Chiral configuration mixing and deep-inelastic scattering of polarized leptons by polarized nucleons*

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In the quark model, a large SU(3)⊗SU(3) configuration mixing appears at $P_z = \infty$ as a relativistic effect due to the high internal velocities for quarks inside hadrons. This mixing describes the relative alignment between the nucleon helicity and the helicities of the quarks. Since the quark-parton model is understood in a frame where the nucleon has a large momentum, we propose to take into account this large mixing in the calculation of the spin-dependent effects in deep-inelastic scattering of a polarized lepton (electron or muon) by a polarized nucleon. The scaling of the spatial wave function is ensured by the Lorentz contraction. Unlike a previous quark-parton model calculation by Gourdin, we take into account the configuration mixing in the estimation of the singlet contribution to the asymmetries. We express our results in terms of the SU(3)⊗SU(3) mixing parameters. Assuming that the nucleon is a superposition of irreducible SU(3)⊗SU(3) representations allowed by the quark model, we can express the singlet contribution in terms of the F, D couplings of the low-lying octet axial-vector matrix elements. The neutron asymmetry appears to be proportional to the deviation of F/D from the SU(6) value 2/3. As expected, the Bjorken sum rule for the difference between the proton and neutron asymmetries is satisfied. In our specific model, where the spin part of the wave function at rest is expressed in terms of free Dirac spinors, we have $F/D = 2/3$, so that we predict a large positive proton asymmetry and a null asymmetry for the neutron, $A^p = \frac{1}{3}|G_A/G_V|$, $A^n = 0$.

I. INTRODUCTION

Electron or muon polarized beams together with polarized targets will be available in the foreseeable future. Therefore, it may be interesting to investigate the predictions of the quark model for the structure functions $g(q^2,\nu)$, $d(q^2,\nu)$, and the resulting asymmetries in deep-inelastic scattering of a polarized lepton by a polarized nucleon.

In the frame of the quark-parton model, various authors[1-3] have investigated the scaling properties of the structure functions and have deduced sum rules previously obtained from current algebra by Bjorken.[4] In these papers the absolute value of the structure functions and asymmetries is computed by assuming the nucleon as composite of three quarks in the usual ($\underline{56}$, $L=0^+$) representation of SU(6). In this way, they obtain a null asymmetry for the neutron and a large, positive asymmetry for the proton, proportional to the SU(6) value of the nucleon axial-vector coupling, $|G_A/G_V|=\frac{5}{3}$. Gourdin[2] has made also a more complete quark-parton model calculation, relating the octet contribution to the asymmetries to the low-lying octet axial-vector matrix elements. His calculation satisfies the Bjorken sum rule and, in this way, he takes into account phenomenologically the configuration mixing for the octet part. As we will show, however, this estimation is not fully consistent since he does not take into account the configuration mixing in the calculation of the singlet contribution.

The quark-parton model is formulated in a frame in which the nucleon has a very large momentum, for instance, the center-of-mass frame of the reaction $e^-p\to e^-X$.[5] It is assumed that, in such a frame, the electromagnetic current interacts with the quarks as if they were pointlike. The structure functions can then be written in terms of the absorptive part of the pointlike forward photon-quark elastic amplitude and the nucleon wave functions at large momentum. Since the interaction is pointlike, the quark structure functions present trivially the scaling property. We will show that the nucleon wave function at $P_z=\infty$ also presents this feature. We will see that, by boosting the hadron wave function at rest to a frame in which it has a momentum $P_z=\infty$, the Lorentz contraction of the wave function ensures that it depends only on the transverse momenta and on fractions of the longitudinal total momentum, i.e., it presents the scaling. This point has been emphasized by Feynman using intuitive arguments.[6]

The structure functions and asymmetries which are involved in scattering of polarized beams by polarized targets will be determined by the spin structure of the nucleon wave function at $P_z=\infty$. As has been emphasized by current algebraists, the nucleon wave function at $P_z=\infty$ does not belong to a pure representation ($\underline{56}$, $L=0^+$) of SU(6) [or, for a definite helicity $h=+\frac{1}{2}$, to the representation

(6,3) of SU(3)⊗SU(3)], but it has a more complicated structure in terms of a linear combination of SU(3)⊗SU(3) irreducible representations. This phenomenon has been called configuration mixing.[7] The configuration mixing lowers the static SU(6) value $|G_A/G_V|=\frac{5}{3}$ in the direction suggested by experiment.

We have shown elsewhere[8] that such a complicated structure at $P_z=\infty$ comes from the simple hypothesis that, even in the hadron center-of-mass frame, quarks are endowed with highly relativistic velocities. This hypothesis is suggested by the harmonic oscillator quark model and the mean level spacing of the hadron spectrum. More precisely, we have assumed that the hadrons can be described by weakly bound states of quarks of effective mass $m_q \simeq \frac{1}{3}M_N$ animated by a relativistic motion. Consequently, we have proposed to replace, in the ($\underline{56}$, $L=0^+$) wave function, Pauli spinors by free Dirac spinors. By boosting at $P_z=\infty$, the Wigner rotations of quark spins provide the complicated structure in terms of SU(3) ⊗SU(3) representations. Moreover, we have obtained a semiquantitative agreement with a phenomenological determination of the mixing parameters by Buccella, De Maria, and Lusignoli.[9] For the nucleon of helicity $+\frac{1}{2}$, the dominant representations are found to be $|(6,3)_8, L_z=0\rangle$, $|(3,\bar{3})_8, L_z=+1\rangle$, and $|(8,1), L_z=-1\rangle$.

