Study of $B_s \to \phi(\rho^0, \omega)$ decays in the standard model and a family nonuniversal Z' model

Ying $\text{Li}\nightharpoonup$, Yue Sun, and Zhi-Tian Zou Department of Physics, [Yantai University,](https://ror.org/01rp41m56) Yantai 264005, China

(Received 15 February 2024; accepted 5 June 2024; published 2 July 2024)

Within the QCD factorization approach, we employ the new results of form factors of $B_s \to \phi$ and calculate the branching fractions, CP asymmetries (CPAs) and polarization fractions of the decay modes $B_s \to \phi(\rho^0, \omega)$ in both the standard model (SM) and the family nonuniversal Z' model. We find that in SM the above observables are relate to two parameters, ρ_H and ϕ_H , which characterize the end-point divergence in the hard spectator-scattering amplitudes. When setting $\rho_H = 0.5$ and $\phi_H \in [-180, 180]^\circ$, the theoretical uncertainties are large. By combining the experimental data, our results can be used to constrain these two parameters. Supposing $\rho_H = 0$, we study the effects of the Z' boson on the concerned observables. With the available branching fraction of $B_s \to \phi \rho^0$, the possible ranges of parameter spaces are obtained. Within the allowed parameter spaces, the branching fraction of $B_s \to \phi \omega$ can be enlarged remarkably. Furthermore, the new introduced weak phase plays important roles in significantly affecting the CPAs and polarization fractions, which are important observables for probing the effects of NP. If these decay modes were measured in the on-going LHC-b and Belle-II experiments in the future, the peculiar deviation from SM could provide a signal of the family nonuniversal $Z[']$ model, which can also be used to constrain the mass of Z' boson in turn.

DOI: [10.1103/PhysRevD.110.013001](https://doi.org/10.1103/PhysRevD.110.013001)

I. INTRODUCTION

It is well known that the hadronic charmless B-meson weak decays play important roles in testing the flavor dynamics of the standard model (SM) and searching for possible effects of new physics (NP) beyond of SM. Particularly, decays dominated by contributions from flavor-changing neutral-currents (FCNC) provide a sensitive probe for NP because their amplitudes are described by loop (or penguin) diagrams where new particles may enter. Moreover, the SM predictions for CP asymmetry (CPA) in several decays are tiny, making them ideal places to look for effects of NP. In past decades, there are already many measurements in rare B decays at the B factories Belle and BABAR, and the LHC experiments, such as $B \to \pi \pi$, $B \to \pi K^*$, $B \to \phi K^*$, and $B_s \to \phi \phi$ decays [\[1\]](#page-8-0).

Although new particles have not been observed directly in the on-going LHC experiments, several experimental results on rare b decays show poor agreement with the corresponding SM predictions. For example, the LHCb collaboration reported deviations in the angular distribution

of the $B \to K^* \mu^+ \mu^-$ decay (the " P'_5 anomaly") and in the branching fractions of the $B_s \to \phi \mu^+ \mu^-$, $B \to K^* \mu^+ \mu^-$, $B \to K \mu^+ \mu^-$, and $B \to K^{(*)} \nu \bar{\nu}$ decays (for recent reviews, see, e.g., [[2](#page-8-1),[3\]](#page-8-2) and references therein). Theoretically, in order to explain the above deviations, on the one hand, we need to calculate all possible corrections in SM including high-order and high-power corrections, and on the other hand we also consider whether these anomalies are due to contributions of NP. It is found that most deviations of semileptonic B decays are related to the FCNC $b \rightarrow s\mu^{+}\mu^{-}$ transition. If NP is indeed at the origin of the anomalies in the $b \to s\mu^+\mu^-$ transition, it is natural to expect signals in other observables induced by other $b \rightarrow s$ transitions, possibly with different realizations though sharing some common features. A natural extension to explore the possible existence of these signals is the charmless hadronic B decays induced by $b \rightarrow s q \bar{q}$ transitions. Based on this strategy, some decays such as $B_s^0 \to K^{(*)} \bar{K}^{(*)}$ [[4](#page-8-3)[,5\]](#page-8-4) and $B_s^0 \rightarrow \phi \phi$ [\[6\]](#page-8-5) have been studied comprehensively. However, unlike semileptonic decays, B meson hadronic decays suffer from larger uncertainties arising from the nonperturbative inputs and corrections from high order and high power, so that it is more difficult to calculate with a high precision. For this reason, we are encouraged to search for observables of some decay modes that have less theoretical uncertainties.

Motivated by the above, we would like to study the process $B_s^0 \to \phi \rho^0$ and $B_s^0 \to \phi \omega$, which are dominated by

[^{*}](#page-0-1) liying@ytu.edu.cn

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

FIG. 1. The possible Feynman diagrams for the $B_s^0 \to \phi \rho^0$, in which (a) is the electroweak penguin diagram, (b) is the suppressed tree diagram and, (c) is the singlet-annihilation diagram.

the FCNC $b \rightarrow s q \bar{q}$ transition. In SM, the possible Feynman diagrams for the $B_s^0 \rightarrow \phi \rho^0$ are presented in Fig. [1](#page-1-0), where Fig. [1\(a\)](#page-1-0) is the electroweak penguin diagram, Fig. [1\(b\)](#page-1-0) is the suppressed tree diagram, and Fig. [1\(c\)](#page-1-0) is the singlet-annihilation diagram. In particular, the last diagram called hairpin diagram is viewed as suppressed by Okubo-Zweig-Iizuka rules, because the ρ^0 meson is produced from at least three gluons, thus this contribution is neglected. Therefore, $B_s^0 \rightarrow \phi \rho^0$ decay is independent of weak annihilation contributions and only sensitive to hard spectator scattering corrections, making it an ideal channel for determining the end-point parameters in the hard spectator scattering amplitudes. Furthermore, the interference between Figs. [1\(a\)](#page-1-0) and [1\(b\)](#page-1-0) might lead to possible CPA in this decay.

In order to calculate the two body charmless nonleptonic B decays, several attractive QCD-inspired approaches, such as QCD factorization (QCDF) [\[7](#page-9-0),[8](#page-9-1)], perturbative QCD $(pQCD)$ [[9](#page-9-2)[,10\]](#page-9-3), and soft-collinear effective theory [\[11,](#page-9-4)[12](#page-9-5)], have been proposed in the past decades. In previous studies, the branching fractions of these decays are shown to be about 10^{-7} in SM, both in QCDF [\[13](#page-9-6)–[17\]](#page-9-7) and in pQCD [\[18\]](#page-9-8). In the experimental side, the branching fraction of $B_s \rightarrow \phi \rho^0$ decay has been measured by the LHCb collaboration [\[19\]](#page-9-9),

$$
\mathcal{B}(B_s \to \phi \rho^0) = (2.7 \pm 0.7_{\text{stat}} \pm 0.2_{\text{syst}}) \times 10^7, \quad (1)
$$

with a significance of about 4σ . However, the polarization fractions and CPA have not been measured until now. In addition, $B_s \to \phi \rho^0$ has been suggested as a tool to measure γ via the mixing-induced CP asymmetry [\[20\]](#page-9-10). Since in the era of LHCb and Belle-II these two processes become interesting objects for tests of isospin-violation and potential NP [\[21\]](#page-9-11), we will in the following study their phenomenology in full detail, in SM and beyond. In the calculation, we will adopt the QCDF approach, since there is no annihilation contribution in these decays.

