

Reaction $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ at 15 GeV/c*

C. P. Horne,[†] S. Hagopian, V. Hagopian, J. E. Lannutti, N. D. Pewitt, D. P. Wilkins, and B. Wind
Department of Physics, The Florida State University, Tallahassee, Florida 32306

J. R. Bensinger

*Brandeis University, Waltham, Massachusetts 02154[‡]
 and The University of Pennsylvania, Philadelphia, Pennsylvania 19104*

(Received 16 October 1974)

About 1800 events of the reaction $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ have been obtained from 387 000 photographs of the SLAC 82-in. bubble chamber corresponding to a $495 \pm 25 \mu\text{b}$ cross section. The increasing cross section of this reaction with beam momentum is explained in terms of minimum momentum-transfer cutoff. The $D^*(2200)$ is observed as both $d\pi^+$ and $d\pi^-$ mass peaks. The $A_1(\rho^0\pi^+)$ is consistent with a Deck-type mechanism. The $A_3(f^0\pi^+)$ is also observed, but the A_4 peak recently claimed is not seen in these data. There is no statistically significant 3π mass enhancement at the A_2 region, but indirect evidence indicates some A_2 production.

I. INTRODUCTION

The coherent reaction $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ restricts the (3π) system to be a pure $I=1$ state. The very small binding energy of the deuteron reduces the probability of large-momentum recoil deuterons, limiting t (the four-momentum transfer squared of the deuteron) to $-t < 0.1 (\text{GeV}/c)^2$ for the vast majority of the events. Small- t events are usually interpreted in terms of t -channel exchange diagrams, and in this reaction the process corresponds to beam diffraction of $\pi^+ \rightarrow (3\pi)^+$. With a coherent deuteron the exchange in the t channel must be $I=0$, $G=+$. This means that the only allowed exchanges are "Pomeron," f^0 and η^0 . The higher the beam momentum of this reaction, the more the Pomeron exchange dominates over lower 0^+ trajectories. These facts make this experiment at 15 GeV/c well suited for a study of pion diffraction ($\pi \rightarrow 3\pi$) via Pomeron exchange.

In Sec. II the experimental procedure is given in some detail. Several other experiments on this same reaction but at lower beam momenta have already been reported,¹⁻¹⁰ and will be compared with these data. In Sec. III the cross section of this experiment is compared with the lower-momenta data, and the rise in cross section with beam momentum is explained in terms of the minimum kinematic cutoff of the recoil deuteron momentum. In Sec. IV these data are compared with single-nucleon-target data to obtain the deuteron form factor. A simple longitudinal phase space (LPS) analysis is discussed in Sec. V to be used for resonance analysis. Although most of the data are of the $\pi \rightarrow 3\pi$ t -channel exchange mode, in Sec. VI small peaks of D^* in the $d\pi^+$ and $d\pi^-$ mass spectra are discussed.

In Sec. VII the observation of the $A_1(\rho^0\pi^+)$ and the $A_3(f^0\pi^+)$, which are the bulk of the data, is discussed. A complete 3π partial-wave analysis has not been done since the number of events is still too small. Even though A_2 production is suppressed (most likely owing to the large form factor of the deuteron), in Sec. VIII a possible indirect observation of the A_2 is discussed.

II. EXPERIMENTAL PROCEDURE

387 000 useful photographs were obtained using the SLAC 82-in. bubble chamber. The rf-separated beam was 96% π^+ , 3% μ^+ , and 1% other. The beam momentum with a nominal value of 15 GeV/c varied over several percent, but the momentum of any beam π^+ was known to better than 0.5%. The scanning criteria selected even-prong events with a stopping track and odd-prong events with a dark positive track. The events were predigitized by measuring two fiducials, the vertex and the end points of all stopping tracks in two views. The fiducial volume for acceptable events was 100 cm long. Since a visible stopping track was required, the corresponding minimum momentum cutoff for the deuteron was between 100 and 120 MeV/c while for a proton it was about 80 to 90 MeV/c.

The scanned and predigitized events were measured on the University of Pennsylvania Hough-Powell Device (HPD). The rms accuracy of the HPD on these photographs is about 2μ . To utilize the precision of the HPD, special effort was made to accurately determine the experiment-dependent parameters, such as optical constants, magnetic field, beam momentum, the index of refraction, and the density of deuterium.