Independently, and following an algebraic approach based on the free-quark current algebra, Melosh[10] has also established the general connection between the configuration mixing and the Wigner rotations of quark spins and its consequences for axial-vector matrix elements, as already suggested in earlier papers of Gell-Mann.[11]

Since we are confident in the spin structure of the nucleon wave function which we have proposed, we think it interesting to predict its consequences for the structure functions and asymmetries for deep-inelastic scattering of a polarized lepton by a polarized nucleon. We will express the predictions in terms of the mixing parameters.

The plan of this paper is as follows. In Sec. II we recall the nucleon wave function at rest, and we show how the Lorentz contraction ensures the scaling as we boost it to $P_z=\infty$. In Sec. III we pay attention to the spin structure of the wave function, and we show how configuration mixing appears as a consequence of the Wigner rotations of quark spins. In Sec. IV we calculate the structure functions and the asymmetries assuming the nucleon to be composed simply of three valence quarks. Section V is devoted to the comparison of the model with previous works.

II. LORENTZ CONTRACTION AND SCALING

On the basis of the harmonic oscillator quark model of hadrons,[12] we adopt the following SU(6) spatial wave function for the nucleon *at rest* (in configuration space):

$$\hat{\psi}_{\vec{P}=0}(\{\vec{r}_i, t_i\}) = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\frac{1}{(2\pi)^{3/2}} \times \exp\left(-iM_N\tfrac{1}{3}\sum_{i=1}^{3} t_i\right)\psi_{\vec{P}=0}(\{\vec{r}_i\}). \tag{2.1}$$

In (2.1), χ' and χ'', and ϕ' and ϕ'' are, respectively, spin and isospin wave functions[13] in such a way that the whole wave function is fully symmetric, corresponding to the ($\underline{56}$, $L=0^+$) supermultiplet. We will come again to the spin structure of this wave function in Sec. III. $\psi_{\vec{P}=0}(\{\vec{r}_i\})$ is the spatial internal wave function for the nucleon at rest, which we assume to be independent of the internal relative time,

$$\psi_{\vec{P}=0}(\{\vec{r}_i\}) = N_0 \exp\left[-\frac{\sum_{i<j}(\vec{r}_i-\vec{r}_j)^2}{6R^2}\right]. \tag{2.2}$$

We assume that the time dependence of the wave function *at rest* comes only from the over-all factor

$$\exp(-iM_N T), \quad T=\tfrac{1}{3}\sum_{i=1}^{3} t_i, \tag{2.3}$$

M_N being the nucleon mass.

In the quark-parton model, the nucleon structure functions are calculated by adding the structure functions for each constituent (incoherent scattering), which depend on $\delta(x-x_i)$, since it is assumed to be pointlike, and by taking the mean value of this sum over the nucleon wave functions at $P_z=\infty$. x_i is the fraction of the longitudinal momentum $P\simeq|P_z|=\infty$ carried by the interacting quark.

Let us now see that if we build up the nucleon wave function at $P_z=\infty$ by applying a pure Lorentz transformation to the nucleon wave function at rest (2.1), the wave function in momentum space depends on longitudinal variables only through the fractions x_i of longitudinal momentum.

The wave function at large momentum has the form

$$\hat{\psi}_{P_z=\infty}(\{\vec{r}_i, t_i\}) = N\prod_{i=1}^{3} S_i(\vec{\beta})\frac{1}{\sqrt{2}}(\chi'\phi'+\chi''\phi'')\frac{1}{(2\pi)^{3/2}}\exp\left[-i\frac{M_N}{3}\sum_{i=1}^{3}\frac{(t_i-\beta r_{iz})}{(1-\beta^2)^{1/2}}\right]\psi_{\vec{P}=0}\left(\left\{\frac{r_{iz}-\beta t_i}{(1-\beta^2)^{1/2}}, \vec{r}_{i\perp}\right\}\right), \tag{2.4}$$

where $\vec{\beta}$ is the nucleon center-of-mass velocity, and $S_i(\vec{\beta})$ are boost matrices which act on quark spinors. We will discuss the effect of these boost matrices on quark spins in the following section.

