Decay-angular-distribution correlated *CP* violation in heavy hadron cascade decays

Yu-Jie Zhao and Zhen-Hua Zhang
School of Nuclear Science and Technology, University of South China, Hengyang, 421001 Hunan, China

Xin-Heng Guo
College of Nuclear Science and Technology, Beijing Normal University, Beijing 100875, China

(Received 16 May 2024; accepted 26 June 2024; published 26 July 2024)

CP violation in baryon decay processes is still undiscovered to date. We present a general analysis of the decay-angular distributions and the corresponding *CP* asymmetries in cascade decays of the type $\mathbb{H} \to R(\to ab)c$, where $\mathbb{H}$ is a heavy hadron that decays through weak interactions $\mathbb{H} \to Rc$ and the resonance R decays strongly via $R \to ab$. Based on the analysis, we propose to search for *CP* violation in the decay-angular distributions in the cascade decay processes $\mathbb{B} \to \mathcal{B}M$, with $\mathcal{B}$ or M subsequently decaying through strong interactions, where $\mathbb{B}$ is the mother baryon, $\mathcal{B}$ and M are the daughter baryon and meson, respectively, and M has to be spin nonzero. We also present some typical decay channels in which the search for such kinds of *CP* asymmetries can be performed.

DOI: 10.1103/PhysRevD.110.013007

I. INTRODUCTION

In the observable Universe, the density of baryons is by far larger than that of antibaryons [1], which is a clear evidence of charge-parity (*CP*) violation and charge-conjugate (*C*) violation, according to Sakharov's criteria for the generation of the baryon asymmetry of the Universe [2]. *CP* violation has been observed in the decays of K [3], D [4], B [5–7], and B_s [8] mesons, all of which are consistent with the description of the Standard Model of particle physics, in which a single *C*-violating weak phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the origin of *CP* violation [9,10]. Nevertheless, *CP* violation in baryon decay processes is still undiscovered in laboratories to date, despite the fact that many efforts have been made.

In baryon decay processes, the nonzero baryon spin provides us with more freedom to construct *CP* violation observables. The *CP* asymmetries induced by the decay asymmetry parameters in the hyperon weak decay transitions $\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-$ such as $\Xi \to \Lambda\pi$ and $\Lambda \to p\pi$ are typical examples. The decay asymmetry parameters of transitions $\frac{1}{2}^+ \to \frac{1}{2}^+ + 0^-$ are induced by the interference of the parity-even S-wave and the parity-odd P-wave amplitudes [11]. They can be measured from the decay-angular distribution if (1) the mother baryon is polarized and/or (2) the daughter baryon decays subsequently via weak interactions. The decay asymmetry parameter-induced *CP* asymmetries in hyperon decays, which have been investigated extensively on the theoretical side [12–15], are expected to be larger than those induced by the branching ratios in the aforementioned hyperon decay channels. Recently, experimental studies on the hyperon decay asymmetry parameter-induced *CP* asymmetries were performed by BESIII and Belle, with precision at about the one-percent level [16,17]. Moreover, decay asymmetry parameter-induced *CP* asymmetries in various charmed baryon decay channels such as $\Lambda_c^+ \to (\Lambda, \Sigma^0)h^+$ and $\Sigma_c^0 \to \Sigma^-\pi^+$ were also investigated by Belle [17,18]. However, they are still too small to be confirmed experimentally at the current stage for both the hyperon and the charmed baryon decays. Theoretical analyses of CPA induced by the decay parameters were also made in bottom baryon decays such as $\Lambda_b^0 \to \Lambda D$ [19,20], while the corresponding experimental study is still absent in bottom baryon decays.

Just like the case of the aforementioned decay asymmetry parameter-induced CPAs, the mechanism of generating *CP* violation via the interference between different canonical amplitudes is a good idea for decay processes with particles of nonzero spin involved and is complementary to CPAs corresponding to the partial decay width. It should be pointed out, however, that the interference between the parity-even and parity-odd amplitudes can only show up in the angular distributions of the final particles when the subsequent decay is also a weak one. In the situation where the corresponding subsequent decay is through strong or electromagnetic interactions which respect parity symmetry, the interference between parity-even and parity-odd amplitudes would simply be absent.[1] Interestingly, for some decay processes, when more than one parity-even and/or parity-odd amplitude enters in the decay, the interference between the amplitudes with the *same parity* can then show up in the decay-angular distributions, even when the subsequent decay is a strong one. We will analyze this in more detail here.

This paper is organized as follows. In Sec. II, we present the general analysis of the decay-angular-distribution-correlated CPAs. In Sec. III, by expressing the decay-angular distribution in terms of the canonical decay amplitudes, the interfering behavior between different canonical decay amplitudes can be seen in a more transparent way. In Sec. IV, we present some suggested decay channels in which the decay-angular-distribution-correlated CPAs are suitable to search for. In the last section, we make our conclusion.

II. GENERAL ANALYSIS OF THE DECAY-ANGULAR DISTRIBUTIONS AND THE CORRELATED *CP* ASYMMETRIES

In general, we will consider a mother hadron $\mathbb{H}$ decaying via a weak decay process $\mathbb{H} \to Rc$ with the intermediate resonance R decaying through strong interactions $R \to ab$. The differential decay width of the aforementioned cascade decay process $\mathbb{H} \to R(\to ab)c$ for unpolarized $\mathbb{H}$ can be expressed as [8][2]

$$\frac{d\Gamma_{\mathbb{H}\to R(\to ab)c}}{ds_{ab}ds_{ac}} = \frac{1}{(2\pi)^3}\frac{1}{32m_{\mathbb{H}}^3}\overline{|\mathcal{M}|^2}, \tag{1}$$

