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Left-right gauge symmetry and an "isoconjugate" model of *CP* violation*

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Left-right symmetry in the starting gauge interactions provides the basis for an "isoconjugate" model of CP violation with the consequence that $\eta_{+-}=\eta_{00}$. The magnitude of CP violation is naturally suppressed at least to the extent that $V+A$ interactions are suppressed in nature compared to the $V-A$ interactions. This allows the possibility that intrinsic CP-violating phase (arising through spontaneous symmetry breaking, for example) may have a maximal character, which may reveal itself at intermediate high energies to possibly disappear at still higher energies. CP violation in leptonic, semileptonic, and $\Delta Y=0$ parity-violating nonleptonic interactions (which could contribute to the electric dipole moment of the neutron) arise on the one hand in fourth order of the weak gauge interactions and on the other via Yukawa interactions between fermions and leftover Higgs mesons as well as via W_L-W_R mixing. The magnitude of the latter contributions may be limited to be less than or of order $G_F\times10^{-4}$ in order that the experimentally observed relation $\eta_{+-}=\eta_{00}$ may hold at least to a few percent. Thus the electric dipole moment of the neutron d_n is expected to be less than or of order 10^{-24} e cm. CP violation in all $|\Delta S|=1$ nonleptonic decays ($Y\to N+\pi$, etc.) should, in general, have the same order of magnitude as $|\eta_{+-}|$.

The desirability of an *"isoconjugate"* relation of the form $[I_3, P^{(-)}]=\frac{1}{2}i\tan\xi\, P^{(+)}$, where $P^{(+)}$ and $P^{(-)}$ are the CP-even and CP-odd interactions contributing to $K_{L,S}\to2\pi$ decays and I_3 is the third component of the $\vec{I}$-spin generator, has been remarked upon.[1] It was shown that the class of theories satisfying the above equation automatically satisfy[2] the relation $\eta_{+-}=\eta_{00}$, with the phase ϕ_{+-} given to a good approximation[3] by $\tan^{-1}(2\Delta m/\Gamma_S)$, where $\Delta m=m_{K_L}-m_{K_S}$ and Γ_S is the width of the short-lived kaon. Both these results are compatible with experiments. The purpose of this note is to point out that a particularly simple scheme of CP violation distinct from previous attempts[4-6] and satisfying the said relation may be realized within a gauge theory of the weak, electromagnetic, *and* strong interactions by assuming that the left- and right-handed gauges[7] enter into the theory in a *symmetrical* fashion,[8] the corresponding currents being coupled to *distinct* gauge mesons W_L and W_R. The observed left-right asymmetry with predominance of left-handed $V-A$ interactions at low energies and breakdown of CP symmetry will arise in such a scheme due to heavier masses of the W_R gauge mesons compared to those of W_L's and the complex character of the fermion mass matrix, respectively, both of which in turn will be attributed to spontaneous symmetry breaking. The main results of such a scheme are the following:

(1) *Regardless* of the detailed structure of the mass matrix, its complex nature transcribed into the gauge interactions straightforwardly leads to the isoconjugate scheme of CP violation mentioned above and hence to the relation $\eta_{+-}=\eta_{00}$.

(2) The magnitude of CP violation is naturally suppressed at least to the extent that the right-handed $V+A$ interactions are known to be suppressed compared to the left-handed ones at present energies. Thus it is possible to allow "intrinsic" CP violation to bear a maximal character.

(3) CP violation in $|\Delta S|=1$ processes (i.e., $K_L\to2\pi$, $Y\to N+\pi$, etc.) should in general exhibit milliweak strength while that in $\Delta S=0$ processes may be considerably weaker than milliweak. Thus the electric dipole moment of the neutron is predicted to be less than or of order 10^{-24} e cm.

We exhibit such a scheme of CP violation within a gauge theory of the weak, electromagnetic, and strong interactions based on the anomaly-free gauge group[8]

$$\mathcal{G}_0\equiv \mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(3)'_{L+R}\times\mathrm{U}(1)'_{L+R}\ ,$$

whose one distinctive feature is its left-right symmetry. Its physical realization may be provided by a sixteenplet of left and sixteenplet of right fermions $\Psi_{L,R}$ comprising quarks and leptons:

