{+1.2}_{-1.6}\pm15]_B \\ & [5238.8^{+1.1}_{-2.2}\pm15]_B \\ & & \cdots \\ & & & \cdots \\ & & & \ddots \\ & & & &$

TABLE XIII. The  $I(J^P)$  quantum numbers, effective potentials, and bound/virtual state solutions of the  $\Xi_{cc}^{(*)}\Sigma^{(*)}, \Xi_{cc}^{(*)}\bar{\Sigma}^{(*)}, \Xi_{cc}^{(*)}\Xi^{(*)}, \Xi_{cc}^{(*)}, \Xi_{cc}^{(*)}\Xi^{(*)}, \Xi_{cc}^{(*)}, \Xi_{c$ 

#### **IV. SUMMARY AND OUTLOOK**

In the past two decades, numerous heavy flavor nearthreshold hadrons have been discovered in experiments, which are considered as promising candidates for molecular tetraquarks and pentaquarks. In this paper, we systematically study the possible molecular hexaquarks in the dibaryon systems. These dibaryon systems are composed of charmed baryons  $[\Sigma_c^{(*)}, \Xi_c^{(\prime,*)}]$ , doubly charmed baryons  $[\Xi_{cc}^{(*)}]$ , and hyperons  $[\Sigma^{(*)}, \Xi^{(*)}]$  combined pairwise. We consider all possible particle-particle and particleantiparticle combinations. We calculate the mass spectrum of molecular hexaquarks in different systems, which can be regarded as the heavy flavor counterparts of the deuteron.

Just as the mass spectrum of conventional hadrons can be described using a set of parameters within the quark potential models, in the molecular picture, there shall also exist an underlying relation between the hadronic molecules of different dihadron systems, rather than each hadronic molecule being an independent entity. Based on our previous research [59], we use the effective potentials at the quark level to describe the residual strong interactions of the S-wave dihadron systems. The interactions in the baryon-baryon and baryon-antibaryon systems are respectively governed by the correlations of

 $q_1q_2$  and  $\bar{q}_1q_2$  (q = u, d), for which we only need two parameters to describe the corresponding interaction strengths, and these two parameters are determined by the well-established states, such as  $P_{\psi}^N$ ,  $T_{cc}$ , and X(3872),  $Z_c(3900)$ , respectively.

It can be easily inferred within our model that if the molecular tetraquarks and pentaquarks exist, then the molecular hexaquarks must also exist. Experimental searches for these hexaquark states will help reveal whether the nature prefers to construct higher-level structural units, i.e., hadronic molecular states, using the colorsinglet conventional hadrons, or whether it merely favors putting the quarks into a bag, i.e., compact multiquark states.

### ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under Grants No. 12105072, No. 12305090, No. 11975033 and No. 12070131001. B. W. was also supported by the Start-up Funds for Young Talents of Hebei University (No. 521100221021). This project is also funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation, Project ID 196253076-TRR 110).

- R. L. Workman *et al.* (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).
- [2] H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, The hiddencharm pentaquark and tetraquark states, Phys. Rep. 639, 1 (2016).
- [3] F.-K. Guo, C. Hanhart, U.-G. Meißner, Q. Wang, Q. Zhao, and B.-S. Zou, Hadronic molecules, Rev. Mod. Phys. 90, 015004 (2018).
- [4] Y.-R. Liu, H.-X. Chen, W. Chen, X. Liu, and S.-L. Zhu, Pentaquark and tetraquark states, Prog. Part. Nucl. Phys. 107, 237 (2019).
- [5] R. F. Lebed, R. E. Mitchell, and E. S. Swanson, Heavyquark QCD exotica, Prog. Part. Nucl. Phys. 93, 143 (2017).
- [6] A. Esposito, A. Pilloni, and A. D. Polosa, Multiquark resonances, Phys. Rep. 668, 1 (2017).
- [7] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C.-P. Shen, C. E. Thomas, A. Vairo, and C.-Z. Yuan, The *XYZ* states: Experimental and theoretical status and perspectives, Phys. Rep. 873, 1 (2020).
- [8] G. Yang, J. Ping, and J. Segovia, Tetra- and penta-quark structures in the constituent quark model, Symmetry 12, 1869 (2020).
- [9] S. Chen, Y. Li, W. Qian, Z. Shen, Y. Xie, Z. Yang, L. Zhang, and Y. Zhang, Heavy flavour physics and *CP* violation at LHCb: A ten-year review, Front. Phys. 18, 44601 (2023).