Although most experimental data is consistent with the SM predictions, most of us believe that SM is just an effective theory of a more fundamental one yet to be discovered. The presence of a Z' boson associated with an additional $U(1)$ ['] gauge symmetry is a well-motivated extension of SM. It should be emphasized that this additional symmetry has not been invented to solve a particular problem of SM, but rather occurs as a by-product in many models like, e.g., grand unified theories, various models of dynamical symmetry breaking, and little-Higgs models. An extensive review about the physics of Z' gauge bosons can be found in [\[22\]](#page-9-12). The most interesting feature is that the family nonuniversal Z' couplings could lead to FCNC in the tree level $[23,24]$ $[23,24]$. The phenomenological effects of Z' in B decays have been studied extensively $[25-29]$ $[25-29]$ $[25-29]$. In this work, we will address the effect of the Z' in the rare decay modes $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$. Because these two decays are all penguin dominated process and mediated by $b \rightarrow s q \bar{q}$, they are expected to be sensitive to the effect of the Z' .

The layout of this paper is as follows. In Sec. [II](#page-1-1) we will present the theoretical predictions of $B_s \to \phi \rho^0$ and $B_s \to$ $\phi\omega$ in SM based on QCDF. Section [III](#page-6-0) is devoted to the contributions of Z' . Some discussions are also given in this section. We will summarize this work at last.

II. PREDICTIONS IN SM

In SM, the effective weak Hamiltonian mediating FCNC transition of the type $b \rightarrow s q \bar{q} (q = u, d)$ has the form [\[30\]](#page-9-17)

$$
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{p=u,c} V_{pb} V_{ps}^* (C_1 O_1^p + C_2 O_2^p) - V_{tb} V_{ts}^* \sum_{i=3}^{10} C_i O_i \right],
$$
\n(2)

where G_F is Fermi coupling constant and $V_{pb}V_{ps}^*$ is the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element. C_i are the Wilson coefficients with the renormalization scale $\mu \sim m_b$. The O_i are the local four fermion operators, and $O_{1,2}^p$ are the left-handed currentcurrent operators, $O_{3,...,6}$ and $O_{7,...,10}$ are the QCD and electroweak penguin operators, respectively, and they can be expressed as follows:

$$
O_{1}^{p} = (\bar{s}b)_{V-A}(\bar{q}p)_{V-A},
$$

\n
$$
O_{2}^{p} = (\bar{s}_{a}b_{\beta})_{V-A}(\bar{q}_{\beta}p_{a})_{V-A},
$$

\n
$$
O_{3} = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}q)_{V-A},
$$

\n
$$
O_{4} = (\bar{s}_{a}b_{\beta})_{V-A} \sum_{q} (\bar{q}_{\beta}q_{a})_{V-A},
$$

\n
$$
O_{5} = (\bar{s}b)_{V-A} \sum_{q'} (\bar{q}q)_{V+A},
$$

\n
$$
O_{6} = (\bar{s}_{a}b_{\beta})_{V-A} \sum_{q} (\bar{q}_{\beta}q_{a})_{V+A},
$$

\n
$$
O_{7} = (\bar{s}b)_{V-A} \sum_{q} \frac{3}{2} e_{q}(\bar{q}q)_{V+A},
$$

\n
$$
O_{8} = (\bar{s}_{a}b_{\beta})_{V-A} \sum_{q} \frac{3}{2} e_{q}(\bar{q}_{\beta}q_{a})_{V+A},
$$

\n
$$
O_{9} = (\bar{s}b)_{V-A} \sum_{q} \frac{3}{2} e_{q}(\bar{q}q)_{V-A},
$$

\n
$$
O_{10} = (\bar{s}_{a}b_{\beta})_{V-A} \sum_{q} \frac{3}{2} e_{q}(\bar{q}_{\beta}q_{a})_{V-A},
$$

\n(3)

where α and β are color indices and e_q are charges of the corresponding quark.

In the QCDF approach, the hadronic matrix element of the decay $\bar{B}_s \to \phi \rho^0$ can be written as

$$
\langle \phi \rho^0 | O_i | \bar{B}_s^0 \rangle = \sum_j F_j^{B_s^0 \to \phi} \int_0^1 dx T_{ij}^I(x) \Phi_\rho(x)
$$

+
$$
\int_0^1 d\xi \int_0^1 dx \int_0^1 dy T_i^I(\xi, x, y)
$$

$$
\times \Phi_{B_s}(\xi) \Phi_\phi(x) \Phi_\rho(y), \tag{4}
$$

where T_{ij}^I and T_i^I are the perturbative short-distance interactions and can be calculated perturbatively. $\Phi_X(x)(X = B_s, \pi, \phi)$ are the universal and nonperturbative distribution amplitudes, which can be estimated by the nonperturbative approaches, such as the light cone QCD sum rules, QCD sum rules, or lattice QCD.

Because the initial state heavy meson has spin 0, the two vector mesons must have the same helicity due to the conservation of angular momentum. Within the framework of QCDF, the effective Hamiltonian matrix elements are written in the form

$$
\langle \phi \rho^0 | \mathcal{H}_{\rm eff} | \bar{B}_s^0 \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(q)} \langle \phi \rho^0 | \mathcal{T}_{\mathcal{A}}^{h,p} | \bar{B}_s^0 \rangle, \qquad (5)
$$

where the superscript h denotes the helicity of the finalstate meson. $T_A^{h,p}$ describes contributions from naive factorization, vertex corrections, penguin contractions, and spectator scattering expressed in terms of the flavor operators $a_i^{p,h}$. Specifically,

$$
\mathcal{T}_{A}^{p,h} = a_{1}^{p,h}(M_{1}M_{2})\delta_{pu}(\bar{s}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_{2}^{p,h}(M_{1}M_{2})\delta_{pu}(\bar{u}b)_{V-A} \otimes (\bar{s}u)_{V-A} + a_{3}^{p,h}(M_{1}M_{2}) \sum (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V-A}
$$

\n
$$
+ a_{4}^{p,h}(M_{1}M_{2}) \sum (\bar{q}b)_{V-A} \otimes (\bar{s}q)_{V-A} + a_{5}^{p,h}(M_{1}M_{2}) \sum (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V-A}
$$

\n
$$
+ a_{6}^{p,h}(M_{1}M_{2}) \sum (-2)(\bar{q}b)_{S-P} \otimes (\bar{s}q)_{S+P} + a_{7}^{p,h}(M_{1}M_{2}) \sum (\bar{s}b)_{V-A} \otimes \frac{3}{2} e_{q}(\bar{q}q)_{V+A}
$$

\n
$$
+ a_{8}^{p,h}(M_{1}M_{2}) \sum (-2)(\bar{q}b)_{S-P} \otimes \frac{3}{2} e_{q}(\bar{s}q)_{S+P} + a_{9}^{p,h}(M_{1}M_{2}) \sum (\bar{s}b)_{V-A} \otimes \frac{3}{2} e_{q}(\bar{q}q)_{V-A}
$$

\n
$$
+ a_{10}^{p,h}(M_{1}M_{2}) \sum (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_{q}(\bar{s}q)_{V-A},
$$

\n(6)

where $(\bar{q}_1q_2)_{V\pm A} \equiv \bar{q}_1\gamma_\mu(1 \pm \gamma_5)q_2$ and $(\bar{q}_1q_2)_{S\pm P} \equiv$ $\overline{q}_1(1 \pm \gamma_5)q_2$, and the summation is over $q = u, d$. The symbol ⊗ indicates that the matrix elements of the operators in T_A are to be evaluated in the factorized form.