For each event the HPD produced approximately 20000 x - y digitizings in each of the three views. These were filtered by an automatic track-following program. The resulting track segments were combined into an event by a pattern-recognition program which worked on a view-by-view basis. The geometry and kinematics programs TVGP (with trackmatch) and SQUAW were used. In order to reduce kinematic ambiguities, the ionization predicted from measured momenta and angles was compared with the ionization on film. This procedure distinguished π^+ 's from protons for most positive tracks with momenta below 1.5 GeV/c. All negative tracks were assumed to be π^- 's.

Approximately 30% of the events failed. In order to recover some of these events, a special program called RESURX was coded, which displayed the filtered points on a storage scope. A human operator compared these displays to the actual photographs projected onto a scanning table and entered instructions to delete extraneous tracks, merge broken tracks, and track-match the three views. This system recovered about $\frac{1}{2}$ of the failed events and yielded a total overall measuring efficiency of 85%.¹¹

Events in the present sample of $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ were further required to fit with a χ^2 probability greater than 1.0%, missing mass squared between -0.1 , and $+0.1$ GeV², and to have at least three constraints. About 30% of the $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ sample remained ambiguous with the one-constraint reaction $\pi^+d \rightarrow \pi^+\pi^+\pi^-pn$, but from checks on the np laboratory opening angle and the np mass, we believe our sample is more than 90% pure.

III. CROSS SECTION

The cross section for the $\pi^+\pi^+\pi^-d$ channel for this experiment was determined by two independent methods, which were consistent within 1%. The first method was to determine the total path length and use the density of liquid deuterium in the relation

$$\sigma = N_s \frac{A}{\rho N_0} \frac{1}{L}, \quad (1)$$

where N_s is the number of $\pi d \rightarrow 3\pi d$ events, corrected for unseen deuterons, $A=2$ is the atomic mass of deuterium, N_0 is Avogadro's number, ρ is the density of deuterium, and L is the total path length. The index of refraction was obtained by measuring top and bottom fiducials with the chamber empty and full. From the index of refraction the density was calculated to be $\rho = 0.129 \pm 0.003$ g/cm³.

The second method used was to normalize the total number of events to the measured π^+d total

cross section at 15 GeV/c, $\sigma_T = 48.2 \pm 0.5$ mb,¹² and to use the relation

$$\sigma_{3\pi d} = \frac{N_s \sigma_T}{N_{\text{tot}}}. \quad (2)$$

The number of unseen deuteron recoil events was estimated by assuming a $d\sigma/dt'$ distribution of the form $\exp(At')$, where $t' = t - t_{\text{min}}$ and t is the four-momentum transfer squared of the deuteron. By fitting the t' distribution in the region $0.02 < -t' < 0.1$ (GeV/c)², A was determined to be 29.0 ± 1.4 (GeV/c)⁻² (see Fig. 1). Using this fit we estimate that 70% of the coherent events were seen by our scanners. With this correction, the cross section for this channel was determined to be 495 ± 25 μ b. The total path length of the data presented corresponds to 5.2 events/ μ b.

Figure 2 presents the published experimental cross sections for the reaction $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ as a function of P_L , the incident laboratory beam momentum. A previous study of the energy dependence of the $\pi^+d \rightarrow \pi^+\pi^+\pi^-d$ cross section was made using the dual diffractive model.¹⁰ The model predicted a cross section falling with increasing energy, reaching a value of 200 μ b or less at 15 GeV/c. This is in strong disagreement with the data, which show a rising cross section. The rising cross section is explained below as a t_{min} effect.

To study the systematic increase in cross section, our data were fitted to the form

$$\frac{d^2\sigma}{dm dt} = \sigma_0(m) \exp(Bt), \quad (3)$$

where m is the 3π mass. Assuming that $\sigma_0(m)$ is independent of energy, these parameters were then used to estimate cross sections at lower en-

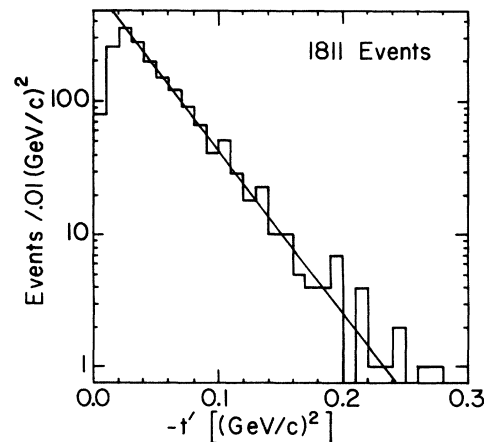


FIG. 1. dN/dt' vs t' for all events; line has form $\exp(29t')$.