- [10] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, An updated review of the new hadron states, Rep. Prog. Phys. 86, 026201 (2023).
- [11] L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, Chiral perturbation theory for heavy hadrons and chiral effective field theory for heavy hadronic molecules, Phys. Rep. 1019, 1 (2023).
- [12] M.-Z. Liu, Y.-W. Pan, Z.-W. Liu, T.-W. Wu, J.-X. Lu, and L.-S. Geng, Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions, arXiv:2404.06399.
- [13] S. K. Choi *et al.* (Belle Collaboration), Observation of a narrow charmonium-like state in exclusive  $B^{\pm} \rightarrow K^{\pm}\pi^{+}\pi^{-}J/\psi$  decays, Phys. Rev. Lett. **91**, 262001 (2003).
- [14] B. Aubert *et al.* (*BABAR* Collaboration), Observation of a broad structure in the  $\pi^+\pi^- J/\psi$  mass spectrum around 4.26-GeV/c<sup>2</sup>, Phys. Rev. Lett. **95**, 142001 (2005).
- [15] M. Ablikim *et al.* (BESIII Collaboration), Observation of a charged charmoniumlike structure in  $e^+e^- \rightarrow \pi^+\pi^- J/\psi$  at  $\sqrt{s} = 4.26$  GeV, Phys. Rev. Lett. **110**, 252001 (2013).
- [16] M. Ablikim *et al.* (BESIII Collaboration), Observation of a charged charmoniumlike structure  $Z_c(4020)$  and search for the  $Z_c(3900)$  in  $e^+e^- \rightarrow \pi^+\pi^-h_c$ , Phys. Rev. Lett. **111**, 242001 (2013).

- [17] A. Bondar *et al.* (Belle Collaboration), Observation of two charged bottomonium-like resonances in  $\Upsilon(5S)$  decays, Phys. Rev. Lett. **108**, 122001 (2012).
- [18] R. Aaij *et al.* (LHCb Collaboration), Observation of a narrow pentaquark state,  $P_c(4312)^+$ , and of two-peak structure of the  $P_c(4450)^+$ , Phys. Rev. Lett. **122**, 222001 (2019).
- [19] R. Aaij *et al.* (LHCb Collaboration), Evidence of a  $J/\psi\Lambda$  structure and observation of excited  $\Xi^-$  states in the  $\Xi_b^- \rightarrow J/\psi\Lambda K^-$  decay, Sci. Bull. **66**, 1278 (2021).
- [20] R. Aaij *et al.* (LHCb Collaboration), Observation of a  $J/\psi\Lambda$  resonance consistent with a strange pentaquark candidate in  $B^- \rightarrow J/\psi\Lambda \bar{p}$  decays, Phys. Rev. Lett. **131**, 031901 (2023).
- [21] R. Aaij *et al.* (LHCb Collaboration), Observation of an exotic narrow doubly charmed tetraquark, Nat. Phys. 18, 751 (2022).
- [22] R. Aaij *et al.* (LHCb Collaboration), Study of the doubly charmed tetraquark  $T_{cc}^+$ , Nat. Commun. **13**, 3351 (2022).
- [23] R. Aaij *et al.* (LHCb Collaboration), A model-independent study of resonant structure in  $B^+ \rightarrow D^+D^-K^+$  decays, Phys. Rev. Lett. **125**, 242001 (2020).
- [24] R. Aaij *et al.* (LHCb Collaboration), Amplitude analysis of the  $B^+ \rightarrow D^+D^-K^+$  decay, Phys. Rev. D **102**, 112003 (2020).
- [25] R. Aaij *et al.* (LHCb Collaboration), First observation of a doubly charged tetraquark and its neutral partner, Phys. Rev. Lett. **131**, 041902 (2023).
- [26] R. Aaij *et al.* (LHCb Collaboration), Amplitude analysis of  $B^0 \rightarrow \bar{D}^0 D_s^+ \pi^-$  and  $B^+ \rightarrow D^- D_s^+ \pi^+$  decays, Phys. Rev. D **108**, 012017 (2023).
- [27] N. Lee, Z.-G. Luo, X.-L. Chen, and S.-L. Zhu, Possible deuteron-like molecular states composed of heavy baryons, Phys. Rev. D 84, 014031 (2011).
- [28] N. Li and S.-L. Zhu, Hadronic molecular states composed of heavy flavor baryons, Phys. Rev. D 86, 014020 (2012).
- [29] W. Meguro, Y.-R. Liu, and M. Oka, Possible  $\Lambda_c \Lambda_c$  molecular bound state, Phys. Lett. B **704**, 547 (2011).
- [30] R. Chen, A. Hosaka, and X. Liu, Heavy molecules and one-σ/ω-exchange model, Phys. Rev. D 96, 116012 (2017).
- [31] B. Yang, L. Meng, and S.-L. Zhu, Hadronic molecular states composed of spin-<sup>3</sup>/<sub>2</sub> singly charmed baryons, Eur. Phys. J. A 55, 21 (2019).
- [32] Y.-W. Pan, M.-Z. Liu, and L.-S. Geng, Triply charmed dibaryons in the one boson exchange model, Phys. Rev. D 102, 054025 (2020).
- [33] X.-Z. Ling, M.-Z. Liu, and L.-S. Geng, Masses and strong decays of open charm hexaquark states  $\Sigma_c^{(*)}\Sigma_c^{(*)}$ , Eur. Phys. J. C **81**, 1090 (2021).
- [34] X.-K. Dong, F.-K. Guo, and B.-S. Zou, A survey of heavyheavy hadronic molecules, Commun. Theor. Phys. 73, 125201 (2021).
- [35] J.-B. Cheng, D.-x. Zheng, Z.-Y. Lin, and S.-L. Zhu, Doublecharm and hidden-charm hexaquark states under the complex scaling method, Phys. Rev. D 107, 054018 (2023).
- [36] S.-Y. Kong, J.-T. Zhu, and J. He, Possible charmed-strange molecular dibaryons, Eur. Phys. J. C 82, 834 (2022).
- [37] D. Song, L.-Q. Song, S.-Y. Kong, and J. He, Possible molecular states from interactions of charmed baryons, Phys. Rev. D 106, 074030 (2022).

