Phenomenological analysis of total cross-section measurements at the Fermi National Accelerator Laboratory*

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The available high-energy total-cross-section data for $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ scattering on protons and deuterons are analyzed. It is found that the diffractive (nonfalling) components of the cross sections are compatible with several different functional forms for the energy dependence, including one in which σ_{K^+p}, σ_{π^+p}, and σ_{pp} will rise asymptotically as ln$s$ with coefficients in the ratios predicted by SU(3) and the naive quark model. The falling components of the cross sections are found to be in agreement with Regge theory. The ρ and ω intercepts are found to be $\alpha_\rho = 0.57$ and $\alpha_\omega = 0.43$. A comparison with forward nondiffractive cross sections verifies the Regge pole phase of the ω but suggests a possible breaking of the ρ pole phase. The data are in strong agreement with universality, are compatible with exchange degeneracy between the f and ω, and indicate a substantial breaking of SU(3). Glauber screening corrections, with inelastic contributions added, are calculated and used to predict the total cross sections for scattering on deuterium. The predictions are in excellent agreement with the data.

I. INTRODUCTION

A new experiment[1] by Carroll *et al.* at the Fermi National Accelerator Laboratory (Fermilab) has yielded very precise values for the total cross sections for $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ on hydrogen and deuterium targets at four points in the momentum range $50 \leq p_{\rm lab} \leq 200$ GeV/c. For the first time a definite rise has been observed in all of the reactions except the $\bar{p}p$. The rise is especially dramatic in the K^+p cross section. Furthermore, the new data provide more exact and higher energy values for the differences of cross sections than have previously been available.

The purpose of this article is to phenomenologically analyze the high-energy data available for hadron-hadron cross sections. In particular, our goals are (a) to parametrize the diffractive components of the cross sections as systematically and compactly as possible; (b) to test whether the falling components of the cross sections are compatible with Regge theory, to determine the parameters, and to compare them with various symmetry predictions; (c) to determine theoretically the screening corrections for scattering on deuterium and then compare the predicted deuterium cross sections with the data.

The plan of this article is as follows: In Sec. II we analyze the secondary (falling) components of the cross sections within the framework of Regge theory. In principle, one would like to determine the residues and intercepts of the f, ρ, ω, and A_2 Regge trajectories directly by forming the linear combinations of cross sections which single out their contributions. This is not feasible in practice, however, because the uncertainties in the Glauber screening corrections needed to determine the neutron-target cross sections are larger than many of the necessary cross-section differences. Therefore, in Sec. II we limit our considerations to proton-target reactions. We first fit the cross-section differences $\Delta(\pi^+p)$, $\Delta(K^+p)$, and $\Delta(pp)$ to determine the ρ and ω intercepts (see Appendix A for notations), yielding $\alpha_\rho = 0.57$ and $\alpha_\omega = 0.43$. The residues are shown to satisfy ω universality very precisely and to be compatible with ρ universality. Furthermore, the data indicate that the φ decouples from nucleons. On the other hand, an SU(3) prediction relating the couplings of the (nondegenerate) ρ and ω trajectories is substantially violated. It is shown that the very rapidly decreasing component of the pp difference can be represented by a threshold modification of the momentum parameter in the Regge power without invoking a low-lying supplementary trajectory. The falling components of the π^+p and pp cross sections are also analyzed, although the actual parameters are dependent upon the functional form assumed for the diffractive component of the cross sections. The π^+p cross section supports f-ω exchange degeneracy and supports an SU(3) prediction relating the couplings of different particles to the same trajectory. The decreasing part of σ_{pp} is parametrized. It could be due to a small breaking of f-ω exchange degeneracy, a low-lying singularity, or some other mechanism.

In Sec. II B we predict the forward differential cross sections for various nondiffractive reactions using the parameters from II A along with certain theoretical assumptions. We find agreement with experiment for the reaction $K_L^0 p \to K_S^0 p$, verifying our parameters and the Regge-pole phase relation for the ω. The predictions are in agreement with previous charge-exchange measurements, but do not agree with a recent high-energy $\pi^- p \to \pi^0 n$ experiment.[2] The Regge-pole picture combined with ρ-A_2 exchange degeneracy has a similar problem in $\pi^- p \to \eta n$ predictions. The predictions are in good agreement with previous experiments, but disagree with recent high-energy data.

