Short-distance singularities in inclusive electroproduction*

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We examine the one-particle inclusive electroproduction process $\gamma(q)+h(p)\to h'(p')$ +anything in the scaling limit $\kappa\equiv q^2$, $\nu\equiv q\cdot p$, $\nu'\equiv q\cdot p'\to\infty$, with $\omega\equiv-\kappa/2\nu$, $\eta\equiv\nu'/\nu$, $u\equiv p\cdot p'$ fixed, using the slant singularity formalism we have recently developed. The exponent of the short-distance singularity on the light cone (which we call slant singularity) is calculated in canonical $\lambda\phi^4$ and quark-gluon theories by combining generalized dimensional analysis and multi-Regge analysis. Experimental consequences are discussed, in particular, multiplicities are evaluated for the various models. A comparison with our predictions in the case of e^+e^- annihilation is made.

I. INTRODUCTION

The process of one-particle inclusive electroproduction

$$\gamma(q)+h(p)\to h'(p')+\text{anything} \tag{1.1}$$

constitutes the next logical step, after the SLAC-MIT experiments,[1] in the experimental investigation of lepton-hadron interactions.[2] Thus the methods developed in treating electroproduction have been utilized to handle this process.[3-9]

An important feature of (1.1) is the presence of a detected single particle in the final state. We have developed an operator analysis[10] appropriate to this class of processes. We considered the quadrilocal operator [$J(x)$ and $S(y)$ are local operators]

$$\tilde{\mathfrak{A}}(x,0;y,z)\equiv R[J(x)S(y)]\,R[J(0)S^\dagger(z)] \tag{1.2}$$

on the light cone (LC) $x^2=0$. The presence of four operators enables $\tilde{\mathfrak{A}}(x,0;y,z)$ to develop singularities as $x^\mu\to 0$ *in addition* to those present as $x^2\to 0$. This is reflected in the presence of singularities in the Fourier transform

$$\mathfrak{A}(x,0;p')\equiv\int dy\,dz\,e^{ip'\cdot(y-z)}\tilde{\mathfrak{A}}(x,0;y,z) \tag{1.3}$$

as $\lambda'\equiv x\cdot p'\to 0$. These short-distance singularities are referred to as *slant singularities.* The power σ of the slant singularity $\lambda'^{-\sigma}$ was called the *slant*, and was shown in Ref. 10 for asymptotically scale-invariant theories to be a linear function of dim$S$, the scale dimension[11] of the operator $S(y)$. The analysis assumed that infrared contributions were nondominant.

This formalism enabled us[10] to treat the interesting electron-positron annihilation process

$$\gamma(q)\to h(p')+\text{anything}. \tag{1.4}$$

The main result was a connection between the h-particle multiplicity and the scale dimension of the source operator corresponding to h. For the particular example of the quark-gluon model with electric current $J^\mu=:\bar{\psi}Q\gamma^\mu\psi:$,

$$\delta=\dim\psi, \tag{1.5}$$

the *spinor* h-particle multiplicity $N_h(\kappa=q^2)$ satisfies

$$N_h(\kappa)\sim\begin{cases}(\sqrt{\kappa}\,)^{d+\delta-5}, & d+\delta-5>0\\ \ln\kappa, & d+\delta-5=0\\ \text{const}, & d+\delta-5<0\end{cases} \tag{1.6}$$

and the *scalar* H-particle multiplicity satisfies

$$N_H(\kappa)\sim\begin{cases}(\sqrt{\kappa}\,)^{D+\delta-11/2}, & D+\delta-\frac{11}{2}>0\\ \ln\kappa, & D+\delta-\frac{11}{2}=0\\ \text{const}, & D+\delta-\frac{11}{2}<0\end{cases} \tag{1.7}$$

where d and D are the dimensions of the spinor and scalar sources, respectively.

It is the purpose of this paper to perform the slant analysis on the process (1.1). The greater complexity of one-particle inclusive electroproduction necessitates a less straightforward analysis, and further dynamical issues come into play. More theoretical ingredients are used in arriving at the prediction of experimental quantities, which should make the process that much more interesting.

We summarize the kinematics and the LC analysis of the process in Sec. II. In Sec. III the slant singularities are investigated, and the analysis is completed with the aid of the Regge formalism. The experimental consequences are deduced in Sec. IV, and the various issues raised are discussed in Sec. V.

II. SCALING IN ONE-PARTICLE INCLUSIVE ELECTROPRODUCTION

The invariant variables in the process (1.1) are

$$\begin{aligned} \kappa &\equiv q^2 < 0\,, \\ \nu &\equiv q\cdot p\,, \\ \nu' &\equiv q\cdot p'\,, \\ u &\equiv p\cdot p'\,, \end{aligned} \tag{2.1}$$

and we shall also use the ratios

$$\begin{aligned} \omega &\equiv -\kappa/2\nu\,, \\ \omega' &\equiv -\kappa/2\nu'\,, \\ \eta &\equiv \nu'/\nu = \omega/\omega'\,. \end{aligned} \tag{2.2}$$

The variables satisfy the kinematical constraints

$$\begin{aligned} (q+p)^2 &> M^2\,, \\ (q+p-p')^2 &> 0\,, \end{aligned} \tag{2.3}$$

or

$$\kappa + 2\nu > 0\,, \tag{2.4}$$

and

$$\kappa + 2\nu - 2\nu' - 2u + M^2 + M'^2 > 0\,, \tag{2.5}$$

where $M^2 = p^2$, $M'^2 = p'^2$.