Now, to take the mean value of an operator we integrate over a surface $t_i = \text{const} = T$. Since $\psi_{\vec{P}=0}$ depends only on the relative coordinates, we get, for $t_i = T$,

$$\tilde{\psi}_{P_z=\infty}(\{\vec{r}_i, t_i = T\}) = N\prod_{i=1}^{3} S_i(\vec{\beta})\,\frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\,\frac{1}{(2\pi)^{3/2}}\exp(-iET)\exp(i\vec{P}\cdot\vec{R})\psi_{\vec{P}=0}\left(\left\{\frac{r_{iz}}{(1-\beta^2)^{1/2}}, \vec{r}_{i\perp}\right\}\right). \tag{2.5}$$

In (2.5), $\vec{R} = \frac{1}{3}\sum_{i=1}^{3}\vec{r}_i$ and T are the center-of-mass position and time, and $\vec{P}$ and E are the center-of-mass momentum and energy of the nucleon. N is a normalization constant fixed by the condition that the charge of the bound state is the same independently of the state of motion.

Now, performing a Fourier transform of the Lorentz-contracted internal wave function

$$\psi_{\vec{P}=0}\left(\left\{\frac{r_{iz}}{(1-\beta^2)^{1/2}}, \vec{r}_{i\perp}\right\}\right) \tag{2.6}$$

we get, in momentum space (since the internal wave function depends only on relative momenta),

$$\tilde{\psi}_{\vec{P}=0}(\{p_{iz}(1-\beta^2)^{1/2}, \vec{p}_{i\perp}\})\,, \tag{2.7}$$

where $\{\vec{p}_i\}$ are the momenta of the quarks in the infinite-momentum frame $(\sum_{i=1}^{3}\vec{p}_i = \vec{P})$. Now, since $P_z \to \infty$, we can write

$$(1-\beta^2)^{1/2} = \left(1 - \frac{P^2}{E^2}\right)^{1/2} = \frac{M_N}{E} \simeq \frac{M_N}{P}\,. \tag{2.8}$$

We see that the internal wave function (2.7) depends only on the transverse momenta $\vec{p}_{i\perp}$ and on fractions $x_i = p_{iz}/P$ of the longitudinal momentum[14]

$$\tilde{\psi}_{\vec{P}=0}\left(\left\{p_{iz}\frac{M}{P}, \vec{p}_{i\perp}\right\}\right) = \tilde{\psi}_{\vec{P}=0}(\{M_N x_i, \vec{p}_{i\perp}\}), \tag{2.9}$$

$\tilde{\psi}(\{\vec{p}_i\})$ being the Fourier transform of the internal spatial wave function in configuration space $\psi(\{\vec{r}_i\})$, it is normalized according to

$$\int |\tilde{\psi}(\{x_i, \vec{p}_{i\perp}\})|^2 \prod_{i=1}^{3} d\vec{p}_{i\perp}\, dx_i\, \delta(1 - \textstyle\sum_{i=1}^{3} x_i) = 1\,. \tag{2.10}$$

Let us now discuss the effect of the Lorentz boosts on the quark spins.

III. WIGNER ROTATIONS OF QUARK SPINS AND CONFIGURATION MIXING

In this section we will describe the spin structure of the nucleon wave function at $P_z = \infty$ which is involved in the determination of the asymmetries which occur with polarized targets.

If the nucleon is assigned to the usual representation $(\underline{56}, L = 0^+)$ of SU(6), this means that, for instance, the nucleon of helicity $+\frac{1}{2}$ belongs to the representation $(6, 3)_8$ of SU(3)⊗SU(3), i.e., two quarks have the helicity aligned to the nucleon helicity, and one quark has the opposite helicity. As emphasized by many authors on phenomenological grounds, the nucleon wave function is in fact a superposition of various SU(3)⊗SU(3) irreducible representations (configuration mixing),[7,9]

$$|N, h = +\tfrac{1}{2}\rangle = \alpha|(6,3)_8, L_z = 0\rangle + \alpha'|(3,6)_8, L_z = +1\rangle + \beta|(3,\bar{3})_8, L_z = +1\rangle + \beta'|(\bar{3},3)_8, L_z = 0\rangle + \gamma|(8,1), L_z = -1\rangle + \gamma'|(1,8), L_z = +2\rangle,$$
$$\alpha^2 + \alpha'^2 + \beta^2 + \beta'^2 + \gamma^2 + \gamma'^2 = 1. \tag{3.1}$$

If configuration mixing is sizable, it will have consequences for the asymmetries, because the various representations in (3.1) indicate the relative alignment between the nucleon and quark helicities. For instance, if $(3,\bar{3})_8$ is present, it means that there is a definite probability of having two quarks with opposite helicity to the nucleon helicity and one quark with the same helicity as the nucleon. Of course, there is an orbital angular momentum between quarks which ensures a resulting helicity $+\frac{1}{2}$ for the nucleon.

Configuration mixing is indeed important. Buccella, De Maria, and Lusignoli have written sum rules which relate the mixing parameters to the baryon-baryon-pseudoscalar-meson coupling constants.[9] We reproduce in Table I the results of their phenomenological analysis.

In Ref. 8 we have given a simple interpretation of this configuration mixing at $P_z = \infty$ as a direct effect of the presence of relativistic quark velocities inside hadrons. A number of features of the hadron spectrum lead in a natural way to the assumption that the quarks inside hadrons are animated by a relativistic motion, even if the hadron is at rest.