- [38] S.-Y. Kong, J.-T. Zhu, and J. He, Possible molecular dibaryons with *csssqq* quarks and their baryon-antibaryon partners, Eur. Phys. J. C **83**, 436 (2023).
- [39] B. Wu, X.-K. Dong, F.-K. Guo, and B.-S. Zou, Possible  $\Sigma_c^* \overline{\Sigma}$  molecular states, Phys. Rev. D **109**, 074041 (2024).
- [40] H. Huang, J. Ping, and F. Wang, Possible *H*-like dibaryon states with heavy quarks, Phys. Rev. C 89, 035201 (2014).
- [41] T. F. Carames and A. Valcarce, Heavy flavor dibaryons, Phys. Rev. D 92, 034015 (2015).
- [42] J. Vijande, A. Valcarce, J. M. Richard, and P. Sorba, Search for doubly-heavy dibaryons in a quark model, Phys. Rev. D 94, 034038 (2016).
- [43] Y.-D. Chen, C.-F. Qiao, P.-N. Shen, and Z.-Q. Zeng, Revisiting the spectrum of baryonium in heavy baryon chiral perturbation theory, Phys. Rev. D 88, 114007 (2013).
- [44] J.-X. Lu, L.-S. Geng, and M. P. Valderrama, Heavy baryonantibaryon molecules in effective field theory, Phys. Rev. D 99, 074026 (2019).
- [45] K. Chen, R. Chen, L. Meng, B. Wang, and S.-L. Zhu, Systematics of the heavy flavor hadronic molecules, Eur. Phys. J. C 82, 581 (2022).
- [46] K. Chen, B. Wang, and S.-L. Zhu, Heavy flavor molecular states with strangeness, Phys. Rev. D 105, 096004 (2022).
- [47] K. Chen, B.-L. Huang, B. Wang, and S.-L. Zhu,  $\Sigma_c \Sigma_c$  interactions in chiral effective field theory, arXiv:2204.13316.
- [48] K. Chen, Z.-Y. Lin, and S.-L. Zhu, Comparison between the  $P_{W}^{N}$  and  $P_{Ws}^{\Lambda}$  systems, Phys. Rev. D **106**, 116017 (2022).
- [49] K. Chen and B. Wang, Flavor-spin symmetry of the  $P_{\psi}^{N}/H_{\Omega_{ccc}}^{N}$  and  $P_{\psi s}^{\Lambda}/H_{\Omega_{ccc}s}^{\Lambda}$  molecular states, Phys. Rev. D **109**, 114028 (2024).
- [50] H.-X. Chen, D. Zhou, W. Chen, X. Liu, and S.-L. Zhu, Searching for hidden-charm baryonium signals in QCD sum rules, Eur. Phys. J. C 76, 602 (2016).
- [51] B.-D. Wan, L. Tang, and C.-F. Qiao, Hidden-bottom and -charm hexaquark states in QCD sum rules, Eur. Phys. J. C 80, 121 (2020).
- [52] Z.-G. Wang, Triply-charmed dibaryon states or two-baryon scattering states from QCD sum rules, Phys. Rev. D 102, 034008 (2020).
- [53] X.-W. Wang, Z.-G. Wang, and G.-L. Yu, Study of  $\Lambda_c \Lambda_c$  dibaryon and  $\Lambda_c \bar{\Lambda}_c$  baryonium states via QCD sum rules, Eur. Phys. J. A **57**, 275 (2021).
- [54] Z. Liu, H.-T. An, Z.-W. Liu, and X. Liu, Where are the hidden-charm hexaquarks?, Phys. Rev. D 105, 034006 (2022).
- [55] Z. Liu, H.-T. An, Z.-W. Liu, and X. Liu, Doubly charmed dibaryon states, Chin. Phys. C 47, 063105 (2023).
- [56] H. Liu, J. He, L. Liu, P. Sun, W. Wang, Y.-B. Yang, and Q.-A. Zhang, Exploring hidden-charm and hidden-strange hexaquark states from lattice QCD, Sci. China Phys. Mech. Astron. 67, 211011 (2024).
- [57] L. Meng, N. Li, and S.-L. Zhu, Deuteron-like states composed of two doubly charmed baryons, Phys. Rev. D 95, 114019 (2017).
- [58] B. Yang, L. Meng, and S.-L. Zhu, Possible molecular states composed of doubly charmed baryons with coupledchannel effect, Eur. Phys. J. A 56, 67 (2020).
- [59] B. Wang, K. Chen, L. Meng, and S.-L. Zhu, Spectrum of the molecular tetraquarks: Unraveling the  $T_{cs0}(2900)$  and  $T_{cs0}^a(2900)$ , Phys. Rev. D **109**, 034027 (2024).