The inclusive reaction is specified[5] by the hadronic tensor

$$\mathcal{W}^{\mu\nu}(q,p,p') \equiv \int dx\, e^{iq\cdot x} \langle p|\mathcal{A}^{\mu\nu}(x,0;p')|p\rangle_{\text{conn}}\,, \tag{2.6}$$

where $\mathcal{A}^{\mu\nu}(x,0;p')$ is the same operator

$$\mathcal{A}^{\mu\nu}(x,0;p') = \int dy\, dz\, e^{ip'\cdot(y-z)} R[J^\mu(x)S(y)] \times R[J^\nu(0)S^\dagger(z)] \tag{2.7}$$

that occurs in e^+e^- annihilation, in which case the hadronic tensor is simply the Fourier transform of $\langle 0|\mathcal{A}^{\mu\nu}(x,0;p)|0\rangle$.

In the scaling limit $\kappa, \nu, \nu' \to \infty$, ω, η, u fixed, LC operator-product expansions (OPE's) determine the behavior of (2.6). We shall consider two renormalizable field-theoretic models: the $\lambda\phi^4$ model with current $j^\mu \equiv i : \phi^\dagger \overleftrightarrow{\partial}^\mu \phi :$ constructed from charged spin-0 constituent fields, and the quark-gluon model with current $J^\mu \equiv : \bar\psi Q \gamma^\mu \psi :$ constructed from charged spin-$\frac{1}{2}$ constituent fields.[12] The canonical LC OPE's are[13-15]

$$j^\mu(x)j^\nu(x') \underset{(x-x')^2\to 0}{\sim} -\Delta_+(x-x')\,\partial_x^\mu \partial_{x'}^\nu B(x,x')\,, \tag{2.8}$$

with

$$B(x,x') \equiv :\phi^\dagger(x)\phi(x') + \phi^\dagger(x')\phi(x):\,, \tag{2.9}$$

and

$$J^\mu(x)J^\nu(0) \underset{x^2\to 0}{\sim} i g^{\mu\nu\alpha\beta}\partial_\alpha \Delta_+(x) b_\beta(x,0)\,, \tag{2.10}$$

with

$$b_\alpha(x,0) \equiv \tfrac{1}{2} :[\bar\psi(x)\gamma_\alpha Q^2 \psi(0) - \bar\psi(0)\gamma_\alpha Q^2\psi(x)]:\,, \tag{2.11}$$

and

$$g^{\mu\nu\alpha\beta} = g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta} - g^{\mu\nu}g^{\alpha\beta}\,. \tag{2.12}$$

For our purposes it is sufficient to consider only the coefficient of $p^\mu p^\nu$ in $\mathcal{W}^{\mu\nu}$:

$$\mathcal{W}^{\mu\nu}(q,p,p') = \mathcal{W}_2(\nu,\omega,\eta,u)p^\mu p^\nu + \cdots\,. \tag{2.13}$$

If an average is taken over the angles of p', then $\mathcal{W}^{\mu\nu}$ can be expanded in the usual way in terms of the two scalar functions $\mathcal{W}_1$ and $\mathcal{W}_2$.[3] Inserting the expressions (2.8), (2.10), valid near the LC, into (2.6), we obtain for the boson model ($\lambda \equiv x\cdot p$)

$$\mathcal{W}_2 \to \frac{2}{2\pi\nu} \int d\alpha\, d\alpha'\, \alpha^2 \delta(\omega - \alpha - \alpha'\eta)\,\tilde B(\alpha,\alpha',u)\,,$$

with

$$\tilde B(\alpha,\alpha',u) = \int d\lambda\, d\lambda'\, e^{-i\alpha\lambda} e^{-i\alpha'\lambda'} B(\lambda,\lambda',u)\,, \tag{2.15}$$

$$B(\lambda,\lambda',u) = \int dy\, dz\, e^{ip'\cdot(y-z)} \langle p|R[\phi^\dagger(x)S(y)]R[\phi(0)S^\dagger(z)] + R[\phi^\dagger(0)S(y)]R[\phi(x)S^\dagger(z)]|p\rangle \Big|_{x^2=0}\,, \tag{2.16}$$

and for the fermion model

$$\mathcal{W}_2 \to \frac{-1}{2\pi\nu} \int d\alpha\, d\alpha'\, \delta(\omega - \alpha - \alpha'\eta)\,\alpha \tilde b(\alpha,\alpha',u)\,, \tag{2.17}$$

with

$$\tilde b(\alpha,\alpha',u) = \int d\lambda\, d\lambda'\, e^{-i\alpha\lambda} e^{-i\alpha'\lambda'} b(\lambda,\lambda',u)\,, \tag{2.18}$$

$$\int dy\,dz\,e^{ip'\cdot(y-z)}\,\tfrac{1}{2}\langle p|R[\bar{\psi}(x)S(y)]\gamma_\beta Q^2R[\psi(0)S^\dagger(z)]-R[\bar{\psi}(0)S(y)]\gamma_\beta Q^2R[\psi(x)S^\dagger(z)]|p\rangle\Big|_{x^2=0}=p_\beta b(\lambda,\lambda',u). \tag{2.19}$$