In Sec. III we consider the diffractive (i.e., nonfalling) components of the K^+p, π^+p, and pp cross sections. We find that *the data are not sufficiently precise to distinguish between several different functional forms for the energy dependence*. One particularly compact parametrization involves a *single logarithm*,

$$\sigma_i^d(s) = C_i \ln\left(\frac{s+m_i}{b_i}\right), \tag{1.1}$$

where σ_i^d is the diffractive cross section for particle i on protons, m_i is a "threshold" factor of order 1000 GeV^2, and b_i is a scale of order 1 GeV^2. The coefficients C_{K^+}, C_{π^+}, and C_p are successfully fixed in the ratio $1:1:\frac{3}{2}$, suggesting that if this parametrization holds true asymptotically, $\sigma_{K^+p}, \sigma_{\pi^+p}$, and σ_{pp} will all grow like $\ln s$ and be in the ratio $1:1:\frac{3}{2}$ [which is predicted by SU(3) and the naive quark-model additivity assumption]. The data are also successfully fitted to other functional forms, such as

$$\sigma_i^d(s) = a_i + b_i \ln s + c_i \ln^2 s\,, \tag{1.2}$$

although no suggestive correlations between the parameters emerge. Unfortunately, the parameters of the falling components of the π^+p and pp cross sections are dependent upon which functional form is used for the diffractive component.

In Sec. IV we consider the deuteron-target cross sections. At Fermilab energies, the conventional elastic Glauber correction does not adequately represent the entire screening effect. Hence, in Sec. IV we determine the inelastic scattering contribution using available data on the inclusive reaction $pp \to p + X$ (which is consistent with a nonvanishing triple-Pomeranchukon coupling). The inelastic contribution is responsible for as much as 20% of the screening corrections. We then combine our estimates of the screening corrections with the proton and neutron cross sections (the latter are computed, assuming exchange degeneracy) to predict the deuteron-target cross sections. The results are in excellent agreement with experiment.

Appendix A summarizes our conventions and notations, and Appendix B is a compendium of theoretical predictions for Regge parameters.

II. TOTAL CROSS-SECTION DIFFERENCES AND SECONDARY REGGE TRAJECTORIES

In this section we describe our fits to the cross-section differences $\Delta(\pi^+p)$, $\Delta(K^+p)$, and $\Delta(pp)$ and test various theoretical predictions concerning secondary trajectories.

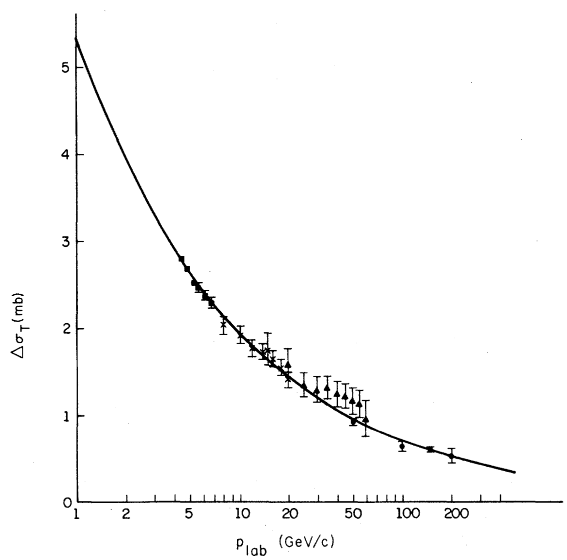

FIG. 1. $\Delta(\pi^+p) = \sigma_{\pi^-p} - \sigma_{\pi^+p}$. The details of the fit to the ρ are described in Tables I and II. The data are from the following references: ■ A. Citron *et al.*, Phys. Rev. 144, 1101 (1966); × K. J. Foley *et al.*, Phys. Rev. Lett. 19, 330 (1967); ▲ S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); 36B, 528 (1971); Nucl. Phys. B65, 1 (1973); ● A. S. Carroll *et al.*, Ref. 1.

