Statistical description of the energy dependence of nonexchange processes*

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Motivated by recent unexpected results on $d\sigma/dt$ ($\pi^- p \to K^+\Sigma^-$) at 0°, we suggest that a statistical picture describes the high-energy as well as the low-energy s dependence of processes without t-channel exchange. A flattening (break) of slope in the fixed-angle s dependence is predicted at the same value of s for all nonexchange reactions with the same incoming channel. In particular, all nonexchange $\pi^- p \to c+d$, $K^- p \to c+d$ reactions will exhibit a break at $s \simeq 7$–8 GeV^2, $s \geq 10$ GeV^2, respectively. Some further predictions are also discussed.

It is well known that differential cross sections for nonexchange processes fall exponentially or as a high power of s at fixed angle. These reactions include

(1) processes with an exotic t-channel exchange, e.g., $\pi^- p \to K^+\Sigma^-$,[1a] $K^- p \to pK^-$,[1b]

(2) other processes without a known t-channel exchange, e.g., $\pi^- p \to \phi n$[1a] [these processes are probably less exotic than the previous ones ($\phi \to \rho\pi$ is possible)], and

(3) large-angle scattering, e.g., $\pi^- p \to \pi^- p$[1c] and $K^0 p \to \pi^+\Lambda$[1d] at 90°.

Once a single exchange is unlikely, we expect double exchange or an intermediate s-channel formation. Recently a rather surprising change of slope has been observed in the reaction $\pi^- p \to K^+\Sigma^-$ at 0°.[1a] It is found that up to $s \simeq 7$–8 GeV^2 $d\sigma/dt \sim s^{-9.6}$ and then the slope decreases sharply to the Regge-type behavior $s^{-1.1\pm0.8}$. A double exchange in the t channel[2] (ρK^* cut) cannot explain this flat behavior since it predicts an $s^{-2.5}$–s^{-3} behavior.

It is suggested here that an s-channel formation of an incoherent superposition of resonances describes both the low-energy and the high-energy s dependence of nonexchange processes. Although a detailed calculation of differential cross sections is impossible at the present stage (especially for inelastic processes) due to the lack of a reliable calculation of the density of states, definite predictions emerge from the above suggestion.

Consider the reaction $a+b \to c+d$ (d may also stand for "anything"), where $a+b$ is nonexotic. The differential cross section in statistical models is written as an incoherent superposition of a large number of intermediate resonances[3]:

$$\frac{d\sigma}{dt} = F(s, m_i, \theta)\frac{\rho_{abcd}(s)}{\rho_{ab}(s)\rho_{cd}(s)} . \tag{1}$$

ρ_{abcd} is the density of states coupled to both $a+b$ and $c+d$, ρ_{ij} is the density of states coupled to $i+j$, and F is a kinematic function which varies smoothly with the energy, the masses, and the scattering angle θ. For elastic scattering the known result[3,4] for "compound elastic" scattering

$$\frac{d\sigma}{dt}\,(\text{elastic}) = F\frac{1}{\rho_{ab}} \tag{2}$$

is recovered.

Since most models predict[5]

$$\rho \sim \sqrt{s}^{\,\alpha} e^{\beta\sqrt{s}}, \tag{3}$$

where α and β are constants, the steep s dependence observed at low energy is predicted; for detailed fits to large-angle elastic scattering see Refs. 3, 4, and 6.

The incoherent sum leading to a steep falloff at low s is expected to change at some higher energy to a coherent sum which leads to a flattening of the s dependence. The coherent contribution is expected to be dual to a Regge-pole exchange (if such an exchange is allowed) or a Regge-cut exchange in the t channel.[7] A change of slope may indeed be observed in $\pi^- p \to \pi^- p$ at 90° (see Ref. 1c) and may in principle be interpreted as a coherent effect.

The inconsistency of the recently observed $s^{-1.1\pm0.8}$ behavior of $\pi^- p \to K^+\Sigma^-$ for $s > 7$ GeV^2 (see Ref. 1a) with a Regge-cut picture contradicts the statistical coherent description. The statistical model is then limited to an extremely narrow energy range, at most for $4 \leq s \lesssim 7$ GeV^2 between the discrete resonances and the onset of flattening.

Since exotic trajectories with $\alpha(0) \simeq \alpha_\rho(0)$ are not found we suggest that both the low-energy and the high-energy s behavior of nonexchange processes are described by an incoherent sum of s-channel resonances. The break (change of slope) in the s dependence of those processes is therefore interpreted as a break in the density of states, thus requiring a density of states which at high s deviates from an exponential in s.