Thus, provided $B(\lambda,\lambda',u)$ and $b(\lambda,\lambda',u)$ exist, in both cases $\nu\mathcal{W}_2$ scales just as in usual electroproduction. The scaling law is

$$\nu\mathcal{W}_2(\nu,\omega,\eta,u)\to\mathcal{F}_2(\omega,\eta,u)\,, \tag{2.20}$$

and similarly,

$$\mathcal{W}_1(\nu,\omega,\eta,u)\to\mathcal{F}_1(\omega,\eta,u)\,. \tag{2.21}$$

The form of the LC OPE's gives[5,17]

$$\mathcal{F}_1(\omega,\eta,u)=\tfrac{1}{2}\omega\mathcal{F}_2(\omega,\eta,u) \tag{2.22}$$

in the fermion model, and

$$\mathcal{F}_1(\omega,\eta,u)=0 \tag{2.23}$$

in the boson model.

From (2.14)–(2.15) and (2.17)–(2.18), we easily find the representations

$$\mathcal{F}_2(\omega,\eta,u)\sim\int d\lambda\,e^{-i\omega\lambda}\left[\omega^2B(\lambda,\eta\lambda,u)+i\,\frac{2\omega^2}{\omega'}B^{(0,1)}(\lambda,\eta\lambda,u)-\frac{\omega^2}{\omega'^2}B^{(0,2)}\lambda,\eta\lambda,u)\right]\quad\text{(boson case)} \tag{2.24}$$

and

$$\mathcal{F}_2(\omega,\eta,u)\sim\int d\lambda\,e^{-i\omega\lambda}\left[\omega b(\lambda,\eta\lambda,u)+i\,\frac{\omega}{\omega'}b^{(0,1)}(\lambda,\eta\lambda,u)\right]\quad\text{(fermion case)}. \tag{2.25}$$

The notation $f^{(m,n)}(\lambda,\lambda',u)$ means $(\partial/\partial\lambda)^m(\partial/\partial\lambda')^n f(\lambda,\lambda',u)$.

Finally, the multiplicity is given asymptotically by

$$N(\nu,\omega)\sim\frac{1}{F_2(\omega)}\int d^4p'\,\delta(p'^2-M'^2)\mathcal{F}_2(\nu,\omega,\eta,u)\,, \tag{2.26}$$

where $F_2(\omega)$ is the usual electroproduction structure function.

III. SLANT ANALYSIS

We shall apply slant analysis to the quantities $B(\lambda,\lambda',u)$ and $b(\lambda,\lambda',u)$ in (2.16) and (2.19), respectively. We recapitulate the method.[10] The R-products are first expanded in an infinite series of operators:

$$R\phi(x)S(y)\to\sum_l (x-y)_R{}^{d_l-d-\delta}\,\mathcal{O}^{(l)}\left(\frac{x+y}{2}\right), \tag{3.1}$$

$$R\phi(0)S^\dagger(z)\to\sum_m(-z)_R{}^{d_m-d-\delta}\,\mathcal{O}^{\dagger(m)}\left(\frac{z}{2}\right), \tag{3.2}$$

where $\delta=\dim\phi$, $d=\dim S$, $d_l=\dim\mathcal{O}^{(l)}$, and R indicates the retarded $i\epsilon$ prescription. Their product is in turn expanded as

$$\mathcal{O}^{(l)}\left(\frac{x+y}{2}\right)\mathcal{O}^{\dagger(m)}\left(\frac{z}{2}\right)\to\sum_n\left(\frac{x+y-z}{2}\right)_W^{d_{lmn}-d_l-d_m}\mathcal{P}^{(lmn)}(0)\,, \tag{3.3}$$

where $d_{lmn}=\dim\mathcal{P}^{(lmn)}$, and W denotes the Wightman $i\epsilon$ prescription.

Substituting (3.1)–(3.3) into (2.16), say, we have

$$B(\lambda,\lambda',u)\to\int dy\,dz\,e^{ip'\cdot(y-z)}\sum_{lmn}(x-y)_R{}^{d_l-d-\delta}\left(\frac{x+y-z}{2}\right)_W^{d_{lmn}-d_l-d_m}(-z)_R{}^{d_m-d-\delta}\langle p|\mathcal{P}^{(lmn)}(0)|p\rangle\,. \tag{3.4}$$

This expansion is symbolic in that the Lorentz structure has not been made explicit. A sum over all possible contractions among the spacetime variables and $\mathcal{P}$, consistent with the dimensions, should be understood. The integrals have been explicitly evaluated in Ref. 10, and $B(\lambda,\lambda',u)$ exhibits slant singularities as $\lambda'\to 0$. The slant singularity would still be the same even if the region of integration in (3.4) were restricted to only some compact neighborhood of the origin. This would mean that we need only assume a finite radius of convergence in the short-distance expansions (3.1)–(3.3). Because only the connected part of the matrix element (2.6) is relevant, we shall seek here only those $\mathcal{P}^{(lmn)}$ that are operators. In the treatment of e^+e^- annihilation of Ref.