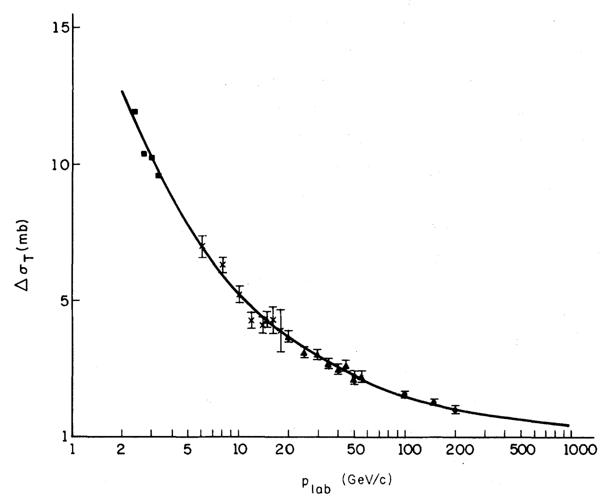

FIG. 2. $\Delta(K^+p) = \sigma_{K^-p} - \sigma_{K^+p}$ fit to determine the ω parameters. The ρ parameters are taken from the fit to $\Delta(\pi^+p)$, and the fit is described in Tables I and II. The data are from the following references: × W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965). ▲ Denisov *et al.* (see data in Fig. 1); ● A. S. Carroll *et al.*, Ref. 1; ■ R. J. Abrams *et al.*, Phys. Rev. D 1, 1917 (1970).

TABLE I. Fits to cross-section differences. All momenta are in GeV/c, and $B^i_{ab}=\beta^i_{ab}p_{\rm lab}{}^{\alpha_i-1}$. χ^2/ν is the total χ^2 divided by the number of degrees of freedom.

Quantity	Value	$p_{\rm lab}$ range	Number of points	χ^2/ν
$\Delta(\pi^+p)=cp_{\rm lab}{}^{-n}$	$c=5.24\pm0.10$ mb $n=0.43\pm0.01$	4.43–200	27	0.82
$\Delta(K^+p)-2B^{\rho}_{Kp}=cp_{\rm lab}{}^{-n}$	$c=15.9\pm0.25$ mb $n=0.57\pm0.01$	3–200	22	0.66
$\Delta(pp)=2\beta^{\rho}_{pp}(p_{\rm lab}-p_0)^{\alpha_\rho-1}+2\beta^{\omega}_{pp}(p_{\rm lab}-p_0)^{\alpha_\omega-1}$	$p_0=0.78\pm0.01$	2.75–200	23	0.65
$\Delta(K^+d)=4\xi_K B^{\omega}_{Kp}$	$\xi_K=0.90\pm0.01$	3.0–200	22	0.84
$\Delta(pd)=4\xi_p\beta^{\omega}_{pp}(p_{\rm lab}-p^d_0)^{\alpha_\omega-1}$	$\xi_p=0.87\pm0.01$ $p^d_0=0.94\pm0.02$	2.50–200	24	0.83

A. Cross-section differences

Consider first the ρ and ω trajectories. We have fitted $\Delta(\pi^+p)$ to a single power, the ρ [see Eq. (A5) and Ref. 3], with the result[4]

$$\beta^{\rho}_{\pi p}=2.62\pm0.05\text{ mb},\quad \alpha_\rho=0.57\pm0.01\,. \tag{2.1}$$

The fit is displayed in Fig. 1. The parameters of the fits are given in Table I, and the extracted Regge parameters in Table II.

$\Delta(K^+p)$ involves two trajectories, the ρ and ω. It would be difficult to fit both simultaneously. Instead, we tentatively accept the ρ universality[5-8] [or SU(3)] prediction (B6) to predict the ρ contribution from (2.1) and then fit the quantity

$$\Delta(K^+p)-2\beta^{\rho}_{Kp}p_{\rm lab}{}^{\alpha_\rho-1}=2\beta^{\omega}_{Kp}p_{\rm lab}{}^{\alpha_\omega-1} \tag{2.2}$$

to determine the ω parameters. The excellent fit, which is shown in Fig. 2, gives

$$\beta^{\omega}_{Kp}=7.95\pm0.13\text{ mb},\quad \alpha_\omega=0.43\pm0.01\,. \tag{2.3}$$

The difference $\Delta(pp)$ involves not only the ρ and ω but also a component which falls rapidly (as $\sim p_{\rm lab}{}^{-1.5}$).

We have found that this falling component can be parametrized simply by writing

$$\Delta(pp)=2\beta^{\rho}_{pp}(p_{\rm lab}-p_0)^{\alpha_\rho-1}+2\beta^{\omega}_{pp}(p_{\rm lab}-p_0)^{\alpha_\omega-1}\,. \tag{2.4}$$

We tentatively use the universality conditions[5-7]

TABLE II. Secondary Regge parameters determined from $\Delta(\pi^+p)$, $\Delta(K^+p)$, and $\Delta(pp)$. Also included are the parameters determined in Sec. III.