where s_{ij} is the invariant mass squared of the particle i and particle j system, $m_{\mathbb{H}}$ is the mass of $\mathbb{H}$, and $\overline{|\mathcal{M}|^2}$ is the spin-averaged decay amplitude squared of the cascade decay $\mathbb{H} \to R(\to ab)c$, which is defined as $\overline{|\mathcal{M}|^2} \equiv \frac{1}{2s_{\mathbb{H}}+1}\sum_{m_z,\lambda_a,\lambda_b,\lambda_c}|\mathcal{M}^{s_{\mathbb{H}},m_z}_{\lambda_a\lambda_b\lambda_c}|^2$. Here, $\mathcal{M}^{s_{\mathbb{H}},m_z}_{\lambda_a\lambda_b\lambda_c}$ is the covariant decay amplitude for the cascade decay $\mathbb{H} \to R(\to ab)c$; $s_{\mathbb{H}}$ is the spin of $\mathbb{H}$ and m_z is its z component (the direction of z is irrelevant here since we are dealing with unpolarized $\mathbb{H}$); and λ_a, λ_b, and λ_c are the helicities of a, b, and c, respectively. The decay amplitude squared can be further expressed as [22]

$$\overline{|\mathcal{M}|^2} = \sum_{\substack{0\le j\le 2s_R\\ j\ \mathrm{even}}} w^{(j)}P_j(c_\theta), \tag{2}$$

where P_j is the jth Legendre polynomial; $c_\theta \equiv \cos\theta$, with θ being the helicity angle of particle a with respect to c (or, equivalently, to $\mathbb{H}$) in the center-of-mass frame of the a & b system; and s_R is the spin quantum number for resonance R. Note that j can only take even values (from 0 to $2s_R$) because the decay of R is through strong interactions which respect the parity symmetry [22].

Obviously, all the weights $w^{(j)}$ describe the decay-angular distributions, i.e., the angular distributions of the final particles. The weight $w^{(j)}$ for the jth Legendre polynomial in Eq. (2) can be expressed as

$$w^{(j)} = \frac{\langle s_R,-s_R;s_R,s_R|s_Rs_Rj0\rangle^2}{|s_{ab}-m_R^2+im_R\Gamma_R|^2}\mathcal{W}^{(j)}\mathcal{S}^{(j)}, \tag{3}$$

where $\mathcal{W}^{(j)}$ and $\mathcal{S}^{(j)}$ contain the information for the decays $\mathbb{H} \to Rc$ and $R \to ab$, respectively, and can be expressed in terms of the helicity decay amplitudes as

$$\mathcal{W}^{(j)} = \sum_\sigma \frac{(-)^{\sigma-s_R}\langle s_R,-\sigma;s_R,\sigma|s_Rs_Rj0\rangle}{\langle s_R,-s_R;s_R,s_R|s_Rs_Rj0\rangle}\sum_{\lambda_c}|\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}|^2 \tag{4}$$

and

$$\mathcal{S}^{(j)} = \sum_{\lambda_a\lambda_b}\frac{(-)^{-\lambda+s_R}\langle s_R,-\lambda;s_R,\lambda|s_Rs_Rj0\rangle}{\langle s_R,-s_R;s_R,s_R|s_Rs_Rj0\rangle}|\mathcal{G}^{s_R}_{\lambda_a\lambda_b}|^2\Big|_{\lambda=\lambda_a-\lambda_b}, \tag{5}$$

with $\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}$ and $\mathcal{G}^{s_R}_{\lambda_a\lambda_b}$ being the helicity decay amplitudes of $\mathbb{H} \to Rc$ and $R \to ab$, respectively; σ being the helicity of R; and "$\langle\cdots|\cdots\rangle$" being the notation for Clebsch-Gordan coefficients. Our definitions of $\mathcal{W}^{(j)}$ and $\mathcal{S}^{(j)}$ are slightly different from those in previous works such as Ref. [22], i.e., with an extra factor $\frac{1}{\langle s_R,-s_R;s_R,s_R|s_Rs_Rj0\rangle}$. The motivation for the introduction of this factor is to guarantee that $\mathcal{W}^{(0)} = \sum_{\sigma,\lambda_c}|\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}|^2$ and $\mathcal{S}^{(0)} = \sum_{\lambda_a\lambda_b}|\mathcal{G}^{s_R}_{\lambda_a\lambda_b}|^2$. Moreover, one can see that all the coefficients in front of the helicity amplitudes squared in $\mathcal{W}^{(j)}$ and $\mathcal{S}^{(j)}$ are now simple rational numbers. One can relax the constraint on j taking even values in Eq. (2) because $\mathcal{S}^{(j)}$ automatically equals 0 for odd j due to the parity symmetry in $R \to ab$ [22].

We will focus on the decay-angular distributions with respect to the helicity angle θ. To this end, we need to integrate out s_{ab}. In the narrow width approximation of R, the differential decay width of the cascade decay $\mathbb{H} \to R(\to ab)c$ can then be expressed as

$$\frac{1}{\Gamma_{\mathbb{H}\to R(\to ab)c}}\frac{d\Gamma_{\mathbb{H}\to R(\to ab)c}}{dc_\theta} = \frac{1}{2}\sum_{\substack{0\le j\le 2s_R\\ j\ \mathrm{even}}}\gamma^{(j)}_{\mathbb{H}\to R(\to ab)c}P_j(c_\theta), \tag{6}$$

[1]The interference between parity-even and parity-odd amplitudes will show up when there are two resonances with similar masses but opposite parities [21,22].

[2]One reason why we only consider the unpolarized case is that the polarizations of the heavy baryons produced on colliders are still too small to be detected [23–25].