$$\Psi_{L,R}=\begin{bmatrix}\mathcal{P}_a^0 & \mathcal{P}_b^0 & \mathcal{P}_c^0\\ \mathfrak{N}_a^0 & \mathfrak{N}_b^0 & \mathfrak{N}_c^0\\ \lambda_a^0 & \lambda_b^0 & \lambda_c^0\\ \chi_a^0 & \chi_b^0 & \chi_c^0\end{bmatrix}_{L,R}+\begin{bmatrix}\nu_e=\mathcal{P}_d^0\\ e^-=\mathfrak{N}_d^0\\ \mu^-=\lambda_d^0\\ \nu_\mu=\chi_d^0\end{bmatrix}_{L,R}\ ,\qquad(1)$$

$$\Psi_L=(2+2,1,3,1)+(2+2,1,1,1)\,,$$
$$\Psi_R=(1,2+2,3,1)+(1,2+2,1,1)\,, \qquad (2)$$

where the representations are characterized by their dimensions. The superscript zeros designate that the corresponding fields are linear combinations of the physical fields to be defined by the mass matrix. The gauge groups $SU(2)_{L,R}$ generate chiral weak interactions, while $SU(3)'_{L+R}$ generates vector strong[9] interactions acting on the 3-component color index (a, b, c) of the quarks (with either integer or fractional charges). The Abelian $U(1)'$ generates[10] a vector gauge interaction, which needs to be introduced to generate electromagnetism.

Our considerations here would be equally applicable to the extended gauge group[8]

$$\mathcal{G}=SU(2)_L\times SU(2)_R\times SU(4)'_{L+R}\,,$$

comprising left-right symmetry as well as baryon-lepton unification. [$SU(4)'$ operates on the four-color index (a, b, c, d).] We conjecture that a qualitative link between breakdown of CP invariance and and tiny but nonzero neutrino[11] masses may emerge in such an extended scheme, which may be considered elsewhere.

For considerations of CP violation, we will be concerned only with the fermion–gauge-boson weak interaction generated by the local group $SU(2)_L \times SU(2)_R$ and the structure of the fermion mass matrix. The former is given by

$$\mathcal{L}_I=\frac{g_L}{2}\sum_{\alpha=a,b,c,d}\;\sum_{i=1,2}(\bar{\psi}_i^\alpha)_L\,\vec{\tau}\gamma_\mu(\psi_i^\alpha)_L(\vec{W}_\mu)_L+(L\to R), \qquad (3)$$

where

$$\psi_1^\alpha=\begin{pmatrix}\mathcal{P}^0_\alpha\\ \mathcal{N}^0_\alpha\end{pmatrix},\quad \psi_2^\alpha=\begin{pmatrix}\chi^0_\alpha\\ \lambda^0_\alpha\end{pmatrix}\;. \qquad (4)$$

Note the left-right *symmetric* nature of the gauge interactions.

For simplicity of writing we consider below the mass matrix of the baryonic quartets only; similar considerations[12] apply to the lepton quartet. Furthermore, we suppress the color index (a, b, c); it is to be understood that identical Yukawa coupling constants and mass parameters apply for all three colors if the relevant Higgs multiplet is a color singlet. A complex mass matrix for the fermions may arise in two alternative ways:

(1) The simplest possibility is that the Higgs-meson–fermion Yukawa coupling constants are complex. Introduce a suitable multiplet $\Phi=(2, 2, 1, 1)$ whose vacuum expectation value may be taken to be

$$\langle\Phi\rangle=\begin{pmatrix}\kappa & 0\\ 0 & \kappa'\end{pmatrix},$$

κ and κ' being real. The general Yukawa coupling permitted by gauge invariance is given by

$$\sum_{i,j=1,2}\bar{\psi}_{iL}(h_{ij}\phi+f_{ij}\tilde{\phi})\psi_{jR}+\text{H.c.}\,,$$

where $\tilde{\phi}=\tau_2\phi^*\tau_2$. This leads to the mass matrix

$$\left[(\bar{\mathcal{P}}^0\bar{\chi}^0)_L\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}\begin{pmatrix}\mathcal{P}^0\\ \chi_0\end{pmatrix}_R+(\bar{\mathcal{N}}^0\bar{\lambda}^0)_L\begin{pmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{pmatrix}\begin{pmatrix}\mathcal{N}^0\\ \lambda^0\end{pmatrix}_R\right]+\text{H.c.}\,, \qquad (5)$$

where $m_{ij}=\kappa h_{ij}+\kappa' f_{ij}$ and $M_{ij}=\kappa f_{ij}+\kappa' h_{ij}$. In general all h_{ij}'s and f_{ij}'s are nonzero and complex.

