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Values of fundamental constants*

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We study how renormalized coupling constants depend on the cutoff in a renormalized quantum field theory. Assuming that the cutoff is connected with quantum gravity effects, relations are found between the fine-structure constant α, the total cross section e^+e^- into hadrons, and the weak-interaction angle θ_w.

It is an open problem whether the values of the fundamental constants of strong, electromagnetic, weak, and gravitational interactions are free parameters or whether they satisfy some inner-consistency condition. In recent times it has been suggested that the requirement of analyticity of first and second kind determines the strong-interaction world.[1]

Unfortunately, the concept of only one admissible coupling constant is foreign to standard quantum field theory; it has been proved that in two-dimensional space-time a $\lambda\phi^4$ theory exists and satisfies all physical requirements for values of the coupling constant which are not too large.[2] Similar results should be valid for any superrenormalizable interaction.[3]

However, if the interaction is only renormalizable, the situation is quite different. A cutoff must be introduced at an intermediate stage and finite results are obtained at any order in perturbation theory only if the bare coupling constant goes to infinity together with the cutoff.[4] This procedure has sometimes been criticized as being too artificial,[5] since it would be much more natural to fix the coupling constant before going to the infinite-cutoff limit. We argue that if finite results are obtained in this unconventional way, the renormalized coupling constant does not depend on the bare one and it is uniquely determined.

This can be simply understood in a theory with only one bare coupling constant G. Let g and Λ denote, respectively, the renormalized coupling constant and the cutoff. The following relation holds:

$$\Lambda \frac{\partial g}{\partial \Lambda}\bigg|_G = -\beta(g), \tag{1}$$

where $\beta(g)$ is the Callan-Symanzik[6,7] function which can be computed in perturbation theory.

If the limit $\lim_{\Lambda\to\infty} g(G,\Lambda) \equiv g_\infty$ exists, then $\beta(g_\infty)$ must be equal to zero[8]; g_∞ cannot be arbitrary since the function β is not identically equal to zero.

Two possibilities are present: (1) $g_\infty = 0$ or (2) $g_\infty \neq 0$. The first one is realized in a great majority of cases $[\beta(g) > 0,\ g \sim 0]$, while the second one is typical of some non-Abelian gauge theories.[9,10] (The latter are the so-called asymptotically free theories.) In making this distinction we have assumed that the bare coupling constant is not too large in order to use perturbation theory as a reliable guide.

In the infinite-cutoff limit only strong interactions survive; if other interactions are present in the Lagrangian, they disappear from the Green's functions and the S matrix. The actual existence of nonstrong interactions suggests that the cutoff is finite, still being very large. In this case the renormalized coupling constant of a nonasymptotically free interaction is different from zero. For example, in pure quantum electrodynamics one finds[11]

$$\alpha = \frac{3\pi}{2\ln(\Lambda/m_e)} + O(\ln^{-2}(\Lambda/m_e)). \tag{2}$$

The dependence on the bare coupling constant appears only in the next-order term; if $\ln(\Lambda/m_e)$ is a very large number, then the value of the nonstrong coupling constant is approximately fixed and is independent of the value of the bare one.

We try to give a physical meaning to the cutoff assuming that quantum gravity provides an automatic cutoff at distances of order 10^{-32} cm (10^{-5} g).[12] The ratio of this distance to any strong- or weak-interaction scale is so small that a good approximation can be obtained without knowing in detail how the cutoff works. As an application we discuss the following model: Strong interactions are approximately symmetric under $SU(4)\times SU(4)$, the only leptons being the electron, the muon, and their neutrinos. Electromagnetic and weak interactions are the same as in the Weinberg model,[13] and the hadrons are coupled to the gauge fields following the Glashow-Iliopoulos-Maiani prescription.[14] For simplicity, Higgs fields[15] are not introduced at this stage.

The hadrons have strong interactions; they must be described by an asymptotically free theory, and their Lagrangian is of the Yang-Mills type. The coupling constant is fixed by the condition $\beta(g)=0$, but its actual value is uncomputable using standard field-theory techniques.

Formulas similar to Eq. (2) can be found for the weak and electromagnetic charges:

$$\frac{g'^2}{4\pi}=\frac{3\pi}{2L}\left(\tfrac{3}{2}+\tfrac{11}{18}A\right)^{-1}+O(L^{-2}),$$
$$\frac{g^2}{4\pi}=\frac{3\pi}{2L}\left(\tfrac{1}{2}A-5\right)^{-1}+O(L^{-2}), \tag{3}$$

where

$$e=\frac{gg'}{g^2+g'^2},$$
$$\tan\theta_w=\frac{g'}{g},$$
$$L=\ln(\Lambda/m_p)\quad (m_p=\text{proton mass}), \tag{4}$$

and

$$A=\frac{9}{10}\left.\frac{\sigma(e^+e^-\to\text{hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}\right|_{m^2\ll s<\Lambda^2}.$$

In this model, we stress that the strong-interaction world is asymptotically scale-invariant, anomalous dimensions[16] are probably present, and A may assume a value quite different from its canonical one[17] which is 3 in the "colored" quark model.

Using as an input the actual value of α and choosing $\Lambda=10^{18}$ GeV ($2L=83$), we predict $\sin^2\theta_w \simeq 0.3$, $R\simeq 20$. Different results are obtained if the model for weak interactions is changed.

Summarizing, we suggest that at distances where quantum gravity becomes relevant, strong, electromagnetic, and weak interactions are described by comparable coupling constants. Quantum fluctuations (vacuum-polarization effects) screen the weak and electromagnetic charges and enhance the strong ones. This mechanism produces relations among the electromagnetic, weak, and gravitational coupling constants. These relations become exact in the limit in which the gravitational coupling constant goes to zero. In the same limit the electromagnetic and weak interactions disappear. Assuming a particular model for the weak interactions,[13,14] we predict the value of the Weinberg angle and of the total cross section for the process of e^+e^- into hadrons. The results do not strongly conflict with the uncertain experimental situation.[18-22]

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