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General three-dimensional superluminal transformations and tachyon kinematics*†

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The existence of a preferred axis in space, called the tachyon corridor, has recently been proposed in order to generalize the one-dimensional causal theory of tachyons to three dimensions. We use this theory to derive the general transformation equations between inertial frames with superluminal relative velocities in a three-dimensional space. The kinematics of tachyons, as well as luxons emitted by them, are then worked out and several characteristics of the tachyon corridor follow from them. These provide the means for determining the direction of the tachyon corridor.

I. INTRODUCTION

It has been shown[1] that a classical three-dimensional causal theory of tachyons can be developed by combining the results of the inversion-non-invariant one-dimensional theory[2,3] with the idea of a tachyon corridor. The one-dimensional theory was constructed by extending the special Lorentz transformations to relative velocities greater than that of light, an approach independently used by Jones,[4] Mariwalla,[5] and Parker.[6] The tachyon corridor is a preferred spatial direction traveling at a preferred velocity, and plays the same role for tachyons as does the time axis for bradyons. That is, the world line of a tachyon along the tachyon corridor is constantly increasing. This characteristic of the tachyon corridor trivially eliminates the possibility of using tachyons to form causal loops and shows that tachyons have primarily cosmological consequences. But the principal reason for postulating the tachyon corridor is to make the three-dimensional theory internally consistent.

The resulting theory is rotationally and Lorentz noninvariant, thus leading to the nonconservation of angular momentum and the violation of the principle of relativity. This must have a bearing on bradyon physics through real and virtual tachyon-bradyon interactions. The detailed evaluation of these effects will have to await the development of a field theory of tachyons, but initial estimates have been given in Ref. 1.

In the one-dimensional theory, the motion of tachyons is unidirectional along the space axis. As such, the only available means to consistently extend the theory to three dimensions was to postulate a preferred direction in space, the tachyon corridor, along which the motion of tachyons is unidirectional. Due to the coupling of Lorentz and rotational invariance, the tachyon corridor travels at a preferred velocity. The necessity of postulating the tachyon corridor is also indicated by the reciprocity[2] between tachyons and bradyons in the one-dimensional theory. Loosely speaking, this can be expressed as an interchange of space-time components on crossing the light barrier. In a one-dimensional space the process of interchange is rather well defined since both space and time have the same dimensionality. But in a three-dimensional space the process is meaningless unless there exists a preferred direction in space which can be interchanged with the time axis. This preferred direction which is called the tachyon corridor is the central postulate of the three-dimensional theory and is to tachyons what the time axis is to bradyons.

The resulting classical three-dimensional causal theory of tachyons, bradyons, and luxons presented in I borrows two fundamental aspects from the BDS (Bilaniuk-Deshpande-Sudarshan) tachyon theory.[7] These are (i) the emission of tachyons in the tachyon state, and (ii) the relation of negative energy to backward motion in time which is at the basis of the reinterpretation principle. Two other attempts at constructing a three-dimensional theory of tachyons have been made, one by Recami and Mignani[8] and the other by Goldoni.[9] Both are based on extending the Lorentz transformations to superluminal inertial frames. In the present paper we will further develop the theory presented in I and derive the resulting general superluminal transformations between inertial frames. Consequently, the kinematics of tachyons, as well as luxons emitted by them, will be derived. Before entering into formal developments, we will present the main characteristics of the three-dimensional theory.

If an observer in an inertial frame measures the velocity of a luxon which is emitted by a tachyon, he will find a value that depends on the direction of travel of the luxon and which, in general, is different from $c = 3\times10^8$ m/sec. On the other hand, each inertial observer finds that tachyonic luxons (luxons emitted by tachyons) will have a velocity c only when they are traveling along one of two directed axes called pseudo-tachyon corridors. The angle Φ between these two pseudo-tachyon corridors determines the absolute state of motion of the observer, as shown in Fig. 1. Any inertial frame of reference for which $\Phi=\pi$ is referred to as a preferred inertial frame. In such a frame of reference the two pseudo-tachyon corridors are collinear, thus defining a unique preferred axis which is the tachyon corridor and along which the velocity of light is c in either direction. An inertial frame of reference having a subluminal or superluminal constant velocity relative to a preferred frame is itself preferred, provided the relative velocity is directed along the tachyon corridor. Thus every member K of the set of preferred frames possesses a tachyon corridor and the velocity of any preferred frame relative to K is along this tachyon corridor. Furthermore, if K and K' are both preferred frames then each one of them finds the origin of the other frame moving along the positive direction of the tachyon corridor. But if K, for example, finds K' moving forward along the time axis, then K' necessarily finds that K is moving backward along the time axis. Under the extended Lorentz transformations between preferred frames the tachyon corridor is covariant. Furthermore, the set of all such transformations forms a group. As expected, it is not possible to separate the existence of a preferred spatial direction from that of a preferred velocity. Thus the tachyon corridor can only exist by traveling at a velocity whose component perpendicular to the tachyon corridor is preferred. The parallel component is arbitrary.

We feel that the one-dimensional causal theory of tachyons[2] is the natural and, due to the severe self-consistency conditions, possibly the only way of extending the one-dimensional Lorentz transformations to relatively superluminal inertial frames. Furthermore, given the one-dimensional theory, the structure of the three-dimensional theory is almost completely determined. To give an example in this regard we note that if the one-dimensional superluminal transformations are taken to be valid along any direction in space, contradictions arise right away in the theory.[1] As a matter of fact it turns out that all we can postulate is the validity of these transformations along only one axis in space, the tachyon corridor, and all the rest follows by necessity. Since we have thus introduced a preferred axis in space, the introduction of a preferred velocity is obligatory.[10]

The derivation of the one-dimensional superluminal transformations along a given axis in space requires the constancy of luxon velocities along both directions of that axis. The tachyon corridor

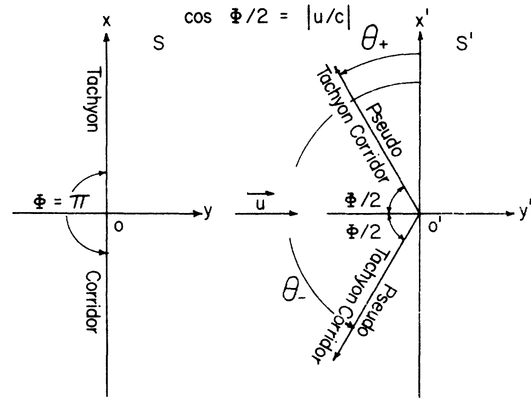

FIG. 1. The angle Φ between the pseudo-tachyon corridors of an inertial frame gives a measure of the absolute velocity of this frame along a direction perpendicular to the tachyon corridor. For a preferred frame $\Phi=\pi$, and the two antiparallel pseudo-tachyon corridors form a preferred axis in space which is the tachyon corridor.

is the only space axis having the desired property. Since, in addition, the tachyon corridor only exists in preferred inertial frames, then the maximum range of validity of the one-dimensional superluminal transformations is the set of all preferred frames. In addition to the constancy of the speed of light along the direction of relative motion, the derivation of the one-dimensional superluminal transformations requires the validity of the principle of relativity. The principle of relativity can be consistently postulated to be valid for the set of all preferred frames. Furthermore, this is the largest set of inertial frames for which it can be consistently postulated. In addition to permitting the derivation of the one-dimensional superluminal transformations along the tachyon corridor, the principle of relativity determines the transformation properties of the two spatial directions of a preferred frame which are perpendicular to the tachyon corridor.[11] Thus if the x axes of two parallel preferred frames K and K' are taken to be along the tachyon corridor, then the transformation properties of the y and z axes are given by $y=y'$ and $z=z'$.

When two transformations along the tachyon corridor are combined, the resulting transformation is also along the tachyon corridor and will be a superluminal transformation if one of the component transformations is superluminal and the other subluminal; otherwise the resulting transformation is subluminal. Thus the set of all special subluminal and superluminal transformations along the tachyon corridor forms a group. On the other hand, when a superluminal transformation along the tachyon corridor is combined with a general Lorentz transformation along a general axis in space, the resulting transformation has a superluminal velocity but is not in general along the tachyon corridor. As such the resulting transformation is neither a Lorentz transformation nor a superluminal transformation between two preferred systems; rather, it belongs to the set of superluminal transformations between a preferred and a nonpreferred inertial frame. This resulting transformation can be combined with yet another general Lorentz transformation to obtain the most general superluminal transformation between two inertial frames. The set of these general superluminal transformations, combined with the group of general Lorentz transformations, forms the group of the extended three-dimensional Lorentz transformations. The construction of a general superluminal transformation is schematically shown in Fig. 2.

In the one-dimensional theory, tachyons could only move forward along the space axis. In the three-dimensional theory, on the other hand, their motion is restricted to a three-dimensional cone which we call the tachyon cone. In a preferred frame the cone axis is the tachyon corridor and the cone half-angle is $\pi/4$. For nonpreferred observers the cone half-angle is greater than $\pi/4$ and approaches $\pi/2$ as the observer's velocity perpendicular to the tachyon corridor approaches the value c. For preferred frames tachyons fill the whole cone while for nonpreferred frames they do not. The tachyon cone is shown in Fig. 3. Finally we point out that the component perpendicular to the tachyon corridor of the relative velocity of two inertial frames can be superluminal only if the component parallel to the tachyon corridor is also superluminal. Superluminal velocities can only be obtained by combining a superluminal velocity along the tachyon corridor with a subluminal velocity perpendicular to it.[1] That is, the superluminal content of any velocity is due to its superluminal component along the tachyon corridor.

