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Higher-order effects in a unified model of strong, weak, and electromagnetic interactions

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In this paper the techniques of renormalization of a spontaneously broken non-Abelian gauge theory are applied to higher-order calculations in the unified model of strong, weak, and electromagnetic interactions due to Pati and Salam. A brief resumé of these techniques and of the model is presented. In the rest of the paper higher-order strong- and weak-interaction processes are calculated to show that the processes which are suppressed in the tree-diagram approximation remain small in higher order and that the differences between hadronic and leptonic masses and weak interactions, which are absent in lowest order, arise from higher-order strong interactions.

I. INTRODUCTION

In a recent paper by Pati and Salam,[1] a model has been proposed in which the strong, weak, and electromagnetic interactions were all treated as a spontaneously broken gauge theory. This differed from a previous model of the same type by Bars, Halpern, and Yoshimura, or de Wit[2] in that the quarks and leptons belong to the same representation of the gauge group. It was pointed out in this paper that this has the consequence that there is no difference in general between the interactions of leptons and quarks, i.e., the leptons interact strongly with each other and with quarks. This would appear to be in direct conflict with the fact that there is no experimental evidence for such interactions. However, the conflict can be resolved by assuming that this lack of evidence is a consequence of low-energy phenomenology and that at sufficiently high energies the strong interactions of leptons would come to light. This is achieved by demanding that the gauge mesons which mediate these interactions are extremely massive.[3]

The suppression at low energies of undesirable processes by requiring that the vector meson which mediates that process be massive is only valid in the tree-diagram approximation. Higher-order corrections do not generally depend on the masses of the gauge particles inside a loop but only on the coupling constant. It is possible, therefore, that a process which is suppressed in the tree-diagram approximation may arise in higher order with too large an amplitude.

In this paper we investigate this problem. Before considering higher-order effects, however, it is necessary to discuss the program for the removal of the infinities by absorbing them into the renormalization of the parameters in the Lagrangian and the fields. As has been pointed out by previous authors,[4] this is not as straightforward as it is in theories whose renormalizability is not a direct result of gauge invariance. Conventionally "renormalized" values of parameters are defined by performing subtractions on the mass shell. In spontaneously broken non-Abelian gauge theories this is impossible since there are more fields, masses, and couplings than there are parameters in the Lagrangian, but since both the bare and renormalized Lagrangians must be gauge-invariant, the only permitted subtraction constants must be renormalizations of the parameters in the Lagrangian. It is not possible, therefore, to define the renormalized parameters in terms of on-shell subtractions, but only in terms of gauge-invariant counterterms which free the theory from unwanted infinities. This technique, known as "intermediate renormalization," is used in all theories with spontaneous symmetry breaking. In Sec. II of this paper we present a resumé of the Ward identities and techniques used to define the subtraction constants in a spontaneously broken gauge theory and we also discuss certain properties of a gauge theory whose gauge group is a direct product of two or more other groups, which will be useful for the discussion of the renormalization of a unified model of strong, weak, and electromagnetic interactions. In Sec. III we present a brief resumé of the model and list the independent renormalization constants. In Sec. IV we use the techniques of renormalization to calculate various higher-order corrections and induced processes. The higher-order corrections give rise to mass differences between the leptons and the quarks. These arise from higher-order strong interactions so that it is only possible to give some indication that these mass differences do indeed arise in higher order. They also produce weak corrections to strong interactions and we show that such corrections only contribute terms of order G_F (the Fermi coupling constant) which violate parity. Induced processes are of two kinds. One is the induced coupling of leptons to the "light" strongly interacting gauge bosons and the other is the induced coupling of the baryon-number-violating currents to the light strongly interacting gauge bosons. It is shown that in both cases the coupling is extremely small so that there is no induced strong interaction of leptons. In Sec. V a summary and some conclusions are presented.

II. RENORMALIZATION OF A SPONTANEOUSLY BROKEN NON-ABELIAN GAUGE THEORY

It has been pointed out by several authors[5] that for a non-Abelian gauge theory the simple Ward identity $Z_1 = Z_2$, relating the vertex subtraction constant to the wave-function renormalization constant, does not hold. Z_1/Z_2 is related to the interactions of the Faddeev-Popov ghosts (there may be particular gauges where these ghosts are absent and $Z_1 = Z_2$).

Further complications occur when one considers a theory in which the gauge symmetry is spontaneously broken. Although such a theory is still renormalizable and gauge-invariant so that formal Ward identities can be derived, these identities are less useful in defining the exact subtraction constants. There are two reasons for this:

(1) The Ward identities relate Feynman diagrams involving Goldstone bosons so that there is one more unknown quantity in the identities.

(2) The subtraction constants cannot be chosen in the conventional manner (by putting all the external momenta on their mass shells) since in order to preserve the gauge invariance of the renormalized Lagrangian *there is only one subtraction constant for each multiplet of particles or coupling constant*, whereas the spontaneous-symmetry-breaking mechanism may be such as to give different particles within a multiplet different masses. *The renormalized quantities are therefore not, in general, the physical quantities* and one must consider carefully the relationship between these renormalized quantities and the physical quantities. There is, therefore, a certain amount of arbitrariness in the choice of the renormalized quantities (one has the choice of which process, if any, is to be subtracted on the mass shell) and different choices, although they will not affect the on-mass-shell vertices (provided that these are expressed in terms of the physical coupling constants), will have a finite effect on the off-shell vertices in higher order (since one is performing an expansion in powers of the renormalized coupling constant).

Further details of this will be given in examples later. It is worth noting at this stage, however, that one expects a finite violation of universality of the physical coupling constants for different processes from higher-order corrections, even though the *gauge invariance implies universality in lowest order.*

A. Product groups

There is one simple identity which can be derived between part of Z_1 and part of Z_2 when the gauge group consists of the product of two or more commuting groups, all of which are non-Abelian,

$$G = G_a \times G_b . \tag{2.1}$$

Let $Z_1^{a,b}$ be the renormalization of a vertex of V_μ^a, a gauge boson of the group G_a from higher-order corrections due to the interactions with gauge bosons, V_μ^b, from G_b only. Z_2^b is the wave-function renormalization constant due to the interactions with V_μ^b only. Now $Z_1^{a,b}/Z_2^b$ is related to the interaction of V_μ^a with the Faddeev-Popov ghosts associated with V_μ^b. But since the groups G_a and G_b commute, there is no such interaction and so

$$Z_1^{a,b} = Z_2^b . \tag{2.2}$$

Similarly,

$$Z_1^{b,a} = Z_2^a . \tag{2.3}$$

This can be expressed qualitatively by observing that since the groups G_a and G_b commute, the gauge bosons V_μ^b are singlets in the space spanned by the generators of G_a so that in that space the gauge bosons V_μ^b behave like a set of U(1) gauge bosons and the Ward identity of quantum electrodynamics may be applied to them.

If the gauge group G is broken spontaneously but the mass matrix does not lead to any large mixing of V_μ^a and V_μ^b, then the identities (2.2) and (2.3) still hold exactly if $Z_1^{a,b}$ is defined at zero momentum transfer and Z_2 is the wave-function renormalization of a massless particle (if V_μ^a and V_μ^b are pure vectors, the last condition may be relaxed provided V_μ^a couples to a conserved current). Therefore, higher-order corrections to a vertex of V_μ^a and a particle of mass m from interactions with gauge bosons V_μ^a, whose mass is M and which couple to the particles with coupling constant g, are of order g^2Q^2/M^2 or g^2m^2/M^2 provided $m^2, Q^2 \ll M^2$. If this last condition is not obeyed, then the corrections are simply of order g^2 (m is the mass of the particles in the external legs, Q^2 is the momentum transfer).