10, by contrast, only $\mathcal{P}^{(lmn)}=c_{lmn}I$ contribute to the vacuum expectation value.

In the present analysis, the Lorentz indices understood to be present in the OPE's (3.1)–(3.3) become crucial. Suppose that spin $\mathcal{P}^{(lmn)}=k$, so that

$$\langle p|\mathcal{P}^{(lmn)}_{\alpha_1\cdots\alpha_k}(0)|p\rangle=a_k^{(lmn)}p_{\alpha_1}\cdots p_{\alpha_k}+\cdots. \quad (3.5)$$

We write the c-number factor in (3.4) as

$$(x-y)^{d_l-d-\delta}\left(\frac{x+y-z}{2}\right)^{d_{lmn}-d_l-d_m}(-z)^{d_m-d-\delta}=\sum_{i,i'}\epsilon_{ii'k}(x,y,z)x^{\alpha_1}\cdots x^{\alpha_i}y^{\alpha_{i+1}}\cdots y^{\alpha_{i'}}z^{\alpha_{i'+1}}\cdots z^{\alpha_k}, \quad (3.6)$$

where $\epsilon_{ii'k}(x,y,z)$ is a scalar function, and

$$\dim\epsilon_{ii'k}(x,y,z)=2d+2\delta-d_{lmn}+k. \quad (3.7)$$

Thus (3.4) becomes

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}\sum_{lmn}\int dy\,dz\,e^{ip'\cdot(y-z)}\sum_k\sum_{i,i'}\epsilon_{ii'k}(x,y,z)\lambda^i(y\cdot p)^{i'-i}(z\cdot p)^{k-i'}a_k^{(lmn)}+\cdots. \quad (3.8)$$

We showed in Ref. 10 that the insertion of one factor of either $(y\cdot p)$ or $(z\cdot p)$ increases the slant of the integral by unity; more specifically, as $\lambda'\to 0$, it is equivalent to multiplying the integral by const$\times(\lambda/\lambda')$. Thus the *most singular* possible behavior of (3.8) as $\lambda'\to 0$ is given by

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}\lambda'^{-\sigma_L}\sum_{k=0}^{\infty}\Gamma_k\left(\frac{\lambda}{\lambda'}\right)^k, \quad (3.9)$$

where σ_L is the slant corresponding to the set of $\mathcal{P}^{(lmn)}$ with a particular *level* L ($\equiv$dim − spin) (see Ref. 15)

$$\sigma_L=d+\delta-\tfrac{1}{2}\,\text{lev}\mathcal{P}^{(lmn)}-3. \quad (3.10)$$

Thus the leading possible term in $B(\lambda,\lambda',u)$ as $\lambda'\to 0$ is u-independent. More generally, we can write

$$\int dy\,dz\,e^{ip'\cdot(y-z)}(x-y)^{d_l-d-\delta}\left(\frac{x+y-z}{2}\right)^{d_{lmn}-d_l-d_m}(-z)^{d_m-d-\delta}\underset{\lambda'\to 0}{\sim}\sum_{j,k}\mathfrak{F}_{jk}(\lambda')x^{\alpha_1}\cdots x^{\alpha_j}p'^{\alpha_{j+1}}\cdots p'^{\alpha_k}, \quad (3.11a)$$

where

$$\mathfrak{F}_{jk}(\lambda')\underset{\lambda'\to 0}{\sim}\text{const}\times(\lambda')^{-\sigma_L-j}, \quad (3.11b)$$

so that a more complete expression for $B(\lambda,\lambda',u)$ is

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}\sum_{lmn}\sum_{jk}a_k^{(lmn)}\mathfrak{F}_{jk}(\lambda')\lambda^j u^{k-j}. \quad (3.12)$$

Equation (3.12) exhibits a double expansion of $B(\lambda,\lambda',u)$ in λ and u valid near $\lambda'=0$. These expressions are, of course, only valid if $\sigma_L\geq 0$. This restriction unfortunately rules out any comparison with perturbation theory.[18]

A remarkable feature of (3.9) is the separation between the slanted factor $\lambda'^{-\sigma_L}$ and the infinite sum over k. The former depends only on the level of the operator $\mathcal{P}^{(lmn)}$; thus the *maximum* slant is carried by the fields $\mathcal{P}$ of the *lowest* level that can contribute to the matrix element (2.6). In contrast the rest of (3.9) is an infinite series in the variable (λ/λ'), whose sum can contain further slant singularities as $\lambda'\to 0$ not determined by dimensional analysis as applied to the slanted piece $\lambda'^{-\sigma_L}$.