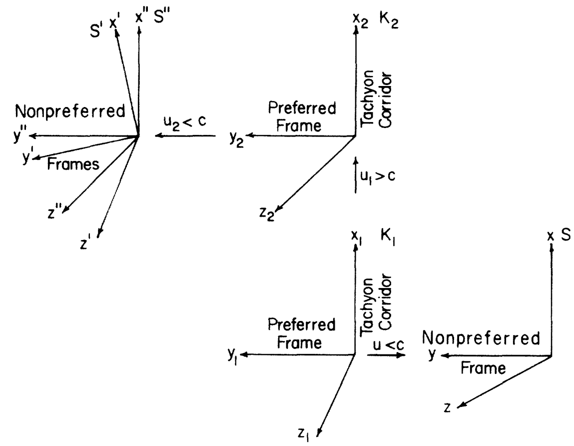

FIG. 2. The most general superluminal transformation in between two inertial frames constructed by four successive transformations: two Lorentz transformations perpendicular to the tachyon corridor, a superluminal transformation along the tachyon corridor, and a space rotation to make the initial and final coordinate systems parallel.

II. TRANSFORMATIONS BETWEEN PREFERRED FRAMES

The transformation equations between preferred inertial frames, oriented with their x axes along the tachyon corridor, have been given in I. In this section we formally state the postulates of the three-dimensional theory that lead to these transformations, and then derive their kinematical consequences for tachyons and luxons.

A. Postulates and derivation

In the following, subluminal and superluminal transformations will be treated simultaneously.

1. The postulates

(i) Preferred frames and the tachyon corridor. There exists a set of preferred inertial frames having their relative velocities along a preferred axis in space, the tachyon corridor.

(ii) The set of inertial frames. (a) Any frame of reference having a subluminal or superluminal constant velocity $\vec{v}$ relative to a preferred inertial

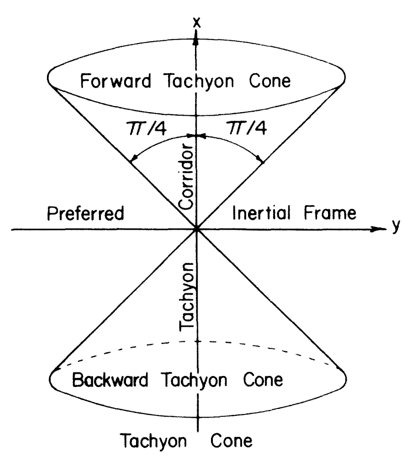

FIG. 3. In a preferred inertial frame the world lines of tachyons are bounded by the tachyon cone. The cone half-angle is $\pi/4$ and its axis is the tachyon corridor. Only luxon world lines can lie on the cone surface.

frame is itself a preferred inertial frame provided that $\vec{v}$ is directed along the tachyon corridor and $|\vec{v}| \neq c$. (b) Any frame of reference having a subluminal velocity relative to a preferred inertial frame is itself an inertial frame. (c) No two inertial frames can have a relative velocity of magnitude c (law of luxon velocities).

(iii) Correspondence with special relativity. In the absence of tachyons and for any subset of inertial frames with subluminal relative velocities, the principle of relativity is valid, the speed of light in vacuum is constant, and space is isotropic.

(iv) Correspondence with the one-dimensional theory. (a) The principle of relativity is valid for the set of preferred frames. (b) A luxon traveling along the tachyon corridor will have a velocity of magnitude c relative to all inertial observers.

(v) Consistency requirements. (a) Among the magnitudes of the relative velocities of three inertial frames, two or none are greater than c (law of tachyon velocities). (b) The time axis is unidirectional with respect to bradyons, but isotropic with respect to tachyons. The tachyon corridor is unidirectional with respect to tachyons but isotropic with respect to bradyons. (c) The two-dimensional space which is orthogonal to the tachyon corridor is isotropic as seen from a preferred inertial frame.

(vi) Spacetime is homogeneous and real in all inertial frames of reference.

The tachyon corridor and the set of preferred frames are intimately connected. On the one hand, the tachyon corridor is a well-defined axis in space only with respect to a preferred frame, and on the other hand, two inertial frames can both be preferred only if their relative velocity is along the tachyon corridor. In a nonpreferred inertial frame, the tachyon corridor splits into two noncollinear directed space axes called pseudo-tachyon corridors. While a luxon traveling along either direction of a tachyon corridor has speed c relative to all observers, the same is true along only one direction of a pseudo-tachyon corridor.

Postulate (i) introduces the intimately connected concepts of the tachyon corridor and the preferred inertial frames. Postulate (ii) introduces the extended set of inertial frames, including the subset of preferred inertial frames. Postulate (iic) is logically necessary as explained in Ref. 2. Postulate (iii) groups the postulates of special relativity in the absence of tachyons and superluminal inertial frames, thus satisfying the principle of correspondence. Postulates (iv), (v), and (vi) permit the derivation of the superluminal transformations in between preferred frames. Postulates (iic), (va), and (vb) are generalizations of analogous postulates in the one-dimensional theory and are imperative for the consistency of the three-dimensional theory. The above set of postulates is internally consistent, but not necessarily minimal. Due to the law of tachyon velocities, the set of inertial frames, as defined by postulate (ii) above, splits into two disjoint subsets. Thus if K and K' are relatively superluminal then every inertial frame is either subluminal relative to K or subluminal relative to K'.

2. The derivation

Consider two parallel preferred inertial frames K and K', with their origins coinciding at time $t=t'=0$. The velocity v of K' relative to K is along the tachyon corridor, with $|v| \neq c$. Let the common $x(x')$ axis be along the tachyon corridor. According to postulate (iva) the principle of relativity is valid for transformations between preferred inertial frames, and thus we have the transformation equations[11] $y=y'$ and $z=z'$. With the relative velocity and the $x(x')$ axes both along the tachyon corridor, postulates (iv) through (vi) satisfy the conditions for the derivation of the transformation properties of x and t as carried out in Ref. 2. Thus the transformation equations between K and K' become[1]

$$x = \mu\gamma(x' + \beta c t'), \tag{1a}$$

$$ct = \mu\gamma(ct' + \beta x'), \tag{1b}$$

$$y = y', \tag{1c}$$

$$z = z', \tag{1d}$$

where

$$\beta = v/c, \tag{2a}$$

$$\gamma = \frac{1}{(|1-\beta^2|)^{1/2}}, \tag{2b}$$

and

$$\mu = \begin{cases} 1 \text{ for subluminal transformations}, \\ \beta/|\beta| \text{ for superluminal transformations}. \end{cases} \tag{2c}$$

The matrix of the transformation which we denote by $Q(\beta)$ is given in the basis (x, y, z, ct) by

$$Q(\beta) = \begin{bmatrix} \mu\gamma & 0 & 0 & \mu\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mu\gamma\beta & 0 & 0 & \mu\gamma \end{bmatrix}. \tag{3}$$

The matrix $Q(\beta)$ is symmetric and obeys the relations

$$Q^T(\beta) = Q(\beta)\,, \tag{4a}$$

$$Q^{-1}(\beta) = Q(-\beta)\,, \tag{4b}$$

and

$$\mathrm{Det}Q(\beta) = \epsilon\,, \tag{5}$$

where

$$\epsilon = \begin{cases} +1 \text{ for subluminal transformations}\,, \\ -1 \text{ for superluminal transformations}\,. \end{cases} \tag{6}$$

If we consider a third preferred frame K'' having dimensionless velocities[12] β_1 and β_2 along the tachyon corridor relative to K' and K, respectively, then from Eqs. (1a) and (1b) we find

$$\beta_2 = \frac{\beta_1 + \beta}{1 + \beta_1 \beta}\,, \tag{7}$$

which leads to

$$(1 - \beta_2{}^2) = \frac{(1 - \beta_1{}^2)(1 - \beta^2)}{(1 + \beta_1 \beta)^2}\,, \tag{8}$$

and shows that two or none of the three velocities β, β_1, and β_2 are superluminal. This in turn means that two or none of the three transformations $Q(\beta)$, $Q(\beta_1)$, and $Q(\beta_2)$ are superluminal transformations.