The decay constant and form factors are defined by [[15](#page-9-18)]

$$
\langle V(p,\varepsilon)|V_{\mu}|0\rangle = f_V m_V \varepsilon_{\mu}^*,\tag{7}
$$

$$
\langle V(p,\varepsilon)|V_{\mu}|B(p_B)\rangle = \frac{2}{m_B + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_B^{\alpha} p^{\beta} V(q^2), \quad (8)
$$

$$
\langle V(p, \varepsilon) | A_{\mu} | B(p_B) \rangle = i \left\{ (m_B + m_V) \varepsilon_{\mu}^* A_1(q^2) - \frac{\varepsilon^* \cdot p_B}{m_B + m_V} (p_B + p)_{\mu} A_2(q^2) - 2m_V \frac{\varepsilon^* \cdot p_B}{q^2} q_{\mu} [A_3(q^2) - A_0(q^2)] \right\},
$$
\n(9)

where $q = p - p'$, $A_3(0) = A_0(0)$, and

$$
A_3(q^2) = \frac{m_B + m_V}{2m_V} A_1(q^2) - \frac{m_B - m_V}{2m_V} A_2(q^2).
$$
 (10)

In this work, the form factors of $B_s \to \phi$ we adopted are from [[31](#page-9-19)] based on the light-cone sum rules. For a decay $B(p_B) \to V_1(\epsilon_1^*, p_1) V_2(\epsilon_2^*, p_2)$, its factorizable matrix elements are thus given as

$$
X_h^{(BV_1, V_2)} \equiv \langle V_2 | J_\mu | 0 \rangle \langle V_1 | J^{\mu'} | B \rangle \tag{11}
$$

$$
=-if_{V_2}m_2\left[(\varepsilon_1^* \cdot \varepsilon_2^*)(m_B + m_{V_1})A_1^{BV_1}(m_{V_2}^2) - (\varepsilon_1^* \cdot p_B)(\varepsilon_2^* \cdot p_B) \frac{2A_2^{BV_1}(m_{V_2}^2)}{m_B + m_{V_1}} + i\varepsilon_{\mu\nu\alpha\beta}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p_B^{\alpha}p_1^{\beta} \frac{2V^{BV_1}(m_{V_2}^2)}{(m_B + m_{V_1})}\right], \quad (12)
$$

where V_1 takes the spectator quark of the B meson and V_2 is the emitted meson. So, the longitudinal ($h = 0$) and transverse $(h = \pm)$ components are obtained as

$$
X_0^{(BV_1,V_2)} = \frac{i f_{V_2}}{2m_{V_1}} \left[(m_B^2 - m_{V_1}^2 - m_{V_2}^2)(m_B + m_{V_1}) A_1^{BV_1}(q^2) - \frac{4m_B^2 p_c^2}{m_B + m_{V_1}} A_2^{BV_1}(q^2) \right],
$$
(13)

$$
X_{\pm}^{(BV_1, V_2)} = -if_{V_2} m_B m_{V_2} \left[\left(1 + \frac{m_{V_1}}{m_B} \right) A_1^{BV_1}(q^2) \mp \frac{2p_c}{m_B + m_{V_1}} V^{BV_1}(q^2) \right]. \tag{14}
$$

In QCDF, $a_i^{p,h}$ in Eq. [\(6\)](#page-2-0) are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections including vertex corrections and hard spectator interactions, and they have the expressions as

$$
a_i^{p,h}(V_1V_2) = \left(C_i + \frac{C_{i\pm 1}}{N_c}\right)N_i^h(V_2) + \frac{C_{i\pm 1}}{N_c}\frac{C_F\alpha_s}{4\pi}\left[V_i^h(V_2) + \frac{4\pi^2}{N_c}H_i^h(V_1V_2)\right] + P_i^{h,p}(V_2),\tag{15}
$$

where $i = 1, ..., 10$. The upper (lower) signs apply when i is odd (even), C_i are the Wilson coefficients, $C_F =$ $(N_c^2 - 1)/(2N_c)$ with $N_c = 3$. The functions $V_i^h(V_2)$ stand for vertex corrections, $H_i^h(V_1V_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark, and $P_i(V_2)$ for penguin contractions. In addition, the expression of the quantities N_i^h reads

$$
N_i^h(V_2) = \begin{cases} 0, & i = 6, 8 \\ 1, & \text{else.} \end{cases}
$$
 (16)

It is noted that there are end-point divergences when we study power corrections in QCDF. When we calculate the hard spectator interactions $H_i^h(V_1V_2)$ at twist-3 order, soft and collinear divergences arise from the soft spectator quark [[7\]](#page-9-0). Since the treatment of end-point divergences is model dependent, these subleading power corrections generally can be studied only in a phenomenological way. In this work, we shall follow [\[7,](#page-9-0)[13\]](#page-9-6) to model the end-point divergence in $H_i^h(V_1V_2)$ as

$$
X_H \equiv \int_0^1 \frac{dx}{1-x} = (1 + \rho_H e^{i\phi_H}) \ln\left(\frac{m_B}{\Lambda_h}\right), \quad (17)
$$

where $\Lambda_h = 500$ MeV is a typical scale, and ρ_H , ϕ_H are the unknown real parameters. The related discussions of (ρ_H, ϕ_H) in $B \to PP, PV$ VV decays are found in Refs. [[14](#page-9-20),[32](#page-9-21)–[34](#page-9-22)].

The amplitudes of $\bar{B}_s^0 \to \phi \rho^0$ and $\bar{B}_s^0 \to \phi \omega$ decays are written as

$$
\mathcal{A}_h(\bar{B}_s^0 \to \phi \rho^0) = \frac{G_F}{2} \sum_{p=u,c} \lambda_p^{(s)} \left[\delta_{pu} a_2^{p,h} + \frac{3}{2} (a_7^{p,h} + a_9^{p,h}) \right] \times X_h^{(\bar{B}_s^0 \phi, \rho)}; \tag{18}
$$

$$
\mathcal{A}_{h}(\bar{B}_{s}^{0} \to \phi \omega) = \frac{G_{F}}{2} \sum_{p=u,c} \lambda_{p}^{(s)} \left[\delta_{pu} a_{2}^{p,h} + 2(a_{3}^{p,h} + a_{5}^{p,h}) + \frac{1}{2} (a_{7}^{p,h} + a_{9}^{p,h}) \right] X_{h}^{(\bar{B}_{s}^{0}\phi,\omega)}.
$$
 (19)

We note that the transverse amplitudes A_{+} are suppressed by a factor m_2/m_B relative to A_0 , as shown in Eqs. [\(13\)](#page-3-0) and [\(14\)](#page-3-1). In addition, the axial-vector and vector contributions to \mathcal{A}_{+} are canceled by each other in the heavy-quark limit, due to an exact form factor relation [\[35\]](#page-9-23). Thus, in quark model [\[36\]](#page-9-24) or naive factorization [[37](#page-9-25)], the hierarchy of helicity amplitudes

$$
\mathcal{A}_0: \ \mathcal{A}_- : \mathcal{A}_+ = 1: \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2 \tag{20}
$$

is therefore expected. This hierarchy can also be explained by the chirality flip [[38](#page-9-26)]. The transverse amplitudes defined in the transversity basis are related to the helicity ones via

$$
\mathcal{A}_{\parallel} = \frac{\mathcal{A}_{+} + \mathcal{A}_{-}}{\sqrt{2}}, \qquad \mathcal{A}_{\perp} = \frac{\mathcal{A}_{+} - \mathcal{A}_{-}}{\sqrt{2}}.
$$
 (21)