ergies by integrating $\exp(Bt)$ from t_{\min} to t_{\max} , multiplying by $\sigma_0(m)$, and then summing over the 3π masses.¹³ The curve so generated, also shown in Fig. 2, agrees fairly well with the data. The excess cross section observed at low P_L could be due to nondiffractive processes. We also compared the published 3π mass distributions at lower energies with the predicted distributions from our data, i.e.,

$$\frac{d\sigma}{dm} = \int_{t_{\min}}^{t_{\max}} \sigma_0(m) e^{Bt} dt, \quad (4)$$

and found quite good agreement. The conclusion is that the cross-section dependence is a t_{\min} effect and is much more important in deuterium than hydrogen because of the steepness of the t slope in deuterium, and the quantity $\sigma_0(m)$ is probably energy-independent.

IV. t DISTRIBUTION

The steep t distribution of coherent data at small t can be interpreted in terms of the t distribution of similar single-nucleon-target data multiplied by an exponential form factor, neglecting double-scattering effects.¹⁴ This form factor is the result of the higher probability of breakup of the compound nucleus with higher momentum transfers.

The $d\sigma/dt$ distribution of these data for $M(3\pi) < 2$ GeV is presented in Fig. 3. The line is a fit to the data for $0.03 < -t < 0.1$ (GeV/c)² with the function $\exp(Bt)$ with $B = (28.0 \pm 1.4)$ (GeV/c)⁻². The restriction on $M(3\pi)$ removes t_{\min} effects. Since there are hardly any events with $-t > 0.1$ (GeV/c)², the effects of double scattering in the deuteron can be neglected.¹⁴ The deuteron differential cross section can be related to the nucleon cross section for beam-dissociation events by the approximate relation

$$\frac{d^2\sigma(d)}{dm dt} = R \exp(Ft) \frac{d^2\sigma(N)}{dm dt}, \quad (5)$$

where $\exp(Ft)$ is the deuteron form factor.¹⁴ To study this relation a comparison of our data with the reaction $\pi^\pm p \rightarrow \pi^\pm \pi^+ \pi^- p$ at 16 GeV/c was made.¹⁵ To select the beam-dissociation events of the $\pi^\pm p$ experiment, only the beam-dissociation sector of the longitudinal phase space (discussed in Sec. V) was used. Since our data are mostly beam dissociation no such selection was performed. The $\pi^\pm p$ experiment quotes only t' distributions, so the comparison shown below is in t' . Both experiments were parametrized in the form $d\sigma/dt' = \sigma_0 \exp(At')$ for various 3π mass regions. Table I lists A for five 3π mass intervals for both the 16 GeV/c $\pi^\pm p$ and this experiment.

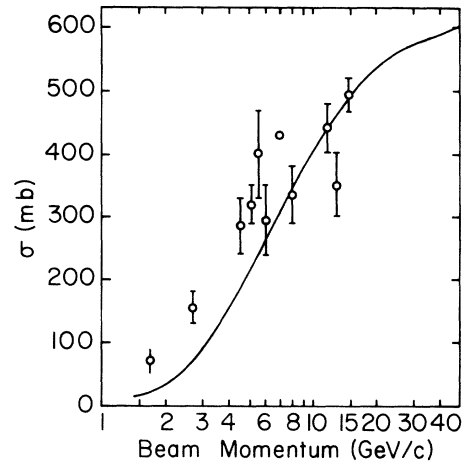


FIG. 2. $\sigma(\pi^+d \rightarrow \pi^+\pi^+\pi^-d)$ vs lab momentum: 1.7 GeV/c—Ref. 1; 2.7 GeV/c—Ref. 2; 4.5 GeV/c—Ref. 3; 5.1 GeV/c—Ref. 4; 5.4 GeV/c—Ref. 5; 6.0 GeV/c—Ref. 6; 7.0 GeV/c—Ref. 7 (error not published); 8.0 GeV/c—Ref. 8; 11.7 GeV/c—Ref. 9; 13.0 GeV/c—Ref. 10; 15.0 GeV/c—this experiment. Curve is calculation using t_{\min} effect normalized to this experiment.