- [60] B. Wang, K. Chen, L. Meng, and S.-L. Zhu, Spectrum of the molecular pentaquarks, Phys. Rev. D 109, 074035 (2024).
- [61] B. Silvestre-Brac, Spectrum and static properties of heavy baryons, Few-Body Syst. **20**, 1 (1996).
- [62] J. Vijande, F. Fernandez, and A. Valcarce, Constituent quark model study of the meson spectra, J. Phys. G 31, 481 (2005).
- [63] A. V. Manohar and M. B. Wise, *Heavy Quark Physics* (Cambridge University Press, New York, 2000), Vol. 10.
- [64] M. J. Savage and M. B. Wise, Spectrum of baryons with two heavy quarks, Phys. Lett. B 248, 177 (1990).
- [65] A. Manohar and H. Georgi, Chiral quarks and the nonrelativistic quark model, Nucl. Phys. B234, 189 (1984).
- [66] C. E. Thomas and F. E. Close, Is X(3872) a molecule?, Phys. Rev. D 78, 034007 (2008).
- [67] W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou,  $\Sigma_c \overline{D}$  and  $\Lambda_c \overline{D}$  states in a chiral quark model, Phys. Rev. C 84, 015203 (2011).
- [68] Z.-C. Yang, Z.-F. Sun, J. He, X. Liu, and S.-L. Zhu, The possible hidden-charm molecular baryons composed of anti-charmed meson and charmed baryon, Chin. Phys. C 36, 6 (2012).

- [69] B. Wang, L. Meng, and S.-L. Zhu, Spectrum of the strange hidden charm molecular pentaquarks in chiral effective field theory, Phys. Rev. D **101**, 034018 (2020).
- [70] H. Polinder, J. Haidenbauer, and U. G. Meissner, Strangeness S = -2 baryon-baryon interactions using chiral effective field theory, Phys. Lett. B **653**, 29 (2007).
- [71] R. Chen, F.-L. Wang, A. Hosaka, and X. Liu, Exotic triplecharm deuteronlike hexaquarks, Phys. Rev. D 97, 114011 (2018).
- [72] P. Junnarkar and N. Mathur, Deuteronlike heavy dibaryons from lattice quantum chromodynamics, Phys. Rev. Lett. 123, 162003 (2019).
- [73] Y.-W. Pan, M.-Z. Liu, F.-Z. Peng, M. Sánchez Sánchez, L.-S. Geng, and M. Pavon Valderrama, Model independent determination of the spins of the  $P_c(4440)$  and  $P_c(4457)$  from the spectroscopy of the triply charmed dibaryons, Phys. Rev. D **102**, 011504 (2020).
- [74] K. Chen, B. Wang, and S.-L. Zhu, Exploration of the doubly charmed molecular pentaquarks, Phys. Rev. D 103, 116017 (2021).
- [75] F.-L. Wang and X. Liu, Surveying the mass spectra and the electromagnetic properties of the  $\Xi_c^{(\prime,*)}D^{(*)}$  molecular pentaquarks, Phys. Rev. D 109, 014043 (2024).