where $\gamma^{(j)}_{\mathbb{H}\to R(\to ab)c}$ can be further parametrized as $\gamma^{(j)}_{\mathbb{H}\to R(\to ab)c} = \lambda^{(j)}_{\mathbb{H}\to Rc}\alpha^{(j)}_{R\to ab}$, with $\lambda^{(j)}_{\mathbb{H}\to Rc}$ and $\alpha^{(j)}_{R\to ab}$ being defined as

$$\lambda^{(j)}_{\mathbb{H}\to Rc} \equiv \frac{\langle s_R, -s_R; s_R, s_R | s_R s_R j0\rangle}{\langle s_R, -s_R; s_R, s_R | s_R s_R 00\rangle} \frac{\mathcal{W}^{(j)}}{\mathcal{W}^{(0)}} \tag{7}$$

and

$$\alpha^{(j)}_{R\to ab} = \frac{\langle s_R, -s_R; s_R, s_R | s_R s_R j0\rangle}{\langle s_R, -s_R; s_R, s_R | s_R s_R 00\rangle} \frac{\mathcal{S}^{(j)}}{\mathcal{S}^{(0)}}, \tag{8}$$

respectively. Here, $\alpha^{(j)}_{R\to ab}$ are a set of decay parameters of $R\to ab$. On the other hand, $\lambda^{(j)}_{\mathbb{H}\to Rc}$ $(j = 1, \ldots, 2s_R)$ describe the polarization of R, as can be seen from Eq. (4).[3] They can be viewed as the generalization of the polarization from spin-half particles to particles of any spins produced in weak decay processes. We say that R is unpolarized if $\lambda^{(j)}_{\mathbb{H}\to Rc} = 0$, for all $j = 1, 2, \ldots, 2s_R$.

The differences between $\lambda^{(j)}_{\mathbb{H}\to Rc}$ and their CP correspondence $\lambda^{(j)}_{\bar{\mathbb{H}}\to \bar{R}\,\bar{c}}$ are measures of CP violation. One can define the decay-angular-distribution-correlated CP asymmetries as [26–28]

$$A^{(j)}_{CP} \equiv \frac{1}{2}\left(\lambda^{(j)}_{\mathbb{H}\to Rc} - \lambda^{(j)}_{\bar{\mathbb{H}}\to \bar{R}\,\bar{c}}\right). \tag{9}$$

Note that $A^{(0)}_{CP} \equiv 0$ by definition. From the experimental side, the direct observables are

$$\tilde{A}^{(j)}_{CP} = A^{(j)}_{CP}\alpha^{(j)}_{R\to ab}, \tag{10}$$

from which one can see that the observability of $A^{(j)}_{CP}$ is restricted by the values of $\alpha^{(j)}_{R\to ab}$. In other words, $\tilde{A}^{(j)}_{CP}$ can be observed only when both $A^{(j)}_{CP}$ and $\alpha^{(j)}_{R\to ab}$ are not too small.

In practice, the following definition of CP asymmetry observables is more easily accessible experimentally. The null points of $P_j(c_\theta)$ divide the interval of $c_\theta \in [-1, 1]$ into $j+1$ subintervals, which can be ordered from right $(c_\theta = +1)$ to left $(c_\theta = -1)$ as the 1st, 2nd, $\cdots$, kth, $\cdots$, $(j+1)$th subintervals. Denoting the event yields of each subinterval as $N^{(j)}_k$ and $\overline{N^{(j)}_k}$ for $k = 1, \ldots, j+1$, the CP asymmetry observables can be defined as

$$\hat{A}^{(j)}_{CP} \equiv \frac{\sum_{k=1}^{j+1}(-)^k\left[N^{(j)}_k - \overline{N^{(j)}_k}\right]}{\sum_k\left[N^{(j)}_k + \overline{N^{(j)}_k}\right]}. \tag{11}$$

It can be easily seen that $\hat{A}^{(0)}_{CP}$ is in fact the conventionally defined CP asymmetries corresponding to the branching ratios, $A_{CP}(\mathbb{H}\to R(\to ab)c)$.

III. INTERFERENCE BEHAVIOR OF CANONICAL DECAY AMPLITUDES IN THE DECAY-ANGULAR DISTRIBUTIONS

Interference between amplitudes with different weak phases may generate large CP asymmetry parameters, provided that there is also a large strong phase difference. Potential final-state interactions may generate such nonperturbative strong phase differences in different canonical partial-wave amplitudes [29]. In addition, the interference behavior is quite obscure in Eq. (4) since there are no interfering terms between different helicity amplitudes for the current situation. Consequently, it will be more transparent to see the interference pattern if we reexpress $\mathcal{W}^{(j)}$ in terms of canonical amplitudes, which reads

$$\mathcal{W}^{(j)} = \sum_{ls,l's'}\rho^{(j)}_{ls,l's'}a^{s_{\mathbb{H}}}_{ls}a^{s_{\mathbb{H}}*}_{l's'}, \tag{12}$$

where $a^{s_{\mathbb{H}}}_{ls}$'s are the canonical decay amplitudes of $\mathbb{H}\to Rc$ and are related to the helicity amplitudes through [30,31]

$$\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c} = \sum_{ls}\left(\frac{2l+1}{2s_{\mathbb{H}}+1}\right)^{\frac{1}{2}}\langle s_R, \sigma; s_c, -\lambda_c | s_R s_c s(\sigma-\lambda_c)\rangle \times \langle l, 0; s, \sigma-\lambda_c | l s s_{\mathbb{H}}(\sigma-\lambda_c)\rangle a^{s_{\mathbb{H}}}_{ls}, \tag{13}$$

or, inversely,

$$a^{s_{\mathbb{H}}}_{ls} = \sum_{\sigma\lambda_c}\left(\frac{2l+1}{2s_{\mathbb{H}}+1}\right)^{\frac{1}{2}}\langle s_R, \sigma; s_c, -\lambda_c | s_R s_c s(\sigma-\lambda_c)\rangle \times \langle l, 0; s, \sigma-\lambda_c | l s s_{\mathbb{H}}(\sigma-\lambda_c)\rangle \mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}, \tag{14}$$

and the rotational-invariant coefficient $\rho^{(j)}_{ls,l's'}$ reads

[3]We can further parametrize $\lambda^{(j)}_{\mathbb{H}\to Rc}$ as $\lambda^{(j)}_{\mathbb{H}\to Rc} = \mathcal{P}^{(j)}_R\kappa^{(j)}_{\mathbb{H}\to Rc}$, where $\mathcal{P}^{(j)}_R$ are the normalized polarization parameters which are defined as $\mathcal{P}^{(j)}_R \equiv \frac{\mathcal{W}^{(j)}}{\mathcal{W}^{(j)}_{\rm abs}}$, and $\kappa^{(j)}_{\mathbb{H}\to Rc}$ can be viewed as the production asymmetry ratio and are defined as $\kappa^{(j)}_{\mathbb{H}\to Rc} \equiv \frac{\mathcal{W}^{(j)}_{\rm abs}}{\mathcal{W}^{(0)}}$, where $\mathcal{W}^{(j)}_{\rm abs} \equiv \sum_{\sigma,\lambda_c}\left|\frac{(-)^{\sigma-s_R}\langle s_R,-\sigma;s_R,\sigma|s_R s_R j0\rangle}{\langle s_R,-s_R;s_R,s_R|s_R s_R j0\rangle}\right||\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}|^2$.