In many cases where the gauge group is a product of two or more groups [e.g., the SU(2)×U(1) theories of weak and electromagnetic interactions] the spontaneous-breaking mechanism introduces large mixing between the groups, and the diagonalized gauge bosons cannot be approximated by singlets in either of the spaces spanned by the generators of the two groups. In such a case, (2.2) and (2.3) are not even approximately valid.

However, in the model in which we are interested in this paper, only the photon spans the space of both groups. For all other gauge bosons the mixing is of order g^2/f^2 or less (g and f are the weak- and strong-coupling constants, respectively). In such a model Eqs. (2.2) and (2.3) are useful for a discussion of weak corrections to strong interactions or alternatively strong corrections to weak interactions, both of which will be considered in greater detail below, and will be verified by direct calculation.

III. THE MODEL

In this section we present a resumé of the unified gauge model of strong, weak, and electromagnetic interactions of Ref. 1. This will consist of listing the fermions, scalar particles, and vector mesons and writing down the bare Lagrangian. Further details are found in Ref. 1.

The fermions form a $(4,1,\bar{4})+(1,4,\bar{4})$ representation of $SU(4)_L\times SU(4)_R\times SU(4')$ as well as two left-handed singlets ξ_L and ξ_L'.

The $(4,1,\bar{4})$ multiplet is

$$\psi_L=\begin{bmatrix}\mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \nu_e\\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & e\\ \lambda_a & \lambda_b & \lambda_c & \mu\\ \chi_a & \chi_b & \chi_c & \nu_\mu\end{bmatrix}_L, \tag{3.1a}$$

whereas the $(1,4,\bar{4})$ multiplet is

$$\psi_R=\begin{bmatrix}\mathcal{P}_a & \mathcal{P}_b & \mathcal{P}_c & \xi\\ \mathfrak{N}_a & \mathfrak{N}_b & \mathfrak{N}_c & e\\ \lambda_a & \lambda_b & \lambda_c & \mu\\ \chi_a & \chi_b & \chi_c & \xi'\end{bmatrix}_R. \tag{3.1b}$$

The gauge group is the subgroup $SU(2)_L\times SU(2)_R\times SU(4')$. The gauge bosons of $SU(4')$ may be written as

$$\begin{bmatrix} V_3+\frac{1}{\sqrt{3}}V_8+\frac{1}{\sqrt{6}}V_{15} & \rho^+ & K^{*+} & X^0\\ \rho^- & \frac{1}{\sqrt{3}}V_8-V_3+\frac{1}{\sqrt{6}}V_{15} & K^{*0} & X^-\\ K^{*-} & \bar{K}^{*0} & \frac{-2}{\sqrt{3}}V_8+\frac{1}{\sqrt{6}}V_{15} & X'^-\\ X^0 & X^+ & X'^+ & -(\frac{3}{2})^{1/2}V_{15}\end{bmatrix}.$$

These particles are responsible for the strong interactions between quarks of the same "valency" but of different "color" (e.g., transitions between $\mathcal{P}_a$ and $\mathcal{P}_b$) and the particles $X^\pm, X'^\pm, X^0, \bar{X}^0$ are responsible for quark-lepton transitions.

The groups $SU(2)_L$ and $SU(2)_R$ are not simply the SU(2) groups of transitions between $\mathcal{P}$ and $\mathfrak{N}$ or λ and χ, but are Cabibbo rotations of them, so that in 4×4 matrix notation their gauge bosons may be written

$$\begin{bmatrix} W^3_{L,R} & W^+_{L,R}\cos\theta_{L,R} & W^+_{L,R}\sin\theta_{L,R} & 0\\ W^-_{L,R}\cos\theta_{L,R} & -W^3_{L,R} & 0 & -W^-_{L,R}\sin\theta_{L,R}\\ W^-_{L,R}\sin\theta_{L,R} & 0 & -W^3_{L,R} & W^-_{L,R}\cos\theta_{L,R}\\ 0 & -W^+_{L,R}\sin\theta_{L,R} & W^+_{L,R}\cos\theta_{L,R} & W^3_{L,R}\end{bmatrix}.$$

We note that θ_L and θ_R need not necessarily be equal, but θ_L is the Cabibbo angle.

The symmetry is broken by three multiplets of scalar fields with nonzero vacuum expectation values:

(i) The first is A, which transforms as $(2+2, 2+2, 1)$ and whose vacuum expectation value is

$$\langle A\rangle=\begin{bmatrix}a_1 & & & \\ & a_2 & & \\ & & a_3 & \\ & & & a_4\end{bmatrix},\quad a_4\gg a_1,a_2,a_3. \tag{3.2}$$

This gives masses to $W_L^\pm$, $W_R^\pm$, and W_L^3-W_R^3.

(ii) Next is B, which transforms as $(1,2+2,\bar{4})$ and whose vacuum expectation value is

$$\langle B\rangle=\begin{bmatrix}0 & & & \\ & 0 & & \\ & & 0 & \\ & & & b\end{bmatrix}. \tag{3.3}$$

This gives masses to $X^\pm$, $X'^\pm$, X^0, $\bar{X}^0$, V_{15}, and $W_R^\pm$. In order to suppress lowest-order quark-lepton transitions and $V+A$ weak interactions,

b is very large.

(iii) Last is C, which transforms as $(2+2,1,\bar{4})$ and whose vacuum expectation value is

$$\langle C\rangle = \begin{bmatrix} c_1 & & & \\ & c_1 & & \\ & & c_1 & \\ & & & c_2 \end{bmatrix}. \tag{3.4}$$

This gives masses to all the W_L's and V's except for the linear combination

$$A^\mu = e\left[\frac{W_L^{3\mu}+W_R^{3\mu}}{g}+\frac{V_Q^\mu}{f}\right], \tag{3.5}$$

where

$$V_Q^\mu = (\tfrac{1}{2})^{1/2}[V_3^\mu + (\tfrac{1}{3})^{1/2}V_8^\mu - (\tfrac{2}{3})^{1/2}V_{15}^\mu]$$

and the electric charge e is given by $1/e^2 = 1/f^2 + 2/g^2$, which is the photon. This also introduces a mixing between $W_L^\pm$, $\rho^\pm$, $K^{*\pm}$, $X^\pm$, and $X'^\pm$.

The fermion masses arise from three terms

$$\tfrac{1}{2}\alpha\bar{\xi}_L\,\mathrm{Tr}B^\dagger\psi_R + \tfrac{1}{2}\alpha'\bar{\xi}_L'\,\mathrm{Tr}B^\dagger M\psi_R + \tfrac{1}{2}\beta\,\mathrm{Tr}\bar{\psi}_L A^\dagger\psi_R + \text{H.c.},$$

where M is the matrix

$$\begin{bmatrix} & & & 1 \\ & \sin 2\theta_R & \cos 2\theta_R & \\ & \cos 2\theta_R & -\sin 2\theta_R & \\ 1 & & & \end{bmatrix}$$

which violates $SU(4)_R$ but preserves the subgroup $SU(2)_L\times SU(2)_R$. The masses of the $\mathcal{P}$, $\mathfrak{N}$, and λ quarks are proportional to a_1, a_2, and a_3, respectively (the charmed quarks, χ, whose masses are proportional to a_4, are much heavier). We see that there is no difference between the quark and lepton masses. It is shown in the next section that it is reasonable to suppose that such a mass difference arises from higher-order strong-interaction corrections to the self-energies of the leptons and quarks. The physical neutrinos are the linear combinations $\alpha b\nu_\mu + \beta a_4\xi_L$ and $\alpha' b\nu_e + \beta a_1\xi_L'$, which are approximately the neutrinos in the left-handed multiplet provided $\alpha b \gg \beta a_4$, $\alpha' b \gg \beta a_1$.