As emphasized by Bogoliubov[15] and Gell-Mann,[7] relativistic corrections reduce the static SU(6) value of the nucleon axial-vector coupling to a value $\frac{5}{3}(1-2\delta)$, where δ is a positive quantity related to the norm of the small components in the quark Dirac spinors. On the other hand, in the harmonic oscillator quark model, the mean level spacing of the hadron spectrum as well as the mean value of the convective term of the quark electromagnetic current[16] suggest also a relativistic internal motion for quarks.

Then, the quarks seem to behave like light quasi-free particles in a smooth potential, endowed with large relativistic velocities. Consequently, we have proposed in Ref. 8 to modify the spin part of the nucleon wave function by replacing the Pauli spinors $\chi_s(i)$ used in the usual formalism, with free Dirac spinors, $\vec{p}_i$ being now the quark momenta in the rest frame,

$$u_s(i)=\begin{pmatrix}\chi_s(i)\\ \mu_q\vec{\sigma}_i\cdot\vec{p}_i\,\chi_s(i)\end{pmatrix}, \tag{3.2}$$

where $\mu_q \simeq 3/2M_N$ is the normal quark magnetic moment; we adopt this simple form in order to reproduce the right order of magnitude for the nucleon total magnetic moment.

TABLE I. Prediction of the naive quark model for the mixing parameters (see Ref. 8) and phenomenological bounds obtained from Adler-Weisberger-type sum rules by Buccella, De Maria, and Lusignoli (Ref. 9).

Mixing parameters	Prediction of the naive quark model		Phenomenological analysis
$\alpha^2(6,3)_8$	0.57	$\alpha^2+\alpha'^2=0.59$	$\alpha^2+\alpha'^2=0.59$
$\alpha'^2(3,6)_8$	0.02		
$\beta^2(3,\bar{3})_8$	0.21	$\beta^2+\beta'^2=0.24$	$0.26\le\beta^2+\beta'^2\le 0.41$
$\beta'^2(\bar{3},3)_8$	0.03		
$\gamma^2(8,1)$	0.14	$\gamma^2+\gamma'^2=0.16$	$\gamma^2+\gamma'^2\le 0.15$
$\gamma'^2(1,8)$	0.02		

The spin wave functions χ' and χ'' written in (2.1) are then understood in terms of the Dirac spinors (3.2).[17] Let us now see what is the spin structure of the wave function at $P_z=\infty$. The effect of the matrix boost $S_i(\vec{\beta})$ on the Dirac spinor (3.2) can be written as a product of a Wigner rotation and a boost which takes the Pauli quark spinor to the helicity spinor at $P_z=\infty$,

$$\frac{1}{\sqrt{2}}\cosh\frac{\omega}{2}\left[1+\mu_q p_{iz}(1-\beta^2)^{1/2}+\mu_q\beta_i\Sigma_i^{(+)}p_i^{(-)}+\mu_q\beta_i\Sigma_i^{(-)}p_i^{(+)}\right]\begin{pmatrix}\chi_i\\ \sigma_{iz}\chi_i\end{pmatrix}, \tag{3.3}$$

where $\tanh\omega=\beta$. The operators $\beta_i\Sigma_i^{(+)}$ and $\beta_i\Sigma_i^{(-)}$ reverse the quark helicity. If the internal quark momenta were negligible compared with its mass, i.e., $\mu_q\langle\vec{p}_i^{\,2}\rangle^{1/2}\simeq 0$, the Wigner rotation in (3.3) would be very small and the nucleon of helicity $+\frac{1}{2}$ would belong to the representation $(6,3)_8$. But since the internal velocities are important, the terms which reverse the helicity in (3.3) are of the same order as the first term. Then, the spin structure is more complex. In fact, by taking the product of the Wigner rotations for the three quarks and resolving the resulting nucleon state in definite $SU(3)\otimes SU(3)$ representations, we have shown in Ref. 8 that all the representations in the expansion (3.1) are present. The dominant representations are in fact $|(6,3)_8, L_z=0\rangle$, $|(3,\bar{3}), L_z=+1\rangle$, and $|(8,1), L_z=-1\rangle$.

Taking $\mu_q=3/2M_N$ and $R^2=11\ \text{GeV}^{-2}$ we have obtained the values of Table I for the mixing parameters, in semiquantitative agreement with the values determined by Buccella, De Maria, and Lusignoli.

With these values we get, for the axial-vector coupling of the nucleon,

$$\left|\frac{G_A}{G_V}\right|=\tfrac{5}{3}(\alpha^2-\alpha'^2)+(\beta^2-\beta'^2)+(\gamma^2-\gamma'^2)$$
$$=1.20. \tag{3.4}$$

Let us recall that in our model we have for the D and F axial couplings in $\frac{1}{2}^+$ baryon semileptonic decays

$$\frac{D}{F+D}=\frac{3}{5}, \tag{3.5}$$

in good agreement with experiment.

