Comments and Addenda

The Comments and Addenda section is for short communications which are not of such urgency as to justify publication in **Physical Review Letters** *and are not appropriate for regular Articles. It includes only the following types of communications: (1) comments on papers previously published in* **The Physical Review** *or* **Physical Review Letters***; (2) addenda to papers previously published in* **The Physical Review** *or* **Physical Review Letters***, in which the additional information can be presented without the need for writing a complete article. Manuscripts intended for this section should be accompanied by a brief abstract for information-retrieval purposes. Accepted manuscripts will follow the same publication schedule as articles in this journal, and galleys will be sent to authors.*

e^+e^- annihilation into hadrons and bridging Higgs particles

Douglas W. McKay and Herman Munczek
Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045
(Received 16 September 1974)

Some of the Higgs particles in unified gauge models should have rather low masses and point couplings to the photon. We estimate their contribution to e^+e^- into hadrons to be of the right order of magnitude and energy dependence.

Of the Higgs particles which bridge the hadronic and leptonic worlds in the Bars-Halpern-Yoshimura (BHY) unified gauge model,[1] several must have rather low masses (~4 GeV) in order to avoid large $\Delta S=2$ effects.[2] Because these "bridging" Higgs scalar fields are the only fields which transform under both the hadronic gauge group and the leptonic gauge group, their electromagnetic coupling is partly vector-meson dominated (hadron behavior) and partly pointlike (lepton behavior). It is the latter coupling which survives at large photon 4-momentum squared. Since, on the other hand, conventional hadrons, because of vector-meson dominance, have form-factor suppression of their coupling to the photon we conjecture that the combination of low mass and undamped coupling to the electromagnetic field could make those Higgs particles quite important in the description of processes involving virtual-photon energies near their production thresholds. The pointlike character of the electromagnetic coupling of the Higgs particles and the importance of the contribution of the production of two Higgs particles and a vector meson to the high-energy behavior of the cross section for $e^+e^-\to$ hadrons in a model calculation was discussed by Bars, Levy, and Halpern.[3]

In this study we present an estimate of the contribution of these Higgs scalars to the total cross section for $e^+e^-\to$ hadrons in the energy range 2–15 GeV. For lack (at the moment) of a specific model about their decay into hadrons, the Higgs particles shall be assumed to be unstable under the strong interaction, while their couplings to the photon shall be taken to be those contained in the BHY interaction Lagrangian. With these simple assumptions we find that, quite encouragingly, the contributions to $\sigma(e^+e^-\to\text{hadrons})$ are very nearly constant from 3 to 10 GeV and of the order of magnitude observed experimentally.[4] In the following we give an account of our assumptions about these scalars, namely their coupling to the photon, their treatment as unstable particles, and the contribution they give to $\sigma(e^+e^-\to\text{hadrons})$. We also briefly discuss implications for hadronic corrections to $g-2$ and for inclusive electroproduction.

As remarked above, recent work[2] has shown that the triplet of complex fields χ_L in the BHY model can give rise to large $\Delta S=2$ effects unless the χ_L masses are constrained to be only a few GeV. The couplings in the Lagrangian which bear on the electromagnetic interaction of these χ_L fields (and their chiral partners χ_R) are[1,2]

$$i\partial_\mu\chi_L^\dagger\left(\tfrac{2}{3}eA_\mu+\frac{h}{\sqrt{2}}V_\mu^L\right)\chi_L+\text{H.c.}$$
$$-\chi_L^\dagger\left(\tfrac{2}{3}eA_\mu+\frac{h}{\sqrt{2}}V_\mu^L\right)^2\chi_L+(L\to R)\ . \quad (1)$$

In this equation, A_μ is the photon field and $V_\mu^{L(R)}$ represent nonet matrices. From the first term in Eq. (1), we can see that there is a direct coupling of the photon to the χ's as if they had charge $\frac{2}{3}e$. However, in this type of theory, which includes vector-meson dominance, V_μ^L and V_μ^R are mixed with A_μ, providing a contribution to the electromagnetic charge which can be implemented by the substitution $(h/\sqrt{2})\ V_\mu^{L(R)}\to QA_\mu$, where

$$Q=\frac{1}{3}\begin{vmatrix}2 & & \\ & -1 & \\ & & -1\end{vmatrix} .$$

This means that the charges of the $\chi^{\alpha}_{L(R)}$ triplets are o, e, e respectively for $\alpha=1,2,3$. As a consequence, the χ-photon vertices will have a momentum dependence only partially damped by the propagator of the intermediate vector meson. The end result is that at large photon 4-momentum squared the dominant contributions will be only those coming from the terms containing explicitly the photon field. It is interesting to see that the χ's behave under those circumstances as if all of them had the same effective charge $\frac{2}{3}e$. We can see, then, from Eq. (1) that the states that couple most strongly to one (large 4-momentum squared) photon are those containing two χ's, and two χ's and a V. (All standard hadron couplings are suppressed in comparison because of the extra $1/q^2$ coming from vector-meson dominance.) These features, combined with the low-mass restriction on the χ's,[2] provide our motivation for investigating the role of the χ's in producing hadron final states in e^+e^- collisions.