We close this section by writing down the bare Lagrangian for all these fields:

$$\begin{aligned}\mathcal{L} = {}& -\tfrac{1}{4}Z_V(\partial_\mu V_\nu^a - \partial_\nu V_\mu^a + if_0 f_{abc}V_\mu^b V_\nu^c)^2 - \tfrac{1}{4}Z_{W_L}(\partial_\mu W_{L\nu}^a - \partial_\nu W_{L\mu}^a + ig_{L0}\epsilon_{abc}W_{L\mu}^b W_{L\nu}^c)^2 \\ & - \tfrac{1}{4}Z_{W_R}(\partial_\mu W_{R\nu}^a - \partial_\nu W_{R\mu}^a + ig_{R0}\epsilon_{abc}W_{R\mu}^b W_{R\nu}^c)^2 + \tfrac{1}{2}Z_A\,\mathrm{Tr}|\partial_\mu A - ig_{L0}\tau\cdot W_{L\mu}A + ig_{R0}A\tau\cdot W_R|^2 \\ & + \tfrac{1}{2}Z_B\,\mathrm{Tr}|\partial_\mu B + ig_{R0}B\tau\circ W_{R\mu} - if_0\lambda\cdot V_\mu B|^2 + \tfrac{1}{2}Z_C\,\mathrm{Tr}|\partial_\mu C + ig_{L0}C\tau\cdot W_{L\mu} - if_0\lambda\cdot V_\mu C|^2 \\ & + Z_L\,\mathrm{Tr}\bar{\psi}_L i\gamma^\mu(\partial_\mu\psi_L - ig_{L0}\tau\cdot W_{L\mu}\psi_L + if_0\psi_L\lambda\cdot V_\mu) + Z_R\,\mathrm{Tr}\bar{\psi}_R i\gamma^\mu(\partial_\mu\psi_R - ig_{R0}\tau\cdot W_{R\mu}\psi_R + if_0\psi_R\lambda\cdot V_\mu) \\ & + Z_\xi\bar{\xi}i\gamma_\mu\partial^\mu\xi + Z_{\xi'}\bar{\xi}'i\gamma_\mu\partial^\mu\xi' + \tfrac{1}{2}\alpha_0\,\mathrm{Tr}(\bar{\xi}B^\dagger\psi_R + \bar{\psi}_R B\xi) + \tfrac{1}{2}\alpha_0'\,\mathrm{Tr}(\bar{\xi}'B^\dagger M\psi_R + \bar{\psi}_R BM\xi') \\ & + \tfrac{1}{2}\beta_0\,\mathrm{Tr}(\bar{\psi}_L A^\dagger\psi_R + \bar{\psi}_R A\psi_L) + V_0(A,B,C),\end{aligned} \tag{3.6}$$

where f_{abc} and ϵ_{abc} are the structure constants of SU(4) and SU(2), respectively, λ_i are the 4-dimensional representation of the generators of SU(4), and τ_i are the special 4-dimensional reducible, Cabibbo-rotated representation of the generators of SU(2). $V_0(A,B,C)$ is the potential formed from the invariants of the gauge group which has a minimum at the required vacuum expectation values.

IV. RENORMALIZATION AND HIGHER-ORDER CORRECTIONS

Before calculating higher-order corrections to physical processes it is necessary to choose a gauge and to add to the Lagrangian the Faddeev-Popov terms.[6] This is achieved by adding to the Lagrangian

$$\begin{aligned}&\frac{1}{\zeta_L}(\partial_\mu W_{L\mu} + \zeta_L M_{W_L}\phi_{W_L})^2 + F_{W_L}^*\delta(\partial_\mu W_{L\mu} + \zeta_L M_{W_L}\phi_{W_L}) \\ &\quad + \frac{1}{\zeta_R}(\partial_\mu W_{R\mu} + \zeta_R M_{W_R}\phi_{W_R})^2 \\ &\quad + F_{W_R}^*\delta(\partial_\mu W_{R\mu} + \zeta_R M_{W_R}\phi_{W_R}) \\ &\qquad + \frac{1}{\zeta_V}(\partial_\mu V_\mu + \zeta_V M_V\phi_V)^2 + F_V^*\delta(\partial_\mu V_\mu + \zeta_V M_V\phi_V),\end{aligned}$$

where ϕ_{W_L}, ϕ_{W_R}, and ϕ_V are the Goldstone bosons associated with W_L, W_R, and V, respectively, F^* are the Faddeev-Popov ghosts, and $\delta(X)$ is the change in X under an infinitesimal gauge transformation F. This gives a Lagrangian for the propagation of F^* into F and the interactions of F and F^* with the vector and scalar mesons. In the bare Lagrangian the Faddeev-Popov ghosts also have a renormalization constant Z_F.

The renormalized Lagrangian is obtained by replacing the wave-function renormalization constants Z by 1 and the bare coupling constants f_0, g_0, etc. by their renormalized values f, g, etc. A wave-function renormalization constant is obtained from the second subtraction on mass shell of any one member of the multiplet. Different choices of the particular member of the multiplet will give the same infinite value of the wave-function renormalization constant but a different finite value. Since there is only one Z for each multiplet, not all the particles will be subtracted on their mass shells. Z_L and Z_R will differ by a finite amount due to the $V-A$ nature of the weak interactions [similarly, if $g_{L0}=g_{R0}$, so that $SU(2)_L \times SU(2)_R$ is a natural symmetry of the theory, the physical coupling constants will differ by a finite quantity]. The coupling-constant renormalizations are calculated from higher-order corrections to one particular vertex (for each coupling constant). They will therefore differ by a finite amount from the physical coupling constants (defined to be the coupling constant for a vertex with all particles on their mass shells), which will in general be different for every vertex. The parameters of $V_0(A, B, C)$ will also be renormalized and this renormalization will determine the renormalizations of the vacuum expectation values of the scalar multiplets (and hence the renormalized masses of the gauge bosons), as well as the masses and self-interactions of the real Higgs scalar particles (the scalars which are not Goldstone bosons). These renormalizations are directly related to the mass renormalizations of the Higgs particles and the gauge bosons, as well as the tadpole corrections (which give the relationship between the vacuum expectation value of the quantized scalar fields and the minimum of the bare potential).

Once all these subtraction constants have been defined, then the renormalization of all the fields, masses, and vertices are defined in terms of them. For example, since the photon is a mixture of W_L, W_R, and V given by Eq. (3.5), the bare photon is given by

$$A_0^\mu = e_0\left(\frac{\sqrt{Z_L}\,W_L^{3\mu}}{g_{L0}} + \frac{\sqrt{Z_R}\,W_R^{3\mu}}{g_{R0}} + \frac{\sqrt{Z_V}\,V_Q^\mu}{f_0}\right), \qquad (4.1)$$

where

$$\frac{1}{e_0^2} = \frac{1}{f_0^2} + \frac{1}{g_{L0}^2} + \frac{1}{g_{R0}^2}.$$

We now apply the techniques of renormalization outlined above to the discussion of the following[7]:

(1) the mass differences between leptons and quarks,

(2) the relationship between the physical and renormalized fields and weak coupling constants for leptons and quarks and a comparison of the corrected weak interactions of leptons and quarks,

(3) the effect of weak-interaction corrections to strong-interaction processes,

(4) induced interactions of leptons with light strongly interacting gauge bosons, and

