Fine-structure constant and entropy in the early universe

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A recent charge-symmetric model of the very early stages of a big-bang cosmology is used to derive a value for the fine-structure constant.

The need to understand the content and time evolution of the universe has attracted the attention of scientists since time immemorial. In attempting to construct a physical model of the early universe, one is immediately faced with the question of the time development of the parameters which appear in the theories which successfully describe systems of laboratory size.

Several ideas have been put forth for dynamical theories with time-varying physical parameters. However, experimental results set upper limits for time variations on the scale of astrophysical or cosmological significance and seem to exclude most of the assumed time variations.[1]

In this work, we shall consider the universe as a closed isolated dynamical system, and we will be interested in studying its constants of the motion. The determination of all the constants of the motion presupposes complete knowledge of the Hamiltonian of the system and, in particular, its inherent symmetries. It is clear that all the constants of the motion need not be independent and some may be related to others. Thus, the knowledge of some of the constants of the motion may be used to generate further constants of the motion. For instance, from Jacobi's identity for classical systems, one obtains that the Poisson bracket of two constants of the motion is itself a constant of the motion. A similar result follows for quantum systems from quantum Poisson brackets.

In setting up a dynamical theory, a rather clear choice is made at the very beginning between the simple assumed constants of the motion, the time-independent parameters of the theory, and the more complicated derived constants of the motion. In what follows, we shall assume the following universal constants of the motion:

$$m_e = \text{mass of the electron}, \tag{1a}$$

$$m_p = \text{mass of the proton}, \tag{1b}$$

$$c = \text{speed of light}, \tag{1c}$$

$$\hbar = \text{Planck's constant}, \tag{1d}$$

$$G = \text{constant of gravitation}, \tag{1e}$$

$$e = \text{unit of electric charge}, \tag{1f}$$

and

$$k = \text{Boltzmann's constant}. \tag{1g}$$

To complete the above list, one should add the coupling constants of the other two known forces in nature—the strong and weak interactions. However, since they will not enter in our considerations, we shall omit them. Note that (1)—with the preceding proviso—constitutes a complete list of universal constants. All other physical quantities depend either on the above constants or relate to physical situations which change with time.

From (1) we can form the following three pure numbers:

$$m_p/m_e = 1836, \quad \text{ratio of proton mass to electron mass}, \tag{2a}$$

$$\alpha \equiv e^2/\hbar c \simeq \tfrac{1}{137}, \quad \text{fine-structure constant}, \tag{2b}$$

and

$$e^2/Gm_e m_p = 2.3\times 10^{39}, \quad \text{ratio of the electrical to the gravitational force between an electron and a proton}. \tag{2c}$$

Note that without a universal constant temperature or constant entropy one may not form a pure number from Boltzmann's constant.

In a recent work,[2] a charge-symmetric model for the very early stages of a big-bang cosmology was presented. In the period $10^{-43} \leqslant t \leqslant 10^{-23}$ sec, time measured from zero at the singular point $R = 0$, $\rho \to \infty$, the entropy per comoving cell of the universe remains constant and is given by

$$S_c = 3\pi\hbar ck/32Gm_{\pi^\pm}^2 = 2.3\times 10^{39}k. \tag{3}$$

The temperature $T_0 = m_{\pi^\pm}c^2/k$ represents the constant temperature of the universe in the period $10^{-23} \leqslant t \lesssim 10^{-7}$ sec. The numerical coincidence between the two large pure numbers (2c) and (3) is striking. We shall form a pure number with (3)—by dividing by k—and suppose that the two ratios of constants of the motion represent the same precise pure number. Hence, equating (2c) and S_c/k, one obtains for the fine-structure constant[3]

$$\alpha = 3\pi m_e m_p / 32 m_{\pi^\pm}^2 = 1/137.94 \pm 0.01 \ . \tag{4}$$

If the hadronic masses m_p and $m_{\pi^\pm}$ have finite nonzero values in the absence of electromagnetic interactions, then (4) implies that the mass of the electron is purely electromagnetic.

It should be noted that both the entropy per cell S_c and the temperature T_0 are constants of the motion over large[4] but finite periods of time. These quantities are, therefore, quasiconstants of the motion. However, since they occur in the very early periods of the "hadron era"—especially S_c—one may argue that their early constancy has left a permanent imprint in the values of the fundamental constants of nature.

Our argument resembles somewhat the original 1937 argument of Dirac.[5] Dirac equates two large pure numbers $\hbar c/G m_p^2 \sim m_p c^2/H\hbar$, with H denoting the time-varying Hubble's "constant." Since $H \sim t^{-1}$, Dirac concludes that $G \sim t^{-1}$. (This time variation is barely consistent with recent solar and stellar evolution data.[1]) However, in our argument we are equating ratios of constants of the motion—and not simply large numbers.

[1]F. J. Dyson, in *Aspects of Quantum Theory*, edited by A. Salam and E. P. Wigner (Cambridge Univ. Press, Cambridge, England, 1972).

[2]M. Alexanian and F. Mejía-Lira, preceding paper, Phys. Rev. D 11, 716 (1975).

[3]For a review of recent theories of the fine-structure constant see S. L. Adler, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 2, p. 115.

[4]E. R. Harrison, Nature 228, 258 (1970).

[5]P. A. M. Dirac, Nature 139, 323 (1937); Proc. R. Soc. A165, 199 (1938).