Because the χ's can have potential energy couplings to the known spin-zero hadrons[5] [assigned, for example, to a $(3^*,3)+(3,3^*)$ representation of $SU(3)\times SU(3)$], and because the masses of the χ's (if they exist) are expected to be considerably larger[2] than those of the known spin-zero hadrons, we view the χ's as unstable particles with possibly large total width for decay into hadron channels. In order to handle production of $\chi\chi$ and $\chi\chi V_{L(R)}$ final states with subsequent decay of χ's into multihadron final states we take the Lagrangian [Eq. (1)] to tell us the point γ-$V\chi\chi$ and γ-$\chi\chi$ couplings and we parametrize the subsequent decay of each χ by a Breit-Wigner function. We neglect interference in the final states. Concentrating on the $V\chi\chi$ contribution, which turns out to be larger than the $\chi\chi$ one by an order of magnitude, we have for the cross section

$$\sigma(Q)=(2\pi)^{-5}\frac{e^2}{3Q^4}18\left(\frac{4}{3}\frac{he}{\sqrt{2}}\right)^2 J(Q) , \tag{2}$$

with

$$J(Q)=\sum_{\lambda}\int d^4k\, d^4q_1\, d^4q_2\, a_\mu(\lambda)\, a_\mu(\lambda)\, \delta^4(k+q_1+q_2-Q) \times\delta(k^2-M_V^2)\,\delta(q_1^2-M^2)\,\delta(q_2^2-M^2) \times\theta(k_0)\,\theta(q_{10})\,\theta(q_{20}) , \tag{3}$$

and

$$a_\mu(\lambda)=-(\delta_{\mu\nu}-Q_\mu Q_\nu/Q^2)\,\epsilon_\nu(\lambda) . \tag{4}$$

Summation over the internal degrees of freedom[6] has been included in Eq. (2). In the overall center of mass, the expression $\sum_\lambda a_\mu(\lambda)\,a_\mu(\lambda)$ depends only on the vector-meson energy. We can then write

$$J=\int d^4k\, d^4t\left(2+\frac{k_0^2}{M_V^2}\right)\delta(k^2-M_V^2)\,\theta(k_0) \times\delta^4(t+k-Q)\,I(t) , \tag{5}$$

where $I(t)$ is the invariant two-body phase space integral for the $\chi\chi$ system. By replacing the mass-shell δ functions according to[7]

$$\delta(q^2-M^2)\rightarrow\frac{M\Gamma/\pi}{(q^2-M^2)^2+M^2\Gamma^2} , \tag{6}$$

we obtain[8]

$$I(t)=\frac{1}{q}[(q^2-4)^2+\gamma^2]^{1/4}\cos\tfrac{1}{2}\alpha\tan^{-1}(4q^2/\gamma) , \tag{7}$$

where $q=t/M$, $\gamma=4\Gamma/M$, and

$$\alpha=\tan^{-1}[\gamma/(q^2-4)]+\pi\,\theta(4-q^2) . \tag{8}$$

This expression for the Breit-Wigner-weighted, two-body phase-space integral approaches the mass-shell results when $\Gamma\rightarrow 0$, as it should, and also when $t\gg M$ for finite Γ, thus showing that it is properly normalized. Therefore, the expression of Eq. (7) appears to be a reasonable approximation to a phase-space integral containing two unstable particles. Folding $I(t)$ into the integral of Eq. (5) we obtain the final expression for the cross section,[9]

$$\sigma(Q)=\alpha^2\left(\frac{h^2}{4\pi}\right)\frac{64}{3\pi^2}\frac{1}{Q^4}J(Q) , \tag{9}$$

where[10]

$$h^2/4\pi=2f_{\rho\pi\pi}{}^2/4\pi\simeq 6 .$$

The main features of this expression are readily seen in Figs. 1 and 2, where $\sigma(e^+e^-\rightarrow\text{hadrons})$ (in nanobarns) is plotted against Q (GeV) for χ masses of 2 and 4 GeV. We have adopted for the γ-$V\chi\chi$ interaction the expression of Eq. (4) because it is the dominant part of the amplitude for production of zero-width χ's near threshold, so we expect our results to be reliable only in the region around $Q=2M+M_V$.[5] For every χ mass and width combination the cross section stays remarkably flat in comparison with the behavior ($\sim 1/Q^2$) of the cross section for annihilation into $\mu^+\mu^-$. This is true also for the cases $M=3$ and 5 GeV, $\gamma=0.5, 1, 2$, which are not shown. This is our main result. It is striking also that the cross-section value is of the correct order of magnitude at 3, 4, and 5 GeV where data presently exist.[4] Lifting the χ mass degeneracy assumption and superposing contribu-