(5) induced transitions due to the mixing of the

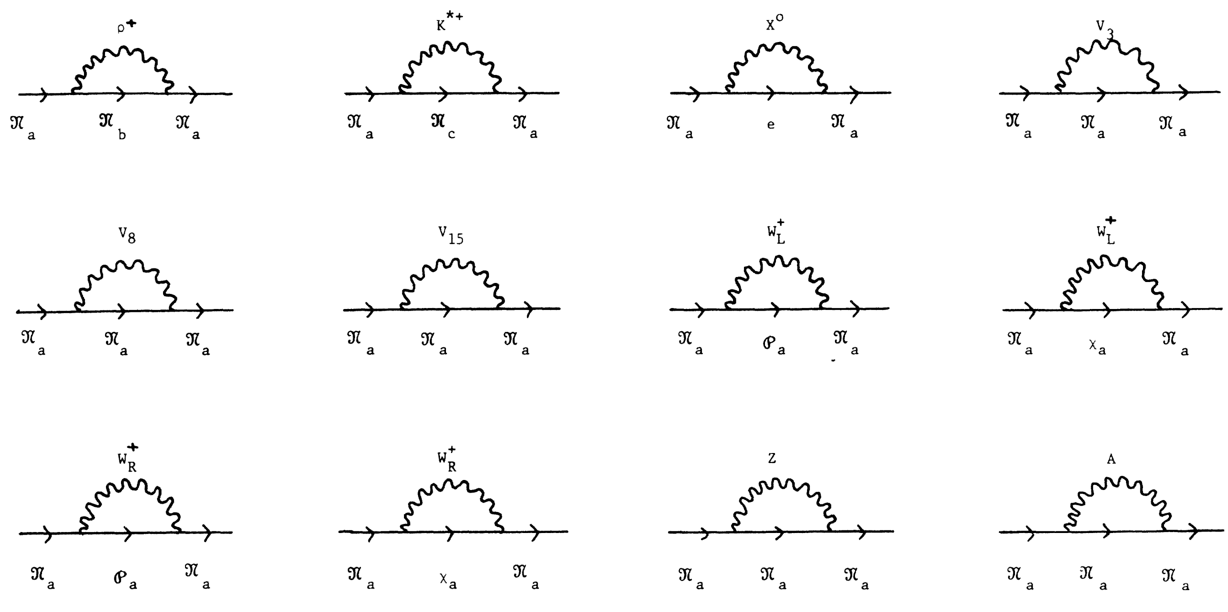

FIG. 1. Self-energy diagrams for $\mathfrak{N}_a$.

gauge bosons V^μ and W_L^μ from their interactions with the scalar multiplet C.

In the first four cases the small mixing between the W_L^μ and V^μ mesons is neglected. This is because such mixings give rise to finite corrections which are proportional to the mixing and therefore negligible.

(1) Quark-lepton mass difference. The self-energy diagrams for $\mathfrak{N}_a$ and e are shown in Figs. 1 and 2. From these diagrams the physical mass renormalizations are for the $\mathfrak{N}_a$ quark (to second order in f^2)

$$\delta m_{\mathfrak{N}} = -m_{\mathfrak{N}} \left\{ \frac{f^2}{4\pi^2}\left[\ln\left(\frac{M_V^2}{\Lambda^2}\right) + \frac{13}{32}\ln\left(\frac{M_X^2}{\Lambda^2}\right) - \frac{105}{64}\right] - \frac{g^2}{32\pi^2}\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) - \frac{7}{2}\right]\right\}, \qquad (4.2)$$

where Λ is the cutoff, M_V is the mass of the gauge mesons $V_1, \ldots V_8$, M_X is the mass of the gauge mesons $V_9, \ldots, V_{15}$, and M_Z is the mass of the weakly interacting neutral gauge boson. The n-dimensional regularization method of 't Hooft and Veltman[8] has been used and the limit as $n \to 4$ has been taken with Λ defined as

$$\Lambda = \lim_{n\to 4} \exp\left[\frac{-1}{(n-4)}\right]. \qquad (4.3)$$

For the electron we have

$$\delta m_e = -m_e \left\{ \frac{f^2}{4\pi^2}\left[\frac{45}{32}\ln\left(\frac{M_X^2}{\Lambda^2}\right) - \frac{105}{64}\right] - \frac{g^2}{32\pi^2}\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) - \frac{7}{2}\right]\right\}. \qquad (4.4)$$

For the Lagrangian to be gauge-invariant both the bare masses and both the renormalized masses must be equal. We therefore make the same subtraction for both (4.2) and (4.3), and we get for the quark-electron mass difference

$$\frac{m_{\mathfrak{N}} - m_e}{m_{\mathfrak{N}}^R} = \frac{f^2}{2\pi^2}\ln\left(\frac{M_X}{M_V}\right), \qquad (4.5)$$

where $m_{\mathfrak{N}}^R$ is some suitably chosen renormalized mass. The expression (4.4) is not an accurate estimate of the mass difference since it relies on a second-order perturbation exapnsion in the strong-coupling constant. Inspired by renormalization-group ideas we might guess that a more accurate value may be

$$\left(\frac{M_X}{M_V}\right)^{f^2/2\pi^2} - 1.$$

It does, however, indicate that the origin of the quark-lepton mass differences may lie in higher-order strong interactions and be related to the fact that the strongly interacting gauge bosons which couple to leptons are much more massive than the ones that couple only to quarks.

(2) Quark-lepton universality of the weak interactions. We shall now give a further example of the problem of choosing subtraction constants by considering higher-order corrections to a leptonic weak process and comparing with higher-order corrections to a hadronic weak process. We begin by considering the fermion wave-function renormalization constants. Once again we assume that a one-loop calculation for the strong interactions provides a reasonable approximation. Again from Figs. 1 and 2 we find that the coefficient of $\not{p}$ for $\mathfrak{N}_a$ is

$$\frac{f^2}{32\pi^2}\left\{\frac{13}{12}\left[\ln\left(\frac{M_X^2}{\Lambda^2}\right) - \frac{1}{2}\right] + \frac{16}{3}\int_0^1 dx\left[(1-x)\ln\left(\frac{M_V^2(1-x) + m_q^2x^2}{\Lambda^2}\right) + \frac{4x(x^2-1)m_q^2}{m_V^2(1-x) + m_q^2x^2}\right]\right\}$$
$$+ \frac{g^2}{32\pi^2}\left\{\left[\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right) - \frac{1}{2}\right]\frac{(1-\gamma^5)}{2} + \left[\ln\left(\frac{M_{W_R}^2}{\Lambda^2}\right) - \frac{1}{2}\right]\frac{(1+\gamma^5)}{2} + \frac{1}{2}\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right) - \frac{1}{2}\right]\right\} + O(G_F m_q^2), \qquad (4.6)$$

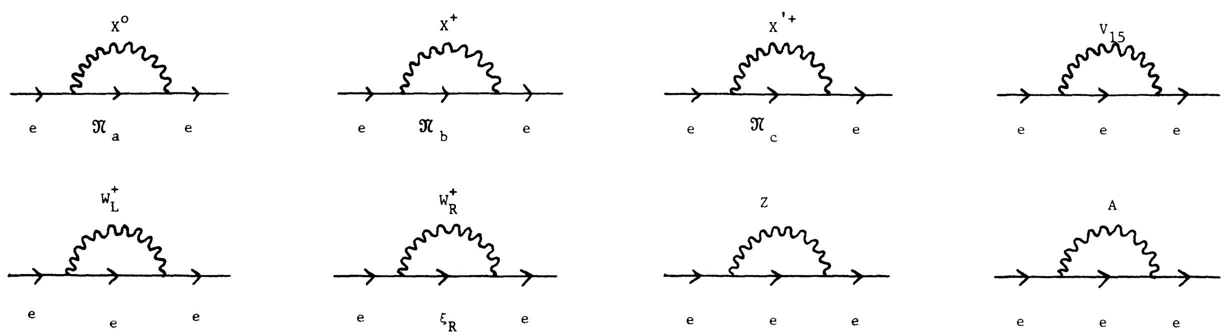

FIG. 2. Self-energy diagrams for the electron.