$$\rho^{(j)}_{ls,l's'} = \frac{\sqrt{(2l+1)(2l'+1)}}{(2s_{\mathbb{H}}+1)\langle s_R,-s_R;s_R,s_R|s_Rs_Rj0\rangle}\sum_{\sigma\lambda_c}(-)^{\sigma-s_R}\langle s_R,-\sigma;s_R,\sigma|s_Rs_Rj0\rangle$$
$$\times\langle l,0;s,\sigma-\lambda_c|lss_{\mathbb{H}}(\sigma-\lambda_c)\rangle\langle s_R,\sigma;s_c,-\lambda_c|s_Rs_cs(\sigma-\lambda_c)\rangle$$
$$\times\langle l',0;s',(\sigma-\lambda_c)|l's's_{\mathbb{H}}(\sigma-\lambda_c)\rangle\langle s_R,\sigma;s_c,-\lambda_c|s_Rs_cs'(\sigma-\lambda_c)\rangle. \tag{15}$$

In general, the presence of the interference between two canonical decay amplitudes in certain $\mathcal{W}^{(j)}$ is constrained by the properties of the coefficients $\rho^{(j)}_{ls,l's'}$. The following two properties of the coefficients $\rho^{(j)}_{ls,l's'}$, which can be proven with the aid of the properties of the Clebsch-Gordan coefficients, turn out to be very important for our analysis here:

(1) For a given value of j satisfying $0\le j\le 2s_R$, the indices of the nonzero elements of $\rho^{(j)}_{ls,l's'}$ fulfill the triangle inequality (necessary condition):

$$|l-l'|\le j\le l+l', \tag{16}$$

$$|s-s'|\le j\le s+s'. \tag{17}$$

(2) Zero elements:

$$\rho^{j}_{ls,l's'}=0,\quad \text{if}\begin{cases} j\text{ is even, one of } l \text{ and } l' \text{ is even and the other is odd;}\\ j \text{ is odd, both } l \text{ and } l' \text{ are even or odd.}\end{cases} \tag{18}$$

The first consequence of property 2 is that—since j can only take even values in Eq. (2)—the interference between parity-even and parity-odd amplitudes is absent in the decay amplitude squared of Eq. (2) when the subsequential decay of R is strong, where the parity of the canonical decay amplitude $a^{s_{\mathbb{H}}}_{ls}$ is determined according to $\Pi_{a^{s_{\mathbb{H}}}_{ls}}=(-)^l\Pi_{\mathbb{H}}\Pi_R\Pi_c$, with Π representing parity. This means that there can only be interference of $a^{s_{\mathbb{H}}}_{ls}$ and $a^{s_{\mathbb{H}}}_{l's'}$ in $\mathcal{W}^{(j)}$ of Eq. (2) when both l and l' are even or odd simultaneously for even j (so that $a^{s_{\mathbb{H}}}_{ls}$ and $a^{s_{\mathbb{H}}}_{l's'}$ have the same parity).

Of course, property 1 can also set constraints on the presence of the interfering terms in $\mathcal{W}^{(j)}$. For example, there will be no interfering terms between different canonical amplitudes in $\mathcal{W}^{(0)}$ because, according to property 1, $l=l'$ and $s=s'$ for $j=0$. Hence, one has $\rho^0_{ls,l's'}=\delta_{ll'}\delta_{ss'}$ when $j=0$, so that $\mathcal{W}^{(0)}=\sum_{\sigma\lambda_c}|\mathcal{F}^{s_{\mathbb{H}}}_{\sigma\lambda_c}|^2=\sum_{ls}|a^{s_{\mathbb{H}}}_{ls}|^2$, from which one can see that there is no interference between different canonical amplitudes in $w^{(0)}$, as expected.

For the weak process $\mathbb{H}\to Rc$, it can be proved that the number of independent canonical decay amplitudes, which is of course the same as that of the independent helicity decay amplitudes, is in total

$$N_{c.a.}=(2s_1+1)(2s_2+1)-\kappa(\kappa+1), \tag{19}$$

where $\kappa\equiv\max\{s_1+s_2-s_3,0\}$. Here, s_1, s_2, and s_3 represent the spins of the particles involved in the weak decay, $\mathbb{H}$, R, and c, which are ordered according to $s_1\le s_2\le s_3$. Moreover, it can be proved that the number of parity-even and parity-odd canonical amplitudes will be either the same (when $N_{c.a.}$ is an even number) or with a difference of 1 (when $N_{c.a.}$ is an odd number). Consequently, in order for the presence of the interferences between the canonical decay amplitudes with the same parity in $w^{(j)}$ for certain j, the number of the independent canonical decay amplitudes should be no less than 3, so that at least two of them share the same parity. The first few combinations of (s_1,s_2,s_3) that fulfill this requirement are (0, 1, 1), (0, 1, 2), $(0,\frac{3}{2},\frac{3}{2})$, (0, 2, 2), $(\frac{1}{2},\frac{1}{2},1)$, $(\frac{1}{2},\frac{1}{2},2)$, $(\frac{1}{2},1,\frac{3}{2})$, and $(\frac{1}{2},\frac{3}{2},2)$, with the number of independent canonical decay amplitudes being 3, 3, 4, 5, 4, 4, 6, and 8, respectively. Phenomenologically, if we confine ourselves only to discussions of unstable $\mathbb{H}$ dominated by weak decays, the first four combinations will be the cases of pseudoscalar heavy meson decays, while the last four combinations will be the cases of heavy baryon decays.