(2) Alternatively, and perhaps more elegantly, a complex mass matrix may also arise through complex vacuum expectation values of scalar fields. In this case, it is necessary to introduce at least two[6] distinct Higgs multiplets, ϕ_1 and ϕ_2, each transforming in the same manner[13] [i.e., (2, 2, 1, 1)]. One may choose (with appropriate choice of potential)

$$\langle\phi_{1,2}\rangle=\begin{pmatrix}\kappa_{1,2}e^{i\xi_{1,2}} & 0\\ 0 & \kappa'_{1,2}e^{i\xi'_{1,2}}\end{pmatrix}. \qquad (6)$$

Allowing for the general gauge-invariant Yukawa interaction,

$$\sum_{i,j}\bar{\psi}_{iL}(h_{ij}\phi_1+h'_{ij}\phi_2+f_{ij}\tilde{\phi}_1+f'_{ij}\tilde{\phi}_2)\psi_{jR}+\text{H.c.}\,,$$

one may again obtain a complex fermion mass matrix of the form (5) with *real* Yukawa coupling constants. The major difference between alternatives (1) and (2) is that CP violation will have a genuine spontaneous origin in the case (2). (The two cases may be distinguishable at higher energies; see remarks later.)

Turning to the mass matrix of the form (5), physical $\mathcal{P}$ and χ fields are obtained by transforming the corresponding (2×2) complex matrix into real diagonal form[14]:

$$U_L\begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix}V_R^{-1}=\begin{pmatrix}m_{\mathcal{P}} & 0\\ 0 & m_\chi\end{pmatrix}, \qquad (7)$$

where U_L and V_R define U(2) transformations on $(\mathcal{P}^0_L, \chi^0_L)$ and $(\mathcal{P}^0_R, \chi^0_R)$ bases, respectively. Even though U_L and V_R define U(2) transformations, the physical consequences of such transformations may be seen to be equivalent to SU(2) transformations of the form

$$\begin{pmatrix}\mathcal{P}^0\\ \chi^0\end{pmatrix}_{L,R}=\begin{pmatrix}\cos\theta_{L,R} & -\sin\theta_{L,R}\,e^{i\delta_{L,R}}\\ \sin\theta_{L,R}\,e^{-i\delta_{L,R}} & \cos\theta_{L,R}\end{pmatrix}\begin{pmatrix}\mathcal{P}\\ \chi\end{pmatrix}_{L,R}, \quad (8)$$

where $\theta_{L,R}$ and $\delta_{L,R}$ are real parameters. Similar transformations will also apply to the $(\mathfrak{N}^0, \lambda^0)_{L,R}$ quarks and observable consequences will be given by the combined transformations in $(\mathcal{P}^0, \chi^0)$ and $(\mathfrak{N}^0, \lambda^0)$ spaces. For ease of writing and without loss of generality, assume that $(\mathfrak{N}^0, \lambda^0)$ are physical fields, i.e., $M_{12}=M_{21}=0$ and M_{11} and M_{22} are real, so that $(\mathfrak{N}^0, \lambda^0)_{L,R}=(\mathfrak{N}, \lambda)_{L,R}$. Substituting Eq. (7) into Eq. (3), and suppressing the color index (a, b, c), the fermion-weak-gauge-boson interactions take the form

$$\begin{aligned}\mathcal{L}_I=\frac{g_L}{\sqrt{2}}[\bar{\mathcal{P}}_L\gamma_\mu(\mathfrak{N}_L\cos\theta_L+e^{i\delta_L}\lambda_L\sin\theta_L)\,W^+_{\mu_L}\\ +\bar{\chi}_L\gamma_\mu(-e^{-i\delta_L}\mathfrak{N}_L\sin\theta_L+\lambda_L\cos\theta_L)\,W^+_{\mu_L}]\\ +\text{H.c.}+(L\to R)+\mathcal{L}_{\text{neutral}}+\mathcal{L}_{\text{lepton}}\ ,\end{aligned} \quad (9)$$

where $\mathcal{L}_{\text{neutral}}$ is composed of neutral-current interactions, is strangeness conserving, and is CP-even. Note that due to $\kappa\kappa'$ interference, which gives rise to W_L-W_R mixing, the $W^\pm_{L,R}$ fields written above are not really the eigenstates of the mass matrix. The mixing term is given by $g_L g_R\kappa\kappa'$ for case (1), and a similar term is given for case (2). For our considerations, we will consistently *assume* that the net effect of such mixing is small and neglect such a mixing. This amounts to choosing κ' to be at least an order of magnitude smaller than κ.[15]