where m_q is a renormalized quark mass, M_{W_L} and M_{W_R} are the masses of W_L^+ and W_R^+, respectively, and G_F is the Fermi weak-coupling constant. It can be seen that since $M_{W_L} \neq M_{W_R}$, $Z_L \neq Z_R$. It has been assumed that the quark masses are much smaller than M_{W_L} so that $G_F m_q^2$ may be neglected. For the lepton we find that the coefficient of $\not{p}$ is

$$\frac{15f^2}{64\pi^2}\left[\ln\left(\frac{M_X^2}{\Lambda^2}\right)-\frac{1}{2}\right]+\frac{g^2}{32\pi^2}\left\{\left[\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right)-\frac{1}{2}\right]\frac{(1-\gamma^5)}{2}+\left[\ln\left(\frac{M_{W_R}^2}{\Lambda^2}\right)-\frac{1}{2}\right]\frac{(1+\gamma^5)}{2}+\frac{1}{4}\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right)-\frac{1}{2}\right]\right\}+O(G_F m_q^2)\,. \tag{4.7}$$

We notice that (4.5) and (4.6) differ by an amount of order $(f^2/6\pi^2)\ln(M_X/M_V)$, and that only one of these expressions may be taken to define Z_L and Z_R. Suppose we choose (4.6); then the renormalized quark wave function creates $(1+\beta)^{1/2}$ physical quarks, where β is the difference between the expressions (4.6) and (4.7). Therefore, in a diagram with external quarks we must divide each external quark line by $(1+\beta)^{1/2}$. For one-loop corrections to diagrams with external quarks this brings us back to defining the effective wave-function renormalization constant as the second subtraction on the mass shell for both leptons and quarks, but care must be taken over the definition of these subtraction constants if one wishes to perform two or more loop calculations.

We now come to the one-loop corrections to the W_L^+-$\mathcal{P}_a$-$\mathcal{N}_a$ (Fig. 3) and W_L^+-e-ν_e (Fig. 4) vertices. For simplicity the dependence on momentum transfer Q^2 will not be written out explicitly. For the W_L-$\mathcal{P}_a$-$\mathcal{N}_a$ vertex we find

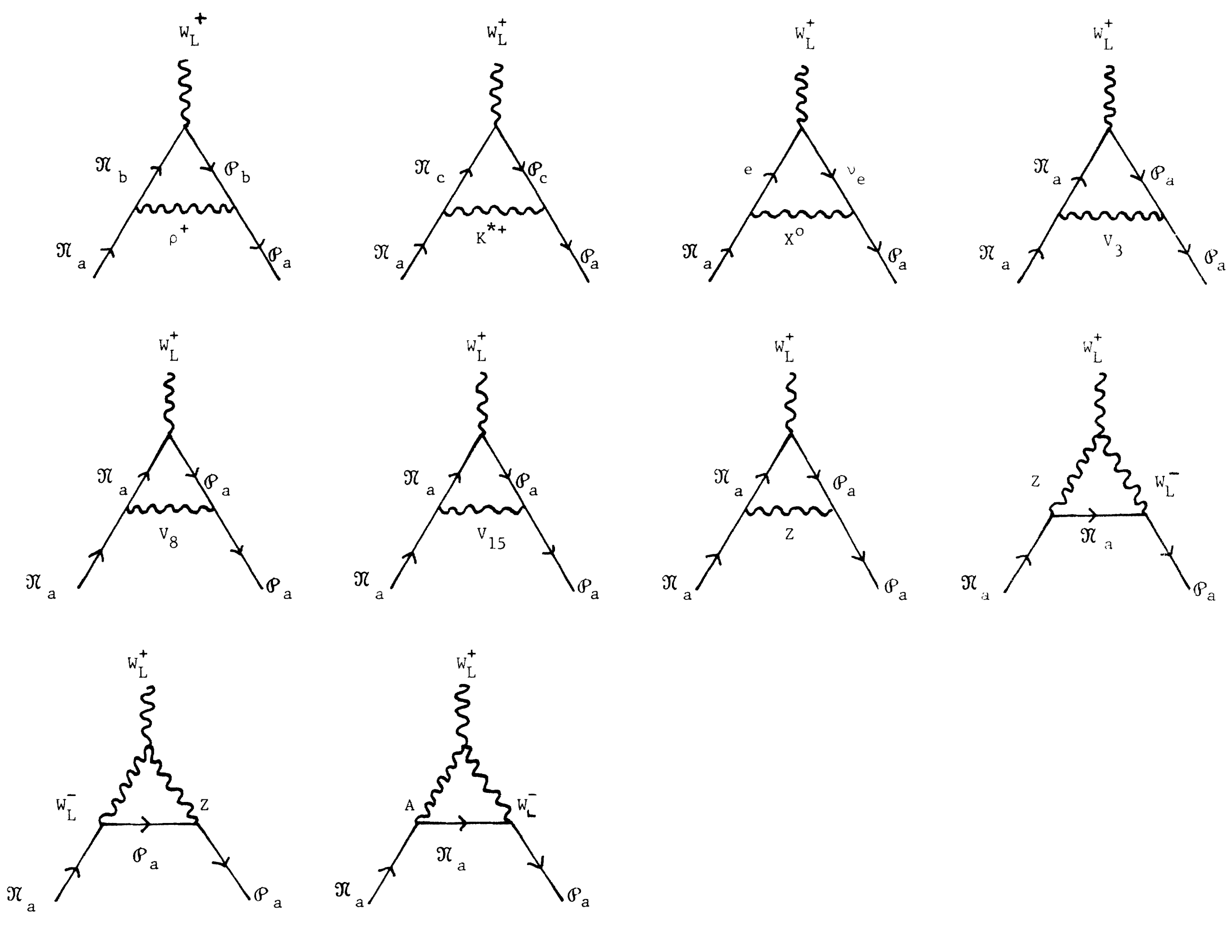

FIG. 3. Corrections to the W_L^+-$\mathcal{P}_a$-$\mathcal{N}_a$ vertex.

$$\frac{f^2}{32\pi^2}g_L\left\{\frac{13}{12}\left[\ln\left(\frac{M_X^2}{\Lambda^2}\right)-\frac{1}{2}\right]+\frac{16}{3}\int_0^1 dx\left[(1-x)\ln\left(\frac{M_V^2(1-x)+m_q^2x}{\Lambda^2}\right)-\frac{4m_q^2}{M_V^2(1-x)+m_q^2x}\right]\right\}\gamma^\mu\frac{(1-\gamma^5)}{2}$$

$$+\frac{f^2}{16\pi^2}g_L\frac{16}{3}\int_0^1 dx\frac{4x^3m_q^2}{M_V^2(1-x)+m_q^2x}\gamma^\mu\frac{(1+\gamma^5)}{2}$$

$$+\frac{g_L^3}{32\pi^2}\left\{\frac{3}{M_{W_L}^2-M_Z^2}\left[M_{W_L}^2\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right)-M_Z^2\ln\left(\frac{M_Z^2}{\Lambda^2}\right)\right]-\frac{7}{2}+\frac{1}{2}\left[\ln\left(\frac{M_Z^2}{\Lambda^2}\right)-\frac{1}{2}\right]\right\}\gamma^\mu\frac{(1-\gamma^5)}{2}$$

$$+Q^2F_q(q_L,f,Q^2)+O(G_Fm_q^2)\,,\qquad(4.8)$$

while for W_L^+-e-ν vertex we find

$$\frac{15f^2g_L}{64\pi^2}\left[\ln\left(\frac{M_X^2}{\Lambda^2}\right)-\frac{1}{2}\right]\gamma^\mu\frac{(1-\gamma^5)}{2}+\frac{g_L^3}{32\pi^2}\left\{\frac{3}{M_{W_L}^2-M_Z^2}\left[M_{W_L}^2\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right)-M_Z^2\ln\left(\frac{M_Z^2}{\Lambda^2}\right)\right]-\frac{13}{4}-\frac{1}{2}\ln\left(\frac{M_Z^2}{\Lambda^2}\right)\right\}\gamma^\mu\frac{(1-\gamma^5)}{2}$$