Let $\vec{w}_\parallel$ and $\vec{w}'_\parallel$ be the components parallel to the tachyon corridor of the velocity of a particle as measured in K and K', respectively, and $\vec{w}_\perp, \vec{w}'_\perp$ be the velocity components perpendicular to the tachyon corridor. Then from Eqs. (1a)–(1d) we have

$$\vec{w}_\parallel = \frac{\vec{w}'_\parallel + \vec{v}}{1 + \vec{w}' \cdot \vec{v}/c^2} \tag{9a}$$

and

$$\vec{w}_\perp = \mu \frac{\vec{w}'_\perp (|1 - v^2/c^2|)^{1/2}}{1 + \vec{w}' \cdot \vec{v}/c^2}\,. \tag{9b}$$

If θ' is the angle that the velocity $\vec{w}'$ in K' makes with the tachyon corridor, then

$$w^2 = \frac{w'^2[\cos^2\theta' + \epsilon(1 - v^2/c^2)\sin^2\theta'] + 2v|\vec{w}'|\cos\theta' + v^2}{[1 + (v|\vec{w}'|/c^2)\cos\theta']^2}\,. \tag{10}$$

Finally, the world interval transforms as

$$[(cdt)^2 - (dx)^2] = \epsilon[(cdt')^2 - (dx')^2]\,, \tag{11a}$$

$$(dy)^2 + (dz)^2 = (dy')^2 + (dz')^2\,. \tag{11b}$$

The interpretation of superluminal transformation equations requires some care. First we note that since superluminal motion along the tachyon corridor is unidirectional, the sign of the relative superluminal velocity of two coordinate systems determines the direction of their relative motion along the time axis. If K' has positive superluminal velocity v relative to K, then K' is moving along the positive direction of the time axis of K. According to Eq. (4b) K will have a velocity $-v$ relative to K' and will thus be moving along the negative time axis of K'. From Eq. (1b) we find that for a particle with a parallel velocity component $w'_\parallel = \Delta x'/\Delta t'$ as measured in K',

$$\frac{\Delta t}{\Delta t'} = \frac{v}{|v|} \frac{(1 + vw'_\parallel/c^2)}{(|1 - v^2/c^2|)^{1/2}} \text{ for } |v| > c\,. \tag{12a}$$

From Eqs. (9a) and (12a) we have for $|w'_\parallel| < c$ (the particle is a bradyon in K') and $|v| > c$

$$\mathrm{sgn}\left(\frac{\Delta t}{\Delta t'}\right) = \mathrm{sgn}(w_\parallel)\,. \tag{12b}$$

Equation (12b) shows that for $(\Delta t') > 0$ (positive-energy bradyon in K') $w_\parallel$ is negative if and only if the tachyon is moving backward in time in K.

B. Kinematic consequences

In this section some of the kinematical consequences of the superluminal transformations will be considered. These will include the tachyon cone, luxon velocities, tachyon velocities, and the measurements of time and length. We will introduce a velocity parameter in preferred inertial frames which we call the "reciprocity" and define it by

$$\vec{\xi} = \left(\frac{dt}{dx}, \frac{dy}{dx}, \frac{dz}{dx}\right), \tag{13a}$$

where it is to be understood that the x axis is along the tachyon corridor. If a tachyon has dimensionless velocity $\vec{\beta}$ then its reciprocity can be written as

$$\vec{\xi} = (1/\beta_x\,, \beta_y/\beta_x\,, \beta_z/\beta_x)\,, \tag{13b}$$

which leads to

$$\xi^2 = \frac{1 + \beta_\perp{}^2}{\beta_\parallel{}^2}\,, \tag{14a}$$

where

$$\beta_\parallel{}^2 = \beta_x{}^2\,,\ \ \beta_\perp{}^2 = \beta_y{}^2 + \beta_z{}^2\,. \tag{14b}$$

For a superluminal transformation between preferred inertial frames, Eqs. (11a), (11b), and (13b) lead to

$$(cdt)^2[1 - \beta^2] = (dx')^2[1 - (\xi')^2]\,, \tag{15}$$

which relates the dimensionless velocity in K to the reciprocity in K' when K and K' are relatively superluminal.

1. The tachyon cone

Another important property of tachyon velocities is that $|\vec{w}_\parallel| > |\vec{w}_\perp|$. That is, superluminality is due to superluminal motion along the tachyon corridor. This can easily be proved from Eqs. (9a) and (9b); with $|v| > c$ we have[1]

$$\frac{w_\parallel^2 - w_\perp^2}{c^2} = \frac{(1 - w'^2/c^2)(v^2/c^2 - 1) + (1 - \vec{v}\cdot\vec{w}'/c^2)^2}{(1 + \vec{v}\cdot\vec{w}'/c^2)^2}. \tag{16}$$

Since $|\vec{w}'| < c$ (the particle is a bradyon in K') and $|\vec{v}| > c$ then the right-hand side of Eq. (16) is positive and we find

$$w_\parallel^2 > w_\perp^2. \tag{17}$$

Thus in a preferred inertial frame, the world line of a tachyon, in addition to being unidirectional along the tachyon corridor, is restricted to a cone whose half-angle is $\pi/4$ and whose axis is the tachyon corridor. Actually, as will be shown later on, only luxon velocities do fill the whole cone asymptotically, while the region of tachyon velocities is somewhat smaller. This can be seen from Eq. (16) by noting that the right-hand side is never smaller than unity. The above cone we refer to as the "tachyon cone."

Finally we would like to point out that a negative-energy bradyon moving backward in time in K' will be a tachyon moving backward along the tachyon corridor in K. This is easily seen from Eq. (1a) which, for $|v| > c$, leads to

$$\left(\frac{\Delta x}{\Delta t'}\right) = \frac{|v|}{(|1 - v^2/c^2|)^{1/2}}\left(1 + \frac{w'_\parallel}{v}\right). \tag{18}$$

For a bradyon in K', $|w'_\parallel| < c$. Thus since $|v| > c$ then $(1 + w'_\parallel/v) > 0$. Hence Δx has the same sign as $\Delta t'$.

2. Luxon velocities

Consider a luxon (zero-mass particle) having velocities $\vec{w}$ and $\vec{w}'$ relative to the preferred inertial frames K and K', respectively. K' has a superluminal velocity v (necessarily along the tachyon corridor) relative to K. The direction of travel of the luxon makes angles θ and θ' with the tachyon corridors in K and K', respectively. Furthermore, we will assume that the luxon is a bradyonic luxon in K', which implies that it is a tachyonic luxon in K and that $|\vec{w}'| = c$. Due to the isotropy of the plane perpendicular to the tachyon corridor in a preferred inertial frame, we can, without any loss of generality, perform a rotation of the coordinate system about the tachyon corridor so that one of the axes perpendicular to the tachyon corridor lies in the plane determined by the tachyon corridor and the direction of motion of the luxon. In spherical coordinates this amounts to restricting the polar angles ϕ and ϕ' to the values 0 or π. We label the velocity components parallel and perpendicular to the tachyon corridor by $w_\parallel, w'_\parallel$ and $w_\perp, w'_\perp$, respectively. We thus have

$$w'_\parallel = c\cos\theta', \quad w'_\perp = c\sin\theta'\cos\phi', \tag{19a}$$

$$w_\perp = w\cos\theta, \quad w_\perp = w\sin\theta\cos\phi, \tag{19b}$$

where ϕ' and ϕ can only take the values 0 or π. Substituting Eq. (19a) in Eqs. (9a) and (9b) we have

$$w_\parallel = \frac{c\cos\theta' + v}{1 + (v/c)\cos\theta'}, \tag{20a}$$

$$w_\perp = \frac{v}{|v|}\frac{c\sin\theta'(v^2/c^2 - 1)^{1/2}}{[1 + (v/c)\cos\theta']}\cos\phi'. \tag{20b}$$

From Eq. (12b) it is seen that $w_\parallel$ is negative only when the luxon is moving backward in time in K. Furthermore, the relative sign of $w_\parallel$ and $w_\perp$ is equal to the sign of $\cos\phi'$. From Eqs. (19b), remembering that $\cos^2\theta = 1$, we get

$$1 - \tan^2\theta = \frac{w_\parallel^2 - w_\perp^2}{w_\parallel^2}, \tag{21}$$

and with $w'^2 = c^2$, Eq. (16) reduces to

$$w_\parallel^2 - w_\perp^2 = c^2; \tag{22}$$

thus

$$1 - \tan^2\theta = \frac{c^2}{w_\parallel^2}, \tag{23}$$

which implies that $\tan^2\theta \leq 1$, the equality sign holding in the limit $w \to \infty$. Thus combining this condition with the result on the sign of $w_\parallel$ we have the following limits on θ':

$0 < \theta < \pi/4$ if luxon is moving forward in time in K';

$3\pi/4 < \theta < \pi$ if luxon is moving backward in time in K'.

(24)

Using Eqs. (19b) we can rewrite Eq. (22), which is the equation of a hyperbola in the $w_\parallel w_\perp$ plane, as

$$\frac{w^2}{c^2} = \frac{1}{\cos^2\theta}. \tag{25}$$

Thus the magnitude of the velocity of a tachyonic luxon in a preferred inertial frame depends on the one parameter θ and hence can be written as $w(\theta)$. We note that

$$w^2(\theta) = w^2(\pi - \theta), \tag{26}$$

which shows that in a preferred inertial frame a luxon has the same speed along both directions of an axis. It should be noted that for the possible range of values of θ as given by Eq. (24) $\cos^2\theta$ is positive, as it should be, in Eq. (25). For $\theta = \pi/4$,

$w^2 \to \infty$ and $w_\perp{}^2 \to w_\parallel{}^2$, as can be seen from Eqs. (25), (21), and (22). Thus in the limit $w^2 \to \infty$ the world line of the luxon is on the surface of the tachyon cone. The other extreme is $w^2 = c^2(\theta = 0, w_\perp = 0)$ where the tachyon is moving along the tachyon corridor. Combining Eqs. (22) and (23) we find

$$\frac{w_\parallel{}^2}{c^2} = \frac{1}{1-\tan^2\theta}, \quad \frac{w_\perp{}^2}{c^2} = \frac{1}{(\cot^2\theta - 1)}. \tag{27}$$

From Eqs. (20a), (20b), and (25) it is seen that the luxon will be moving along the tachyon corridor in K and with speed c if and only if it is moving along the tachyon corridor in K', that is, for $\theta' = 0$ or π. Furthermore, since the luxon has speed c relative to both K and K', it will have speed c relative to every inertial frame that is subluminal relative to K or subluminal relative to K'. But since the relative velocity of K and K' is superluminal, then due to the law of tachyon velocities every inertial frame is either subluminal relative to K or subluminal relative to K'. Hence a luxon traveling along either direction of the tachyon corridor in a preferred inertial frame will be traveling along the tachyon corridor in every other preferred frame and will have speed c relative to all inertial observers. This result is consistent with postulate (v).