IV. ASYMMETRIES

In calculating the asymmetries we will assume the nucleon to be simply composed of three valence quarks. We know that this scheme is too simple, since for $x\to 0$ the $q\bar{q}$ pairs have an important diffractive contribution and that the gluons carry an important part of the nucleon longitudinal momentum. However, in order to make simple predictions for the polarization effects we think it better to keep the simplest model for the moment.

Assuming that the particles are polarized along the direction of motion, the asymmetry is defined by[1]

$$\frac{d\sigma^{\uparrow\downarrow}-d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow}+d\sigma^{\uparrow\uparrow}}, \tag{4.1}$$

where $d\sigma^{\uparrow\uparrow}$ is the cross section when the spins of the lepton (electron or muon) and the nucleon are parallel, and $d\sigma^{\uparrow\downarrow}$ when the polarizations are antiparallel. The spin dependence of the differential cross section is given by[1]

$$\frac{d^2\sigma^{\uparrow\downarrow}}{d\Omega\, dE_2}-\frac{d\sigma^{\uparrow\uparrow}}{d\Omega\, dE_2}=\frac{\alpha^2}{\pi}\frac{E_2}{M_N E_1}\frac{1}{Q^2}[(E_1+E_2\cos\theta)\,d(q^2,\nu)+(E_1-E_2\cos\theta)\,g(q^2,\nu)]. \tag{4.2}$$

In (4.2), the lepton mass is neglected, E_1 and E_2 are the initial and final lepton energies in the laboratory system, and θ is the lepton scattering angle.

The structure functions $d(q^2,\nu)$ and $g(q^2,\nu)$ are defined from the decomposition of the current commutator between polarized proton states of covariant spin ξ_μ,

$$iW^A_{\mu\nu}(P,q)=\tfrac{1}{2}\int d^4x\, e^{iq\cdot x}\{\langle P,\xi|[j_\mu(x),j_\nu(0)]|P,\xi\rangle - (\xi\to-\xi)\}, \tag{4.3}$$

into covariants,

$$W^A_{\mu\nu}=\epsilon_{\mu\nu\rho\sigma}q^\rho\xi^\sigma d(q^2,\nu)+(\xi\cdot q)\epsilon_{\mu\nu\rho\sigma}q^\rho P^\sigma g(q^2,\nu). \tag{4.4}$$

Let us see that we get, in our model,

$$g(q^2,\nu)=0. \tag{4.5}$$

The nucleon wave function at $P_z=\infty$ is given by a combination of free-quark helicity spinors, as indicated in (3.3). The polynomials indicate the relative alignment between the nucleon and the quark helicities. Computing the commutator (4.4) for a free-quark Dirac spinor, we get (assuming a minimal electromagnetic coupling, i.e., no anomalous magnetic moment)

$$W^A_{\mu\nu}(i)=\frac{(2\pi)^4}{2m_q}\delta^4(p_i+q-p_i')\epsilon_{\mu\nu\rho\sigma}q^\rho\xi^\sigma, \tag{4.6}$$

where p_i and p_i' are the initial and final quark momenta. For a quark we find then a structure function of the type $d(q^2,\nu)$. The polynomials in the transverse momenta do not change this form. Therefore we get $g(q^2,\nu)=0$.

The asymptotic form of the spin-dependent cross section in the lepton-proton center-of-mass system is given by[1]

$$\tfrac{1}{2}(d\sigma^{\uparrow\downarrow}-d\sigma^{\uparrow\uparrow})\underset{P\to\infty}{\simeq}\alpha^2P^{-1}(\pi M_N Q^2)^{-1}\times[d(q^2,\nu)+M_N\nu g(q^2,\nu)]\,d^3k_2 \tag{4.7}$$

and the spin-dependent cross section for the lepton–point-quark system,

$$\tfrac{1}{2}(d\sigma_i^{\uparrow\downarrow}-d\sigma_i^{\uparrow\uparrow})=2\alpha^2P^{-1}e_i^{\,2}(M_NQ^2\nu)^{-1}\delta(x-x_i)\,d^3k_2. \tag{4.8}$$

Then, to calculate the structure function $d(q^2,\nu)$ we must simply assume incoherent scattering and take into account the relative alignment between the quarks and nucleon helicities given by the decomposition (3.1).

In the Bjorken limit, we get, in our model,[18]

$$\frac{1}{2\pi}[\nu d(q^2,\nu)]_{p,n}=A^{p,n}G(x), \tag{4.9}$$

where $G(x)$ is the probability for a quark to have a fraction x of the longitudinal momentum. We find, for the proton and the neutron,

$$A^p=\tfrac{5}{9}(\alpha^2-\alpha'^2)-\tfrac{1}{9}(\beta^2-\beta'^2)+(\gamma^2-\gamma'^2), \tag{4.10}$$

$$A^n=-\tfrac{4}{9}(\beta^2-\beta'^2)+\tfrac{2}{3}(\gamma^2-\gamma'^2). \tag{4.11}$$