The difference between the $A(\pi d)$ and $A(\pi^\pm p)$ is fairly constant and the average of this difference corresponding to the deuteron form factor is

$$F = 20 \pm 3 \text{ (GeV/c)}^{-2}.$$

This value is in agreement with other determinations of the deuteron form factor.¹⁶

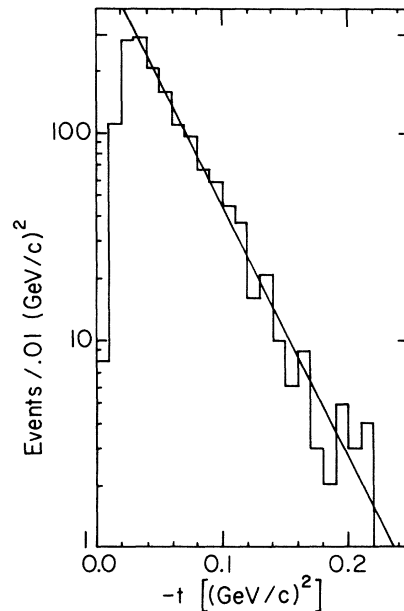


FIG. 3. dN/dt vs t for $M(3\pi) < 2$ GeV; line has form $\exp(28t)$.

TABLE I. A of $\pi^\pm p \rightarrow \pi^\pm \pi^+ \pi^- p$ at 16 GeV/c (Ref. 15) and this experiment, where $d\sigma/dt' = \sigma_0(m) \exp(At')$. For details see text.

$M(3\pi)$ (GeV)	$\pi^- p$	A [(GeV/c) $^{-2}$]		$\pi^+ d$	$A(\pi^+ d)$ $-A(\pi^\pm p)$
		$\pi^+ p$	$\pi^\pm p$ (av)		
0.6–1.00	14.6 ± 1.8	11.3 ± 0.5	13.0 ± 0.9	33 ± 4	20 ± 4
1.0–1.12	11.5 ± 0.8	9.6 ± 0.7	10.6 ± 0.5	26 ± 4	15 ± 4
1.12–1.28	9.8 ± 0.7	7.6 ± 0.6	8.7 ± 0.5	28 ± 3	19 ± 3
1.28–1.40	7.1 ± 0.5	5.0 ± 0.6	6.1 ± 0.4	30 ± 4	24 ± 4
1.40–3.00	7.2 ± 0.7	5.7 ± 0.3	6.5 ± 0.4	27 ± 2	21 ± 2

V. LONGITUDINAL PHASE SPACE ANALYSES

Longitudinal phase space (LPS) analyses have been used successfully in the past to isolate reaction mechanisms.^{15,17} In the $\pi^+ d \rightarrow \pi_f^+ \pi_s^+ \pi^- d$ reaction, π_f^+ (the fast π^+) is always in the forward hemisphere and the d is always backward, but the π_s^+ (the slow π^+) and π^- can be either backward or forward.

Figure 4 presents the data in the $x(\pi_s^+)$, $x(\pi^-)$ coordinates where $x = p_{\parallel}/p_{\text{beam}}$ in the center of mass frame. Each quadrant corresponds to a different process, as is illustrated also in Fig. 4. Most of the events fall into the beam-dissociation region (Q1), and even those events not in Q1 fall near its boundary. We will discuss some of the effects of the LPS analysis in the following sections.

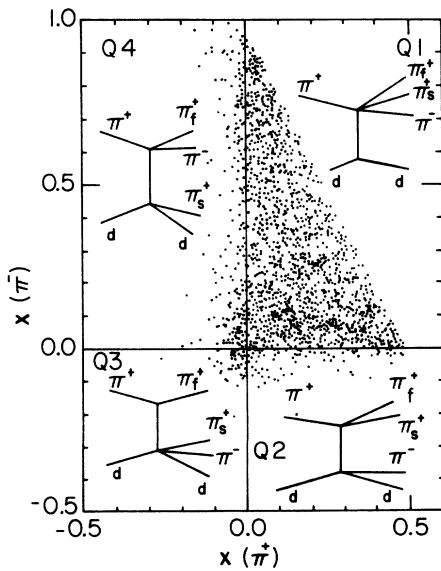


FIG. 4. Longitudinal phase space plot where x is approximately the Feynman variable, $x = p_{\parallel}/p_{\text{beam}}$ in the center-of-mass frame.

VI. D^* PRODUCTION

Figure 5 shows the $d\pi^+$ and $d\pi^-$ mass plots. The D^* peak at 2200 MeV is well known, although the interpretation is not yet settled. Two possible explanations are a genuine $d\pi$ resonance or a bound state of ΔN . The latter was suggested by the fact that the D^* mass is just the sum of the masses of a Δ and a nucleon.¹⁸ Recently, Podolsky presented¹⁹ a third interpretation in which a peak in the D^{*++} region is produced by a Deck-type mechanism shown in Fig. 6(a). But this model predicts no peak in the D^{*0} region. In our data a D^{*0} peak is present but is smaller than D^{*++} , so this model cannot completely explain our data.