To fix the dynamically determined slant singularities we incorporate further dynamical information supplied by Regge theory. In the usual way, it follows from (3.5), (3.8), and (3.9) that the sum

$$\sum_{k=0}^{\infty}\Gamma_k\Lambda^k\equiv\mathfrak{R}(\Lambda) \quad (3.13)$$

defines a function which satisfies

$$\mathfrak{R}(\Lambda)\underset{\Lambda\to\infty}{\sim}B_\alpha\Lambda^\alpha, \quad (3.14)$$

where α is the $t=0$ intercept of the leading Regge pole which can contribute.[19] Because of the occurrence of λ in the series (3.9) in the ratio (λ/λ'), we immediately deduce that

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}B_\alpha(\lambda')^{-\sigma_L-\alpha}\lambda^\alpha. \quad (3.15)$$

Note that the dependence of this leading possible singularity in λ' on the other variables λ and u is completely specified. Note also that we have implicitly assumed that the leading singularity in B can be obtained by the summation of the leading singularity of each term in (3.12). This is reasonable since nonleading contributions are less singular by integer powers.

The fermion analog of (3.9) is

$$b(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}\lambda'^{-\sigma_L}\sum_{k=0}^{\infty}\gamma_k\left(\frac{\lambda}{\lambda'}\right)^k, \tag{3.16}$$

and here Regge theory gives

$$\sum_{k=0}^{\infty}\gamma_k\Lambda^k\underset{\Lambda\to\infty}{\sim}b_\alpha\Lambda^{\alpha-1}, \tag{3.17}$$

so that

$$b(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}b_\alpha(\lambda')^{-\sigma_L-\alpha+1}\lambda^{\alpha-1}. \tag{3.18}$$

Thus the infinitely complicated slant behavior in the dynamical piece is parametrized for us in nature in the form of α, the leading Regge intercept. We then conclude that the slants σ_T are given by

$$\text{boson case: } \sigma_T=\sigma_L+\alpha, \tag{3.19}$$

$$\text{fermion case: } \sigma_T=\sigma_L+\alpha-1. \tag{3.20}$$

The lowest level of the operators that can contribute to this process is two,[20] as in electroproduction. Thus, for scalar sources in the boson theory,

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}B_\alpha\lambda'^{-\Sigma}\lambda^\alpha, \tag{3.21}$$

$$\Sigma=\Sigma_2+\alpha, \tag{3.22}$$

$$\Sigma_2=D+\Delta-4, \tag{3.23}$$

with $D=\dim S$ (scalar source), and $\Delta=\dim\phi$. In the fermion theory,

$$b(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}b_\alpha\lambda'^{-\Sigma}\lambda^{\alpha-1}, \tag{3.24}$$

$$\Sigma=\Sigma_2+\alpha-1, \tag{3.25}$$

$$\Sigma_2=D+\delta-\tfrac{9}{2} \tag{3.26}$$

for a scalar source S, and

$$b(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}b_\alpha\lambda'^{-\sigma}\lambda^{\alpha-1}, \tag{3.27}$$

$$\sigma=\sigma_2+\alpha-1, \tag{3.28}$$

$$\sigma_2=d+\delta-4 \tag{3.29}$$

for a spinor source s of $\dim s=d$, and $\delta=\dim\psi$. Thus for $\alpha=1$ (Pomeron), the slant is actually determined purely by dimensional analysis ($\sigma_T=\sigma_L$) in the fermion quark-gluon model, while it is increased by one above that value ($\sigma_T=\sigma_L+1$) in the boson $\lambda\phi^4$ model. All this assumes the absence of dominant infrared effects.

In the above discussions, we have assumed that the leading possible singularity (3.11b) actually accurs in (3.11a). This, of course, need not be the case. The most general possibility,

$$\mathfrak{F}_{jk}(\lambda')\underset{\lambda'\to 0}{\sim}\lambda'^{-\sigma_L-\tau_{jk}},\quad \tau_{jk}\le j \tag{3.30}$$

can be treated in a similar way. The case in which $\tau_{jk}\equiv 0$ is especially interesting. This would amount to the vanishing of the first j leading possible singularities for each j. Although it at first appears otherwise, this contingency need not rest on a remarkable accident. The singularities λ'^{-j} in (3.11a) arise from a large-distance mechanism in (3.11a). Although this does not represent a strict infrared effect in the sense discussed in Ref. 10 [e.g., it remains even if the large distance contributions in the integrations in (3.11a) are cut off], there is the suggestion that the resulting singularities $(\lambda')^{-j}$, $(\lambda')^{-j+1}$, ... lack the physical significance held by the $(\lambda')^{-\sigma_L}$ singularity.

These extra singularities are, in fact, absent in the parton model.[21] The methods employed in Ref. 22 and 23 can be easily used to show that in the parton model[24]

$$\int dy\,dz\,e^{ip'\cdot(y-z)}R[J_\mu(x)S(y)]R[J_\nu(0)S^\dagger(z)]$$

$$\underset{x^2\to 0}{\sim}\int dy\,dz\,e^{ip'\cdot(y-z)}\langle 0|R[\bar\psi(x)S(y)]\gamma_\mu R[\psi(0)S^\dagger(z)]|0\rangle :\bar\psi(x)\gamma_\nu\psi(0):+\cdots$$

$$\underset{x\to 0}{\sim}(\text{const})p_\mu(\lambda')^{-\sigma_0}:\bar\psi(x)\gamma_\nu\psi(0):+\cdots=p_\mu(\lambda')^{-\sigma_0}\sum_n x^{\alpha_1}\cdots x^{\alpha_n}\mathcal{O}_{\nu\alpha_1\cdots\alpha_n}(0)+\cdots, \tag{3.31}$$

where

$$\sigma_0=\begin{cases} d+\delta-2 & \text{(spinor source)}\\ D+\delta-\tfrac{5}{2} & \text{(scalar source)}\end{cases} \tag{3.32}$$

is the level zero slant appropriate to deep inelastic annihilation. Comparison with (3.11) reveals that $\mathfrak{F}_{jk}(\lambda')\underset{\lambda'\to 0}{\sim}\delta_{k0}\lambda'^{-\sigma_0}$ in the parton model.