Take the typical weak transition of the type $\mathbb{B}\to\mathcal{B}V$ ($\frac{1}{2}^+\to\frac{1}{2}^++1^-$), such as $\Lambda_b\to\Lambda\rho^0$, as an example, where the vector meson decays subsequently to two pseudoscalar mesons through strong interactions. It can be easily seen that there are four independent weak decay canonical amplitudes in total, according to Eq. (19). There are, in total, three $\mathcal{W}^{(j)}$'s for $j=0$, 1, and 2, which can be respectively expressed in terms of the helicity amplitudes as well as the canonical ones as

$$\mathcal{W}^{(0)}=|\mathcal{F}_{1\frac{1}{2}}|^2+|\mathcal{F}_{-1-\frac{1}{2}}|^2+|\mathcal{F}_{0\frac{1}{2}}|^2+|\mathcal{F}_{0-\frac{1}{2}}|^2, \tag{20}$$

$$\mathcal{W}^{(1)} = |\mathcal{F}_{1\frac{1}{2}}|^2 - |\mathcal{F}_{-1-\frac{1}{2}}|^2, \tag{21}$$

$$\mathcal{W}^{(2)} = |\mathcal{F}_{1\frac{1}{2}}|^2 + |\mathcal{F}_{-1-\frac{1}{2}}|^2 - 2\left(|\mathcal{F}_{0\frac{1}{2}}|^2 + |\mathcal{F}_{0-\frac{1}{2}}|^2\right), \tag{22}$$

and

$$\mathcal{W}^{(0)} = \left(|a_{0\frac{1}{2}}|^2 + |a_{1\frac{1}{2}}|^2 + |a_{1\frac{3}{2}}|^2 + |a_{2\frac{3}{2}}|^2\right), \tag{23}$$

$$\mathcal{W}^{(1)} = \frac{2}{\sqrt{3}}\left[\sqrt{2}\Re\left(a_{0\frac{1}{2}}a^*_{1\frac{1}{2}}\right) + \Re\left(a_{0\frac{1}{2}}a^*_{1\frac{3}{2}}\right) + \Re\left(a_{1\frac{1}{2}}a^*_{2\frac{3}{2}}\right) + \sqrt{\frac{1}{2}}\Re\left(a_{1\frac{3}{2}}a^*_{2\frac{3}{2}}\right)\right], \tag{24}$$

$$\mathcal{W}^{(2)} = -\left(|a_{1\frac{3}{2}}|^2 + |a_{2\frac{3}{2}}|^2\right) - 2\sqrt{3}\Re\left(a_{0\frac{1}{2}}a^*_{2\frac{3}{2}} + a_{1\frac{1}{2}}a^*_{1\frac{3}{2}}\right), \tag{25}$$

where the symbol $\Re(\cdots)$ means that we take the real part of the term in the parentheses. From the helicity forms of $\mathcal{W}^{(1)}$ and $\mathcal{W}^{(2)}$, one can clearly see that they describe the polarization of the vector meson V: Note that $\mathcal{W}^{(1)}$ represents the asymmetry between helicity $+1$ and -1 of V, while $\mathcal{W}^{(2)}$ represents the asymmetry of transverse and longitudinal polarizations of V. The factor 2 in front of the longitudinal polarization parts is important and also understandable. It reflects the simple fact that the degrees of freedom for the transverse polarizations ($\sigma = \pm 1$) and the longitudinal parts ($\sigma = 0$) are different. On the other hand, the interference behavior between different amplitudes is easier to see from the canonical forms of $\mathcal{W}^{(1)}$ and $\mathcal{W}^{(2)}$.

The differential decay width can then be expressed as[4]

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta} = \frac{1}{2} + \frac{\mathcal{W}^{(2)}\mathcal{S}^{(2)}}{2\mathcal{W}^{(0)}\mathcal{S}^{(0)}}P_2(c_\theta) = \frac{1}{2} - \frac{1}{\sqrt{2}}\lambda^{(2)}_{\mathbb{B}\to\mathcal{B}V}P_2(c_\theta). \tag{26}$$

Note that $\mathcal{S}^{(0)}$ and $\mathcal{S}^{(2)}$ are correlated according to $\mathcal{S}^{(2)}/\mathcal{S}^{(0)} = -\sqrt{2}$. The reason for the correlation is understandable. Since the vector meson decays into two pseudoscalar mesons, there can only be one independent helicity amplitude. Both $\mathcal{S}^{(0)}$ and $\mathcal{S}^{(2)}$ must be proportional to the square of this helicity amplitude; hence, they are correlated. If the relative strong phase differences between $a_{0\frac{1}{2}}$ and $a_{2\frac{3}{2}}$ and/or $a_{1\frac{1}{2}}$ and $a_{1\frac{3}{2}}$ are not small, the contributions of the corresponding interfering terms to the *CP* asymmetries associated with the decay-angular distributions $A^{(2)}_{CP}$ could be large.

To see the *CP* asymmetry behavior in more detail, let us first parametrize the canonical decay amplitudes $a^{s_{\mathbb{H}}}_{ls}$ and their *CP* counterparts $\overline{a^{s_{\mathbb{H}}}_{ls}}$ as

$$a^{s_{\mathbb{H}}}_{ls} = \left(a^T_{ls} + a^P_{ls}e^{i(\phi_{\rm w}+\delta^{PT}_{ls})}\right)e^{i\delta_{ls}}, \tag{27}$$

$$\overline{a^{s_{\mathbb{H}}}_{ls}} = \left(a^T_{ls} + a^P_{ls}e^{i(-\phi_{\rm w}+\delta^{PT}_{ls})}\right)e^{i\delta_{ls}}, \tag{28}$$

where a^T_{ls} and a^P_{ls} are the tree and penguin parts of the canonical decay amplitudes, respectively; $\phi_{\rm w}$ and δ^{PT}_{ls} are, respectively, the weak and the strong phase difference between the tree and penguin parts of the same canonical decay amplitude a_{ls}; and δ_{ls} is the overall strong phase of the canonical amplitudes $a^{s_{\mathbb{H}}}_{ls}$. The *CP* asymmetry parameters will behave as

$$A^{(j)}_{CP} \sim 2\sin\phi_{\rm w}\sum_{ls,l's'}\rho^{(j)}_{ls,l's'}\left\{a^T_{ls}a^P_{l's'}\sin\left[(\delta_{ls}-\delta_{l's'}) - \delta^{PT}_{l's'}\right] - a^P_{ls}a^T_{l's'}\sin\left[(\delta_{ls}-\delta_{l's'}) + \delta^{PT}_{ls}\right]\right\}. \tag{29}$$