Quantity	Value	Comment
α_ρ	0.57 ± 0.01	
α_ω	0.43 ± 0.01	
$\frac{1}{2}\beta^{\rho}_{\pi p}=\beta^{\rho}_{Kp}=\beta^{\rho}_{pp}$	1.31 ± 0.03 mb	$\beta^{\rho}_{\pi p}$ determined from $\Delta(\pi^+p)$
$\frac{1}{2}\gamma^{\rho}_{\pi}=\gamma^{\rho}_{K}=\gamma^{\rho}_{p}$	1.14 ± 0.01 (mb)$^{1/2}$	
$3\beta^{\omega}_{Kp}=\beta^{\omega}_{pp}$	23.9 ± 0.4 mb	β^{ω}_{Kp} determined from $\Delta(K^+p)$
$\gamma^{\omega}_{p}=3\gamma^{\omega}_{K}$	4.88 ± 0.04 (mb)$^{1/2}$	
$\beta^{f}_{\pi p}$	16.8 ± 0.8 mb	assumes $\alpha_f=\alpha_\omega$
γ^{f}_{π}	3.44 ± 0.18 (mb)$^{1/2}$	assumes $\gamma^{f}_{p}=\gamma^{\omega}_{p}$
α_{pp}	0.42 ± 0.05	effective trajectory for σ_{pp} falloff from parametrization (3.2)
β_{pp}	11.1 ± 0.3 mb	effective residue for σ_{pp} falloff from parametrization (3.2)

(B6) and (B7) to predict β_{pp}^{ρ} and β_{pp}^{ω} from (2.1) and (2.3) and perform a fit to determine p_0. The result is

$$p_0 = 0.78 \pm 0.01 \text{ GeV}/c, \qquad (2.5)$$

which is a reasonable value for a threshold effect. The fit, shown in Fig. 3, is remarkably successful. This is especially true when one considers that above 20 GeV/c Fig. 3 is really a prediction, based on universality, from $\Delta(\pi^+p)$ and $\Delta(K^+p)$.

From the success of these fits we conclude that *ω universality, $3\gamma_K^{\omega} = \gamma_p^{\omega}$, is very accurate,* probably to better than 10%. Furthermore, the success of ω universality indicates *that the ϕ does in fact decouple from nucleons*[9] (i.e., $\gamma_p^{\phi} \lesssim \gamma_K^{\omega}/10$).

The fits are also *compatible* with the ρ universality relations (B6), but since the ρ residues are so much smaller than the ω residues (Table II), the fits to $\Delta(K^+p)$ and $\Delta(pp)$ cannot be regarded as a sensitive test of (B6).

The ratio $\alpha_\rho/\alpha_\omega$ is given by

$$\frac{\alpha_\rho}{\alpha_\omega} = 1.32 \pm 0.04, \qquad (2.6)$$

indicating a 30% breaking of the SU(3) prediction (B21) [$\alpha_\rho = \alpha_\omega$ also follows from meson-meson exchange degeneracy[10-14] (B4) when combined with (B1)].

Since $\alpha_\rho \neq \alpha_\omega$, any test of the SU(3) relation[8,15-17]

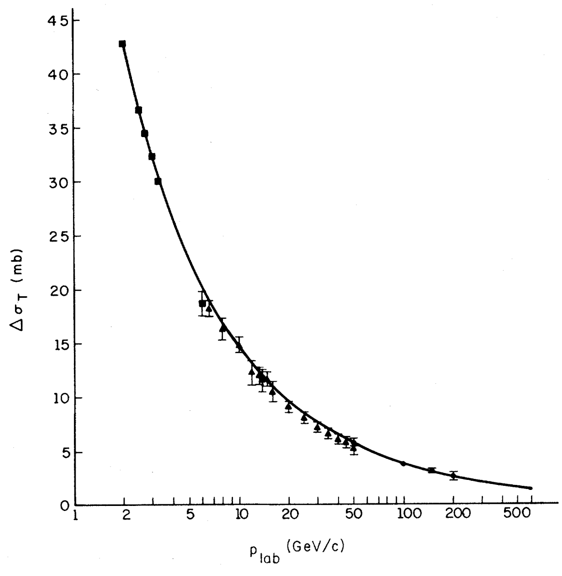

FIG. 3. $\Delta(pp) = \sigma_{\bar{p}p} - \sigma_{pp}$ fit to $2\beta_{pp}^{\rho}(p_{lab} - p_0)^{\alpha_\rho - 1} + 2\beta_{pp}^{\omega}(p_{lab} - p_0)^{\alpha_\omega - 1}$. The residues are predicted from $\Delta(\pi^+p)$ and $\Delta(K^+p)$ and the fit is to p_0. The details are in Tables I and II and the data are from the following references: ■ D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1968); R. J. Abrams *et al.*, Phys. Rev. D 1, 1917 (1970); other symbols and corresponding references appear in Fig. 2.