In the calculation of these relations we have used Table II and the matrix elements

$$\langle\phi''^{+}_{123}|\hat e_3^{\,2}|\phi''^{+}_{123}\rangle=\tfrac{2}{9},\quad \langle\phi''^{0}_{123}|\hat e_3^{\,2}|\phi''^{0}_{123}\rangle=\tfrac{1}{3},$$
$$\langle\phi'^{+}_{123}|\hat e_3^{\,2}|\phi'^{+}_{123}\rangle=\tfrac{4}{9},\quad \langle\phi'^{0}_{123}|\hat e_3^{\,2}|\phi'^{0}_{123}\rangle=\tfrac{1}{9}, \tag{4.12}$$

$$\langle\phi''^{+}_{231}|\hat e_3^{\,2}|\phi''_{231}\rangle=\langle\phi''_{312}|\hat e_3^{\,2}|\phi''_{312}\rangle=\tfrac{1}{4}\langle\phi''_{123}|\hat e_3^{\,2}|\phi''_{123}\rangle+\tfrac{3}{4}\langle\phi'_{123}|\hat e_3^{\,2}|\phi'_{123}\rangle, \tag{4.13}$$

$$\langle\phi'_{231}|\hat e_3^{\,2}|\phi'_{231}\rangle=\langle\phi'_{312}|\hat e_3^{\,2}|\phi'_{312}\rangle=\tfrac{3}{4}\langle\phi''_{123}|\hat e_3^{\,2}|\phi''_{123}\rangle+\tfrac{1}{4}\langle\phi'_{123}|\hat e_3^{\,2}|\phi'_{123}\rangle.$$

In Table II we have expressed the SU(3) ⊗ SU(3) representations in terms of quark helicity spinors and the SU(3) quark wave functions ϕ' and ϕ''. ϕ''_{ijk} and ϕ'_{ijk} are, respectively, symmetric and antisymmetric with regard to i, j. (ij, k) means that quarks i and j have helicity $+\frac{1}{2}$ and quark k has helicity $-\frac{1}{2}$. Note that relations (4.10) and (4.11) are quite general and depend only on the hypothesis that the nucleon wave function can be written as a superposition of the SU(3) ⊗ SU(3) irreducible representations allowed by the quark model.

If configuration mixing were absent, we would obtain the following proton and neutron asymmetries:

TABLE II. SU(3)⊗SU(3) representations for the nucleon of helicity $+\frac{1}{2}$.

$(6,3)_8$	$\frac{1}{\sqrt{3}}\sum_P \phi''_{123}(12,3)\vert L_z=0\rangle$
$(3,6)_8$	$\frac{1}{\sqrt{3}}\sum_P \phi''_{123}(3,12)\vert L_z=+1\rangle$
$(3,\bar{3})_8$	$\frac{1}{\sqrt{3}}\sum_P \phi'_{123}(3,12)\vert L_z=+1\rangle$
$(\bar{3},3)_8$	$\frac{1}{\sqrt{3}}\sum_P \phi'_{123}(12,3)\vert L_z=0\rangle$
$(8,1)$	$\frac{1}{\sqrt{2}}[\phi''_{123}\vert L_z=-1\rangle+\phi'_{123}\vert L_z=-1\rangle](123,0)$
$(1,8)$	$\frac{1}{\sqrt{2}}[\phi''_{123}\vert L_z=+2\rangle+\phi'_{123}\vert L_z=+2\rangle](0,123)$

$$A^p=\tfrac{5}{9},\quad A^n=0\ . \tag{4.14}$$

Let us now see that we can write, from (4.10) and (4.11), the asymmetries in terms of the F, D couplings of the low-lying octet axial-vector matrix elements.

In terms of the mixing parameters, F and D read[19]

$$F=\tfrac{2}{3}(\alpha^2-\alpha'^2)+(\gamma^2-\gamma'^2)\ ,$$
$$D=(\alpha^2-\alpha'^2)+(\beta^2-\beta'^2)\ . \tag{4.15}$$

From (4.15) we get

$$\tfrac{2}{3}D-F=\tfrac{2}{3}(\beta^2-\beta'^2)-(\gamma^2-\gamma'^2) \tag{4.16}$$

so that A^p and A^n can be written in terms of F and D (Ref. 20):

$$A^p=\tfrac{1}{3}(F+D)-\tfrac{2}{3}(\tfrac{2}{3}D-F),\quad A^n=-\tfrac{2}{3}(\tfrac{2}{3}D-F)\ . \tag{4.17}$$

The neutron asymmetry is then proportional to the deviation of F/D from the SU(6) result $F/D=\frac{2}{3}$. Since $G(x)$ in (4.9) is normalized to one, the difference A^p-A^n satisfies, as expected, the Bjorken sum rule,

$$A^p-A^n=\frac{1}{3}\left|\frac{G_A}{G_V}\right|\simeq 0.40\ . \tag{4.18}$$

We obtain a large positive asymmetry for the proton and a small asymmetry for the neutron. Taking $\alpha=D/(F+D)\simeq 0.67$, we get a small *negative* neutron asymmetry.