In the general $\tau_{jk}=0$ case, (3.12) becomes

$$B(\lambda,\lambda',u)\underset{\lambda'\to 0}{\sim}\lambda'^{-\sigma_L}\sum_{k=0}^{\infty}\sum_{j=0}^{k}c_{jk}\lambda^j u^{k-j}\equiv\lambda'^{-\sigma_L}B(\lambda,u). \tag{3.33}$$

Now the coefficient $B(\lambda,u)$, defined by the above power series, is not determined. As before, Regge theory gives

$$B(\lambda,u)\underset{\lambda\to\infty}{\sim}\bar B_\alpha\lambda^\alpha, \tag{3.34}$$

so that the large-λ behavior is independent of u. In the fermion case,

$$b(\lambda, \lambda', u) \underset{\lambda' \to 0}{\sim} (\lambda')^{-\sigma_L} b(\lambda, u), \quad (3.35)$$

with

$$b(\lambda, u) \underset{\lambda \to \infty}{\sim} \bar{b}_\alpha \lambda^{\alpha-1}. \quad (3.36)$$

Comparison of (3.15) and (3.33) reveals that in the boson case with Pomeron ($\alpha = 1$) dominance, the leading possible slant singularity (3.15) is a power greater than that given by (3.33). On the other hand, in the fermionic case with Pomeron dominance, the leading possible singularity (3.18) is the same as that given by (3.35). Note also that (3.35) agrees with (3.18) for large λ.

IV. EXPERIMENTAL CONSEQUENCES

The slant singularities we have discussed correspond in momentum space to the singularities in $\mathfrak{F}_2$ as $\eta \to 0$ with $\omega \neq 0$ and u fixed. This statement follows simply by rewriting (2.14) and (2.17) as

$$\mathfrak{F}_2(\omega, \eta, u) \sim -\int d\lambda\, e^{-i\omega\lambda} B^{(2,0)}(\lambda, \eta\lambda, u) \quad \text{(boson case)}, \quad (4.1)$$

and

$$\mathfrak{F}_2(\omega, \eta, u) \sim -i\int d\lambda\, e^{-i\omega\lambda} b^{(1,0)}(\lambda, \eta\lambda, u) \quad \text{(fermion case)}. \quad (4.2)$$

An immediate consequence of the form of (3.9), (3.16) is that at the point $\eta = 0$, $\mathfrak{F}_2(\omega, \eta, u)$ has no dependence on u at all. This should be an experimentally meaningful statement.

Another interesting consequence is that the behavior of $\mathfrak{F}_2(\omega, \eta, u)$ in ω and η is also completely specified at that point. Substituting the slant singularities (3.21), (3.24), (3.27) into (4.1) or (4.2), we get, in the boson theory,

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} \text{const} \times \omega^{D+\Delta-3} \eta^{-(D+\Delta+\alpha-4)}, \quad (4.3)$$

and in the fermion theory,

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} \text{const} \times \omega^{D+\delta-9/2} \eta^{-(D+\delta+\alpha-11/2)} \quad (4.4)$$

for a scalar source, and

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} \text{const} \times \omega^{d+\delta-4} \eta^{-(d+\delta+\alpha-5)} \quad (4.5)$$

for a spinor source. These behaviors are in principle verifiable experimentally.

In the case of partonlike behavior ($\tau_{jk} = 0$), on the other hand, we would obtain from (3.33)–(3.36) that in the boson theory

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} H(\omega, u) \eta^{-(D+\Delta-4)}, \quad (4.6a)$$

$$H(\omega, u) \underset{\omega \to 0}{\sim} \text{const} \times \omega^{D+\Delta-\alpha-5}, \quad (4.6b)$$

and that in the fermion theory

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} H(\omega, u) \eta^{-(D+\delta-9/2)}, \quad (4.7a)$$

$$H(\omega, u) \underset{\omega \to 0}{\sim} \text{const} \times \omega^{D+\delta-\alpha-9/2} \quad (4.7b)$$

for a scalar source, and

$$\mathfrak{F}_2(\omega, \eta, u) \underset{\eta \to 0}{\sim} h(\omega, u) \eta^{-(d+\delta-4)}, \quad (4.8a)$$

$$h(\omega, u) \underset{\omega \to 0}{\sim} \text{const} \times \omega^{d+\delta-\alpha-4} \quad (4.8b)$$

for a spinor source. The behaviors (4.3)–(4.5) are easily distinguishable experimentally from the partonlike behaviors (4.6)–(4.8).