The $|\Delta S|=1$ charm-conserving effective nonleptonic Hamiltonian following from Eq. (9) is then given by

$$\begin{aligned}\mathcal{H}_{\text{wk}}^{|\Delta S|=1}=(f_L e^{-i\delta_L}+f_R e^{-i\delta_R})S\\ +(f_L e^{-i\delta_L}-f_R e^{-i\delta_R})P+\text{H.c.},\end{aligned} \quad (10)$$

where

$$\begin{aligned}S=(V_{\pi^-}V_{K^+}+A_{\pi^-}A_{K^+})_{\text{symm}}\\ -(V'_{\pi^-}V'_{K^+}+A'_{\pi^-}A'_{K^+})_{\text{symm}}\ ,\\ P=(V_{\pi^-}A_{K^+}+A_{\pi^-}V_{K^+})_{\text{symm}}\\ -(V'_{\pi^-}A'_{K^+}+A'_{\pi^-}V'_{K^+})_{\text{symm}}\ ;\end{aligned} \quad (11)$$

here $V_{\pi^-}=\bar{\mathcal{P}}\gamma_\mu\mathfrak{N}$, $A_{\pi^-}=\bar{\mathcal{P}}\gamma_\mu\gamma_5\mathfrak{N}$, $V'_{\pi^-}=\bar{\chi}\gamma_\mu\mathfrak{N}$, $V_{K^+}=\bar{\lambda}\gamma_\mu\mathcal{P}$, etc. We define $(V_{\pi^-}V_{K^+})_{\text{symm}}\equiv(V_{\pi^-}V_{K^+}+V_{K^+}V_{\pi^-})$, and $f_{L,R}\equiv g_{L,R}{}^2\sin2\theta_{L,R}/4m_{W_{L,R}}{}^2$. Redefining the phase of the λ field[16] (which has no observable consequence), we may express (10) as

$$H_{\text{wk}}^{|\Delta S|=1}=S^{(+)}+[P^{(+)}+P^{(-)}]\ , \quad (12)$$

where

$$\begin{aligned}&S^{(+)}=G_S(S+S^\dagger)\ ,\\ &P^{(+)}=G_P\cos\xi(P+P^\dagger),\quad P^{(-)}=iG_P\sin\xi(P-P^\dagger)\ ,\\ &G_S=f_L(1+\epsilon^2+2\epsilon\cos\delta)^{1/2},\quad \delta=\delta_R-\delta_L\\ &G_P=f_L(1+\epsilon^2-2\epsilon\cos\delta)^{1/2}\ ,\\ &\tan\xi=\frac{2\epsilon\sin\delta}{1-\epsilon^2},\quad \epsilon=(f_R/f_L)\ .\end{aligned} \quad (13)$$

Clearly the interaction given by (12) is in general CP-violating. Note that it would be CP-conserving if either f_L or f_R were zero and/or $\delta=\delta_L-\delta_R=0$. The scheme possesses CP violation not only through the gauge interactions (12), but also through the Yukawa interaction between fermions and leftover Higgs mesons. We will *assume* that the effects of the latter are suppressed compared to those of the former by the masses of the Higgs mesons (see estimates later). We are led to make such an assumption in view of the simple results (i.e., $\eta_{+-}=\eta_{00}$) that follow from Eq. (12) in agreement with experiments, which will be affected in general in the presence of a Higgs-boson contribution.

We now remark that the parity-violating CP-even $(P^{(+)})$ and CP-odd $(P^{(-)})$ operators satisfy the desired relation

$$[I_3, P^{(-)}]=\tfrac{1}{2}i\tan\xi\,P^{(+)} \quad (14)$$

and hence[2] $\eta_{+-}=\eta_{00}$. Using Eq. (14) we may also *calculate* η_{+-}, which for small $\tan\xi$ is given[17] by

$$\eta_{+-}\simeq i\tan\xi\,(1-y), \quad (15)$$

where $y=A_2/(A_1+A_2)$, with $A_i\equiv\int dt\langle K^0|T(\Omega_i(t)\Omega_i(0))|\bar{K}^0\rangle$ and $\Omega_1=S^{(+)}$, $\Omega_2=P^{(+)}$. Thus y is a *complex* number with magnitude of order 1 and, barring accidental cancellation, $|\eta_{+-}|$ is of order $\tan\xi$. Thus [see Eq. (13)], the suppression of CP violation may arise entirely due to the smallness of (f_R/f_L), leaving the possibility that "intrinsic" CP violation in weak processes may be maximal (i.e., $\delta=\frac{1}{2}\pi$, say), which ought to manifest itself at intermediate high energies (i.e., energies comparable to M_{W_R}). At still higher energies and momentum transfers ($\gg M_{W_R}$), one may expect CP violation [if it is of spontaneous origin, as in case (2)] as well as parity violation (if the bare coupling constants $g_L^{(0)}$ and $g_R^{(0)}$ are equal) to disappear[18] altogether.