In our specific model, where the spin part of the nucleon wave function at rest is written in terms of free Dirac spinors, we preserve the static SU(6) result $F/D=\frac{2}{3}$,[8] so that we predict

$$A^p=\frac{1}{3}\left|\frac{G_A}{G_V}\right|\ ,\quad A^n=0\ . \tag{4.19}$$

V. COMPARISON WITH PREVIOUS WORKS

As we have pointed out, Kuti and Weisskopf,[1] Gourdin,[2] and Close, Gilman, and Karliner[3] have estimated the asymmetry for proton and neutron targets assuming that the nucleon is assigned to the ($\underline{56}$, $L=0^+$) representation of the static SU(6). In this way they get the results (4.14). We have shown that the presence of configuration mixing changes the situation significantly.

Our work differs from a quark-parton model calculation by Gourdin.[2] He obtains the relations

$$A^p=\tfrac{2}{3}f_1+\tfrac{1}{3}F+\tfrac{1}{9}D\ ,$$
$$A^n=\tfrac{2}{3}f_1-\tfrac{2}{9}D\ , \tag{5.1}$$

where F and D are the octet contributions and f_1 is a singlet part. Gourdin takes into account configuration mixing phenomenologically in the estimation of the octet contributions, since he takes F and D from experiment, satisfying in this way the Bjorken sum rule.

The singlet contribution f_1 is given by

$$f_1=\tfrac{1}{3}\sum_{j\sigma}\epsilon_j\sigma D_{j\sigma}\ , \tag{5.2}$$

where $\frac{1}{3}$ stands for the quark baryonic number, ϵ_j is equal to ±1 according to whether the parton is a quark or an antiquark. $D_{j\sigma}$ is the mean value, in the hadron, of the number of quarks of type j having a spin parallel ($\sigma=+1$) or antiparallel ($\sigma=-1$) to the hadron spin. Gourdin estimates f_1 by assuming the spin of the gluons to be uncorrelated with the nucleon spin, but he makes also an implicit assumption which is no longer valid when configuration mixing is present. The hypothesis is that the probability to have a quark polarized along the nucleon helicity is twice the probability of having a quark with helicity opposite to that of the nucleon. This means that he assumes, in evaluating the singlet part, that the nucleon of helicity $+\frac{1}{2}$ belongs to the $(6,3)_8$ representation of SU(3)⊗SU(3). In this way he obtains

$$f_1=\tfrac{1}{3}\ . \tag{5.3}$$

However, when configuration mixing is present, f_1 is no longer equal to $\frac{1}{3}$. We obtain, from (5.2) and Table II

$$f_1=\tfrac{1}{3}(\alpha^2-\alpha'^2)-\tfrac{1}{3}(\beta^2-\beta'^2)+(\gamma^2-\gamma'^2) \tag{5.4}$$

and in terms of F and D, f_1 is given by

$$f_1=-\tfrac{1}{3}D+F\ . \tag{5.5}$$

With the values of the mixing angles of Table I, we get

$$f_1 = 0.24 \ . \tag{5.6}$$

In conclusion, our method, starting from a definite spin wave function at $P_z = \infty$, takes into account automatically the complications of the configuration mixing for both the singlet and octet parts.

After this work was finished, J. Weyers pointed out to us a recent work by Close, Osborn, and Thomson[21] on the Melosh transformation. They examine, in particular, its consequences for the asymmetries. They find for the proton and neutron asymmetries,

$$A^{\gamma p} = \tfrac{5}{9} X \quad (-1 \leq X \leq +1), \quad A^{\gamma n} = 0 \ . \tag{5.7}$$

Note that the result $A^{\gamma n} = 0$ is the same one that we find and, as we have pointed out, it is related to the result $F/D = \frac{2}{3}$, which holds also in the Melosh approach (for the relation between our work and the one of Melosh, see Ref. 8). We can understand also the bounds for the proton asymmetry on the basis of our formulas. From expression (4.17) if we take the constraint $(\gamma^2 - \gamma'^2) = \frac{2}{3}(\beta^2 - \beta'^2)$ which comes from $F/D = \frac{2}{3}$, and *we leave the mixing parameters completely free,* we get indeed

$$-\tfrac{5}{9} \leq A^{p} \leq +\tfrac{5}{9}, \quad A^{n} = 0 \ . \tag{5.8}$$

If, more generally, we do not adopt the constraint $(\gamma^2 - \gamma'^2) = \frac{2}{3}(\beta^2 - \beta'^2)$, we get the larger following bounds:

$$-1 \leq A^{p} \leq +1, \quad -\tfrac{2}{3} \leq A^{n} \leq +\tfrac{2}{3} \ . \tag{5.9}$$

On the other hand, if we use the bounds on the mixing parameters obtained by Buccella *et al.*[9] from Adler-Weisberger-type sum rules, we get

$$-0.50 \leq A^{p} \leq +0.50, \quad -0.21 \leq A^{n} \leq +0.21 \ . \tag{5.10}$$

However, the relation between the mixing parameters and the axial-vector matrix elements of the low-lying octet restricts much more the asymmetries, as given by (4.17), a result very close to our theoretical prediction

$$A^{p} = \frac{1}{3}\left|\frac{G_A}{G_V}\right| , \quad A^{n} = 0 \ . \tag{5.11}$$

Concerning the work of Close *et al.*, we think that the parameter X cannot be considered as free. On the basis of our work, we think that it must be equal to the quantity η of Melosh, $|G_A/G_V| = \frac{5}{3}\eta$, $\eta \simeq 1/\sqrt{2}$. Then, the Bjorken sum rule is satisfied.