With the amplitudes, we then obtain the branching fraction of $\bar{B}_s^0 \to V_1 V_2$ as

$$
\mathcal{B}(\bar{B}_s^0 \to V_1 V_2) = \frac{\tau_{B_s} |p_c|}{8\pi m_{B_s}^2} (|\mathcal{A}_L|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2), \quad (22)
$$

where τ_{B_s} is the lifetime of the B_s meson and $|p_c|$ is the absolute value of two final-state hadrons' momentum in the B_s rest frame. Then, three polarization fractions are then defined as

$$
f_{\alpha} \equiv \frac{|\mathcal{A}_{\alpha}|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_{\parallel}|^2 + |\mathcal{A}_{\perp}|^2}
$$
 (23)

with $\alpha = L, \parallel, \perp$. In addition, we can also define the direct CPAs as

$$
A_{CP} = \frac{|\mathcal{A}(\bar{B}_s \to \bar{V}_1 \bar{V}_2)|^2 - |\mathcal{A}(B_s \to V_1 V_2)|^2}{|\mathcal{A}(\bar{B}_s \to \bar{V}_1 \bar{V}_2)|^2 + |\mathcal{A}(B_s \to V_1 V_2)|^2};\tag{24}
$$

$$
A_{CP}^{\alpha} = \frac{f_{\alpha}(\bar{B}_s \to \bar{V}_1 \bar{V}_2) - f_{\alpha}(B_s \to V_1 V_2)}{f_{\alpha} \bar{B}_s \to \bar{V}_1 \bar{V}_2) + f_{\alpha}(B_s \to V_1 V_2)}.
$$
(25)

Using the input parameters listed in Table [I](#page-4-0), we now calculate the branching fractions, polarization fractions, and direct CPAs. As aforementioned, two of the most important parameters are ρ_H and ϕ_H . We first follow [\[13\]](#page-9-6) and adopt the default values $\rho_H = 0$ and $\phi_H = 0$. Three observables of decay $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$ in SM are presented as

$$
\mathcal{B}(B_s \to \phi \rho^0) = (4.1 \pm 0.5) \times 10^{-7}, \n\mathcal{B}(B_s \to \phi \omega) = (1.5^{+0.6}_{-0.5}) \times 10^{-7},
$$
\n(26)

$$
f_L(B_s \to \phi \rho^0) = (89.5^{+0.9}_{-1.0})\%,
$$

$$
f_L(B_s \to \phi \omega) = (82.1^{+2.4}_{-2.8})\%,
$$
 (27)

$$
A_{CP}(B_s \to \phi \rho^0) = (32.8^{+0.1}_{-0.6})\%,
$$

\n
$$
A_{CP}(B_s \to \phi \omega) = (-0.6^{+3.8}_{-2.1})\%,
$$
\n(28)

where the errors are from the uncertainties of the nonperturbative parameters. It can be seen that the longitudinal polarization fractions and the CPAs are not sensitive to the nonperturbative inputs, such as decay constants, form factors, and moments in the distribution amplitudes. In addition, we find that for $B_s \to \phi \rho^0$ the major uncertainties are from the form factors, while the decay constants dominate the uncertainties in $B_s \rightarrow \phi \omega$. The branching fractions are in agreement with the previous predictions of [\[13\]](#page-9-6), and are larger than those of [\[15\]](#page-9-18), because the form factors of $B_s \rightarrow \phi$ we used are larger than theirs. Comparing to the experimental data [[19](#page-9-9)] shown in Eq. [\(1\),](#page-1-2) our result is a bit larger than the data. The other observables have not been measured until now.

In Ref. [\[14\]](#page-9-20), the authors had fitted $\rho_H = 0.5$ within all $B_{u,d,s} \to VV$ data. Within these ranges, we suppose $\phi_H \in [-180^\circ, 180^\circ]$ and obtain the results as

$$
\mathcal{B}(B_s \to \phi \rho^0) = (4.2^{+0.7+0.6}_{-0.6-0.5}) \times 10^{-7}, \n\mathcal{B}(B_s \to \phi \omega) = (2.9^{+1.1+0.0}_{-0.9-2.2}) \times 10^{-7},
$$
\n(29)

B meson parameters B B_{s}	m_R (GeV) 5.366	$\tau_B(ps)$ 1.472	f_R (MeV) 230 ± 20	λ_R (MeV) 300 ± 100		
Light vector mesons V ρ ϕ ω	f_V (MeV) 216 ± 3 233 ± 5 197 ± 3	f_V^{\perp} (MeV) 165 ± 9 191 ± 9 148 ± 9	a_1^V Ω $\overline{0}$ Ω	a_2^V 0.15 ± 0.07 0.18 ± 0.05 0.15 ± 0.07	$\overline{0}$ Ω Ω	$a_1^{\perp,V}$ $a_2^{\perp,V}$ 0.14 ± 0.02 0.14 ± 0.02 0.14 ± 0.02
Form factors at $q^2 = 0$ $A_1^{B_s \phi}$ 0.296 ± 0.03	$A_2^{B_s \phi}$ 0.25 ± 0.03	$V^{B_s \phi}$ 0.387 ± 0.03				
Quark masses $m_b(m_b)$ (GeV) 4.2	0.91	0.3	$m_c(m_b)$ (GeV) $m_c^{\text{pole}}/m_b^{\text{pole}}$ $m_s(2.1 \text{ GeV})(\text{GeV})$ 0.095 ± 0.020			
Wolfenstein parameters A 0.825	λ 0.2265	$\bar{\rho}$ 0.1598	$\bar{\eta}$ 0.3499	γ 67.6		

TABLE I. Summary of input parameters.

$$
f_L(B_s \to \phi \rho^0) = (86.5^{+1.0+8.0}_{-2.2-5.3})\%
$$

$$
f_L(B_s \to \phi \omega) = (75.7^{+2.3+13.4}_{-2.7-16.1})\%
$$
, (30)

$$
A_{CP}(B_s \to \phi \rho^0) = (31.4^{+0.8+15.6}_{-2.8-14.7})\%,
$$

\n
$$
A_{CP}(B_s \to \phi \omega) = (-2.3^{+1.6+14.3}_{-0.8-8.9})\%,
$$
\n(31)

where the first errors are from the uncertainties of the nonperturbative parameters, and the second ones come from the phase ϕ_H . From the results, it is obvious that the strong phase ϕ_H takes larger uncertainties, especially for decay $B_s \rightarrow \phi \omega$. Moreover, when the ϕ_H is in the range [35, 160]^o, the branching fraction of $B_s \to \phi \rho^0$ agrees with the data [\[19\]](#page-9-9). In QCDF, vertex corrections V_i , penguin contractions P_i , and hard spectator scattering H_i all take strong phases. The contributions from vertex corrections and penguins are calculable. However, the end-point singularities appearing in the hard spectator scattering will take large uncertainties.

In order to show the effects of the parameters (ρ_H, ϕ_H) , we set $\rho_H = 0$ and $\rho_H = 0.5$ and plot the variations of the branching fractions, CPAs and longitudinal polarization fractions of $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$ decays with the phase ϕ_H in Figs. [2](#page-5-0) and [3,](#page-5-1) respectively. In fact, if ρ_H becomes larger, the uncertainties taken by ϕ_H will be even larger. In all figures, only uncertainties of the form factors (decay constants) in $B_s \to \phi \rho^0$ ($B_s \to \phi \omega$) are considered.

From Eqs. [\(18\)](#page-3-2) and [\(19\),](#page-3-3) one finds that these two decay modes are characterized by an interplay of the flavor-singlet QCD penguin amplitude, the color-allowed electroweak penguin amplitude, and the color- and CKM-suppressed tree amplitude. For $B_s \rightarrow \phi \omega$ decay, due to a partial cancellation between the QCD and electroweak penguin contributions, the CKM suppressed tree amplitude is the largest partial amplitude, which is very sensitive to the hard spectator scattering. The roles of tree and penguin amplitudes are reversed in the $B_s \to \phi \rho^0$ decay. In addition, in $B_s \rightarrow \phi \omega$ decay, the nonfactorization of transverse spectator scattering is important. This is the reason that QCD corrections become much important, even though they are α_s suppressed, as shown in the figures.