3. *Tachyon velocities*

The introduction of tachyons as bradyons in a superluminal inertial frame imposes restrictions on the possibilities of faster-than-light motion, and guarantees the validity of the law of tachyon velocities, thus rendering the three-dimensional theory consistent. For the case of transformations in between inertial frames we will present the results mainly in the form of three theorems.

To set up the problem, consider the three preferred frames K_1, K_2, and K_3. Let K_3 have a subluminal velocity u relative to K_1, and let K_2 have a superluminal velocity v relative to K_1. Both the velocities u and v are along the tachyon corridor.

Theorem (i). If β and ξ are the dimensionless velocity and reciprocity of a particle respectively, then the conditions $\beta^2 < 1$ and $\xi^2 < 1$ are mutually exclusive.

Proof. From Eqs. (14a) and (14b) we find that

$$1 - \xi^2 > 0 \Longleftrightarrow \beta_\perp{}^2 < \beta_\parallel{}^2 - 1; \tag{28}$$

on the other hand, since $\beta^2 = \beta_\parallel{}^2 + \beta_\perp{}^2$ then

$$1 - \beta^2 > 0 \Longleftrightarrow \beta_\perp{}^2 < 1 - \beta_\parallel{}^2. \tag{29}$$

Thus the conditions $\beta^2 < 1$ and $\xi^2 < 1$ taken simultaneously would imply that $\beta_\perp{}^2 < -|1 - \beta_\parallel{}^2|$, which is never satisfied for real $\beta_\perp$. It is important to note that the theorem does not exclude the possibility of β^2 and ξ^2 as being both greater than unity. It is actually easy to see that this will be the case for $\beta_\perp{}^2 > |1 - \beta_\parallel{}^2|$.

Theorem (ii). Under a superluminal transformation in between two preferred frames K_1 and K_2 the following is true:

(a) A bradyon in K_2 will be a tachyon satisfying the condition $\xi^2 < 1$ in K_1.

(b) A tachyon in K_2 will be a bradyon in K_1 if and only if it satisfies the condition $\xi^2 < 1$ in K_2.

Proof. (a) Consider a particle having a subluminal dimensionless velocity $\vec{\beta}'$ relative to K_2. The particle's dimensionless velocity and reciprocity relative to K_1 are $\vec{\beta}$ and $\vec{\xi}$, respectively. Using Eqs. (1a)–(1d), (9a), and (9b) we find

$$\left(\frac{v}{c} + \beta'_\parallel\right)^2 (1 - \xi^2) = [1 - (\beta')^2]\left[\left(\frac{v}{c}\right)^2 - 1\right]. \tag{30}$$

Remembering that $|v/c| > 1$, we find that for $|\beta'| < 1$ the right-hand side is positive. Hence $\xi^2 < 1$. Furthermore, due to theorem (i), this implies that $\beta^2 > 1$. That is, in K_1, the particle is a tachyon obeying the condition $\xi^2 < 1$. (b) Consider a particle having a superluminal dimensionless velocity $\vec{\beta}'$ and reciprocity $\vec{\xi}'$ relative to K_2. Its dimensionless velocity relative to K_1 is $\vec{\beta}$. On combining Eqs. (14a) and (14b) for ξ' with Eqs. (9a) and (9b) we find

$$\left(1 + \beta'_\parallel \frac{v}{c}\right)^2 (1 - \beta^2) = (\beta'_\parallel)^2[1 - (\xi')^2]\left[\left(\frac{v}{c}\right)^2 - 1\right]. \tag{31}$$

From Eq. (31) it is seen that, for $|v/c| > 1$, $\beta^2 < 1$ if and only if $(\xi')^2 < 1$.

Since tachyons in the present theory are introduced as bradyons in superluminal inertial frames, then theorem (iia) shows that, in a preferred frame, the possible velocities of tachyons are restricted by the condition

$$\xi^2 < 1. \tag{32}$$

With this condition, theorem (iib) then proves the validity of the law of tachyon velocities for the case of superluminal transformations in between preferred frames. Theorem (ii) is also indicated by the transformation properties of the world interval as given by Eq. (15).

Theorem (iii). The condition $\xi^2 < 1$ is invariant under subluminal transformations in between preferred frames of reference.

Proof. Consider a particle having a superluminal dimensionless velocity $\vec{\beta}'$ and reciprocity $\vec{\xi}'$ relative to K_3, while its dimensionless velocity and reciprocity relative to K_1 are $\vec{\beta}$ and $\vec{\xi}$, respectively. Remembering that the velocity u of K_3 relative to K_1 is subluminal, and combining Eqs. (9a) and (9b) with Eq. (14a) we find

$$\left(\beta'_{\parallel}+\frac{u}{c}\right)^2(1-\xi^2)=(\beta'_{\parallel})^2\left[1-\left(\frac{u}{c}\right)^2\right][1-(\xi')^2]. \tag{33}$$

For $|u/c|<1$, Eq. (33) shows that $(\xi')^2<1$ implies, and is implied by, $(\xi)^2<1$.

Theorems (i), (ii), and (iii), when combined with the fact that the addition of two subluminal velocities produces a subluminal velocity, complete the proof of the law of tachyon velocities for transformations in between preferred inertial frames. Furthermore, theorem (iii) shows that condition (32) on the velocity of tachyons in a preferred inertial frame is consistent with the principle of relativity. This is important for the consistency of the theory since the principle of relativity is postulated to hold for transformations in between preferred inertial frames.

Our results can be summarized by stating that, in preferred inertial frames, tachyon reciprocities are less than unity in absolute value. This Lorentz-invariant condition is a consequence of the introduction of tachyons as bradyons in superluminal reference frames and in turn guarantees the validity of the law of tachyon velocities. The allowed tachyon, bradyon, and luxon velocities are shown in Fig. 4. In a $\beta_{\parallel}$-$\beta_{\perp}$ plane, bradyon velocities lie inside the circle

$$\beta_{\parallel}{}^2+\beta_{\perp}{}^2=1, \tag{34a}$$

while tachyon velocities are bounded by the two branches of the hyperbola

$$\beta_{\parallel}{}^2-\beta_{\perp}{}^2=1. \tag{34b}$$

The asymptotes pass through the origin and make an angle of $\pi/4$ with the $\beta_{\parallel}$ axis. The branch along $\beta_{\parallel}>0$ is for forward motion in time, and that along $\beta_{\parallel}<0$ is for backward motion in time.

4. Length and time

Consider two preferred inertial frames K_0 and K having their x axis and relative velocity along the tachyon corridor. The velocity of K_0 relative to K is v. Then from Eqs. (1a)-(1d) we have

$$\tau=\frac{\mu\tau_0}{(|1-v^2/c^2|)^{1/2}}, \tag{35}$$

$$l_{\parallel}=\epsilon\mu l^0_{\parallel}(|1-v^2/c^2|)^{1/2}, \tag{36a}$$

$$l_{\perp}=l^0_{\perp}, \tag{36b}$$

where τ_0 is a time interval as measured by a K_0 observer on a clock at rest in K_0, and τ is the same time interval as measured by an observer in K. $l^0_{\parallel}$ and $l^0_{\perp}$ are, respectively, the components parallel and perpendicular to the tachyon corridor, of a rod, as measured in its rest system K_0. $l_{\parallel}$ and $l_{\perp}$ are its components as measured in K. μ and ϵ are defined by Eqs. (2c) and (6), respectively. Let θ_0 be the angle that the rod makes with the tachyon corridor in K_0, and θ the angle that the rod makes with the tachyon corridor in K; then from Eqs. (36a) and (36b) we find

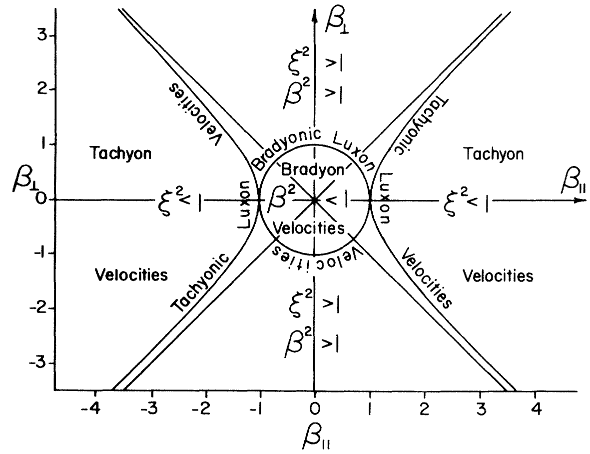

FIG. 4. In a preferred inertial frame tachyon velocities obey the condition $\xi^2<1$ and are thus bounded by the branches of the hyperbola $\beta_{\parallel}{}^2-\beta_{\perp}{}^2=1$. Tachyonic luxon velocities are given by $\xi^2=1$ and thus lie on the two branches of the hyperbola. Bradyonic luxon velocities lie on the circle $\beta^2=1$, and bradyon velocities lie inside this circle. No particle can have a velocity in the region $\beta^2>1$, $\xi^2>1$.

$$\tan\theta=\frac{\epsilon\mu\tan\theta}{(|1-v^2/c^2|)^{1/2}}. \tag{37}$$

For $|v/c|>1$, $\mu=v/|v|$ and Eq. (35) shows that the observer in K will find the clocks of K either moving forward in time, if $v/c>1$, or moving backward in time, if $v/c<-1$. This is perfectly consistent with the general approach of the theory according to which a negative superluminal relative velocity along the tachyon corridor implies backward relative motion in time. For $v/c>1$, $\epsilon=-1$ and Eq. (36a) shows that $l_{\parallel}$ is negative. This, of course, does not mean that the rod has negative length but rather that the spatial ordering of its two ends in K and K_0 is different. The situation can be further clarified by studying the variation of θ with v according to Eq. (37). The variation is shown in Fig. 5.