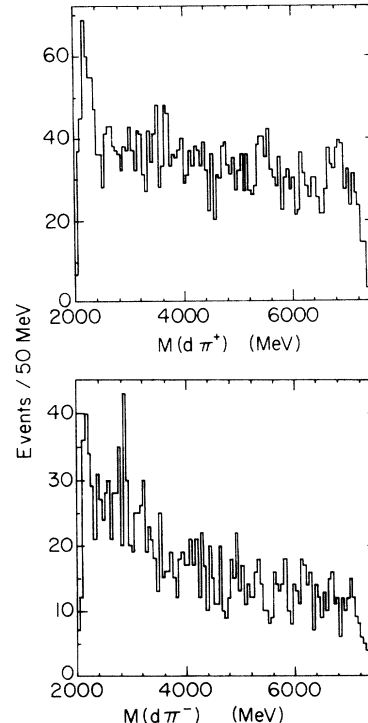


FIG. 5. $d\pi^+$ and $d\pi^-$ invariant-mass distributions.

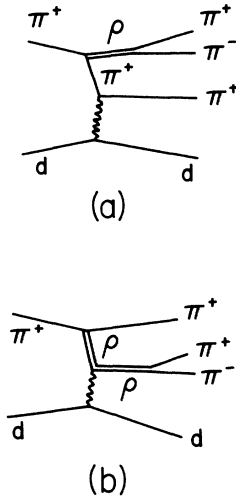


FIG. 6. Diagrams describing possible 3π production mechanisms.

Figures 7(a) and 7(b) show the Gottfried-Jackson scattering angle, the scattering angle of the deuteron in the $d\pi$ system, for D^{*++} and D^{*0} . The forward-peak distributions are not characteristic of resonant behavior, so this gives greater credence to the ΔN interpretation.

The four-standard-deviation peak in the $d\pi^-$ mass spectrum in Fig. 5(b) at 2900 MeV does not appear to be a resonance. This is implied by the absence of a $d\pi^+$ peak at the same mass, by an extremely forward angular distribution which does not change in the 2900-MeV mass region, and by the fact that events in the peak come from the sector of LPS corresponding to beam dissociation.

Recently an enhancement in the $d\pi^+\pi^-$ mass spectrum (D^{**}) around 2300 MeV has been reported in a 15-GeV π^-d experiment.²⁰ The uncut $d\pi^+\pi^-$ distribution for our data shows no low $d\pi^+\pi^-$ mass enhancement. However, if we select on the target dissociation sector of LPS, i.e., Q3, the masses are all confined to the D^{**} region. Selecting on D^{*0} or D^{*++} also yields an enhancement in the D^{**} region. This enhancement has fairly obvious kinematical interpretations, since the d , π_s^+ , and π^- have low momenta in the laboratory, so it is not clear that it is significant.

VII. THE 3π MASS DISTRIBUTION: THE A_1 AND A_3

Figure 4 shows that more than three fourths of the events in this reaction are in quadrant 1 of the LPS plot, where all pions are forward in the center-of-mass frame. Thus this reaction is useful

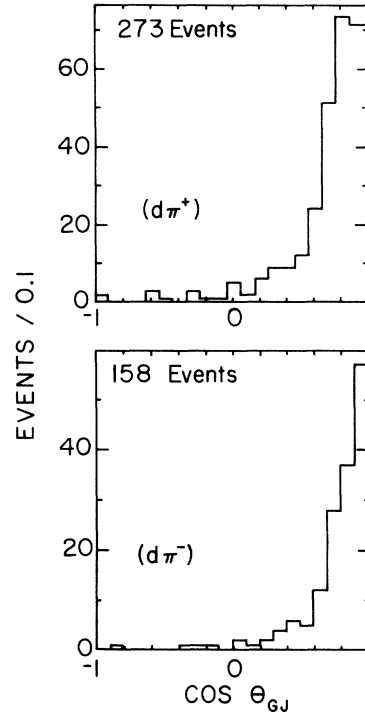


FIG. 7. Gottfried-Jackson scattering angle of the deuteron for D^{*++} and D^{*0} ; $M(D^*) < 2.3$ GeV.

in studying the diffraction of beam pions into three pions. Figure 8 shows the $\pi^+\pi^+\pi^-$ invariant-mass distribution for all events. The general features are the broad A_1 enhancement at 1100 MeV with a width of about 300 MeV, and the A_3 at about 1650 with a width of about 200 MeV. Higher masses are suppressed relative to hydrogen data owing to the steep slope of the differential cross section.