In the future, if these above observables could be measured with high precision, our theoretical results will be useful for determining the range of (ρ_H, ϕ_H) in turn. It is known to us that in B decays, weak annihilation also plays important roles in explaining CP asymmetries and large transverse polarization fractions in some decays. Similarly, the end-point singularity is also parametrized by ρ_A and ϕ_A . Though in most literature $\rho_A = \rho_H$ and $\phi_A = \phi_H$ are supposed for simplicity, these relations cannot be proved. As aforementioned, both $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$ are irrelevant to the annihilation contributions, and suitable for determining the parameters of hard spectator scattering. Once the parameters (ρ_H, ϕ_H) were determined, the future measurements of $B_s \to K^*K^*$, $\phi\phi$ would be helpful to

FIG. 2. The possible regions of CP averaged branching fraction, CPA and longitudinal polarization fraction of decay $B_s \to \phi \rho^0$ as a function of the phase θ_H , the red regions represent results from the $\rho = 0.5$, and the horizontal (gray) regions are results of $\rho = 0$.

FIG. 3. The possible regions of CP averaged branching fraction, CPA and longitudinal polarization fraction of decay $B_s \to \phi \omega$ as a function of the phase θ_H , the red regions represent results from the $\rho = 0.5$, and the horizontal (gray) regions are results of $\rho = 0$.

study the dynamics of annihilations and further determine the parameters (ρ_A, ϕ_A) .

III. EFFECTS OF NEW PHYSICS

Though when $\rho_H = 0.5$, the SM prediction of branching fraction of $B_s \to \phi \rho^0$ can explain the data, we have chances to search for the effects of NP, because ρ_H and ϕ_H have not been completely confirmed yet. In this section, we will study the effects of an extra Z' gauge boson in these decays $B_s \to \phi(\rho^0, \omega)$. Ignoring the mixing between Z^0 and Z' , we write the couplings of the Z' to fermions as $[22]$ $[22]$ $[22]$

$$
J_{Z'}^{\mu} = g' \sum_{i} \bar{\psi}_{i} \gamma^{\mu} [\epsilon_{i}^{\psi_{L}} P_{L} + \epsilon_{i}^{\psi_{R}} P_{R}] \psi_{i}, \qquad (32)
$$

where *i* is the family index and ψ labels the fermions and $P_{L,R} = (1 \mp \gamma_5)/2$. In some string and grand unified theory models $[39-44]$ $[39-44]$ $[39-44]$ $[39-44]$, the Z' couplings are not required to be family universal. When rotating to the physical basis, FCNCs generally appear at tree level in both left-handed and right-handed sectors, explicitly, as

$$
B^{L} = V_{\psi_{L}} \epsilon^{\psi_{L}} V_{\psi_{L}}^{\dagger}, \qquad B^{R} = V_{\psi_{R}} \epsilon^{\psi_{R}} V_{\psi_{R}}^{\dagger}.
$$
 (33)

For simplicity, we suppose that the right-handed couplings are flavor diagonal and $B_{sb}^R = 0$. As a result, the Z' part of the effective Hamiltonian for $b \rightarrow s\bar{q}q(q = u, d)$ transition has the form

$$
\mathcal{H}_{\text{eff}}^{Z'} = \frac{2G_F}{\sqrt{2}} \left(\frac{g'M_Z}{g_1 M_{Z'}} \right)^2 B_{sb}^L(\bar{s}b)_{V-A} \times \sum_q [B_{qq}^L(\bar{q}q)_{V-A} + B_{qq}^R(\bar{q}q)_{V+A}], \quad (34)
$$

where $g_1 = e/(\sin \theta_W \cos \theta_W)$ and $M_{Z'}$ is the Z' mass. Compared with the effective weak Hamiltonian of SM shown in Eq. [\(2\)](#page-1-3), the above Hamiltonian Eq. [\(34\)](#page-6-1) can be rearranged as

$$
\mathcal{H}_{\text{eff}}^{Z'} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^*
$$
\n
$$
\times \sum_{q} (\Delta C_3 O_3^q + \Delta C_5 O_5^q + \Delta C_7 O_7^q + \Delta C_9 O_9^q), \quad (35)
$$

where O_i^q ($i = 3, 5, 7, 9$) are the effective operators of SM. ΔC_i are the modifications to the corresponding SM Wilson coefficients caused by the Z' boson, which are expressed as

$$
\Delta C_3 = -\frac{2}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^L + 2B_{dd}^L),
$$

\n
$$
\Delta C_5 = -\frac{2}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^R + 2B_{dd}^R),
$$

\n
$$
\Delta C_7 = -\frac{4}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^R - B_{dd}^R),
$$

\n
$$
\Delta C_9 = -\frac{4}{3V_{tb}V_{ts}^*} \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 B_{sb}^L (B_{uu}^L - B_{dd}^L),
$$
\n(36)

in terms of the model parameters at the M_W scale. It is found that the Z' contributes not only to the electroweak (EW) penguin operators but also to the QCD penguin ones. In particular, we suppose that new physics is manifest in the EW penguins by setting $B_{uu}^{L,R} = -2B_{dd}^{L,R}$ as done in [[45](#page-10-0)]. Because of the Hermiticity of the effective Hamiltonian, the diagonal elements of the effective coupling matrices $B_{qq}^{L,R}$ are real. However, for the off-diagonal one B_{sb}^L , it could be a complex with a new weak phase ϕ_{sb}^L . As a result, the resulting Z' contributions to the Wilson coefficients are then written as

$$
\Delta C_{3,5} \simeq 0,
$$

\n
$$
\Delta C_{9,7} = 4 \frac{|V_{tb} V_{ts}^*|}{V_{tb} V_{ts}^*} \xi_{sb}^{L,R} e^{-i\phi_{sb}^L},
$$
\n(37)

with

$$
\xi_{sb}^{L,R} = \left(\frac{g'M_Z}{g_1 M_{Z'}}\right)^2 \left|\frac{B_{sb}^L B_{dd}^{L,R}}{V_{tb} V_{ts}^*}\right|.
$$
 (38)

The next step is to constrain the ranges of the new defined parameters $\xi_{sb}^{L,R}$. In generally, we suppose that both the $U(1)_Y$ and $U'(1)$ gauge groups originate from the same grand unified theories, and $g'/g_1 \sim 1$ is expected. The direct search for Z' boson is one of important physics programs of current and future high-energy colliders. However, the direct signal of the new Z' boson has not been observed in the current experiments such as CMS and ATLAS, implying that the mass of Z' would be larger than the TeV scale. In this work, we set $M_{Z'} \geq 3$ TeV conservatively. The family nonuniversal Z' leads to $\Delta B = 2$ and $\Delta S = 2$ FCNC, so that $B_s^0 - \bar{B}_s^0$ mixing happens at the tree level. Then, the mass difference Δm_{B_s} presents the most strong constraint to the models with the Z' boson, and $|B_{sb}^L|$ ~ $|V_{tb}V_{ts}^*|$ is theoretically required [[46](#page-10-1)–[48](#page-10-2)]. Meanwhile, in order to explain CPAs of $B \to K\pi$ and branching fractions of $B \to K\phi$ and $B \to K^*\phi$, the diagonal elements should satisfy $|B_{ss}^{L,R}| \sim |B_{dd}^{L,R}| \sim \mathcal{O}(1)$ [[25](#page-9-15),[26](#page-9-29)]. In addition, this parameter region is of interest for collider detection. Based on the above results obtained, we shall use

$$
|\xi| = |\xi_{db}^{L,R}| = |\xi_{sb}^{L,R}| \in (1 \sim 2) \times 10^{-3}.
$$
 (39)