III. TRANSFORMATIONS BETWEEN PREFERRED AND NONPREFERRED FRAMES

The usual methods of generalizing the special Lorentz transformations make use of the isotropy of three-dimensional space and the principle of relativity. Since the presence of tachyons introduces a preferred axis and preferred inertial frames the methods of special relativity cannot, without modification, be used to generalize the superluminal transformations between preferred frames to all inertial frames.

A. Transformations perpendicular to the tachyon corridor

Having studied the properties of transformations along the tachyon corridor, we will now study the transformations perpendicular to the tachyon corridor, between a preferred inertial frame K and a nonpreferred inertial frame S'. As has already been shown, the introduction of tachyons as bradyons in a superluminal preferred inertial frame implies that the superluminal content of any velocity is due to its superluminal content along the tachyon corridor. Since the velocity of S' relative to K has no component along the tachyon corridor, then it is necessarily subluminal, and hence K and S' are related by an ordinary Lorentz transformation. The three main points that we will consider here are (i) the transformation properties of the tachyon corridor, (ii) the determination of the absolute value of the velocity of S', and (iii) the possible range of tachyon velocities in S'. To set up the problem we consider the two parallel inertial frames K and S'. K is preferred and has its x axis along the tachyon corridor, while S' is nonpreferred and has a velocity $\vec{u}$ along the y axis[13] relative to K.

1. The pseudo-tachyon corridor

Consider two luxons moving in opposite directions along the tachyon corridor in K with velocities $+c$ and $-c$. As measured by S' these luxons will have velocities $\vec{c}\,'_+$ and $\vec{c}\,'_-$, respectively, and will be traveling along directions making angles θ_+ and θ_- with the x axis. The velocity transformation equations give

$$(\vec{c}\,'_\pm)_z = 0\,, \quad (\vec{c}\,'_\pm)_y = -u_y\,, \quad (\vec{c}\,'_\pm)_x = \pm c(1-u^2/c^2)^{1/2}\,. \tag{38}$$

Thus $\theta_\pm$ is given by

$$\sin\theta_\pm = |u/c|\,, \quad \cos\theta_\pm = \pm(1-u^2/c^2)^{1/2}\,. \tag{39a}$$

Hence

$$\theta_- = \pi - \theta_+\,. \tag{39b}$$

The two directed axes along the direction of travel of these two luxons in S' are the two pseudo-tachyon corridors of S'. The angle between the two pseudo-tachyon corridors is given by

$$\Phi = \theta_- - \theta_+ = \pi - 2\theta_+\,, \tag{40a}$$

with

$$\cos(\Phi/2) = |u/c| \tag{40b}$$

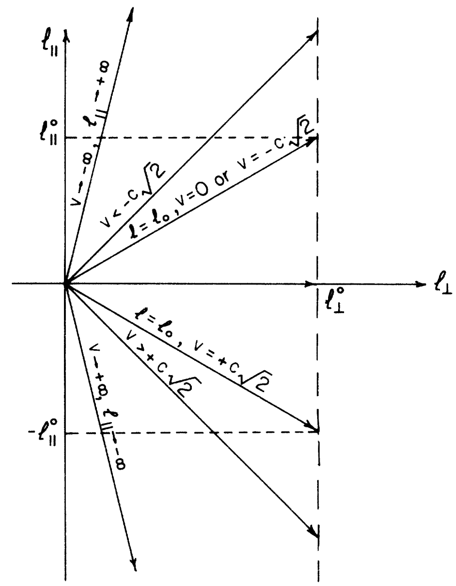

FIG. 5. Variation of the length and orientation of a rigid rod as a function of its velocity as seen by a preferred observer. The rest system of the rod is also preferred. In its rest system the rod has length l_0 and projections parallel and perpendicular to the tachyon corridor of $l_\parallel^0$ and $l_\perp^0$, respectively.

for a preferred inertial frame $u=0$ and $\Phi=\pi$; thus the two pseudo-tachyon corridors are collinear and form the tachyon corridor.

As has already been shown, a luxon will have a speed c relative to all inertial observers if and only if it is measured by a preferred observer to be traveling along the tachyon corridor. Due to the above discussion this result can be generalized to read: A luxon will have a speed c relative to all inertial observers if and only if it is measured by an inertial observer to be traveling along the positive direction of one of two directed axes, in the observer's frame, which are the pseudo-tachyon corridors. For further reference we note that the two pseudo-tachyon corridors and the velocity vector $\vec{u}$ are directed in such a way as to make obtuse angles when they are taken two by two, and lie in a plane that is parallel to the tachyon corridor. Furthermore, $\vec{u}$ bisects the angle between the two pseudo-tachyon corridors.

2. The absolute velocity

Since the principle of relativity is valid for transformations between preferred frames, then velocities along the tachyon corridor are relative. Hence, only velocities perpendicular to the tachyon corridor are absolute and thus inherently measurable. The most convenient choice is to take the velocity of the tachyon corridor as zero. Then the absolute velocity of an inertial frame becomes its velocity perpendicular to the tachyon corridor. That is, the absolute velocity of S' is $\vec{u}$. The previous discussion on pseudo-tachyon corridors provides a procedure for internally determining $\vec{u}$. To do so, an observer in K' will first have to experimentally determine the directions along which tachyonic luxons have a velocity c. That is, he has to determine the two pseudo-tachyon corridors of his frame. Then the angle Φ between these two tachyon corridors will give his absolute speed $|\vec{u}|$ through Eq. (40b). The direction of $\vec{u}$ is determined by the fact that $\vec{u}$ lies in the plane of the pseudo-tachyon corridors, bisects the angle between them, and is directed in a way to make obtuse angles with both of them.

The absolute velocity and pseudo-tachyon corridors make nonpreferred inertial frames more anisotropic than preferred ones, for while in a preferred frame the two-dimensional space perpendicular to the tachyon corridor is isotropic, in a nonpreferred frame, the only space symmetry which is left other than homogeneity of space-time is reflection in the plane determined by the pseudo-tachyon corridors.

3. Permitted velocities in S'

In a preferred inertial frame the permitted tachyon velocities are given by the condition $\xi^2<1$, and tachyonic luxon velocities are given by $\xi^2=1$, where $\vec{\xi}$ is the reciprocity as defined by Eq. (13a). Then the permitted range of tachyon and tachyonic luxon velocities in S' is the transform from K to S' of the region $\xi^2<1$. Consider a particle having a superluminal velocity $\vec{w}$ in K and $\vec{w}\,'$ in S'. Remembering that the relative velocity $\vec{u}$ is along the y axis, we have the two equations

$$\left(1-\frac{w'^2}{c^2}\right)=\frac{(1-w^2/c^2)(1-u^2/c^2)}{(1-\vec{u}\cdot\vec{w}/c)^2} \tag{41}$$

and

$$w_x'=\left(\frac{dt'}{dt}\right)w_x\,. \tag{42}$$

Equation (41) is actually true for a general Lorentz transformation in between two inertial frames irrespective of the direction of the relative velocity $\vec{u}$, and shows that the law of tachyon velocities is valid for velocity transformations between relatively subluminal inertial frames. Since $u^2/c^2<1$, then it is seen from Eq. (41) that the particle is a tachyon in S' if and only if it is a tachyon in K. Equation (42), on the other hand, shows that no causal loops can be formed in S', since the unidirectionality of motion along the tachyon corridor in K implies unidirectionality along the x axis in S'. This can be seen as follows: In the preferred frame K it has already been shown that the component of the world line of a tachyon along the tachyon corridor is constantly increasing. This implies that $w_x dt>0$. Thus, due to Eq. (42), $w_x' dt'>0$, which in turn implies that the component of the world line of a tachyon along the x' axis is constantly increasing. It is important to note that if we consider a third inertial frame S'' having a velocity $\vec{v}$ along the x' axis relative to S', then $w_x'' dt''$ is not always positive, and thus the component of the world line of a tachyon along x'' is not constantly increasing. This does not, however, produce inconsistencies in the theory since the x'' axis of S'' is not equivalent to the x' axis of S'. This is related to the Thomas precession and the anisotropy of space and can be seen by considering another preferred frame K^0 relative to which the velocity of S'' is perpendicular to the tachyon corridor. Then S'' and K^0 are not parallel and thus x'' is not parallel to the tachyon corridor of K^0.