The nature of the A_1 and A_3 enhancements has been the subject of controversy for several years.

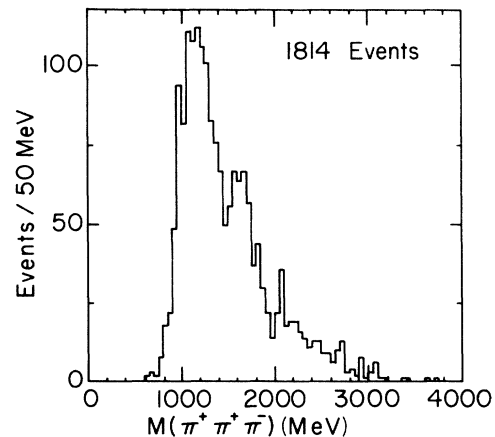


FIG. 8. $\pi^+\pi^+\pi^-$ invariant-mass distribution.

SU(3) theory predicts a 1^-1^+ resonance, and the A_1 is the only known candidate. However, a Deck-type mechanism can also describe the A_1 peak without invoking a resonance.²¹ Most of the A_1 analyses have been done with proton-target experiments, which have backgrounds from Δ production and from a strong A_2 signal close to the A_1 . We estimate that A_2 production is less than 5% of our data (see Sec. VIII). Unfortunately, we do not have enough data to be able to perform a complete phase-shift analysis on the 3π final state, but it is hoped that such an analysis can be done after more data are obtained. Figure 9 is the $\pi^+ \pi^-$ mass plot. This spectrum shows a very strong ρ^0 peak and a smaller f^0 peak. No statistically significant g^0 peak is observed.

When the $\pi^+ \pi^-$ mass spectrum is plotted for the events in the A_1 mass region almost every event has one $\pi^+ \pi^-$ mass in the ρ^0 mass region. Since there are two π^+ 's per event, a selection can be made in terms of the laboratory momentum of the π^+ . Let π_f^+ be the higher-momentum π^+ and π_s^+ the lower-momentum π^+ . (A selection on longitudinal momentum instead of total momentum does not change any of the discussion given below.) Figure 10 shows the invariant-mass distributions of $\pi_f^+ \pi^-$ and $\pi_s^+ \pi^-$. The ρ^0 signal comes mostly from the $\pi_f^+ \pi^-$, although there is a definite signal from the $\pi_s^+ \pi^-$ mass spectrum. One question that can be answered is which of the two Deck mechanisms shown in Fig. 6 dominates the A_1 production. Figure 6(a) favors a faster ρ^0 in the laboratory frame compared to Fig. 6(b). Since most of the ρ^0 's produced are fast and are associated with the π_f^+ , 6(a) is favored over 6(b). In addition the Gottfried-Jackson scattering angular distribution between the beam π^+ and the π^+ of the ρ^0 is very similar to the $\pi\pi$ vertex angular distributions of $\pi N \rightarrow \rho^0 N$. This im-

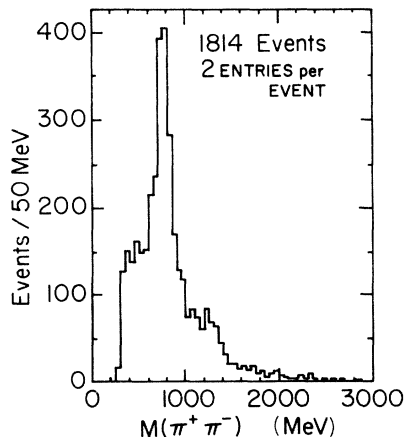


FIG. 9. $\pi^+ \pi^-$ invariant-mass distribution, two entries per event.

plies that the ρ^0 in our data is consistent with a $\pi\pi$ scattering vertex for ρ^0 production, which is not expected from Fig. 6(b).