	$\mu = m_h$		$\mu_h = \sqrt{\Lambda_h m_b}$		
Wilson coefficients	C_i^{SM}	$\Delta C_i^{Z'}$	C_i^{SM}	$\Delta C_i^{Z'}$	
C_1	1.075	$-0.006\xi^L$	1.166	$-0.008\xi^L$	
C_2	-0.170	$-0.009\xi^L$	-0.336	$-0.014\xi^{L}$	
C_3	0.013	$0.05\xi^{L} - 0.01\xi^{R}$	0.025	$0.11\xi^L - 0.02\xi^R$	
C_4	-0.033	$-0.13\xi^{L} + 0.01\xi^{R}$	-0.057	$-0.24\xi^{L} + 0.02\xi^{R}$	
C_5	0.008	$0.03\xi^{L} + 0.01\xi^{R}$	0.011	$0.03\xi^{L} + 0.02\xi^{R}$	
C_6	-0.038	$-0.15\xi^{L} + 0.01\xi^{R}$	-0.076	$-0.32\xi^{L} + 0.04\xi^{R}$	
C_7/α_{em}	-0.015	4.18 ξ^{L} – 473 ξ^{R}	-0.034	$5.7\xi^{L} - 459\xi^{R}$	
C_8/α_{em}	0.045	$1.18\xi^{L} - 166\xi^{R}$	0.089	3.2 ξ^{L} – 355 ξ^{R}	
C_9/α_{em}	-1.119	$-561\xi^{L} + 4.52\xi^{R}$	-1.228	$-611\xi^{L} + 6.7\xi^{R}$	
C_{10}/α_{em}	0.190	$118\xi^{L} - 0.5\xi^{R}$	0.356	$207\xi^{L} - 1.4\xi^{R}$	

TABLE II. The Wilson coefficients C_i within the SM and with the contribution from the Z' boson included in the naive dimensional regularization scheme at the scale $\mu = m_b$ and $\mu_h = \sqrt{\Lambda_h m_b}$ [[27](#page-9-30)].

We also note that the other SM Wilson coefficients also receive contributions from the Z' boson through renormalization group (RG) evolution. Given that there is no significant RG running effect between $M_{Z'}$ and M_W scale, the RG evolution of the modified Wilson coefficients is exactly the same as the ones in the SM [[30](#page-9-17)]. The Wilson coefficients at m_b and $\sqrt{\Lambda_h m_b}$ scale have been presented in Table [II](#page-7-0).

To illustrate the effect of the Z' boson, by setting $\rho_H = 0$ and $\xi = (1, 2) \times 10^{-3}$, one can get the variations of the CP averaged branching fractions, CPAs and longitudinal polarization fractions of decays $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$ as a function of the new weak phase ϕ_{sb} , as shown in Figs. [4](#page-7-1) and [5](#page-7-2), where the horizontal lines are the center values predicted in SM. From the left panel of Fig. [4,](#page-7-1) we find that large $\xi > 0.003$, namely a lighter Z' boson, is

FIG. 4. The CP averaged branching fractions, CPAs and longitudinal polarization fractions of decay $B_s \to \phi \rho^0$ as a function of the new weak phase ϕ_{sb} , the dotted (red) and solid (black) lines represent results from the $\xi = 0.001, 0.002$, and the solid horizontal (green) lines are the center values of SM.

FIG. 5. The CP averaged branching fractions, CPAs and longitudinal polarization fractions of decay $B_s \to \phi \omega$ as a function of the new weak phase ϕ_{sb} , the dotted (red) and solid (black) lines represent results from the $\xi = 0.001, 0.002$, and the solid lines the horizontal (green) lines are the center values of SM.

ruled out. When $\xi = 0.001$, $\phi_{sb} \in (-40, 80)^\circ$ is favored. With this range, the branching fraction of $B_s \to \phi \omega$ would be $(0.8 \sim 1.2) \times 10^{-6}$, which is larger than that of SM by 1 order of magnitude, as shown in the left panel of Fig. [5](#page-7-2). As aforementioned, the newly introduced weak phase ϕ_{sb} in the off-diagonal element of B_{sb}^L plays a major role in changing CPA. The CPAs are much sensitive to ξ and ϕ_{sb} . For $B_s \rightarrow \phi \rho^0$, if $\xi = 0.002$, the range of CPA is $-56\% \sim 38\%$. If $\xi = 0.001$, its CPA could reach $\pm 85\%$ for some special ϕ_{sb} . The CPA of $B_s \to \phi \omega$ shares the same characteristics with $B_s \to \phi \rho$. These remarkable changes shown in the center panels of Figs. [4](#page-7-1) and [5](#page-7-2) will be important signals in testing the model.

For the decay $B \to \phi \rho^0$, the longitudinal polarization fraction f_L is not sensitive to nonperturbative parameters if we adopt $\rho_H = 0$ and $\phi_H = 0$, as shown in the right panel of Fig. [2](#page-5-0). When including the contribution of Z' , it could decrease to 0.33, when $\xi = 0.001$ and $\phi_{sb} = 27^\circ$. If $\phi_{sb} = -10^{\circ}$, it could be enhanced to 0.97. For the decay $B \to \phi \omega$, if $\phi_{sb} \in (-40, 80)^\circ$, the f_L is in the range (0.84, 1), which is larger than the prediction of SM. In addition, we also note that when $\xi = 0.002$, the change is not as remarkable as the case of $\xi = 0.001$, which can be seen in the right panels of Figs. [4](#page-7-1) and [5.](#page-7-2) The reason is that, when $\xi = 0.001$, the contribution of Z' is comparable with that of SM, however when $\xi = 0.002$, the contribution of Z' becomes larger than that of SM and dominates the amplitude. If we only consider the effect of Z' , $f_L =$ 0.91 for both decays. In the future, these observables could be used to probe the effect of new physics. If the Z' were detected in the colliders directly, these decays would also be useful to constrain the couplings.

IV. SUMMARY

In this work, we calculated the branching fractions, CP asymmetries, and polarization fractions of the decay mode $B_s \rightarrow \phi \rho^0$ and $B_s \rightarrow \phi \omega$ within the QCD factorization approach in both the SM and the family nonuniversal Z' model. This approach is suitable because these decay modes have no contributions from annihilation diagrams. In SM, both decays are sensitive to two parameters, ρ_H and ϕ_H , which are from end-point singularities in the hard spectator scattering.

Using the latest results of form factors of $B_s \to \phi$, we obtained $\mathcal{B}(B_s \to \phi \rho^0) = (4.1 \pm 0.5) \times 10^{-7}$ with $\rho_H = 0$, and is $(4.2^{+0.7+0.6}_{-0.6-0.5}) \times 10^{-7}$ with $\rho_H = 0.5$ and $\phi_H \in [-180, 180]^\circ$. These results are a bit larger than the current experimental data. In addition, the longitudinal polarization fraction f_L is 89.4% and 86.3% for different values of ρ_H . Because of the interference between tree and penguin contributions, the CPA A_{CP} is about 30%. The future measurements of f_L and A_{CP} are helpful to determine the parameters (ρ_H , ϕ_H). The decay $B_s \rightarrow \phi \omega$ was also studied.