A flattening of the slope of ρ is known in nuclear physics[8] and has already been suggested in particle physics.[6] The physical reason for such a flattening is as follows: Suppose that the intermediate state is an excited "fireball." One can look at the excited levels as resulting from excitation of single constituents, which eventually causes the blocking of single-particle levels.[9] In superconductivity (which is possible if the constituents are fermions, and which applies to solid state and nuclear physics) there is an abrupt change of slope in ρ, such that $d\rho/ds$ is discontinuous at the point of the break, i.e., a phase transition occurs.[10] Below the break there is rapid increase of the density of states, similar to an exponential increase, followed by the break and a flatter ρ.

A detailed model for ρ based on the above picture has been suggested in Ref. 6. Since it is not clear how to include quantum numbers in the calculation we cannot proceed in applying the model to processes other than elastic.

Even without a complete knowledge of ρ one can obtain predictions based on the following assumptions:

(1) Differential cross sections for nonexchange processes are given by Eq. (1).

(2) The density of states exhibits a break; at the point of the break $d\rho/ds$ is discontinuous (ρ_{ij} and ρ_{kl} may show a break at different points).[11]

We then predict that all nonexchange processes with the same incoming channel exhibit a break (a flattening of slope) at the same value of s; the energy dependence may vary from process to process. If the break in $d\sigma/dt(\pi^-p\to K^+\Sigma^-)_{0°}$ occurs at $s\simeq 7$–8 GeV^2 as indicated by the recent results,[1a] all nonexchange reactions with π^-p as the incoming channel should change slope at $s\simeq 7$–8 GeV^2. This is in agreement with the data on $d\sigma/dt(\pi^-p\to\pi^-p)_{90°}$,[1c,12] and with recent data on $\sigma(\pi^-p\to\phi n)$ (see Ref. 1a) which show a break at $s\simeq 8$ GeV^2; above the break $\sigma(\pi^-p\to\phi n)\sim s^{-1.3\pm0.3}$. A check of the above prediction awaits more data on both 0° ($\pi^-p\to\pi^+\Delta^-$, $\pi^-p\to\phi n$, etc.) and 90° ($\pi^-p\to\pi^-p$, $\pi^-p\to K^0\Lambda$, etc.) cross sections. Since $d\sigma/dt(K^-p\to\pi^+\Sigma^-, K^+\Xi^-)_{0°}$ show no break for $s\lesssim 10$ GeV^2,[1a] all K^-p nonexchange processes should change slope at $s\gtrsim 10$ GeV^2. This is compatible with rather scarce data on $d\sigma/dt(K^-p\to K^-p)_{180°}$ (see Ref. 1b), which seems to change slope somewhere around $s\simeq 10$ GeV^2. Again, more data at 0° ($K^-p\to\pi^+\Sigma^-$, etc.), 90° ($K^-p\to\bar{K}^0n$, $K^-p\to K^-p$, etc.) and 180° ($K^-p\to K^-p$) are required. There are even fewer data on the $\bar{p}p$ channel; the data on $d\sigma/dt(\bar{p}p\to\bar{p}p)_{90°}$ (see Ref. 1c) and $d\sigma/dt(\bar{p}p\to\bar{p}p)_{180°}$ (see Ref. 1e) indicate that a break does not appear for $s<10$ GeV^2; thus all nonexchange $\bar{p}p$ reactions ($\bar{p}p\to K^+K^-$ at 0°, $\bar{p}p\to\bar{p}p$ at 90° and 180°, etc.) will flatten at some $s\gtrsim 10$ GeV^2.[13] These reactions are expected to be even more "statistical" than the other nonannihilation processes.

If we restrict the states (e.g., to a specific angular momentum[14]) the density of states flattens. Since not all states that contribute to ab will contribute to cd (e.g., resonances do not couple with the same strength to πN as to $K\Sigma$; note that such a difference appears in the exponent) it is reasonable that ρ_{abcd} will be flatter than ρ_{ab}, ρ_{cd} if $ab\neq cd$. Thus inelastic nonexchange reactions are predicted to exhibit a flatter s dependence than elastic nonexchange reactions. This prediction is confirmed when one compares $d\sigma/dt(\pi^-p\to K^+\Sigma^-)_{0°}\sim s^{-1.1}$ (see Ref. 1a) and $d\sigma/dt(\pi^-p\to\pi^-p)_{90°}\sim s^{-8}$ (see Ref. 1c) (both for $s>7$ GeV^2).