We should remark that we have chosen the phase of the λ field[16] such that CP violation in the $|\Delta S|=1$ nonleptonic Hamiltonian is entirely in the parity-violating part [see Eq. (12)]. With this choice of phase, CP violation in $K_{L,S}\to3\pi$ decays arises only through the contributions of $P^{(-)}$ to the $(K_1$-$K_2)$ mass matrix, and one thus obtains the relation $\eta_{+-0}=\eta_{000}=\rho$ as in a superweak theory, where ρ is the CP-even mixing parameter in K_L, i.e.,

$$K_L = (K_2 + \rho K_1)/(1 + |\rho|^2)$$

and

$$\eta_{ijk} = M(K_S^0 \to \pi^i \pi^j \pi^k)/M^0(K_L^0 \to \pi^i \pi^j \pi^k).$$

Alternatively, we may have chosen the phase of the λ field such that CP violation would be shifted entirely into the parity-conserving sector with a relation analogous to Eq. (14) between $S^{(-)}$ and $S^{(+)}$. In this case CP violation in $K_{L,S} \to 2\pi$ decays would arise only through the mass matrix, while that in $K_{L,S} \to 3\pi$ decays would arise both through the mass matrix and the decay amplitudes. All observable consequences are, of course, *independent* of the choice of phase. It is worth noting that *unlike* the superweak theory, in the present scheme $\eta_{+-0} \neq \eta_{+-}$. In fact (see Ref. 2), one obtains in the present scheme[19] $(\eta_{+-} - \eta_{+-0}) = [(R+\rho)/(1-\rho R)] - \rho = R = i\tan\xi$ (neglecting ρR compared to unity).

It is easy to see from Eq. (9) that the CP-violating part of the gauge interactions is purely[20] $\Delta S = 1$ and therefore its contribution to the electric dipole moment of the neutron d_n is small ($\sim e G_F^2 \sin^2\theta \sin\xi\, m_p^3 \approx 10^{-29}$ e cm). The Higgs-boson interactions contain $\Delta Y = 0$ P- and T-violating parts. However, as their contributions affect the relation $\eta_{+-} = \eta_{00}$, which is experimentally valid up to 5–10%, one may expect the effective strength of such interactions (h^2/m_σ^2) to be[21] $\lesssim G_F \times 10^{-4}$. Thus their contributions to d_n is of order $eh^2 m_p/m_\sigma^2 \lesssim 10^{-24}$ e cm.

Analogous to $\Delta Y = 0$ nonleptonic transitions, CP violation in leptonic and semileptonic processes (including $\Delta Q = 0$ and $|\Delta S| = 1$ transitions) may arise in the scheme only via exchange of Higgs mesons and W_L-W_R mixing, whose effective strengths are, however, limited to be $\lesssim G_F \times 10^{-4}$ (for reasons mentioned above).[20,21]

In summary, from a theoretical point of view, the unique feature of the scheme is the isoconjugate relation (14), and we stress that the fact that the left- and right-handed currents enter into the theory in a *symmetrical* fashion, being coupled to *distinct* gauge mesons W_L and W_R, has played an *essential* role[22,23] in its realization. Having assumed that the leftover Higgs mesons are sufficiently massive, the only parameter of the scheme relevant for CP violation at present energies is $\tan\xi$.

We should remark that parameters of the fermion mass matrix, which characterize the scales of SU(2), SU(3), SU(4), and chiral $SU(4)_L \times SU(4)_R$ breaking and Cabibbo rotation on the one hand, as well as CP violation on the other, are not at present calculable deviations from "natural" symmetries. This is a pressing problem and to our knowledge is not resolved in a satisfactory manner in any full-fledged realistic scheme. Perhaps the solution may lie in a better understanding of the mechanism of spontaneous symmetry breaking, in particular if it is dynamical. It is to be hoped that the eventual nature of such symmetry breaking, while it will constrain and thereby interrelate the parameters of the theory, does not alter the character of CP violation in the scheme as presented here. We should stress once again that the nature of CP violation in the scheme as reflected by Eqs. (9) and (14) follows straightforwardly from the gauge structure and does *not* depend upon precise values of the parameters (once the relevant parameters are nonzero). Furthermore, it should be noted that despite the arbitrariness in $\sin\delta$, the CP-violating parameter $\tan\xi \simeq (f_R/f_L)\sin\delta$ is *guaranteed to be small by the experimental fact that* (f_R/f_L) *is known to be small.*