Because when we adopt $\rho_H = 0.5$, the effects of Z' will be buried in the large uncertainties taken from ϕ_H , we only study its effects in $B_s \to \phi \rho^0$ and $B_s \to \phi \omega$ by setting $\rho_H = 0$. In comparison with experimental data, $\xi > 0.003$ (light Z') is ruled out. By setting $\xi = 0.001$, the range $\phi_{sb} \in (-40, 80)$ ° is favored. The branching fraction of $B_s \rightarrow \phi \omega$ may be enlarged by 1 order of magnitude by the Z' boson within the allowed parameter space. The f_L of $B_s \rightarrow \phi \rho^0$ decreases to 0.33 with $\phi_{sb} = 27^\circ$, and reaches to 0.97 with $\phi_{sb} = -10^{\circ}$. Furthermore, as the direct CPA is concerned, it can reach −85% with suitable parameter space. The contribution of Z' to $B_s \to \phi \omega$ was also calculated. All above observables could be measured in the on-going LHC-b experiment and Belle-II, and the future measurements with high precision will provide a plate to test the nonuniversal Z' model, and can be used to constrain the mass of the Z' boson in turn.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation of China under Grant No. 12375089, and the Natural Science Foundation of Shandong province under Grants No. ZR2022ZD26 and No. ZR2022MA035.

- [1] R. L. Workman et al. (Particle Data Group), Review of particle physics, [Prog. Theor. Exp. Phys.](https://doi.org/10.1093/ptep/ptac097) 2022, 083C01 (2022).
- [2] Y. Li and C. D. Lü, Recent anomalies in B physics, [Sci.](https://doi.org/10.1016/j.scib.2018.02.003) Bull. 63[, 267 \(2018\).](https://doi.org/10.1016/j.scib.2018.02.003)
- [3] S. Bifani, S.Descotes-Genon, A.Romero Vidal, and M. H. Schune, Review of lepton universality tests in *B* decays, J. Phys. G 46[, 023001 \(2019\).](https://doi.org/10.1088/1361-6471/aaf5de)
- [4] M. Algueró, A. Crivellin, S. Descotes-Genon, J. Matias, and M. Novoa-Brunet, A new B-flavour anomaly in

 $B_{d,s} \to K^{*0} \bar{K}^{*0}$: Anatomy and interpretation, [J. High En](https://doi.org/10.1007/JHEP04(2021)066)[ergy Phys. 04 \(2021\) 066.](https://doi.org/10.1007/JHEP04(2021)066)