It is naturally expected (although this is only a possibility) that the relative strong phases between different canonical decay amplitudes could be relatively large, compared to the strong phase difference of the tree and penguin parts with different weak phases within the same canonical decay amplitude,

$$\delta_{ls} - \delta_{l's'} > \delta^{PT}_{ls} \sim \delta^{PT}_{l's'}. \tag{30}$$

This means that the *CP* asymmetry parameter(s) $A^{(j)}_{CP}$ will have a good chance of being dominated by the interference terms between different canonical decay amplitudes:

$$A^{(j)}_{CP} \sim 2\sin\phi_{\rm w}\sum_{ls\neq l's'}\rho^{(j)}_{ls,l's'}\left(a^T_{ls}a^P_{l's'} - a^P_{ls}a^T_{l's'}\right)\sin(\delta_{ls}-\delta_{l's'}). \tag{31}$$

Since there is no interference between different canonical decay amplitudes in $\hat{A}^{(0)}_{CP}$, it is likely that $A^{(j)}_{CP}$ for $j \neq 0$ will be larger than $\hat{A}^{(0)}_{CP}$, provided that the relative strong phases between different canonical decay amplitudes $\delta_{ls} - \delta_{l's'}$ are large. It should be emphasized that, in order to get large $\hat{A}^{(j)}_{CP}$'s, the relative strong phases between different

[4]The behavior in the expression seems to be different from the one used in the literature such as in [32]. One can see from Eq. (13) of Ref. [32] that there are interference terms of *S*- and *D*-waves in the integrated decay width Γ, while there is no such term (interference between $a_{0\frac{1}{2}}$ and $a_{2\frac{3}{2}}$ in our notation) in Eq. (24). This is because the definitions of the canonical amplitudes are different. In the current paper, a_{ls} are the canonical amplitudes in the sense that they transform irreducibly under SO(3), while the *S*- and *D*-waves in Ref. [32] are not.

canonical decay amplitudes is not enough; the weak phase $\phi_{\rm w}$ and the term $a^T_{ls}a^P_{l's'} - a^P_{ls}a^T_{l's'}$ should not be too small either.

While the presence of the strong phase difference between different partial-wave amplitudes is an important necessary condition for large CP asymmetry corresponding to the decay-angular distributions, it should be pointed out that this is not a sufficient condition. To see this, let us again use the decay $\mathbb{B} \to \mathcal{B}V$ as an example. Theoretical analyses of this type of decay have been performed via the generalized factorization approach (GFA) for bottom baryon decays [33–35]. Based on helicity decay amplitudes obtained from GFA, it can be shown that the canonical decay amplitudes and their CP conjugates take the forms $a_{ls} = \mathcal{C}_V K_{ls}$ and $\overline{a_{ls}} = \overline{\mathcal{C}_V} K_{ls}$, where K_{ls} is the kinematical part which depends on l and s, and the parameters $\mathcal{C}_V$ and $\overline{\mathcal{C}_V}$ contain the CKM matrix elements and are universal for all a_{ls}. In particular, $\mathcal{C}_V$ and $\overline{\mathcal{C}_V}$ are independent of l and s. While we can generate a nonzero direct CP asymmetry corresponding to the branching ratios, the CP asymmetries corresponding to the decay-angular distributions, which are defined in Eq. (11), are predicted to be exactly zero by GFA because the universal $\mathcal{C}_V$ ($\overline{\mathcal{C}_V}$) is canceled out in $\lambda^{(j)}_{\mathbb{H}\to Rc}$ ($\lambda^{(j)}_{\bar{\mathbb{H}}\to \bar{R}\,\bar{c}}$) so that $\lambda^{(j)}_{\mathbb{H}\to Rc} = \lambda^{(j)}_{\bar{\mathbb{H}}\to \bar{R}\,\bar{c}}$. In other words, there can be only one CP asymmetry in the approach of GFA, i.e., CP asymmetry corresponding to the branching ratios, $\tilde{A}^{(0)}_{CP}$. Consequently, it is crucially important to go beyond GFA for the predictions of CP asymmetry corresponding to the decay-angular distributions.

IV. SUGGESTED CHANNELS FOR SEARCHING FOR DECAY-ANGULAR-DISTRIBUTION-CORRELATED *CP* ASYMMETRIES IN BARYON DECAYS

It is very hard to make a concrete prediction of any decay-distribution-correlated CP asymmetries in heavy baryon cascade decays. Nevertheless, our analyses provide some guidelines for searching for such CP asymmetries in certain decay processes. According to the above analysis, two of the guidelines are especially important. The first one is that the number of independent canonical decay amplitudes $N_{c.a.}$ should be no less than 3, which will constrain the spins of the particle involved according to Eq. (19). The second one is that j can only take even values from 0 to $2s_R$, which implies that s_R should be no less than 1 in order for the nontrivial decay-angular distributions ($j \geq 2$) to appear. Of course, the properties of the coefficients $\rho^{(j)}_{ls,l's'}$ can provide us with more detailed information on the interference of different canonical decay amplitudes in $\mathcal{W}^{(j)}$.