As regards the experimental consequences, the present scheme is distinct from a superweak theory; the two schemes coincide only in the predictions $\eta_{+-} = \eta_{00}$ and $\eta_{+-0} = \eta_{000}$. They may be distinguished by measurements of (i) the phase ϕ_{+-}, (ii) the strength of CP violation in processes such as $Y \to N + \pi$ decays, and (iii) most important, the electric dipole moment of the neutron, if it turns out to lie between 10^{-24} and 10^{-28} e cm. The present scheme may also be distinguished from the usual milliweak theories (see Ref. 1 for example, and other references cited therein), since the latter generally predict a neutron electric dipole moment d_n of order 10^{-22} to 10^{-23} e cm. It is also distinct from other gauge-theory models[4-6] of CP violation, since it combines the predictions $\eta_{+-} = \eta_{00}$ with a *small* electric dipole moment of the neutron ($d_n \lesssim 10^{-24}$ e cm).[24]

Added note: Considerably after the submission of our paper it was brought to our attention by Professor L. Wolfenstein that a somewhat similar model of CP violation has also been proposed recently by Frenkel and Ebel.[25] There is one major difference: Our lepton sector is left-right symmetric as much as the hadron sector is; i.e., the right-handed leptons as well as the hadrons couple to W_R's; owing to this, if any U(1) is introduced into the gauge group to generate the photon, it turns out to be pure vector gauge and the complete gauge group is anomaly-free. In the case of Frenkel and Ebel the known leptons are coupled only to $SU(2)_L$ but not to $SU(2)_R$; hence their charges *need* contribution from an Abelian U(1) gauge, which is not pure vector. This leads to anomalies, unless one introduces heavy leptons to cancel the anomalies. Some of the consequences of the model in the hadronic sector, i.e., the isoconjugate relation and the exactness of $\eta_{+-} = \eta_{00}$, $\eta_{+-} = \eta_{000}$ as being properties of the gauge interactions of the model, are not noted in Ref. 25.

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[1]R. N. Mohapatra and J. C. Pati, Phys. Rev. D 8, 2317 (1973).

[2]To repeat the arguments of Ref. 1, take matrix elements of both sides of $[I_3, P^{(-)}] = \frac{1}{2} i \tan\xi P^{(+)}$ between $\langle \pi^i \pi^j|$ and $|K_1\rangle$ states, where $(i, j) = (+-)$ or $(0,0)$ and $CP|K_1\rangle = |K_1\rangle$ and $CP|K_2\rangle = -|K_2\rangle$. Since $I_3|\pi^i\pi^j\rangle = 0$ and $I_3|K_1\rangle = -\frac{1}{2}|K_2\rangle$, the ratio $R \equiv \langle \pi^i\pi^j|P^{(-)}|K_2\rangle / \langle \pi^i\pi^j|P^{(+)}|K_1\rangle = i \tan\xi$ *independent* of (i, j), from which it follows that $\eta_{+-} = \eta_{00} = (R + \rho)/(1 - \rho R)$, where ρ is the CP-even mixing parameter in K_L, i.e., $K_L = (K_2 + \rho K_1)/(1 + |\rho|^2)$.

[3]It holds to the extent that one may neglect $|3\pi\rangle$ compared to the $|2\pi\rangle$ real intermediate state in the ratio of off-diagonal to diagonal elements of the K_1-K_2 width matrix; on the other hand, $\tan\phi_{+-} = 2\Delta m/\Gamma_s$ is an exact prediction of the superweak theory [L. Wolfenstein, Phys. Rev. Lett. 13, 562 (1964)].

[4]R. N. Mohapatra, Phys. Rev. D 6, 2023 (1972).

[5]A. Pais, Phys. Rev. D 8, 625 (1973).

[6]T. D. Lee, Phys. Rep. 9C, 143 (1974).

[7]Right-handed currents (without, however, left-right symmetry) were suggested in Ref. 4 for introducing CP violation into gauge theories.

[8]Such a left-right symmetric gauge structure has been proposed by J. C. Pati and Abdus Salam, Phys. Rev. Lett. 31, 661 (1973); Phys. Rev. D 10, 275 (1974). The details of the Higgs mechanism giving adequate masses (i.e., $m_{W_R} \gg m_{W_L}$, etc.) are given in this reference.

[9]J. C. Pati and Abdus Salam, Phys. Rev. D 8, 1240 (1973).