$$+Q^2F_e(g_L,f,Q^2)+O(G_Fm_q^2)\,.\qquad(4.9)$$

After accounting for the wave-function renormalization [and the $(1+\beta)^{1/2}$ factor for the external quarks] we are left with the following for quarks:

$$\frac{2f^2g_L}{3\pi^2}\int_0^1 dx\frac{x^3m_q^2\gamma^5}{M_V^2(1-x)+m_q^2x}+\frac{g_L^3}{32\pi^2}\left\{\frac{3}{M_{W_L}^2-M_Z^2}\left[M_{W_L}^2\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right)-M_Z^2\ln\left(\frac{M_Z^2}{\Lambda^2}\right)\right]-\ln\left(\frac{M_W^2}{\Lambda^2}\right)-3\right\}\gamma^\mu\frac{(1-\gamma^5)}{2}$$

$$+Q^2F_q(g,f,Q)+O(G_Fm_q^2)\,,\qquad(4.10)$$

and for leptons

$$\frac{g_L^3}{32\pi^2}\left\{\frac{3}{M_{W_L}^2-M_Z^2}\left[M_{W_L}^2\ln\left(\frac{M_{W_L}^2}{\Lambda^2}\right)-M_Z^2\ln\left(\frac{M_Z^2}{\Lambda^2}\right)\right]-\ln\left(\frac{M_W^2}{\Lambda^2}\right)-3\right\}\gamma^\mu\frac{(1-\gamma^5)}{2}+Q^2F_e(g_L,f,Q^2)+O(G_Fm_q^2)\,.\qquad(4.11)$$

Examination of (4.10) and (4.11) shows that whereas the infinite parts of the strong-interaction contributions to Z_1 and Z_2 are equal, the same is not true for the weak-interaction corrections. Moreover, since the expressions (4.10) and (4.11) are not identical and since there is only one subtraction term $(g_{L0}-g)\gamma^\mu\frac{1}{2}(1-\gamma^5)$ for both of them, there is a violation of weak-interaction universal-

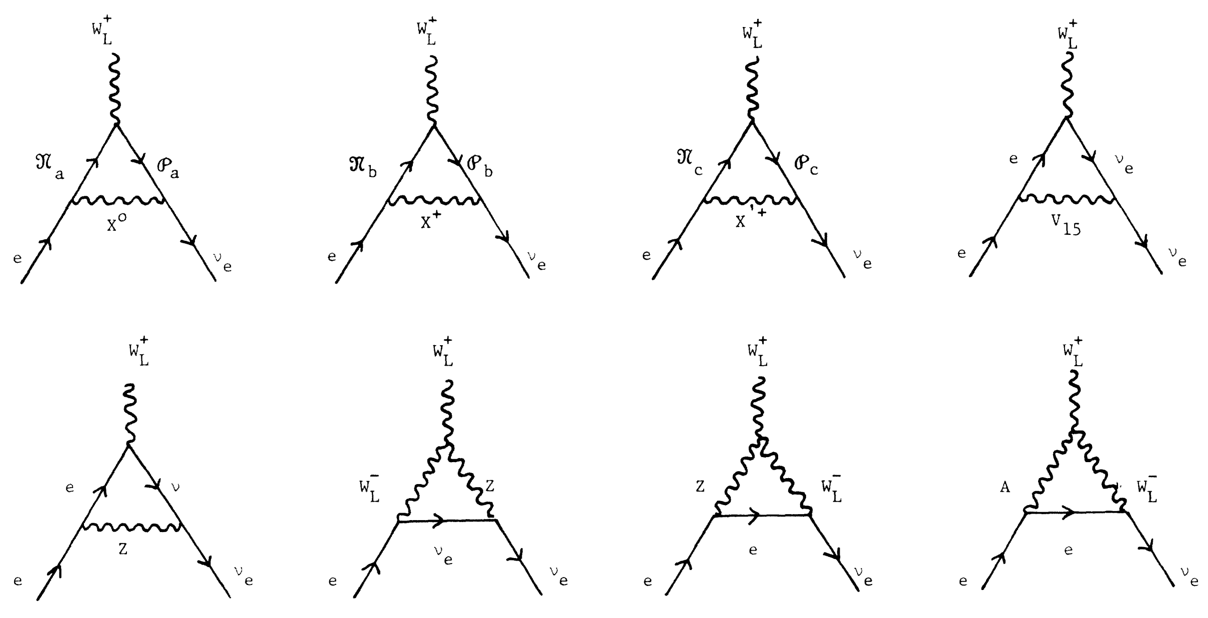

FIG. 4. Corrections to the W_L^+-e-ν_e vertex.

ity between quarks and leptons which at zero momentum transfer is

$$\frac{2f^2g_L}{3\pi^2}\int_0^1 dx\frac{x^3m_q{}^2\gamma^5}{M_V{}^2(1-x)+m_q{}^2x}+O(G_Fm_q{}^2). \tag{4.12}$$

Since m_q and M_V are of the same order of magnitude this would appear to be large if f is large. However, this is a problem of making a perturbation expansion in the strong-interaction coupling constant which would be there in any strong-interaction field theory. We can, however, draw the qualitative conclusion that since the weak interaction is a gauge theory which limits the number of permitted counterterms and since the strong interactions of leptons and quarks are different, then the strong interactions will contribute to a violation of the universality of leptonic and hadronic weak interactions which is too large to be explained by higher-order weak and electromagnetic interactions.

A different choice of subtraction constant would correspond to a different relationship between the renormalized coupling constant and the magnitude of the weak interaction at zero momentum transfer. The dependence on the momentum transfer is a function of the renormalized coupling constant so that a different choice of these will affect higher-order corrections only. To a one-loop approximation the dependence on momentum transfer is independent of the choice of the subtraction constant.

For small momentum transfers a diagram in which the heaviest internal particle has mass M has a momentum-transfer dependence f^2Q^2/M^2 or g^2Q^2/M^2 for strong- or weak-interaction corrections, respectively. From this we can see that the strong interactions of leptons introduce contributions to their electromagnetic form factors which are of order $f^2/M_X{}^2$ (since only X, X', X^0, $\overline{X}^0$, and V_{15} interact strongly with leptons). These contributions are much smaller than G_F (since M_X is very large) and so they introduce no new measurable quantity. The electromagnetic form factors of the quarks will be of order $f^2/M_V{}^2$ for small momentum transfers, but this cannot be computed exactly for realistic values of f.

(3) Weak corrections to strong interactions. A further example of the use of Eqs. (2.2) and (2.3) is the calculation of weak-interaction corrections to a strong vertex. The diagrams for this are shown in Fig. 5. The contribution from these diagrams is given by

$$\frac{g_L{}^2f}{32\pi^2}\left\{\left[\ln\left(\frac{M_{W_L}{}^2}{\Lambda^2}\right)-\frac{1}{2}\right]\gamma^\mu\frac{(1-\gamma^5)}{2}+\left[\ln\left(\frac{M_{W_R}{}^2}{\Lambda^2}\right)-\frac{1}{2}\right]\gamma^\mu\frac{(1+\gamma^5)}{2}+\frac{1}{2}\ln\left(\frac{M_Z{}^2}{\Lambda^2}\right)\gamma^\mu\right\}+O(G_Fm_q{}^2) \tag{4.13}$$

at zero momentum transfer. After taking into account the wave-function renormalization constants, we are left with terms which are of order $fG_Fm_q{}^2$, so that provided $m_q{}^2, Q^2 \ll M_{W_L}{}^2$ the parity violation from higher-order weak corrections to a strong-interaction vertex is of order G_F and not g^2 as may have been expected.[9] This is consistent with Eqs. (2.2) and (2.3) and the fact that the gauge group of the strong interaction commutes with the weak-interaction gauge group with very little mixing induced by the spontaneous breaking mechanism.