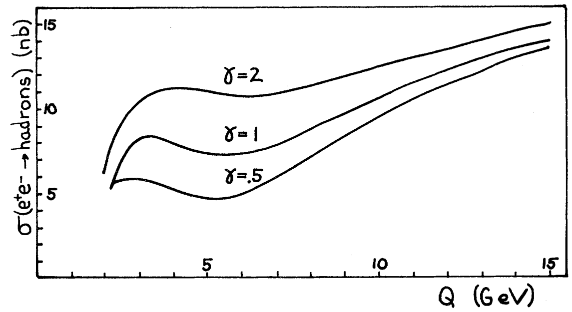

FIG. 1. Estimated cross sections for $M=2$ GeV and several values of $\gamma=4\Gamma/M$.

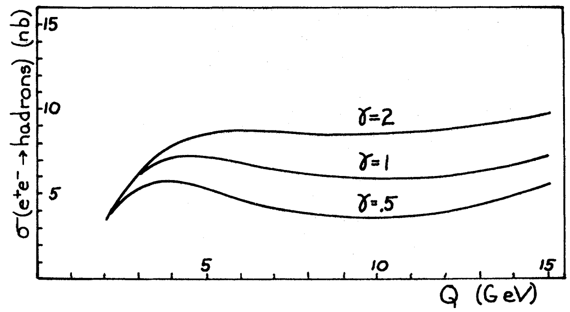

FIG. 2. Estimated cross section for $M=4$ GeV and several values of $\gamma=4\Gamma/M$.

tions of various masses and widths for the Higgs particles obviously does not alter either the flatness of the cross section or its approximate magnitude of 5–10 nb. Clearly the rather low cross-section values we obtain can be enhanced by a superposition of the tail of low-energy contributions and by contributions from additional "mixed symmetry" Higgs scalars which could happen to have relatively small masses[11, 12] (2–5 GeV). Additional spin-one hadrons and corresponding additional Higgs particles can also be incorporated in this type of gauge theory[12] and would lead to an increase in the calculated cross section.

In conclusion, our simple considerations based on the BHY unified gauge model lead to semiquantitative agreement with the $e^+e^-\to$hadrons total-cross-section data at 3, 4, and 5 GeV. The calculated behavior is a direct result of the peculiar features of the mixed-symmetry Higgs particles.

The features of the χ particles which we have discussed in this paper have bearing, of course, on the hadronic corrections to the muon g factor and on the behavior of the structure functions which describe inclusive electroproduction. Using our cross-section expression (9) and integrating only over the region $4 \leq Q^2 \leq 200$ GeV^2, in which we expect it to be the dominant contribution to $\chi\chi V$ production, we find that the correction to $g-2$ is less than the corrections estimated from low-lying vector-meson states,[13] which are themselves an order of magnitude below present experimental limits. The problem of estimating the very-high-energy $(Q \gg M)$ limit of the $\chi\chi V$ production cross sections is being studied.

Comparison of the model with presently available inclusive electroproduction data appears to be just out of reach. The mechanism explored in this paper could show testable effects in electroproduction when $|Q^2|>9$ GeV^2 and simultaneously $W^2=Q^2+2m\nu+m^2>25$ GeV^2,[14] whereas the recent detailed SLAC data[15] go up to only $W^2\approx 15$ GeV^2 at $|Q^2|=4$ GeV^2. We are presently estimating the contribution of χ production to the inelastic structure function to determine if effects emerge which are testable by higher-energy experiments.

We thank Charles Eklund, Dean Halderson, and Larry Loos for advice and help in carrying out the computer calculations.

---

[1]I. Bars, M. Halpern, and M. Yoshimura, Phys. Rev. Lett. 29, 969 (1972); Phys. Rev. D 7, 1233 (1973).

[2]H. Munczek, Phys. Rev. D 10, 1345 (1974).

[3]I. Bars, M. B. Halpern, and D. J. Levy, Phys. Rev. D 9, 400 (1974).