[10]The corresponding fermion current may be taken to be

$$\left(\tfrac{1}{24}\right)^{1/2}\left[\sum_{\alpha=a,b,c} (\bar{\mathcal{P}}^0_\alpha \mathcal{P}^0_\alpha + \bar{\mathfrak{N}}^0_\alpha \mathfrak{N}^0_\alpha + \bar{\lambda}^0_\alpha \lambda^0_\alpha + \bar{\chi}^0_\alpha \chi^0_\alpha)_{L+R} - 3(\bar{\nu}_e \nu_e + \bar{e} e + \bar{\mu}\mu + \bar{\nu}_\mu \nu_\mu)_{L+R}\right],$$

generated by the 15th generator of SU(4)′ (see Ref. 8).

[11]Note that CP violation in the q_α sector vanishes as $(m_{q_\alpha})_{\alpha=a,b,c} \to 0$ for any $q = (\mathcal{P}, \mathfrak{N}, \lambda$, or $\chi)$, since one can independently redefine the phases of $q_{\alpha L}$ and $q_{\alpha R}$. On the other hand, quarks and lepton masses are related (see Ref. 8) within the extended gauge group, expecially if the Higgs multiplet is a color singlet. This leads us to conjecture that CP violation in this extended scheme is linked on the one hand to nonvanishing quark *and* lepton masses (including neutrinos) and to Cabibbo angles $\theta_{L,R}$ on the other. (See later that CP violation in gauge interactions would disappear if θ_L or $\theta_R \to 0$.)

[12]Note that the Yukawa coupling parameters and therefore the mass matrices for the baryonic (a, b, c) and leptonic quartets are unrelated (related) if the gauge group is $\mathcal{G}_0$ $(\mathcal{G})$.

[13]As emphasized in Ref. 6, one needs invariant couplings of the form $a(\phi_1^\dagger\phi_2) + b(\phi_1^\dagger\phi_2)^2$ + H.c. to retain observable consequences of complex vacuum expectation values. This necessitates ϕ_1 and ϕ_2 having the same representation. [We should remark that duplication of Higgs fields is by no means unnatural if they are to be regarded as $(\bar{\psi}_L\psi_R)$ composites, which give rise to four $\phi_i = (2,2,1,1)$.] Defining the fields $\phi_3 \equiv \tau_2\phi_1^*\tau_2$ and $\phi_4 \equiv \tau_2\phi_2^*\tau_2$, it is easy to verify that the full potential, which gives the desired pattern of vacuum expectation values $\langle\phi_{1,2}\rangle$ (chosen in the text) for appropriate choice of signs of the coefficients, is given by

$$\sum_{i,j,k,l=1,2,3,4}\Big[a_{ij}\,\mathrm{Tr}(\phi_i^\dagger\phi_j) + b_{ijkl}\mathrm{Tr}(\phi_i^\dagger\phi_j\phi_k^\dagger\phi_l) + c_{ijkl}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)\mathrm{Tr}(\phi_k^\dagger\phi_l) + d_{ij}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)\,(T_L^\dagger T_L + T_R^\dagger T_R)\Big]$$
$$+ \sum_{\alpha=L,R}[\mu_\alpha^2(T_\alpha^\dagger T_\alpha) + f'(T_\alpha^\dagger T_\alpha)^2] + e'(T_L^\dagger T_L)(T_R^\dagger T_R) + \text{H.c.},$$

where we have introduced two additional fields $T_L = (2,1,1,1)$ and $T_R = (1,2,1,1)$ to provide desired mass differences between $W_L^\pm$ and $W_R^\pm$ (solely) due to differences between $\mu_L{}^2$ and $\mu_R{}^2$, all other terms being $L \longleftrightarrow R$ symmetric. Note that such a pattern is not affected in the presence of SU(3)′ or SU(4)′ color-gauge interactions. We thank Professor A. Pais for discussions on the choice of the potential.

[14]The reader may convince himself that such U(2) transformations can always be found so as to transform an arbitrary 2×2 complex matrix to real diagonal form.

[15]This follows from the observed validity of the relation $\eta_{+-} = \eta_{00}$ to a few percent. Note that the CP-nonconserving amplitude due to the *mixing term* introduces corrections of order (κ'/κ) to the above relation, which holds exactly in the absence of such a mixing. The choice $\kappa' \ll \kappa$ (with κ' and $\kappa \neq 0$) is compatible with the general Higgs potential (Ref. 13) and the requirement of renormalizability of the theory.