(4) Induced leptonic strong interactions. The diagrams which lead to leptonic coupling to the light strongly interacting gauge bosons $V_1, \ldots, V_8$ are shown in Fig. 6. Actually only V_3 and V_8 can couple to leptons in this way. The contribution from each diagram is

$$\frac{f^3}{32\pi^2}\ln\left(\frac{M_X{}^2}{\Lambda^2}\right)\lambda_{3,8}^{ii}, \tag{4.14}$$

where $\lambda_{3,8}$ is some linear combination of λ_3 and λ_8. The total contribution from all these diagrams is therefore $\mathrm{Tr}(\lambda_{3,8})=0$. In practice the cancellation is not exact since the masses of $X^\pm$, $X'^\pm$, X^0, and $\overline{X}^0$ differ slightly due to the mixing with $W_L^\pm$ by an amount proportional to the mixing $(g/f)c_1c_2$,

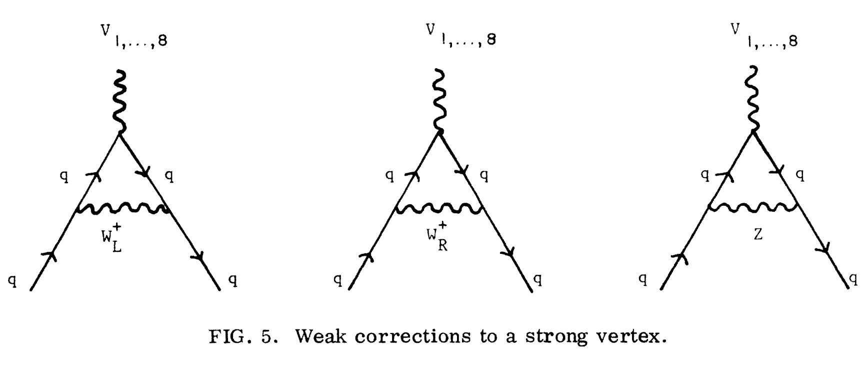

FIG. 5. Weak corrections to a strong vertex.

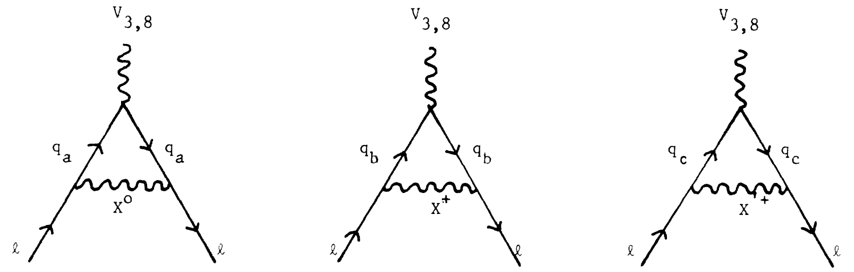

FIG. 6. Induced strong interactions of leptons.

so we are left with a nonvanishing contribution from the diagrams which is of order $(f^2g/32\pi^2)(c_1c_2/M_X^2)$ which is very small ($\lesssim 10^{-10}$) and does not therefore give rise to any new lepton interactions (this is no larger than the interaction of leptons with V_{15}). This result is expected from the fact that the lepton currents are singlets of color SU(3)′ and would only be expected to couple to members of an SU(3)′ octet of vector bosons with a strength proportional to the breaking of SU(3)′.

(5) Higher-order interactions through the mixing. There are three mixings which give rise to interesting processes from higher-order graphs, namely W_L^+-ρ^- or W_L^+-K^{*-} mixings, X^+-ρ^-, X'^+-ρ^-, X^+-K^{*-}, or X'^+-K^{*-} mixings, and W_L^+-X^- or W_L^+-X'^- mixings. In all cases the processes induced by these mixings are finite because the diagonalized gauge bosons couple with equal and opposite strength to the mixing current so that the infinite parts cancel each other.

Typical W_L^+-ρ^- or W_L^+-K^{*-} mixing diagrams are shown in Fig. 7. They give rise to violations of charm, strangeness, parity, and color conservation. All these diagrams are proportional to the mixing, which is so small that the amplitude for this process is much smaller than G_F, so that it is only the violation of color (the conversion of an a-type quark to a b-type quark) which gives rise to a new physical process which is absent in tree diagrams. The amplitude for such a process is

$$\cos\theta_C \frac{f^2g^2}{16\pi^2}\frac{c_1c_2}{M_{W_L}{}^2} \times\left[-3\not{p}\,\frac{(1-\gamma^5)}{2}+4m_q\frac{(1-\gamma^5)}{2}\right]\ln\left(\frac{M_V{}^2}{M_{W_L}{}^2}\right) . \quad (4.15)$$

By attaching strongly interacting particles to these diagrams we get an amplitude for quark-quark scattering which violates color conservation:

$$\cos\theta_C\frac{f^4g^2}{32\pi^2}\frac{c_1c_2}{M_{W_L}{}^2} \times\langle\bar{q}_a|\gamma^\mu|q_a\rangle\langle\bar{q}_a|\gamma^\mu\tfrac{1}{2}(1-\gamma^5)|q_b\rangle\ln\left(\frac{M_V{}^2}{M_{W_L}{}^2}\right) . \quad (4.16)$$

Typical X^+-ρ^-, X'^+-ρ^-, X^+-K^{*-}, X'^+-K^{*-} mixing diagrams are shown in Fig. 8. These can give rise to violations of lepton number and baryon number as well as to the violations caused by W_L^+-ρ^- mixing. However, since these mixings are much smaller than W_L^+-X^-, W_L^+-X'^- mixing, we can neglect this mixing and confine ourselves to a discussion of W_L^+-X^- and W_L^+-X'^- mixing only.

Self-energy-type diagrams which give rise to transitions between quarks and leptons are shown in Fig. 9. The contribution from these diagrams is

$$\frac{f^2g^2}{16\pi^2}\cos\theta_C\frac{c_1c_2}{M_X{}^2} \times\left[-3\not{p}\,\frac{(1-\gamma^5)}{2}+4m_q\frac{(1-\gamma^5)}{2}\right]\ln\left(\frac{M_{W_L}{}^2}{M_X{}^2}\right) . \quad (4.17)$$

Since this is finite there is no need to renormalize it. If, however, one chooses to do so by rediagonalizing the fermions, then the coupling of these rediagonalized fermions to $V_1,\ldots,V_8$ or to $W_L^\pm, W_R^\pm, Z$ will have a direct quark-lepton transition identical to that obtained by attaching $V_1,\ldots,V_8$, $W_L^\pm, W_R^\pm, Z$ lines to the diagrams in Fig. 9. By attaching a strongly interacting gauge boson $V_1,\ldots,V_8$ to the diagrams in Fig. 9 one

FIG. 7. W_L^+-ρ^- mixing in higher order.