Given these formulas, it is a simple matter to evaluate the more easily measured multiplicities with (2.20). The calculation is best done with the Sudakov[25] parametrization

$$p' = \alpha q + \beta p + \pi_\perp, \quad (4.9)$$

so that

$$d^4p' = \nu\, d\alpha\, d\beta\, d^2\pi_\perp, \quad (4.10)$$

and $\pi_\perp$ is spacelike, with $\pi_\perp{}^2 = -p_\perp'^2$. The integration variables are related to the variables we use by

$$\alpha = \frac{1}{\nu(1+2\omega/\nu)}(u - \eta), \quad (4.11)$$

$$\beta = \frac{1}{1+2\omega/\nu}\left(\eta + \frac{2\omega}{\nu} u\right), \quad (4.12)$$

$$-\pi_\perp{}^2 = \kappa\alpha^2 + \beta^2 + 2\alpha\beta\nu - M'^2, \quad (4.13)$$

which become in the scaling limit

$$\alpha \sim \frac{1}{\nu}(u - \eta) + \mathcal{O}\left(\frac{1}{\nu^2}\right), \quad (4.14)$$

$$\beta \sim \eta + \frac{1}{\nu} 2\omega(u - \eta) + \mathcal{O}\left(\frac{1}{\nu^2}\right), \quad (4.15)$$

$$-\pi_\perp{}^2 \sim 2\eta u - \eta^2 - M'^2 + \frac{1}{\nu}[2\omega(u-\eta)^2 - 4\omega] + \mathcal{O}(1/\nu^2). \quad (4.16)$$

Thus the contribution to (2.26) in the scaling limit is evaluated by changing variables to be

$$N(\nu, \omega) \sim \frac{1}{F_2(\omega)} \int d\eta\, du\, \theta(2\eta u - \eta^2 - M'^2) \times \mathfrak{F}_2(\omega, \eta, u), \quad (4.17)$$

where the θ function expresses $p_\perp'^2 > 0$. It is easy to see from (2.5) that

$$MM' \leq u \leq \nu(1 - \omega - \eta) = u_{\max}. \quad (4.18)$$

The range of the η integration is determined by the θ function, which gives

$$\eta_0 = u - (u^2 - M'^2)^{1/2} < \eta < u + (u^2 - M'^2)^{1/2} . \quad (4.19)$$

For $\eta \to 0$,

$$\eta_0 \cong \frac{M'^2}{2u} \geq \frac{M'^2}{2u_{\max}} \cong \frac{M'^2}{2\nu(1-\omega)} . \quad (4.20)$$

Equation (4.17) then becomes

$$N(\nu, \omega) \sim \frac{1}{F_2(\omega)} \int_{M'^2/2\nu(1-\omega)}^{\eta_{\max}} d\eta \int_{MM'}^{\nu(1-\omega-\eta)} du\, \mathfrak{F}_2(\omega, \eta, u) . \quad (4.21)$$

Our framework gives no information on the behavior of $\mathfrak{F}_2$ as $u \to \infty$. We shall assume that the u integration is convergent. This is suggested by (4.16), where large u corresponds to large transverse momentum, and $p_\perp'^2$ integrations are generally taken to converge. If the assumption is violated, our results for multiplicities would be modified by factors from the u integration.[26] Using the behaviors (3.21), (3.24), (3.27), we have in the boson theory

$$N(\nu, \omega) \sim \frac{1}{F_2(\omega)} \omega^{D+\Delta-3} \begin{cases} [(1-\omega)\nu]^{D+\Delta+\alpha-5}, & D+\Delta+\alpha-5>0 \\ \ln[(1-\omega)\nu], & D+\Delta+\alpha-5=0 \\ \text{const}, & D+\Delta+\alpha-5<0 \end{cases} \quad (4.22)$$

and in the fermion theory

$$N(\nu, \omega) \sim \frac{1}{F_2(\omega)} \omega^{D+\delta-9/2} \begin{cases} [(1-\omega)\nu]^{D+\delta+\alpha-13/2}, & D+\delta+\alpha-\frac{13}{2}>0 \\ \ln[(1-\omega)\nu], & D+\delta+\alpha-\frac{13}{2}=0 \\ \text{const}, & D+\delta+\alpha-\frac{13}{2}<0 \end{cases} \quad (4.23)$$

for a scalar source, and

$$N(\nu, \omega) \sim \frac{1}{F_2(\omega)} \omega^{d+\delta-4} \begin{cases} [(1-\omega)\nu]^{d+\delta+\alpha-6}, & d+\delta+\alpha-6>0 \\ \ln[(1-\omega)\nu], & d+\delta+\alpha-6=0 \\ \text{const}, & d+\delta+\alpha-6<0 \end{cases} \quad (4.24)$$

for a spinor source. On the other hand, taking the behaviors (4.6)–(4.8), we have in the boson theory

$$N(\nu, \omega) \sim \frac{\overline{H}(\omega)}{F_2(\omega)} \begin{cases} [(1-\omega)\nu]^{D+\Delta-5}, & D+\Delta-5>0 \\ \ln[(1-\omega)\nu], & D+\Delta-5=0 \\ \text{const}, & D+\Delta-5<0 \end{cases} \quad (4.25)$$