- [5] Y. Li, G. H. Zhao, Y. J. Sun, and Z. T. Zou, Family nonuniversal Z' effects on $B_{d,s} \to K^{*0} \bar{K}^{*0}$ decays in perturbative QCD approach, Phys. Rev. D 106[, 093009 \(2022\).](https://doi.org/10.1103/PhysRevD.106.093009)
- [6] T. Kapoor and E. Kou, New physics search via CP observables in $B_s^0 \rightarrow \phi \phi$ decays with left- and right-handed chromomagnetic operators, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.108.096002) 108, 096002 [\(2023\).](https://doi.org/10.1103/PhysRevD.108.096002)
- [7] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, OCD factorization for $B \to \pi \pi$ decays: Strong phases and CP violation in the heavy quark limit, [Phys. Rev. Lett.](https://doi.org/10.1103/PhysRevLett.83.1914) 83, [1914 \(1999\)](https://doi.org/10.1103/PhysRevLett.83.1914).
- [8] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, QCD factorization for exclusive, nonleptonic B meson decays: General arguments and the case of heavy light final states, [Nucl. Phys. B](https://doi.org/10.1016/S0550-3213(00)00559-9) 591, 313 (2000).
- [9] Y. Y. Keum, H. n. Li, and A. I. Sanda, Fat penguins and imaginary penguins in perturbative QCD, [Phys. Lett. B](https://doi.org/10.1016/S0370-2693(01)00247-7) 504, [6 \(2001\).](https://doi.org/10.1016/S0370-2693(01)00247-7)
- [10] C. D. Lu, K. Ukai, and M. Z. Yang, Branching ratio and CP violation of $B \to \pi \pi$ decays in perturbative OCD approach, Phys. Rev. D 63[, 074009 \(2001\)](https://doi.org/10.1103/PhysRevD.63.074009).
- [11] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, An effective field theory for collinear and soft gluons: Heavy to light decays, Phys. Rev. D 63[, 114020 \(2001\)](https://doi.org/10.1103/PhysRevD.63.114020).
- [12] C. W. Bauer, D. Pirjol, and I. W. Stewart, Soft collinear factorization in effective field theory, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.65.054022) 65, [054022 \(2002\).](https://doi.org/10.1103/PhysRevD.65.054022)
- [13] M. Beneke, J. Rohrer, and D. Yang, Branching fractions, polarisation and asymmetries of $\rightarrow VV$ decays, [Nucl. Phys.](https://doi.org/10.1016/j.nuclphysb.2007.03.020) B 774[, 64 \(2007\)](https://doi.org/10.1016/j.nuclphysb.2007.03.020).
- [14] Q. Chang, X. Li, X. Q. Li, and J. Sun, Study of the weak annihilation contributions in charmless $B_s \rightarrow VV$ decays, [Eur. Phys. J. C](https://doi.org/10.1140/epjc/s10052-017-4980-9) 77, 415 (2017).
- [15] H. Y. Cheng and C. K. Chua, QCD factorization for charmless hadronic B_s decays revisited, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.80.114026) 80, 114026 [\(2009\).](https://doi.org/10.1103/PhysRevD.80.114026)
- [16] G. Faisel and J. Tandean, Rare nonleptonic \bar{B}_s^0 decays as probes of new physics behind $b \rightarrow s \mu \bar{\mu}$ anomalies, [Phys.](https://doi.org/10.1103/PhysRevD.99.075007) Rev. D 99[, 075007 \(2019\)](https://doi.org/10.1103/PhysRevD.99.075007).
- [17] G. Faisel and J. Tandean, Connecting $b \to s\ell\bar{\ell}$ anomalies to enhanced rare nonleptonic \bar{B}_s^0 decays in Z' model, [J. High](https://doi.org/10.1007/JHEP02(2018)074) [Energy Phys. 02 \(2018\) 074.](https://doi.org/10.1007/JHEP02(2018)074)
- [18] A. Ali, G. Kramer, Y. Li, C. D. Lu, Y. L. Shen, W. Wang, and Y. M. Wang, Charmless non-leptonic B_s decays to PP , PV and VV final states in the pQCD approach, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.76.074018) 76[, 074018 \(2007\).](https://doi.org/10.1103/PhysRevD.76.074018)
- [19] R. Aaij et al. (LHCb Collaboration), Observation of the decay $B_s^0 \to \phi \pi^+ \pi^-$ and evidence for $B^0 \to \phi \pi^+ \pi^-$, [Phys.](https://doi.org/10.1103/PhysRevD.95.012006) Rev. D 95[, 012006 \(2017\)](https://doi.org/10.1103/PhysRevD.95.012006).
- [20] R. Fleischer, Search for the angle γ in the electroweak penguin dominated decay $B_s \to \pi^0 \phi$, [Phys. Lett. B](https://doi.org/10.1016/0370-2693(94)91275-0) 332, 419 [\(1994\).](https://doi.org/10.1016/0370-2693(94)91275-0)
- [21] E. Kou et al. (Belle-II Collaboration), The Belle II physics book, [Prog. Theor. Exp. Phys.](https://doi.org/10.1093/ptep/ptz106) 2019, 123C01 (2019).
- [22] P. Langacker, The physics of heavy Z' gauge bosons, [Rev.](https://doi.org/10.1103/RevModPhys.81.1199) Mod. Phys. 81[, 1199 \(2009\)](https://doi.org/10.1103/RevModPhys.81.1199).
- [23] G. Buchalla, G. Burdman, C. T. Hill, and D. Kominis, GIM violation and new dynamics of the third generation, [Phys.](https://doi.org/10.1103/PhysRevD.53.5185) Rev. D 53[, 5185 \(1996\).](https://doi.org/10.1103/PhysRevD.53.5185)
- $[24]$ E. Nardi, Z' , new fermions and flavor changing processes. Constraints on $E(6)$ models from $\mu \rightarrow eee$, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.48.1240) 48, [1240 \(1993\)](https://doi.org/10.1103/PhysRevD.48.1240).
- [25] V. Barger, L. L. Everett, J. Jiang, P. Langacker, T. Liu, and C. E. M. Wagner, $b \rightarrow s$ transitions in family-dependent $U(1)$ ' models, [J. High Energy Phys. 12 \(2009\) 048.](https://doi.org/10.1088/1126-6708/2009/12/048)
- [26] K. Cheung, C. W. Chiang, N. G. Deshpande, and J. Jiang, Constraints on flavor-changing Z' models by B_s mixing, Z' production, and $B_s \rightarrow \mu^+\mu^-$, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2007.07.032) 652, 285 (2007).
- [27] Q. Chang, X. Q. Li, and Y. D. Yang, Constraints on the nonuniversal Z-prime couplings from $B \to \pi K$, πK^* and ρK decays, [J. High Energy Phys. 05 \(2009\) 056.](https://doi.org/10.1088/1126-6708/2009/05/056)
- [28] J. Hua, C. S. Kim, and Y. Li, Testing the non-universal Z' model in $B_s \rightarrow \phi \pi^0$ decay, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2010.06.004) 690, 508 (2010).
- [29] J. Hua, C. S. Kim, and Y. Li, Annihilation-type charmless radiative decays of B meson in non-universal Z' model, [Eur.](https://doi.org/10.1140/epjc/s10052-010-1395-2) Phys. J. C 69[, 139 \(2010\).](https://doi.org/10.1140/epjc/s10052-010-1395-2)
- [30] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Weak decays beyond leading logarithms, [Rev. Mod. Phys.](https://doi.org/10.1103/RevModPhys.68.1125) 68, [1125 \(1996\)](https://doi.org/10.1103/RevModPhys.68.1125).
- [31] A. Bharucha, D. M. Straub, and R. Zwicky, $B \to V \ell^+ \ell^-$ in the standard model from light-cone sum rules, [J. High](https://doi.org/10.1007/JHEP08(2016)098) [Energy Phys. 08 \(2016\) 098.](https://doi.org/10.1007/JHEP08(2016)098)
- [32] Q. Chang, X. Hu, J. Sun, and Y. Yang, Probing spectator scattering and annihilation corrections in $B_s \rightarrow PV$ decays, Phys. Rev. D 91[, 074026 \(2015\)](https://doi.org/10.1103/PhysRevD.91.074026).
- [33] J. Sun, Q. Chang, X. Hu, and Y. Yang, Constraints on hard spectator scattering and annihilation corrections in $B_{u,d} \rightarrow$ PV decays within QCD factorization, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2015.03.001) 743, 444 [\(2015\).](https://doi.org/10.1016/j.physletb.2015.03.001)
- [34] Q. Chang, J. Sun, Y. Yang, and X. Li, Spectator scattering and annihilation contributions as a solution to the πK and $\pi \pi$ puzzles within QCD factorization approach, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.90.054019) 90[, 054019 \(2014\).](https://doi.org/10.1103/PhysRevD.90.054019)
- [35] M. Beneke and T. Feldmann, Symmetry breaking corrections to heavy to light B meson form-factors at large recoil, [Nucl. Phys. B](https://doi.org/10.1016/S0550-3213(00)00585-X) 592, 3 (2001).
- [36] J.G. Korner and G.R. Goldstein, Quark and particle helicities in hadronic charmed particle decays, [Phys. Lett.](https://doi.org/10.1016/0370-2693(79)90085-6) B 89[, 105 \(1979\)](https://doi.org/10.1016/0370-2693(79)90085-6).
- [37] G. Kramer and W.F. Palmer, Branching ratios and CP asymmetries in the decay $B \to VV$, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.45.193) 45, 193 [\(1992\).](https://doi.org/10.1103/PhysRevD.45.193)
- [38] A. L. Kagan, Polarization in $B \to VV$ decays, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2004.09.030) 601[, 151 \(2004\)](https://doi.org/10.1016/j.physletb.2004.09.030).
- [39] S. Chaudhuri, S. W. Chung, G. Hockney, and J. D. Lykken, String consistency for unified model building, [Nucl. Phys. B](https://doi.org/10.1016/0550-3213(95)00147-7) 456[, 89 \(1995\).](https://doi.org/10.1016/0550-3213(95)00147-7)
- [40] G. Cleaver, M. Cvetic, J. R. Espinosa, L. L. Everett, P. Langacker, and J. Wang, Physics implications of flat directions in free fermionic superstring models 1. Mass spectrum and couplings, Phys. Rev. D 59[, 055005 \(1999\).](https://doi.org/10.1103/PhysRevD.59.055005)
- [41] M. Cvetic, G. Shiu, and A.M. Uranga, Three family supersymmetric standard-like models from intersecting brane worlds, Phys. Rev. Lett. 87[, 201801 \(2001\)](https://doi.org/10.1103/PhysRevLett.87.201801).
- [42] M. Cvetic, P. Langacker, and G. Shiu, Phenomenology of a three family standard like string model, [Phys. Rev. D](https://doi.org/10.1103/PhysRevD.66.066004) 66, [066004 \(2002\).](https://doi.org/10.1103/PhysRevD.66.066004)
- [43] T. K. Kuo and N. Nakagawa, An $Sp_L(6) \times U_{\gamma}(1)$ extension of the electroweak theory, Phys. Rev. D 30[, 2011 \(1984\).](https://doi.org/10.1103/PhysRevD.30.2011)
- [44] V. D. Barger, N. G. Deshpande, T. Kuo, A. Bagneid, S. Pakvasa, and K. Whisnant, Discovery limits of new gauge bosons of $Sp_L(6) \times U(1)$, [Int. J. Mod. Phys. A](https://doi.org/10.1142/S0217751X87000697) 02, 1327 [\(1987\).](https://doi.org/10.1142/S0217751X87000697)
- [45] A. J. Buras, R. Fleischer, S. Recksiegel, and F. Schwab, $B \to \pi\pi$, new physics in $B \to \pi K$ and implications for rare K and B decays, Phys. Rev. Lett. 92[, 101804 \(2004\)](https://doi.org/10.1103/PhysRevLett.92.101804).
- [46] V. Barger, C. W. Chiang, J. Jiang, and P. Langacker, $B_s - \bar{B}_s$ mixing in Z' models with flavor-changing neutral currents, [Phys. Lett. B](https://doi.org/10.1016/j.physletb.2004.06.105) 596, 229 (2004).
- [47] Q. Chang, X. Q. Li, and Y. D. Yang, Family non-universal Z' effects on $\bar{B}_q - B_q$ mixing, $B \to X_s \mu^+ \mu^-$ and $B_s \to \mu^+ \mu^$ decays, [J. High Energy Phys. 02 \(2010\) 082.](https://doi.org/10.1007/JHEP02(2010)082)
- [48] X. Q. Li, Y. M. Li, G. R. Lu, and F. Su, $B_s^0 - \bar{B}_s^0$ mixing in a family non-universal Z' model revisited, [J. High Energy](https://doi.org/10.1007/JHEP05(2012)049) [Phys. 05 \(2012\) 049.](https://doi.org/10.1007/JHEP05(2012)049)