The above guidelines suggest that we can study the decay-angular distributions and the corresponding CP asymmetries in the following two immediate situations. The first one is that $\mathbb{H}$ is a pseudoscalar heavy meson, which will be denoted as $\mathbb{M}$ for this situation. From Eq. (19) one can see that, in this situation, the necessary condition for $N_{c.a.} \geq 3$ is that the spins of both R and c should be nonzero and larger than half. As aforementioned, typical examples for the spin combinations (s_1, s_2, s_3) of $\mathbb{M}$, R, and c that fulfill this requirement are (0, 1, 1), (0, 1, 2), $(0, \frac{3}{2}, \frac{3}{2})$, (0, 2, 2). The simplest decays which can be used to perform the search for decay-angular-distribution-correlated CP asymmetries are (1) $\mathbb{M} \to V_1V_2$, with V_1 or V_2 decaying strongly to two pseudoscalar mesons; (2) $\mathbb{M} \to B_1^*B_2^*$, with B_1^* and B_2^* being spin-one-and-a-half baryons and one of them decaying via strong interactions. Since our main concern is heavy baryon decay processes, we will not go through the heavy meson decay processes any further [36].

The second situation, which is our main concern in this paper, is that $\mathbb{H}$ represents a spin-half heavy baryon, which will be denoted as $\mathbb{B}$ in what follows. In this case, since either R or c is a baryon, in order for $N_{c.a.} \geq 3$, Eq. (19) indicates that the other particle must be a spin-nonzero meson. In view of the above constraints, we propose to search for decay-distribution-correlated CP violation in cascade decays of the following types: (1) $\mathbb{B} \to \mathcal{B}M$, $M \to M_1M_2$, with the spin of the meson M being nonzero; (2) $\mathbb{B} \to \mathcal{B}M$, $\mathcal{B} \to \mathcal{B}'M'$, with the spin of the baryon resonance $\mathcal{B}$ being larger than $\frac{1}{2}$, and the spin of M being nonzero. Here, $\mathcal{B}$ and $\mathcal{B}'$ represent light baryons, and M, M', M_1, and M_2 represent light mesons.

The first type seems to be more common and more applicable, among which the most relevant decay type is of the form $\mathbb{B} \to \mathcal{B}V$, with a subsequent strong decay of the vector resonance $V \to P_1P_2$. Typical decays of the form $\mathbb{B} \to \mathcal{B}V$ include (1) $b \to du\bar{u}$ transitions: $\Lambda_b^0 \to p\rho(770)^+$, $\Lambda_b^0 \to N(1520)^*\rho(770)^+$; (2) $b \to su\bar{u}$ transitions: $\Lambda_b^0 \to \Lambda\rho(770)^0$, $\Lambda_b^0 \to pK^*(892)^-$, $\Lambda_b^0 \to N(1520)K^*$; (3) $c \to ud\bar{d}$ transitions: $\Lambda_c^+ \to p\rho(770)^0$, $\Xi_c^+ \to p\overline{K^*}(892)^0$; and (4) $c \to us\bar{s}$ transitions: $\Lambda_c^+ \to p\phi$, $\Lambda_c^+ \to \Sigma^+K^*(892)^0$.

For the second type, $\mathbb{B} \to \mathcal{B}M, \mathcal{B} \to \mathcal{B}'M'$, where the spin of the intermediate baryon resonance is no less than $\frac{3}{2}$ and the spin of M is no less than 1, typical decays include (1) $c \to ud\bar{d}$ transitions: $\Lambda_c^+ \to N(1520)^*\rho(770)^0$, $\Xi_c^+ \to N(1520)^*\overline{K^*}(892)^0$; (2) $c \to us\bar{s}$ transitions: $\Lambda_c^+ \to N(1520)^*\phi$, $\Lambda_c^+ \to \Sigma^+K^*(892)^0$; (3) the $b \to du\bar{u}$ transition: $\Lambda_b^0 \to N(1520)^*\rho(770)^+$; and (4) the $b \to su\bar{u}$ transition: $\Lambda_b^0 \to N(1520)K^*$. For most of the cases, the study of $A^{(j)}$ and/or $\hat{A}^{(j)}$ for $j = 2$ will be enough since all the spins of the resonances mentioned in the above examples are less than 2.

V. SUMMARY AND CONCLUSION

CP violation in baryon decay processes has not been observed yet. In this paper, the decay-angular-distribution-correlated CP asymmetries for cascade decays of unpolarized heavy hadrons are analyzed. By expressing the differential decay width in terms of the canonical decay amplitudes, we analyze the general condition for the presence of the interfering terms between different canonical decay amplitudes. The presence of the interfering terms is important for the generation of large decay-angular-distribution-correlated *CP* asymmetries.

We focus mainly on one typical type of decay, in which the heavy baryon decays weakly into two daughter hadrons, with one of them decaying strongly into two granddaughter hadrons. The analysis indicates that when the two daughter hadrons are both spin nonzero, there will be at least two parity-even and parity-odd canonical amplitudes. The interference between the canonical amplitudes with the same parity properties will be present in the angular distributions of the final particles. With a possible large strong phase between different canonical amplitudes, it is possible that a large *CP* asymmetry corresponding to the decay-angular distributions may emerge. We also present some typical decay channels in which the search for such *CP* asymmetries can be performed.

ACKNOWLEDGMENTS

We thank Professor Hai-Yang Cheng, Professor Hsiang-nan Li, and Dr. Chia-Wei Liu for valuable discussions. This work was supported by the National Natural Science Foundation of China under Grants No. 12192261 and No. 12275024, the Natural Science Foundation of Hunan Province under Grant No. 2022JJ30483, and Scientific Research Fund of Hunan Provincial Education Department under Grant No. 22A0319.

[1] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, Astron. Astrophys. **641**, A6 (2020); Astron. Astrophys. **652**, C4(E) (2021).

[2] A. D. Sakharov, Violation of *CP* invariance, C asymmetry, and baryon asymmetry of the universe, Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32 (1967).

[3] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Evidence for the 2π decay of the K_2^0 meson, Phys. Rev. Lett. **13**, 138 (1964).

[4] R. Aaij *et al.* (LHCb Collaboration), Observation of *CP* violation in charm decays, Phys. Rev. Lett. **122**, 211803 (2019).

[5] B. Aubert *et al.* (*BABAR* Collaboration), Measurement of *CP* violating asymmetries in B^0 decays to *CP* eigenstates, Phys. Rev. Lett. **86**, 2515 (2001).