The A_3 is very similar to the A_1 except that it is $f^0 \pi$. A $\pi^+ \pi^-$ mass plot for events in the A_3 region shows both the f^0 and ρ^0 signal, but the adjacent 3π mass intervals also have the ρ^0 signal, and a subtraction leaves just the f^0 signal. We note that even though A_1 is associated almost uniquely with the ρ^0 , the ρ^0 events are produced for most 3π masses. Figure 11 shows the number of events with at least one $\pi^+ \pi^-$ mass combination in the ρ^0 region as a function of 3π invariant mass. It is clear from this plot that ρ^0 production is falling off smoothly with increasing 3π mass above the A_1 region. There is an appreciable amount of ρ^0 at the A_3 mass but no rise in the ρ^0 cross section in the A_3 mass region.

We do not see any evidence for an enhancement at $M(3\pi) = 1950$ MeV, as reported by the charge-symmetric experiment $\pi^- d$ at 15 GeV/c,²² and in fact the distribution shown in Fig. 8 has a small dip at 1950 MeV. Depending on the way one draws

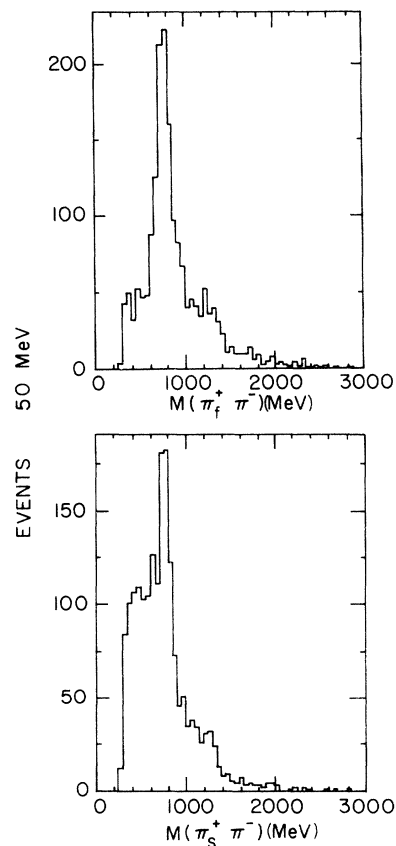


FIG. 10. $\pi_f^+ \pi^-$ and $\pi_s^+ \pi^-$ invariant-mass distributions where momentum of $\pi_f^+ >$ momentum of π_s^+ .

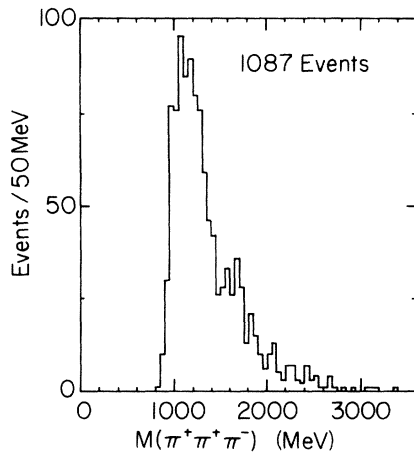


FIG. 11. $\pi^+\pi^+\pi^-$ invariant-mass distribution for events with at least one $M(\pi^+\pi^-)$ between 660 and 860 MeV.

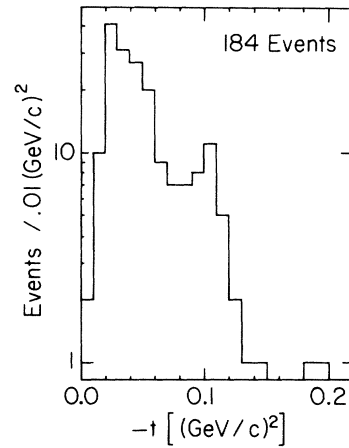


FIG. 12. t distribution for events in A_2 mass range; $1260 < M(3\pi) < 1360$ MeV.

background, there may be a small peak in the T region at 2100 MeV.

VIII. A_2 PRODUCTION

Coherent A_2 production off deuterons was first observed in a 7-GeV/c π^+d experiment.⁷ The A_2 has also been seen in coherent production off heavier nuclei.²³ It is characteristic of A_2 production to have a turnover in the differential cross section due to the change in helicity at the 3π vertex. In hydrogen-target experiments the A_2 observation is enhanced by selecting events with larger $|t|$ values, for example, $-t > 0.075$ (GeV/c)² in a 6-GeV/c π^-p experiment.²⁴ The production of A_2 coherently off deuterons is different from production off single nucleons because large- t values are less likely.