Using the velocity transformation equations and the condition $\xi^2<1$ we find that the allowed region of tachyon velocities in S' is given by

$$\left(\frac{w_x'^2}{c^2}-\frac{w_z'^2}{c^2}\right)\left(1-\frac{u^2}{c^2}\right)-\left[\left(\frac{w_y'}{c}+\frac{u_y}{c}\right)^2+\left(1+\frac{u_y w_y'}{c}\right)^2\right]\geqslant 0\,. \tag{43}$$

B. Transformations along an arbitrary axis

In this section we will derive the superluminal transformation equations between a nonpreferred inertial frame S' and a preferred inertial frame K_1 having its x axis along the tachyon corridor. S' is parallel to K_1 and has a velocity $\vec{w}$ relative to it.

1. The direct transformation from S' to K_1

The derivation of the direct transformation is carried out in two steps: (i) By the composition of successive transformations we evaluate the matrix T, which transforms S' to K_1, for the special case where $\vec{w}=(w_x,w_y,0)$ is in the xy plane. (ii) Having obtained the transformation matrix for the special case $\vec{w}=(w_x,w_y,0)$ we use the isotropy of the yz plane in the preferred frame K_1 to generalize T to the case where $\vec{w}$ has an arbitrary orientation. For this purpose, two additional inertial frames are introduced (see Fig. 2): K_2, which is a preferred frame parallel to K_1 and has a velocity $u_1=w_x$ along the tachyon corridor relative to K_1, and S'', which is a nonpreferred frame parallel to K_2 and has the same origin as S'. Thus, according to the velocity transformation equations (9a) and (9b) the velocity of S' relative to K_2 is along the common $y''(y_2)$ axes. We designate this velocity by u_2. Finally S' is obtained from S'' by a rotation of angle θ about the z'' axis. The angle θ is determined by the condition that S' be parallel to K_1.

Strict parallelism between two inertial frames is only possible when the relative velocity is along one of the coordinate axes, while in the general case the coordinate axes of one Cartesian system as measured by the other do not form a mutually perpendicular triplet. Thus in special relativity parallelism in the case of general transformations is defined by the condition that under equal rotations the two coordinate systems can be related by a special Lorentz transformation. In the case of superluminal transformations, the extended Lorentz transformations only hold along the tachyon corridor, and hence a preferred and a nonpreferred inertial frame cannot be related by an extended transformation [the matrix $Q(\beta)$] no matter what spatial orientation they may have. Another equivalent way of formulating the criterion of parallelism in special relativity is to require that $T^{-1}(\vec{w}) = T(-\vec{w})$, where $\vec{w}$ is the relative velocity between the initial and final coordinate systems, and $T(\vec{w})$ the corresponding transformation matrix. This is a consequence of the principle of relativity. In the case of superluminal transformations the initial and final frames of reference are not equivalent, in general, and hence this criterion of parallelism cannot be used. Still another equivalent way of stating the special relativistic criterion for parallelism is to require that the direction cosines of the relative velocity vector are the same (except for a difference in sign) in both inertial frames. This criterion of parallelism is applicable in the case of superluminal transformations, although it is not imperative as in special relativity. Nevertheless, it is indicated by the principle of correspondence.

The resulting transformation matrix is given by

$$T(\vec{w};\epsilon) = \begin{bmatrix} \epsilon\dfrac{w}{w'} + \dfrac{\mu\gamma-\epsilon}{ww'}w_x^2 & \dfrac{\mu\gamma-\epsilon}{ww'}w_x w_y & \dfrac{\mu\gamma-\epsilon}{ww'}w_x w_z & \dfrac{\mu\gamma}{c}w_x \\ \dfrac{\mu\gamma-1}{ww'}w_y w_x & \left(\dfrac{w}{w'}-1\right)\dfrac{w_y^2}{w_\perp^2} + \dfrac{\mu\gamma-1}{ww'}w_y^2 + 1 & \left(\dfrac{w}{w'}-1\right)\dfrac{w_y w_z}{w_\perp^2} + \dfrac{\mu\gamma-1}{ww'}w_y w_z & \dfrac{\mu\gamma}{c}w_y \\ \dfrac{\mu\gamma-1}{ww'}w_z w_x & \left(\dfrac{w}{w'}-1\right)\dfrac{w_z w_y}{w_\perp^2} + \dfrac{\mu\gamma-1}{ww'}w_z w_y & \left(\dfrac{w}{w'}-1\right)\dfrac{w_z^2}{w_\perp^2} + \dfrac{\mu\gamma-1}{ww'}w_z^2 + 1 & \dfrac{\mu\gamma}{c}w_z \\ \dfrac{\mu\gamma w'}{cw}w_x & \dfrac{\mu\gamma}{c}\dfrac{w'}{w}w_y & \dfrac{\mu\gamma}{c}\dfrac{w'}{w}w_z & \mu\gamma \end{bmatrix}, \qquad (44)$$

where as usual

$$\epsilon = \begin{cases} +1 \text{ for } \vec{w} \text{ subluminal}, \\ -1 \text{ for } \vec{w} \text{ superluminal}. \end{cases} \qquad (45a)$$

μ can be written as

$$\mu = \left(\frac{w_x}{|w_x|}\right)\left(\frac{1-\epsilon}{2}\right) + \left(\frac{1+\epsilon}{2}\right) \qquad (45b)$$

and

$$\gamma = \frac{1}{[\epsilon(1-w_x^2/c^2) - w_y^2/c^2 - w_z^2/c^2]^{1/2}}. \qquad (46)$$

Finally $\vec{w}'$ is the velocity of K_1 relative to S'. Since K_1 and S' are not equivalent inertial frames we cannot use the principle of relativity to deduce that $\vec{w}' = -\vec{w}$. It is actually not difficult to show that $\vec{w}'$ is given by

$$w'^2 = w_\parallel^2 + \epsilon w_\perp^2 \qquad (47)$$

and that $\vec{w}'$ is superluminal if and only if $\vec{w}$ is superluminal. We note that

$$\operatorname{Det} T(\vec{w};\epsilon) = \epsilon, \qquad (48)$$

and that γ can be written as

$$\gamma = \frac{\sqrt{\epsilon}}{(1-w'^2/c^2)^{1/2}}. \qquad (49)$$

The velocity transformation equations can be derived by using Eq. (44) and are given by

$$\vec{\beta}_\parallel = \frac{\epsilon(w/w')\vec{\beta}'_\parallel + \{\mu\gamma/c + [(\mu\gamma-\epsilon)/ww'](\vec{w}\cdot\vec{\beta}')\}\vec{w}_\parallel}{\mu\gamma[1+(w'/w)\vec{w}\cdot\vec{\beta}'/c]}, \qquad (50a)$$

$$\vec{\beta}_\perp = \frac{\vec{\beta}'_\perp + \{\mu\gamma/c + [(\mu\gamma-1)/ww'](\vec{w}\cdot\vec{\beta}') + (w/w'-1)(\vec{w}\cdot\vec{\beta}')w_\perp^2\}\vec{w}_\perp}{\mu\gamma[1+(w'/w)\vec{w}\cdot\vec{\beta}'/c]}, \qquad (50b)$$

where $\vec{\beta}=\vec{\beta}_{\parallel}+\vec{\beta}_{\perp}$ and $\vec{\beta}'=\vec{\beta}'_{\parallel}+\vec{\beta}'_{\perp}$ are the dimensionless velocities of a particle as measured in K_1 and S', respectively, and the notation $\parallel$ and $\perp$ refers to the x axis, in K_1, and to the x' axis in S' (rather than to the relative velocity vector $\vec{w}$).

For a subluminal velocity $\vec{w}$, $\mu=\epsilon=w'/w=1$ and Eqs. (44), (49), and (50) reduce to the familiar results of special relativity, and the matrix $T(\vec{w};\epsilon)$ reduces to the matrix $L(\vec{w})$ of the proper homogeneous Lorentz transformation between two inertial frames. For a superluminal velocity $\vec{w}$, $\epsilon=-1$, $\mu=w_x/|w_x|$, and

$$w'^2=w^2\cos 2\Theta\,, \tag{51}$$

where Θ is the angle that the velocity vector $\vec{w}$ makes with the tachyon corridor. In this case γ can be written as

$$\gamma=\frac{1}{[(w^2/c^2)\cos 2\Theta-1]^{1/2}}\,. \tag{52a}$$

Alternatively, if $\vec{\xi}$ is the reciprocity of S' relative to K, then $\mu\gamma$ can be written as

$$\mu\gamma=\frac{(c/w_x)}{(1-\xi^2)^{1/2}}\,. \tag{52b}$$

2. *The inverse transformation*

The transformation from the preferred inertial frame K_1 to the nonpreferred inertial frame S', where S' is parallel to K_1, and K_1 has its x axis along the tachyon corridor, is carried out by the matrix $T'(\vec{w}';\epsilon)=T^{-1}(\vec{w};\epsilon)$, where $\vec{w}'$ is the velocity of K_1 relative to S'. Inverting T we obtain

$$T^{-1}(\vec{w};\epsilon)=[\sigma T(-\vec{w};\epsilon)]^T\,, \tag{53a}$$

where

$$\sigma=\begin{bmatrix}\epsilon & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & \epsilon\end{bmatrix}\,. \tag{53b}$$

To obtain the matrix $T'(\vec{w}';\epsilon)$ the parameters of T^{-1} must be transformed to those of T'. The relations between the two sets of parameters can be derived starting with the condition of parallelism, which is

$$\frac{\vec{w}}{w}=-\frac{\vec{w}'}{w'}\,. \tag{54}$$

Combining Eqs. (54) and (47) we obtain

$$\frac{w^2}{w'^2}=\frac{w'^2}{w''^2}\,, \tag{55a}$$

where the velocity parameter w'' is defined by

$$w''^2=w'^2_{\parallel}+\epsilon w'^2_{\perp}\,. \tag{55b}$$

Equations (54), (55a), and (55b) then give

$$\vec{w}=\frac{w'}{w''}\vec{w}'\,, \tag{55c}$$

where w' and w'' are absolute values. Due to Eq. (54), for superluminal transformations $\mu'=w'_x/|w'_x|=-\mu$. Since $\vec{w}'$ is superluminal if and only if $\vec{w}$ is superluminal, ϵ has the same significance relative to $\vec{w}'$ as it has relative to $\vec{w}$. Furthermore, γ as given by Eq. (49) is already expressed in terms of $\vec{w}'$.