and in the fermion theory

$$N(\nu, \omega) \sim \frac{\overline{H}(\omega)}{F_2(\omega)} \begin{cases} [(1-\omega)\nu]^{D+\delta-11/2}, & D+\delta-\frac{11}{2}>0 \\ \ln[(1-\omega)\nu], & D+\delta-\frac{11}{2}=0 \\ \text{const}, & D+\delta-\frac{11}{2}<0 \end{cases} \quad (4.26)$$

for a scalar source, and

$$N(\nu, \omega) \sim \frac{\overline{h}(\omega)}{F_2(\omega)} \begin{cases} [(1-\omega)\nu]^{d+\delta-5}, & d+\delta-5>0 \\ \ln[(1-\omega)\nu], & d+\delta-5=0 \\ \text{const}, & d+\delta-5<0 \end{cases} \quad (4.27)$$

for a spinor source. Here

$$\overline{H}(\omega) = \int du\, H(\omega, u),$$

$$\overline{h}(\omega) = \int du\, h(\omega, u).$$

Experimental determinations of these multiplicities for large ν and fixed ω can clearly distinguish among the various models and assumptions we have dealt with.[27] (We again recall our assumption of nondominant infrared effects.)

V. DISCUSSION

It is particularly interesting to compare these operator predictions with those made for e^+e^- annihilation. In the latter case, in the boson theory

$$N(\kappa) \sim (\sqrt{\kappa}\,)^{D+\Delta-5}, \tag{5.1}$$

and in the fermion theory

$$N(\kappa) \sim (\sqrt{\kappa}\,)^{D+\delta-11/2} \tag{5.2}$$

for a scalar source, and

$$N(\kappa) \sim (\sqrt{\kappa}\,)^{d+\delta-5} \tag{5.3}$$

for a spinor source. If we take the behaviors (4.22)–(4.24), then for the fermion theory, with $\alpha = 1$, the exponents are *identical* for the scalar and spinor multiplicities in both annihilation and electroproduction. On the other hand, for the boson theory, they differ by *one*. In reality, energy-momentum conservation requires that the exponents be $<\frac{1}{2}$ for (4.22)–(4.27), and <1 for (5.1)–(5.3). *Thus, if the same slant singularity contributes to both processes*, we have these interesting correlations of the multiplicities in the two processes in these two models. For example, in the fermion theory, logarithmic multiplicities in one process implies the same for the other. On the other hand, for the boson theory, for any multiplicity in electroproduction, the multiplicity in annihilation is necessarily finite. Of course this need not be the case; presumably if a given singularity in the boson theory gives rise to logarithmic multiplicities in annihilation, that singularity is automatically decoupled from electroproduction, thus conserving energy-momentum. It must be said that in this respect the fermion theory is more pleasant.

By contrast, if we take the partonlike behaviors (4.25)–(4.27), then the exponents are identical in all cases for annihilation and electroproduction (for fixed ω). In this case, electroproduction would be strictly similar to annihilation as a probe of source dimensions.

The one-particle inclusive electroproduction process thus provides another measuring device for the minimal dimensions of hadronic sources. The greater complexity of the process is seen to reveal more of the underlying theory in terms of the different occurrences of α in different theories. Experimental data on this process, as on annihilation, should thus illuminate many aspects of the dynamics of lepton-hadron interactions.

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[1]They measure $\gamma(q) + h(p) \to$ anything. See the review of H. Kendall, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N.Y., 1972), p. 247.

[2]For preliminary experimental results, see K. Berkelman, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (see Ref. 1), and in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 41.

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[11]As discussed in Ref. 10, it is actually the minimal dimension of S that is involved here.

[12]Precise definitions of these currents must of course be given. See for example R. A. Brandt and W.-C. Ng, Nuovo Cimento 13A, 1025 (1973).

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[16]Although these quantities will not exist in general in purely canonical theories, they will exist in the models of Ref. 12.

[17]These relations also obtain in parton models. See for example Refs. 3 and 4.

[18]This is in contrast to annihilation, where the relatively larger slants enable us to check our analysis in perturbation theory.

[19]The relevance of the ordinary two-body Regge trajectories in three-to-three amplitudes of the type relevant here has been discussed by A. H. Mueller, Phys. Rev. D 2, 2963 (1970). A sum over the leading Regge

trajectories (Pomeron, tensor mesons, etc.) should, here and elsewhere, be understood. Particular Regge residues, b_α, may of course vanish. Fixed poles may also be present, but these should lie below the leading Regge trajectories.

[20]This is equivalent to canonical scaling. See Refs. 13–15.

[21]See Refs, 3, 4, and 6 for the applications of the parton model to inclusive electroproduction.

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[24]Only the second-leading LC singularity contributes in the parton model. See Ref. 23.

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[26]It follows from (4.3)–(4.5) that the u cutoff is not present at $\eta=0$. If the cutoff is of the form $e^{-\eta\mu} \sim e^{-(M'^2+\vec{p}_\perp'^2)}$, then the multiplicities are actually increased by a power of ν. In this case, however, the multiplicities would probably violate energy conservation and so that leading slant would decouple, and our results would not change.

[27]One easily checks by our methods that contributions to the multiplicities from nonleading LC singularities are never more important than the leading contributions we have calculated.