Note added in proof. Some recent works have reached conclusions similar to ours. J. Ellis and R. Jaffe [Phys. Rev. D 9, 1444 (1974)] have deduced our results (4.17) under the weaker form of sum rules for the scaling limit of $\nu d(q^2, \nu)$, using the free-quark current algebra, the quark parton model, and the assumption that the $\lambda\bar{\lambda}$ contribution to the $q\bar{q}$ sea is not polarized. L. M. Sehgal [Phys. Rev. D 10, 1663 (1974)], using the sum rules of the quark parton model, has related the mean values of the total quark spin S_z and the total quark orbital angular momentum L_z (for a given nucleon helicity $J_z = S_z + L_z$) to the F and D axial couplings of the low-lying $\frac{1}{2}^+$ octet. In a very complete work on the consequences of the Melosh transformation for polarized electroproduction, F. E. Close [Nucl. Phys. B80, 269 (1974)] has deduced the same sum rules as we and Fllis and Jaffe and studied the possible x dependence of the asymmetries. He assumes that a more general (but purely *ad hoc*) Melosh transformation introduces such x dependence.

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[1]J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[2]M. Gourdin, Nucl. Phys. B38, 418 (1972); A. J. G. Hey and J. Mandula, Phys. Rev. D 5, 2610 (1972); D. A. Dicus, R. Jackiw, and V. L. Teplitz, *ibid.* 4, 1733 (1971); D. Wray, Weizmann Institute report, 1972 (unpublished); C. Nash, Nucl. Phys. B31, 419 (1971).

[3]F. E. Close, F. J. Gilman, and I. Karliner, Phys. Rev. D 6, 2533 (1972).

[4]J. D. Bjorken, Phys. Rev. D 1, 1376 (1970).

[5]R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969); J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969).

[6]R. P. Feynman, in *High Energy Collisions,* edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1969), p. 237.

[7]R. Dashen and M. Gell-Mann, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1966,* edited by A. Perlmutter, J. Wojtaszek, E. C. G. Sudarshan, and B. Kurşunoğlu (Freeman, San Francisco, Calif., 1966).

[8]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 9, 2636 (1974).

[9]F. Buccella, M. De Maria, and M. Lusignoli, Nucl. Phys. B6, 430 (1968).

[10]H. J. Melosh, Caltech thesis (unpublished); Phys. Rev. D 9, 1095 (1974).

[11]M. Gell-Mann, in *Strong and Weak Interactions: Present Problems,* proceedings of the School of Physics "Ettore Majorana," 1966, edited by A. Zichichi (Academic, New York, 1967), p. 202; in *Hadrons and Their*

Interactions, proceedings of the School of the Physics "Ettore Majorana," 1967, edited by A. Zichichi (Academic, New York, 1968), p. 169.

[12]D. Faiman and A. W. Hendry, Phys. Rev. 173, 1720 (1968); 180, 1572 (1968).

[13]A. N. Mitra and M. Ross, Phys. Rev. 158, 1630 (1967).

[14]The Lorentz contraction of Gaussian wave function was introduced by A. L. Licht and A. Pagnamenta [Phys. Rev. D 2, 1150 (1970)] in order to describe the observed dipole shape of the nucleon electromagnetic form factors.

[15]P. N. Bogoliubov, Ann. Inst. Henri Poincaré 8, 163 (1968).

[16]L. A. Copley, G. Karl, and E. Obryk, Nucl. Phys. B13, 303 (1969).

[17]Note, incidentally, that the SU(6) classification of hadrons has not been changed. The nucleon belongs to a 56 supermultiplet of an SU(6) group. The point is that now the spin generators of such a group are no longer the sum of the Pauli matrices $\sum_i \vec{\sigma}(i)$, but rather the sum of the Foldy-Wouthuysen mean spin operators of the quarks, $\sum_i \vec{S}_i^{(M)}$.

[18]Note that we neglect the *small* x dependence which comes from the boost of the quarks spins through the term $\mu_q p_{iz}(1-\beta^2)^{1/2} \cong \mu_q M_N(x_i - \frac{1}{3})$. The most significant effect of the Wigner rotations is purely transverse.

[19]F. Buccella, H. Kleinert, C. A. Savoy, E. Celeghini, and E. Sorace, Nuovo Cimento 69, 133 (1970).

[20]If we assume that in general the factorization (4.9), with $A^{p,n}$ independent of x, does not hold and that each SU(3)⊗SU(3) representation provides a different x dependence, the relations (4.17) are still valid in the weaker form of sum rules for the Bjorken limit of $\nu d(q^2, \nu)$ integrated over x.

[21]F. E. Close, H. Osborn, and A. M. Thomson, Nucl. Phys. B77, 281 (1974).