From Eqs. (48) and (53a) we find

$$\operatorname{Det}T'(\vec{w}';\epsilon)=\epsilon\,. \tag{56}$$

For $|w'|<c$, $-\mu'=\epsilon=w'/w''=1$ and the transformation matrix $T'(\vec{w}';\epsilon)$ reduces to the general homogeneous Lorentz transformation $L(\vec{w}')$. On the other hand, if $\vec{w}'$ is superluminal, then $\epsilon=-1$, $\mu'=w'_x/|w'_x|$, and

$$w''^2=w'^2\cos 2\Theta\,, \tag{57}$$

where Θ is the angle that the velocity vector $-\vec{w}'$ makes with the x' axis of S'. Due to the condition of parallelism, this is the same angle Θ that the velocity vector $\vec{w}$ makes with the tachyon corridor. Finally the velocity transformation equations are given by

$$\beta'_x=\frac{(w/w')\beta_x+[(-\epsilon\mu\gamma/c)w'/w+(\epsilon\mu\gamma/ww')(w_x\beta_x+\epsilon\vec{w}_\perp\cdot\vec{\beta}_\perp)-\vec{w}\cdot\vec{\beta}/ww']w_x}{-\epsilon\mu\gamma[(w_x/c)\beta_x+\epsilon\vec{w}_\perp\cdot\vec{\beta}_\perp/c-1]}\,, \tag{58a}$$

$$\vec{\beta}'_\perp=\frac{\vec{\beta}_\perp+[-(\epsilon\mu\gamma/c)w'/w+(\epsilon\mu\gamma/ww')(w_x\beta_x+\epsilon\vec{w}_\perp\cdot\vec{\beta}_\perp)-\vec{w}\cdot\vec{\beta}/ww'+(w/w'-1)\vec{w}_\perp\cdot\vec{\beta}_\perp/w^2_\perp]\vec{w}_\perp}{-\epsilon\mu\gamma[(w_x/c)\beta_x+\epsilon\vec{w}_\perp\cdot\vec{\beta}_\perp/c-1]}\,. \tag{58b}$$

C. The special transformation

By the special transformation is meant a transformation between a preferred and a nonpreferred inertial frame with the relative velocity vector along a common coordinate axis. When the relative velocity vector is along the tachyon corridor this special transformation reduces to the extended transformation given by Eq. (3), whereas for subluminal velocities the special transformation reduces to the special Lorentz transformation.

1. *The transformation matrix*

To obtain the special transformation matrix we need to orient both the x axis of K_1 and the x' axis of S' along the direction of the relative velocity vector. This can be accomplished by two successive rotations. The first rotation about the $x(x')$ axis places the relative velocity vector $\vec{w}$ and $\vec{w}'$ in the xy and $x'y'$ planes, respectively. The second rotation is about the new $z(z')$ axes by an angle θ. The new orientations of K_1 and S' will be referred to as K_{sp} and S'_{sp}, respectively. For $\theta=\Theta$ (where Θ is the angle between $\vec{w}$ and the tachyon corridor) the relative velocity of S'_{sp} relative to K_{sp} is positive, while for $\theta=\Theta+\pi$ it is negative. We denote this velocity by w (contrary to our convention up to this point) and use $|w|$ for the absolute value. The rotation matrix about the z axis will be denoted by $R_z(\theta)$.

The special transformation matrix $P(w,\Theta)$ from S'_{sp} to K_{sp} is then given by

$$P(w,\Theta)=R_z(\theta)T(w_{\parallel},w_{\perp},0;\epsilon)R_z^{-1}(\theta)\,. \tag{59}$$

Explicitly this is

$$P(w,\Theta)=\begin{bmatrix} \mu\gamma|w/w'| & (1-\epsilon)|w/w'|\cos\Theta\sin\Theta & 0 & (\mu\gamma/c)w \\ 0 & |w'/w| & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (\mu\gamma/c)|w'/w|w & 0 & 0 & \mu\gamma \end{bmatrix}. \tag{60}$$

For $|w|<c$, the transformation matrix P reduces to the special Lorentz transformation. On the other hand, for $|w|>c$ it becomes

$$P_{\text{super}}(w,\Theta)=\begin{bmatrix} \mu\gamma(\sec 2\Theta)^{1/2} & \sin 2\Theta(\sec 2\Theta)^{1/2} & 0 & \mu\gamma(w/c) \\ 0 & (\cos 2\Theta)^{1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mu\gamma(w/c)(\cos 2\Theta)^{1/2} & 0 & 0 & \mu\gamma \end{bmatrix}. \tag{61}$$

The inverse special transformation from K_{sp} to S'_{sp} for superluminal transformations is given by

$$P'_{\text{super}}(w',\Theta)=\begin{bmatrix} \mu'\gamma(\cos 2\Theta)^{1/2} & -\mu'\gamma\sin 2\Theta(\sec 2\Theta)^{1/2} & 0 & \mu'\gamma(w'/c) \\ 0 & (\sec 2\Theta)^{1/2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mu'\gamma(w'/c)(\cos 2\Theta)^{1/2} & -\mu'\gamma(w'/c)(\sec 2\Theta')^{1/2}\sin 2\Theta & 0 & \mu'\gamma \end{bmatrix}. \tag{62}$$

It should be noted that the angle that the velocity vector $\vec{w}'$ makes with the x' axis of S' is $\pi+\Theta$ rather than Θ. The asymmetry between the y and z axes in the transformations (60), (61), and (62) is due to the fact that the y axis has been chosen to lie in the plane determined by the relative velocity vector and the tachyon corridor, while the z axis is perpendicular to this plane.

2. *Length and time measurements*

Let (l_x^0, l_y^0, l_z^0) be the components of a rod as measured in its rest system S'_{sp}, and (l_x, l_y, l_z) its components as measured by an observer in K_{sp}. Furthermore, let τ_0 and τ be the measurements of S'_{sp} and K_{sp}, respectively, of a time interval as recorded by a clock which is stationary relative to S'_{sp}. Then the transformation matrix $P_{\text{super}}(w,\Theta)$ leads to

$$l_x=l_x^0(\sec 2\Theta)^{1/2}\left[-\frac{\mu}{\gamma}+\frac{l_y^0}{l_x^0}\sin 2\Theta\right], \tag{63a}$$

$$l_y=l_y^0(\cos 2\Theta)^{1/2}, \tag{63b}$$

$$l_z=l_z^0, \tag{63c}$$

$$\tau=\mu\gamma\tau^0\,. \tag{63d}$$

For $\Theta=0$ or π the above equations reduce to Eqs. (35), (36a), and (36b). Equations (63) apply to the case where the rod and clock are stationary in a nonpreferred inertial frame S'_{sp} and are measured from an inertial frame K_{sp}. If, on the other hand, the rod and clock are stationary in K_{sp} and are measured from S'_{sp}, then we find by using the matrix $P'_{\text{super}}(w',\Theta)$

$$l'_x = -l_x^0 \frac{\mu'}{\gamma}(\cos 2\Theta)^{1/2}\left[1-\left(\frac{l_y^0}{l_x^0}\right)\tan 2\Theta\right], \quad (64a)$$

$$l'_y = l_y^0(\sec 2\Theta)^{1/2}, \quad (64b)$$

$$l'_z = l_z^0, \quad (64c)$$

$$\tau' = \mu'\gamma\tau^0, \quad (64d)$$

which again reduce to Eqs. (35), (36a), and (36b) for $\Theta = 0$ or π. The asymmetry between the transformation properties of y and z is due to the special orientation of the coordinate axes by virtue of which both the relative velocity and tachyon corridor lie in the xy plane.