[6] K. Abe *et al.* (Belle Collaboration), Observation of large *CP* violation in the neutral B meson system, Phys. Rev. Lett. **87**, 091802 (2001).

[7] R. Aaij *et al.* (LHCb Collaboration), First observation of *CP* violation in the decays of B_s^0 mesons, Phys. Rev. Lett. **110**, 221601 (2013).

[8] R. L. Workman (Particle Data Group), Review of particle physics, Prog. Theor. Exp. Phys. **2022**, 083C01 (2022).

[9] N. Cabibbo and A. Maksymowicz, Angular correlations in K_{e4} decays and determination of low-energy $\pi-\pi$ phase shifts, Phys. Rev. **137**, B438 (1965); Phys. Rev. **168**, 1926(E) (1968).

[10] M. Kobayashi and T. Maskawa, *CP* violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. **49**, 652 (1973).

[11] T. D. Lee and C.-N. Yang, General partial wave analysis of the decay of a hyperon of spin $1/2$, Phys. Rev. **108**, 1645 (1957).

[12] J. F. Donoghue and S. Pakvasa, Signals of *CP* nonconservation in hyperon decay, Phys. Rev. Lett. **55**, 162 (1985).

[13] J. F. Donoghue, X.-G. He, and S. Pakvasa, Hyperon decays and *CP* nonconservation, Phys. Rev. D **34**, 833 (1986).

[14] I. I. Bigi, X.-W. Kang, and H.-B. Li, *CP* asymmetries in strange baryon decays, Chin. Phys. C **42**, 013101 (2018).

[15] J.-P. Wang and F.-S. Yu, Probing hyperon *CP* violation from charmed baryon decays, Phys. Lett. B **849**, 138460 (2024).

[16] M. Ablikim *et al.* (BESIII Collaboration), Polarization and entanglement in baryon-antibaryon pair production in electron-positron annihilation, Nat. Phys. **15**, 631 (2019).

[17] L. K. Li *et al.* (Belle Collaboration), Search for *CP* violation and measurement of branching fractions and decay asymmetry parameters for $\Lambda_c^+ \to \Lambda h^+$ and $\Lambda_c^+ \to \Sigma^0 h^+$ ($h = K,\pi$), Sci. Bull. **68**, 583 (2023).

[18] S.-S. Tang, L.-K. Li, X.-Y. Zhou, and C.-P. Shen, Recent measurements of decay asymmetry parameter and *CP* asymmetry for charmed baryon decays at Belle, Symmetry **15**, 91 (2023).

[19] A. K. Giri, R. Mohanta, and M. P. Khanna, Possibility of extracting the weak phase γ from $\Lambda_b \to \Lambda D^0$ decays, Phys. Rev. D **65**, 073029 (2002).

[20] S. Zhang, Y. Jiang, Z. Chen, and W. Qian, Sensitivity studies on the CKM angle γ in $\Lambda_b^0 \to D\Lambda$ decays, arXiv: 2112.12954.

[21] Z.-H. Zhang, X.-H. Guo, and Y.-D. Yang, *CP* violation in $B^\pm \to \pi^\pm\pi^+\pi^-$ in the region with low invariant mass of one $\pi^+\pi^-$ pair, Phys. Rev. D **87**, 076007 (2013).

[22] Z.-H. Zhang and J.-J. Qi, Analysis of angular distribution asymmetries and the associated *CP* asymmetries in three-body decays of bottom baryons, Eur. Phys. J. C **83**, 133 (2023).

[23] R. Aaij *et al.* (LHCb Collaboration), Measurements of the $\Lambda_b^0 \to J/\psi\Lambda$ decay amplitudes and the Λ_b^0 polarisation in pp collisions at $\sqrt{s} = 7$ TeV, Phys. Lett. B **724**, 27 (2013).

[24] R. Aaij *et al.* (LHCb Collaboration), Measurement of the $\Lambda_b^0 \to J/\psi\Lambda$ angular distribution and the Λ_b^0 polarisation in pp collisions, J. High Energy Phys. 06 (2020) 110.

[25] P. Avery *et al.* (CLEO Collaboration), Measurement of the Λ_c^+ decay asymmetry parameter, Phys. Rev. Lett. **65**, 2842 (1990).

[26] Z.-H. Zhang and X.-H. Guo, A novel strategy for searching for CP violations in the baryon sector, J. High Energy Phys. 07 (2021) 177.

[27] Y.-R. Wei and Z.-H. Zhang, The forward-backward asymmetry induced CP asymmetry of $B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$, Phys. Rev. D **106**, 113002 (2022).

[28] Z.-H. Zhang, Searching for CP violation through two-dimensional angular distributions in four-body decays of bottom and charmed baryons, Phys. Rev. D **107**, L011301 (2023).

[29] K. M. Watson, The effect of final state interactions on reaction cross-sections, Phys. Rev. **88**, 1163 (1952).

[30] M. Jacob and G. C. Wick, On the general theory of collisions for particles with spin, Ann. Phys. (N.Y.) **7**, 404 (1959).

[31] S. U. Chung, Spin formalisms, 10.5170/CERN-1971-008 (1971).

[32] S. Pakvasa, S. P. Rosen, and S. F. Tuan, Parity violation and flavor selection rules in charmed baryon decays, Phys. Rev. D **42**, 3746 (1990).

[33] O. Leitner and Z. J. Ajaltouni, Testing CP and time reversal symmetries with $\Lambda_b \to \Lambda V(1-)$ decays, Nucl. Phys. B, Proc. Suppl. **174**, 169 (2007).

[34] C. Q. Geng, C.-W. Liu, and T.-H. Tsai, Charmed baryon weak decays with vector mesons, Phys. Rev. D **101**, 053002 (2020).

[35] C.-Q. Geng and C.-W. Liu, Time-reversal asymmetries and angular distributions in $\Lambda_b \to \Lambda$V, J. High Energy Phys. 11 (2021) 104.

[36] I. Dunietz, H. R. Quinn, A. Snyder, W. Toki, and H. J. Lipkin, How to extract CP violating asymmetries from angular correlations, Phys. Rev. D **43**, 2193 (1991).