[4]G. Tarnopolsky *et al.*, Phys. Rev. Lett. 32, 432 (1974); A. Litke *et al.*, *ibid.* 30, 1189 (1973); M. Gritti *et al.*, Nuovo Cimento 13A, 593 (1973).

[5]Among the host of terms which comprise the most general $U(3)\times U(3)\times SU(2)_L\times Y_L$-invariant potential energy in the BHY model are couplings of χ's to conventional hadron spin-zero particles. For example, in the notation of Ref. 2, part of the invariant potential energy is

$$a\,\mathrm{Tr}[\Sigma\Sigma^+(D_L T_L^\dagger + T_L D_L^\dagger)] + b\,\mathrm{Tr}[\Sigma\Sigma^\dagger D_L D_L^\dagger] + c\,\mathrm{Tr}[\Sigma\Sigma^\dagger T_L T_L^\dagger] + d\,\mathrm{Tr}[D_L T_L^\dagger + T_L D_L^\dagger] + (L\to R),$$

where Σ is the $(3^*,3)+(3,3^*)$ representation matrix and D_L and T_L are the BHY "M" particles written in their three-by-two irreducible matrix form. The χ_L fields form the first column of the D matrix, and the nonvanishing expectation values appear in the second column of D and in both columns of T. The constants a, b, c, and d can easily be arranged so that the Σ mass matrix is diagonal, there are no terms linear in the χ fields, and there are $\Sigma\Sigma^\dagger\chi$ couplings proportional to a. Analyzing the most general solutions of the extremum problem for the invariant potential will be extremely complex, especially in view of the possibility of adding "heavy pions" to retain a successful description of $\pi^0 \to \gamma\gamma$ (see Ref. 1).

[6]We take all of the χ_L's and χ_R's degenerate in mass and denote this mass by M. The masses of hadronic spin-one mesons, denoted by M_V, will be taken to be degenerate at 1 GeV.

[7]See, for example, H. Pilkuhn, *The Interactions of Hadrons* (North-Holland, Amsterdam, 1967).

[8]Before making the substitution of Eq. (6) it is convenient to make use of the singular functions in $I(t)$ in order to factorize the δ-function integrations, i.e., in the center-of-mass system

$$\begin{aligned} I(t) &= \int d^4q_1 d^4q_2 \delta^4(t-q_1-q_2)\delta(q_1{}^2-M^2) \\ &\quad\times \delta(q_2{}^2-M^2)\theta(q_{10})\theta(q_{20} \\ &= 4\pi\int_0^\infty \delta(\tfrac{1}{4}t^2-M^2-\xi^2)\xi^2 d\xi \\ &\quad\times \int_0^{t_0}\delta(t^2-2\xi_0 t_0)d\xi_0 \\ &\to \frac{4}{\pi}M^2\Gamma^2\int_0^\infty \frac{\xi^2 d\xi}{(\frac{1}{4}t^2-M^2-\xi^2)^2+M^2\Gamma^2} \\ &\quad\times \int_0^{t_0}\frac{d\xi_0}{(t^2-2\xi_0 t_0)^2+M^2\Gamma^2}, \end{aligned}$$

which gives Eq. (7).

[9]We quote only the $V\chi\chi$ case since we have estimated the $\chi\chi$ case to be an order of magnitude smaller.

[10]See, for example, L. M. Brown, H. Munczek, and P. Singer, Phys. Rev. Lett. 21, 707 (1968).

[11]We have only those Higgs particles, and their chiral partners, which *must* have small mass. I. Bars [Nucl. Phys. B64, 163 (1973)] has emphasized the interest in finding tests for the existence or nonexistence of these objects. To get a rough idea of the contributions that the remaining Higgs scalars could make, one can take all of them to have the same mass and width as the χ's, and the additional contribution to the total e^+e^- cross section is $\frac{3}{8}$ times the χ cross section; see Eq. (9) and Figs. 1 and 2.

[12]I. Bars, M. B. Halpern, and K. D. Lane, Nucl. Phys. B65, 518 (1973); M. B. Halpern, *ibid.* B66, 78 (1973).

[13]M. Gourdin and E. De Rafael, Nucl. Phys. B10, 667 (1969).

[14]The $|Q|^2$ and W^2 values quoted would ensure an order-of-magnitude suppression of the usual vector-meson-dominated hadronic form factors relative to point couplings and would ensure sufficient final-state energy to produce two χ's and a proton for the case of $M=2$ GeV, the smallest M value which we have considered.

[15]A. Bodek *et al.*, SLAC Report No. SLAC-PUB-1442, 1974 (unpublished).