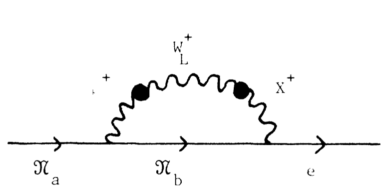

FIG. 8. X^+-ρ^- mixing in higher order.

can get an amplitude for the process $q+q\to q+l$ which is approximately

$$\cos\theta_C \frac{f^4 g^2}{32\pi^2}\frac{c_1 c_2}{M_X{}^2}\frac{1}{t-M_V{}^2} \times \ln\left(\frac{M_{W_L}{}^2}{M_X{}^2}\right)\langle \bar{q}|\gamma^\mu \tfrac{1}{2}(1-\gamma^5)|\,l\rangle\langle\bar{q}|\gamma^\mu|q\rangle \quad (4.18)$$

(t is the momentum transfer squared), which is of the order of $<10^{-10}$. This has to be cubed if it is to be applied to the process $B+\pi\to\pi+3l$, giving an amplitude $\lesssim 10^{-30}$ and a cross section $\lesssim 10^{-59}$ μb. Although this is very small, it is considerably larger than the amplitude for such a process obtained from the tree-diagram approximation which gives 10^{-14} for $q+q\to q+l$ (a cross section of 10^{-83} μb for $B+\pi\to\pi+3l$). We now see why higher-order corrections are important since they can give rise to processes with a larger amplitude than that obtained from tree diagrams (even though they are still far too small to be detected experimentally at the present time).

Another example of this arises when we note that one of the strongly interacting particles $\frac{1}{2}(V_8+\sqrt{3}\,V_3)$ has a small mixing g^2/f^2 with Z. Attaching this particle to the diagrams in Fig. 9 gives a contribution to quark decay into leptons which is

$$\cos\theta_C \frac{f^2 g^4}{196\pi^2}\frac{1}{t-M_V{}^2}\frac{c_1 c_2}{M_X{}^2}\ln\left(\frac{M_{W_L}{}^2}{M_X{}^2}\right) \times\langle q|\gamma^\mu\tfrac{1}{2}(1-\gamma^5)|\,l\rangle\langle\bar{l}|\gamma^\mu\tfrac{1}{2}(1-\gamma^5)|\,l\rangle. \quad (4.19)$$

This is of order 10^{-12}, which although very small is larger than the tree-diagram contribution, which is of order 10^{-13}. This makes a difference of a factor of 10^{-6} in the lifetime of the proton.[10]

One notices that only the b- and c-type quarks can decay directly into leptons. The a-type quark can only decay in two stages into leptons. Since the amplitude for an a-type quark to transform into a b- or c-type quark is of order G_F [see Eq. (4.14)], the amplitude for the decay of an a-type quark is 10^{-17}, although this is zero in the tree-diagram approximation. This is important since we believe that the known hadrons are color singlets so that their wave function is of the form $\epsilon_{abc}q^a q^b q^c$ and that such a state cannot decay completely into leptons from tree diagrams only, but it can do so from higher-order processes which arise from the mixing.

V. SUMMARY AND CONCLUSIONS

We begin this section by listing the theorems which have been used to renormalize the model of Ref. 1 and to investigate the effects of higher-order corrections, and to construct the scalar potential.

(1) For a massless Yang-Mills theory

$$\left[\frac{Z_1}{Z_2}\right]_{\text{Yang-Mills}} = \left[\frac{Z_1}{Z_2}\right]_{\text{matter}} = \left[\frac{Z_1}{Z_2}\right]_{\text{Faddeev-Popov}},$$

(although $[Z_1/Z_2]$ is not equal to 1).

(2) For a massless Yang-Mills theory in which the gauge group is a product of two commuting groups $G_a\times G_b$, and if Z_2^a and Z_2^b are defined to be wave-function renormalizations due to the interactions of the Yang-Mills fields of the groups G_a and G_b, respectively, and if $Z_1^{a,b}$ is defined to be the correction to a vertex of a Yang-Mills field from G_a due to interactions of the Yang-Mills fields from the group G_b (with $Z_1^{b,a}$ similarly defined), then

$$\frac{Z_1^{a,b}}{Z_2^b} = \frac{Z_1^{b,a}}{Z_2^a} = 1 \text{ in all gauges}.$$

(3) For a spontaneously broken symmetry where Z_1 and Z_2 cannot generally be defined at the physical point (on-mass-shell), the theorems (1) and (2) hold for the infinite parts. If Z_1 is defined at zero momentum transfer with the other external legs on their mass shells, and Z_2 is defined at the physical point, then the finite corrections to theorems (1) and (2) are of order g^2m^2/M^2 provided $m\ll M$, where m is the mass of the external leg, M is the mass of the Yang-Mills field inside the loop, and g is the coupling constant associated

FIG. 9. X^+-$W_L{}^-$ mixing in higher order.

with the gauge theory. If, furthermore, the gauge theory has only vector interactions then these finite corrections are of order $g^2(m_1^2 - m_2^2)/M^2$, provided $m_1^2 - m_2^2 \ll M^2$, where m_1 and m_2 are the masses of the two external legs in the vertex.

(4) In a spontaneously broken gauge symmetry the particular choice of subtraction point only affects the momentum-transfer behavior of a vertex in higher orders than the one-loop approximation.

(5) In a spontaneously broken gauge symmetry any mixing terms, couplings, or mass differences which are absent in lowest order will be finite and calculable in higher order provided that such mixings, couplings, or mass differences arise only from interactions with gauge bosons and not through interactions with the scalar particles. If, however, these mixings, couplings, or mass differences are absent in lowest order because of a special choice of the parameters in the scalar potential (leading to a particular choice of vacuum expectation values), then they will acquire infinite contributions in higher order. These infinite contributions can always be absorbed into the renormalization of the parameters in the scalar potential.

These theorems were applied in Sec. IV to various higher-order processes and corrections, and were verified by direct calculation. The first two calculations involved higher-order strong interactions. Since perturbation results are invalid for theories with a strong-coupling constant, the results of these calculations are qualitative rather than quantitative in nature. They suggested that the quark-lepton mass difference and the violations of quark-lepton weak-interaction universality are a consequence of the higher-order strong interactions of leptons and quarks. The differences arise from the differences of the masses of the strongly interacting particles which contribute to the higher-order corrections and the fact that gauge invariance demands that the same subtraction constant be used for the leptonic and the hadronic process. In the case of the mass difference we obtained a result of the order of $(f^2/2\pi^2)\ln(M_X/M_V)$, whereas for the violation of universality the result was of the order of $f^2/6\pi^2$ [the $\ln(M_X/M_V)$ term canceled by virtue of theorem (2)]. Measurement of nuclear beta decay and hyperon decay[11] suggests that the violation of universality is about 5%, so that $f^2/6\pi^2$ may be of order 0.05.[12] This calculation may be quantitatively accurate, whereas the lepton-quark mass difference is large because of the factor $\ln(M_X/M_V)$.

The other calculations in Sec. IV involved either higher-order weak interactions or interactions involving the mass mixing between gauge bosons. It was shown that parity violation in strong-interaction processes from higher-order weak corrections was of the order G_F, and that the violation of $SU(4)_L \times SU(4)_R$ quantum numbers or the coupling of leptons to the light gauge bosons of SU(4)′ from higher-order processes which were absent in lowest order was proportional to the mixing parameter and therefore harmless. It was also shown that in processes in which baryon number is violated, although the matrix element for such a process is still extremely small, the contribution from higher-order corrections involving the mixing was larger than the contribution from the lowest-order tree diagrams.

From these calculations it is concluded that higher-order corrections do not give rise to unacceptably large matrix elements for processes which were suppressed in lowest order and that the relatively large desirable difference between leptonic and hadronic physics, which was absent in lowest order, arises from higher-order strong interactions.

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gauge ($\zeta_L = \zeta_R = \zeta_V = 1$). Furthermore, all interactions with scalar particles are neglected. This is justified by the fact that only the multiplet A interacts with fermions with coupling constant $G_F^{1/2} m_q$ (G_F is the Fermi weak coupling constant, m_q is a typical quark mass), so that all such corrections will be of order $G_F m_q^2$, which is negligible.

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