Let us end with the following remarks:

(a) More data are certainly needed to show that the energy dependence of nonexchange processes disagrees with Regge-cut predictions. Indeed, it is possible to fit the last several points in $d\sigma/dt(\pi^-p\to K^+\Sigma^-)_{0°}$ with a form compatible with a Regge cut, and to argue that a nonstatistical ρ trajectory contributes to $\pi^-p\to\phi n$. However, the absence of line-reversal symmetry between the 0° cross sections for $\pi^-p\to K^+\Sigma^-$ and $K^-p\to\pi^+\Sigma^-$ (see Ref. 1a) is another indication that double or single exchange cannot describe the above processes.

(b) Another prediction of the statistical interpretation above is that in nonexchange processes the s dependence of $d\sigma/dt$ is approximately independent of the angle (as long as we are far from an exchange region). This prediction deviates greatly from the Regge-cut prediction, and is an important test of the model.

(c) It is not clear to us how to describe a nonexchange reaction with an exotic s channel ($K^+p\to K^+p$, $pp\to pp$ at 90°, etc.). If these processes involve an intermediate multicluster formation, then—since clusters with different ρ_{ij} appear—more than one break is predicted in their s dependence. This is in agreement with $d\sigma/dt(pp\to pp)_{90°}$ where a series of breaks appear.[1f]

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[1]Data referred to in the text are compiled and presented in the following papers. (a) $\pi^- p \to K^+\Sigma^-$, $K^- p \to \pi^+\Sigma^-$, $K^- p \to K^- p \to K^+\Xi^-$ at 0°: C. W. Akerlof *et al.*, Phys. Rev. Lett. 33, 119 (1974); for $\pi^- p \to \phi n$ see also D. S. Ayres *et al.*, *ibid.* 32, 1463 (1974). (b) $K^- p \to K^- p$ at 180°: V. Chabaud *et al.*, Phys. Lett. 38B, 445 (1972). (c) $\pi^\pm p \to \pi^\pm p$, $\bar{p}p \to \bar{p}p$, $K^- p \to K^- p$ at 90°: See Ref. 3. (d) $\bar{K}^0 p \to \pi^+\Lambda$, $K_L^0 p \to K_S^0 p$, $\bar{K}^0 p \to \pi^+\Sigma^0$, $\pi^- p \to \pi^0 n$ at 90°: G. W. Brandenburg *et al.*, Phys. Lett. 44B, 305 (1973). (e) $\bar{p}p \to \bar{p}p$ at 180°: V. Chabaud *et al.*, *ibid.* 38B, 441 (1972). (f) $pp \to pp$ at 90°: Particle Data Group, LBL Report No. UCRL-20000NN, 1970 (unpublished).

[2]R. J. N. Phillips, Phys. Lett. 24B, 342 (1967); C. Michael, *ibid.* 29B, 230 (1969); C. Quigg, Nucl. Phys. B23, 269 (1971).

[3]G. Eilam, Y. Gell, B. Margolis, and W. J. Meggs, Phys. Rev. D 8, 2871 (1973).

[4]G. Fast and R. Hagedorn, Nuovo Cimento 27, 208 (1963); G. Fast, R. Hagedorn, and L. W. Jones, *ibid.* 27, 856 (1963).

[5]R. Hagedorn, Nuovo Cimento Suppl. 3, 147 (1965); S. Fubini and G. Veneziano, Nuovo Cimento 64A, 84 (1969); K. Bardakci and S. Mandelstam, Phys. Rev. 184, 1640 (1969); S. Frautschi, Phys. Rev. D 3, 2821 (1971); W. Nahm, Nucl. Phys. B45, 525 (1972).

[6]G. Eilam and Y. Gell, Phys. Rev. D 8, 3930 (1973).

[7]S. Frautschi, Nuovo Cimento 12A, 133 (1972); B. Margolis, W. J. Meggs, and R. K. Logan, Phys. Rev. D 8, 199 (1973).

[8]See, for instance, the data in K. Tsukada *et al.*, Nucl. Phys. 78, 369 (1966).

[9]L. G. Moretto, Nucl. Phys. A185, 145 (1972).

[10]In nuclear physics the transition is not as sharp as in a superconductor due to the finite number of particle; see T. Ericson, Adv. Phys. 9, 425 (1960).

[11]In a constituent model such as the one described in Ref. 6 the location of the break depends on the mass of the lowest state and on the binding energy of the relevant constituents.

[12]It is predicted in Ref. 6 that $d\sigma/dt\,(\pi^- p \to \pi^- p)_{90°}$ exhibits a break at $s \simeq 7$ GeV^2.

[13]It is predicted in Ref. 6 that $d\sigma/dt\,(\bar{p}p \to \bar{p}p)_{90°}$ exhibits a break at $s \simeq 17$ GeV^2.

[14]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969); C. B. Chiu and R. L. Heimann, Phys. Rev. D 4, 3184 (1971).