The 3π mass distribution presented in Fig. 8 does not show a statistically significant A_2 signal separate from the dominant A_1 . Unfortunately, a momentum-transfer cut of $-t > 0.075$ (GeV/c)² removes most of the events, so the resulting mass distribution is not statistically significant. The only evidence that some A_2 might be produced comes from the t -distribution studies. Figure 12

shows the t distribution for 3π masses in the A_2 region [$1260 < M(3\pi) < 1360$ MeV]. There is a definite shoulder in this distribution, while no similar shoulders are seen at adjacent 3π mass ranges. If the shoulder is assumed to be the result of A_2 production, a cross section for $\pi^+d \rightarrow A_2^+d$ at 15 GeV/c is estimated to be roughly 5 to 25 μb .

ACKNOWLEDGMENTS

The authors would like to thank W. Selove, G. P. Yost, J. Albright, R. Knop, J. Richey, W. Kononenko, and the authors of ERGON for their valuable assistance at various points in the course of this experiment. The efforts of the technical staffs at Florida State University and the University of Pennsylvania were much appreciated. The support and assistance of J. Ballam, R. Watt, and the 82-in. bubble-chamber crew were indispensable. Gratitude is extended to P. K. Williams, J. D. Kimel, and G. Chew for valuable discussions.

*Work supported in part by the U. S. Atomic Energy Commission.

†Present address: Schlumberger-Doll Research Center, Ridgefield, Connecticut 06877.

‡Present address.

¹T. C. Bacon *et al.*, Phys. Rev. **157**, 1263 (1967).

²R. J. Miller *et al.*, Phys. Rev. **178**, 2061 (1969).

³A. Forino *et al.*, Phys. Lett. **19**, 68 (1965).

⁴R. Vanderhaghen *et al.*, Nucl. Phys. **B13**, 329 (1969).

⁵B. J. Derrey *et al.*, Phys. Rev. D **3**, 635 (1971).

⁶G. Vegni *et al.*, Phys. Lett. **19**, 526 (1965).

⁷D. Harrison *et al.*, Phys. Rev. D **5**, 2730 (1972).

⁸A. M. Cnops *et al.*, Phys. Rev. Lett. **21**, 1609 (1968).

⁹G. Vegni, in *Proceedings of the Topical Seminar on the Interaction of Elementary Particles with Nuclei*, edited by G. Bellini, L. Bertocchi, and S. Bonetti (I.N.F.N., Trieste, Italy, 1970), p. 103.

¹⁰K. Paler *et al.*, Nucl. Phys. **B33**, 13 (1971); Phys.

Rev. Lett. 26, 1675 (1971).

- ¹¹N. D. Pewitt, Ph.D. thesis, Florida State University, 1974 (unpublished).
- ¹²S. P. Denisov, Phys. Lett. 36B, 415 (1971).
- ¹³Suggested by G. Chew (private communication).
- ¹⁴D. P. Sidhu and C. Quigg, Phys. Rev. D 7, 755 (1973); V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966); L. Bertocchi and L. Caneschi, Nuovo Cimento 52A, 295 (1967).
- ¹⁵J. V. Beaupré *et al.*, Phys. Lett. 41B, 392 (1972).
- ¹⁶F. Bradamante *et al.*, Phys. Lett. 31B, 87 (1970).
- ¹⁷J. Ballam *et al.*, Phys. Rev. D 4, 1946 (1971).
- ¹⁸M. A. Abolins, Phys. Rev. Lett. 15, 125 (1965); I. Butterworth *et al.*, *ibid.* 15, 500 (1965).
- ¹⁹W. J. Podolsky *et al.*, Bull. Am. Phys. Soc. 19, 568 (1974).
- ²⁰G. P. Yost *et al.*, Bull. Am. Phys. Soc. 19, 81 (1974).
- ²¹E. L. Berger, Phys. Rev. 166, 1525 (1968); Y.-Y. Yam, Phys. Rev. D 8, 1572 (1973); G. Ascoli and L. M. Jones, *ibid.* 8, 3894 (1973). See also *Experimental Meson Spectroscopy—1974*, proceedings of the International Conference on Experimental Meson Spectroscopy, Boston, 1974, edited by D. A. Garelick (A.I.P., New York, 1974) for more recent discussions.
- ²²P. L. Bastien *et al.*, in *High Energy Physics and Nuclear Structure, Proceedings of the Fifth International Conference on High Energy Physics and Nuclear Structure, Uppsala, Sweden, 1973*, edited by G. Tibell (North-Holland, Amsterdam, 1974), p. 143.
- ²³U. Kruse *et al.*, Phys. Rev. Lett. 32, 1328 (1974).
- ²⁴D. J. Crennell *et al.*, Phys. Rev. Lett. 24, 781 (1970).