IV. THE MOST GENERAL SUPERLUMINAL TRANSFORMATION

In this section we will derive the superluminal transformation in between two parallel intertial frames, without imposing any restrictions on the magnitude and orientation of their relative velocity. Due to lack of isotropy and insufficient symmetry in the problem at hand, none of the methods we have used thus far is directly applicable to the derivation of the general transformation. A direct approach to the problem, using successive transformations, including a rotation about a general axis in space, is necessary. The transformation we are seeking is a generalized Lorentz transformation from the inertial frame S' to the inertial frame S, where S and S' are parallel. The velocity of S' relative to S is $\vec{v}$ and that of S relative to S' is $\vec{v}'$. Due to the anisotropy of space the transformation equations will depend on the orientation of S and S'. On the other hand, given the transformation equations for one orientation, those for another orientation are easily obtainable through the rotation matrix that connects the two orientations. Thus we will solve the problem for a given standard orientation, whereby the absolute velocity $\vec{u}$ of S is along the negative y axis, and the xy plane of S is parallel to the plane determined by the two pseudotachyon corridors of S.

To set up the problem, consider, in addition to S and S' two other intermediate coordinate systems, K_1, and S'', as shown in Fig. 2. K_1 is a preferred inertial frame with its x axis along the tachyon corridor. We require that K_1 be parallel to S. Due to the special orientation of S, this implies that the velocity of S relative to K_1 is perpendicular to the tachyon corridor; it is actually given by $\vec{u}$, the absolute velocity of S. S'' is parallel to K_1, at rest relative to S', and shares the same origin with S'. The relative velocities of the four coordinate systems involved are shown in Table I.

The transformation matrix $G(\vec{v}, u, \epsilon)$ from S' to S can be written as the product of three transformation matrices,

$$G(\vec{v}, u, \epsilon) = L_y(-u)T(\vec{w}; \epsilon)R_{\hat{n}}^{-1}(\theta), \quad (65)$$

where $R_{\hat{n}}^{-1}(\theta)$ transforms S' into S'', $T(\vec{w};\epsilon)$ transforms S'' to K_1, and $L_y(-u)$ transforms K_1 to S. $L_y(-u)$ is the usual special Lorentz transformation along the y axis. $T(\vec{w};\epsilon)$ is the transformation from the inertial frame S'' to the preferred frame K_1, and is given by Eq. (44). Finally $R_{\hat{n}}(\theta)$ is the rotation matrix that transforms S'' to S'. This matrix rotates the coordinate axes by θ about the unit vector $\hat{n}$. Equivalently, this matrix rotates vectors by angle $-\theta$ about $\hat{n}$. Hence[14]

$$R_{\hat{n}}(\theta) = e^{-\theta N} = I\cos\theta + (1-\cos\theta)\hat{n}\hat{n}^T - N\sin\theta, \quad (66a)$$

where $\hat{n}^T$ is the transpose of $\hat{n}$, I is a unit matrix, and N the generator of the rotation. These are given by

$$\hat{n} = \frac{\vec{v}'\times\vec{v}''}{|\vec{v}''\times\vec{v}'|}, \quad \sin\theta = \frac{|\vec{v}'\times\vec{v}''|}{|\vec{v}'||\vec{v}''|}, \quad \cos\theta = \frac{\vec{v}'\cdot\vec{v}''}{|\vec{v}'||\vec{v}''|}, \quad (66b)$$

and

$$N = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}. \quad (66c)$$

Equations (65) and (66) determine the transformation matrix $G(\vec{v}, u, \epsilon)$ provided that $\cos\theta$, $\sin\theta$, $\hat{n}$, and $\vec{w}$ can be expressed in terms of $\vec{v}$, u, and ϵ. This is accomplished by using the condition of parallelism and the velocity transformation equations. We note that $\hat{n}$ is determined by the relative orientations of $\vec{u}$ and $\vec{v}$, while θ depends in addition on the magnitudes of $\vec{u}$ and $\vec{v}$.

TABLE I. The relative velocities of the four inertial frames, S, S', S'', and K_1 involved in deriving the general superluminal transformation.

relative to \ velocity of	S	S'	K_1	S''
S	0	$\vec{v}$	$-\vec{u}$	$\vec{v}$
S'	$\vec{v}'$	0	$\vec{w}''$	0
K_1	$\vec{u}$	$\vec{w}$	0	$\vec{w}$
S''	$\vec{v}''$	0	$\vec{w}'$	0

Correspondence considerations suggest that γ be defined by

$$G_{44} = \mu\gamma, \tag{67a}$$

with

$$\mu = \frac{v_x}{|v_x|}\left(\frac{1-\epsilon}{2}\right) + \left(\frac{1+\epsilon}{2}\right). \tag{67b}$$

This leads to

$$\gamma = \frac{[1-(u^2/c^2)]^{1/2}\sqrt{\epsilon}}{[(1-\epsilon u^2/c^2)(1-\epsilon v_y{}^2/c^2) - (1-u^2/c^2)(v_x{}^2/c^2 + \epsilon v_z{}^2/c^2) + (2uv_y/c^2)(1-\epsilon)]^{1/2}} . \tag{68}$$

It is easy to show that the γ of Eq. (68) reduces, under appropriate conditions, to the γ functions considered previously. If we introduce the parameter ζ by

$$\zeta^2 = \frac{1}{c^2}(v_x{}^2 + \epsilon v_y{}^2 + \epsilon v_z{}^2) - (1-\epsilon)\left|\frac{(u^2/c^2)(v_y{}^2/c^2 - \epsilon) + 2uv_y/c^2}{(1-u^2/c^2)}\right| , \tag{69}$$

then the general expression for γ reduces to

$$\gamma = \frac{\sqrt{\epsilon}}{(1-\zeta^2)^{1/2}} . \tag{70}$$

For S inertial, $\zeta^2 \to v'^2/c^2$, for S' inertial, $\zeta^2 \to v^2/c^2$, and for subluminal transformations, $\zeta^2 \to v^2/c^2$.

With γ defined according to Eq. (68) the fourth column of G can be easily calculated. To obtain the other twelve elements of $G(\vec{v}, u, \epsilon)$ we define the matrix $F(\vec{v}, u, \epsilon)$ by

$$F(\vec{v}, u, \epsilon) = L_y(-u)T(\vec{w}; \epsilon) . \tag{71}$$

The elements of F can easily be obtained in compact form. In addition, we define the vectors $\vec{f}^{(\nu)}$ and $\vec{g}^{(\nu)}$ by

$$\vec{f}^{(\nu)} = (F_{\nu 1}, F_{\nu 2}, F_{\nu 3}), \quad \nu = 1, 2, 3, 4, \tag{72a}$$

$$\vec{g}^{(\nu)} = (G_{\nu 1}, G_{\nu 2}, G_{\nu 3}), \quad \nu = 1, 2, 3, 4 . \tag{72b}$$

The vectors $\vec{g}^{(\nu)}$ are then related to the vectors $\vec{f}^{(\nu)}$ by

$$\vec{g}^{(\nu)} = \vec{f}^{(\nu)}\cos\theta + (1-\cos\theta)(\vec{f}^{(\nu)} \cdot \hat{n})\hat{n} + \sin\theta[\vec{f}^{(\nu)} \times \hat{n}], \quad \nu = 1, 2, 3, 4 \tag{73}$$

where $\hat{n}$, $\sin\theta$, and $\cos\theta$ are given by Eq. (66). The elements of $G(\vec{v}, u, \epsilon)$ are then given by

$$G_{44} = \mu\gamma , \tag{74a}$$

$$G_{i4} = \mu\gamma\frac{v_i}{c}, \quad i = 1, 2, 3 \tag{74b}$$

$$G_{ij} = g_j^{(i)} \quad \begin{cases} i = 1, 2, 3, 4, \\ j = 1, 2, 3 . \end{cases} \tag{74c}$$

V. CONCLUSION

In deriving the general superluminal Lorentz transformations based on the three-dimensional tachyon theory of I, we have found three main cases to consider: (i) The transformation between two preferred frames. In this case the relative velocity is along the tachyon corridor and the transformation matrix is simple in form. (ii) The transformation between a preferred frame and a nonpreferred frame. In this case the relative velocity is superluminal if and only if its component along the tachyon corridor is superluminal. Furthermore, the velocities of the direct and inverse superluminal transformations are not equal even though they are both superluminal. This can be explained by the fact that the two inertial frames are not equivalent. The transformation matrix in this case depends on the relative velocity vector and the angle that this vector makes with the tachyon corridor. The matrix is more complicated than that of the general Lorentz transformation but can still be written in compact and explicit form. (iii) The transformation between two inertial nonpreferred frames. In this case, the transformation matrix is a function of the relative velocity, its orientation relative to the tachyon corridor, and the absolute velocity of one of the two inertial frames. It should be noted that the inverse transformation is not in general simply related to the direct transformation. This is due to the fact that the two inertial frames involved are not equivalent on two accounts: Their orientations relative to the tachyon corridor are different, and their absolute velocities are different. The transformation matrix in this case is rather complicated and cannot be expressed in a compact form in terms of the basic set of parameters on which it depends. Nonetheless, we have calculated all elements explicitly and compactly in terms of the natural parameters of the problem.

Whether tachyons exist or not, this work shows how valuable it is to have a principle of relativity, at least in bradyon physics, that guarantees the equivalence of all inertial frames.

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[12]By dimensionless velocity is meant the velocity in units of c. (c is the velocity of a bradyonic luxon in vacuum.)

[13]Since the two-dimensional space perpendicular to the tachyon corridor is isotropic in a preferred inertial frame, choosing $\vec{u}$ along the y axis in order to simplify the problem does not lead to any loss of generality.

[14]See, for example, Ben Noble, *Applied Linear Algebra* (Prentice-Hall, New York, 1969), pp. 421 and 422.