[16]I.e.,

$$\lambda_{L,R} \to e^{i\phi}\lambda_{L,R};$$
$$\tan\phi = (f_L\sin\delta_L + f_R\sin\delta_R)/(f_L\cos\delta_L + f_R\cos\delta_R).$$

[17]This follows by noting that $(p^2/q^2) = \langle\bar{K}^0|T|K^0\rangle/\langle K^0|T|\bar{K}^0\rangle = M^{(-)}/M^{(+)}$, where $M^{(\pm)} \equiv A_1 + (1 \pm 2i\tan\xi - \tan^2\xi)A_2$ and $(p/q) = (1+\rho)/(1-\rho)$. See Ref. 2.

[18]Note that the Higgs sector cannot be completely left-right symmetric in order that M_{W_R} be different from M_{W_L} after spontaneous symmetry breaking. However, if the asymmetry is only through mass terms of the scalar fields (see footnote 13), it will be unimportant at high energies and in such a case $(g_L - g_R)$ would be

computable. This will be discussed elsewhere. Note that if the Higgs mechanism is eventually replaced by dynamical symmetry breaking, then parity violation would be entirely of spontaneous origin (assuming $g_L^{(0)} = g_R^{(0)}$. Spontaneous parity violation has been considered by P. Fayet [Orsay report, 1974 (unpublished)] in a different context.

[19]We thank Professor L. Wolfenstein for pointing out that the difference between η_{+-} and η_{+-0} provides, at least in principle, an avenue for measuring the most relevant parameter of our scheme ($\tan\xi$) in a model-independent manner.

[20]There would be a $\Delta Y = 0$ P- and T-violating term in the gauge interactions due to W_L- W_R mixing which is proportional to $g_L g_R \kappa\kappa'$; this contribution is limited to be $\lesssim 10^{-24}$ e cm if (κ'/κ) is less than (1/10), which (as explained in footnote 15) is required to preserve the relation $\eta_{+-} = \eta_{00}$.

[21]Since the Yukawa coupling constants (h's and f's) are of order $e(m_q/m_{W_L})$, if one requires Higgs-boson exchange amplitudes to be $\lesssim G_F \times 10^{-4}$, it would follow that $m_\sigma \gtrsim (m_q/m_{W_L}) \times 3 \times 10^3 \approx 30$ GeV if $(m_q/m_W) \simeq 10^{-2}$ (say). Such scales of masses for the Higgs mesons are at least not unreasonable, considering that m_W's are of order 100 GeV.

[22]Note that an alternative scheme (based on phase angles between vector and axial-vector currents *outside* the gauge-theory approach) also leads to the relation (14) and $\eta_{+-} = \eta_{00}$ only for a special relation between $\Delta S = 0$ and $\Delta S = 1$ phase angles as pointed out in Ref. 1. Such a scheme still differs from the present note in the structure of $S^{(\pm)}$ and does not predict $\eta_{+-0} = \eta_{000}$. Furthermore, it should lead to a neutron electric dipole moment d_n of order 10^{-23} e cm.

[23]We note that if we chose

$$\begin{pmatrix} \mathcal{P}^0 \\ \chi^0 \end{pmatrix}_R$$

and

$$\begin{pmatrix} \mathfrak{N}^0 \\ \lambda^0 \end{pmatrix}_R$$

as $SU(2)_R$ doublets with

$$\begin{pmatrix} \mathcal{P}^0 \\ \mathfrak{N}^0 \end{pmatrix}_L$$

and

$$\begin{pmatrix} \chi^0 \\ \lambda^0 \end{pmatrix}_L$$

as the $SU(2)_L$ doublets, we may again introduce *CP* violation into the gauge interactions (after spontaneous symmetry breaking) in a manner similar to that discussed in the text; the resulting scheme, however, does not satisfy the isoconjugate relation (14). Furthermore, assuming that $\mathfrak{N}^0_{R,L}$ and $\lambda^0_{R,L}$ are related to the physical fields by Cabibbo rotations, one obtains $\Delta S = 2$ transitions in order f_R. One must thus choose

$$f_R = (g_R{}^2/4 m_{W_R}{}^2)(\sin 2\theta_R) < G_F{}^2 m_p{}^2$$

to avoid conflict with $(m_{K_L} - m_{K_S})$. In this case the gauge interactions provide a genuine superweak scheme of *CP* violation. Note that in this case, one *must* introduce an Abelian U(1) contribution to electric charge [in spite of the full SU(4)′-color gauging] since the $SU(2)_R$ mentioned above can not contribute to electric charge. The resulting scheme is still anomaly-free.

[24]See L. Wolfenstein, Nucl. Phys. B77, 375 (1974) for a review on estimates of the electric dipole moment of the neutron in various gauge-theory and other models.

[25]J. Frenkel and M. Ebel, Univ. of Wisconsin report (unpublished).