$\frac{1}{2}\gamma_\pi^{\rho} = \gamma_K^{\omega}$ [Eq. (B20)] is obscured by the ambiguity that the ratio $\frac{1}{2}\gamma_\pi^{\rho}/\gamma_K^{\omega}$ is dependent on the parameter s_0 used to scale s in the Regge power. For $s_0 = 2m_N \times 1$ GeV (which we have used)

$$\frac{\gamma_\pi^{\rho}}{2\gamma_K^{\omega}} = 0.70 \pm 0.01, \qquad (2.7)$$

while SU(3) predicts the ratio to be unity [Eq. (B20)]. *This indicates a 30% breaking of the SU(3) prediction for the factorized residues and a 50% breaking in the SU(3) predictions for the full residues.* Had we used $s_0 = 1$ GeV^2, the ratio (2.7) would have been around 0.64. A scale of $s_0 = 285$ GeV^2 would restore the SU(3) predictions for the residues: The symmetry breaking would then be contained in the factors $(s/s_0)^{\alpha_i - 1}$.

The Johnson-Treiman[18] and Freund[19] relations (B26) and (B27), which are implied by the union of SU(3) with universality, work much better in the Fermilab region than at lower energies.[20] This is due to a compensation between the SU(3) breaking in the residues and in the $p_{lab}^{\alpha_i - 1}$ factors. From the discussion in the previous paragraph the two relations would agree exactly with the present fit at the point $s = 285$ GeV^2 ($p_{lab} = 152$ GeV/c).

If we take the SU(3) relation (B23) seriously, the d_V/f_V ratio for the vector trajectory couplings to baryons is $d_V/f_V = -0.25 \pm 0.01$ for $s_0 = 2m \times 1$ GeV. Universality predicts the ratio to be 0, but the deviation is clearly due to SU(3) breaking and not a failure of universality.

We now turn to the f and A_2 trajectories and the question of exchange degeneracy.[10-14] We shall see in Sec. IV that the ρ-A_2 exchange degeneracy predictions

$$\sigma_{K^+p} = \sigma_{K^+n}, \quad \sigma_{pp} = \sigma_{pn} \qquad (2.8)$$

are compatible with the data. However, from Table II we see that the ρ (and A_2) contribution to these cross sections is insignificant compared to the ω (and f) contribution. Hence, the success of (2.8) is almost automatic at reasonably high energies.

The second prediction, that σ_{K^+p} and σ_{pp} should have no falling components, is much more stringent because it tests f-ω exchange degeneracy as well as ρ-A_2 exchange degeneracy. The K^+p cross section shows no hint of a falling component above 3 GeV/c. However, there is *a quite substantial drop in σ_{pp}* (about 9 mb between $p_{lab} = 2.0$ and $p_{lab} = 50$ GeV/c). Unfortunately, due to the rising component in σ_{pp} *it is impossible to uniquely determine the energy dependence of this falling component.* It could be part of the diffractive component of the cross section (see Sec. III), behaving as a logarithm, or it could be a secondary term behaving as a power. If we parametrize this component as $\beta_{pp} p_{\rm lab}{}^{\alpha_{pp}-1}$, the data are compatible with any value of α_{pp} between 0.3 and 0.5. A "best fit" yields

$$\beta_{pp} = 11.1 \pm 0.3 \text{ mb}, \qquad \alpha_{pp} = 0.42 \pm 0.05. \tag{2.9}$$

(The details are described in Sec. III.)

There are several possible explanations for this component; it could be the effect of a cut or of a low-lying trajectory. It is probably *not* due to the f' or ϕ not decoupling, as a large effect would then be expected in σ_{K^+p} [see (B20) and (B23)]. The (somewhat uncertain) value of α_{pp} in (2.9) suggests a breaking of f-ω exchange degeneracy. If this were the case, and if the breaking were due entirely to the residues, then to account for

$$\beta_{pp} = (\gamma_p^f)^2 - (\gamma_p^\omega)^2 = 11.1 \text{ mb} \tag{2.10}$$

we would require (Table II)

$$\frac{\gamma_p^f}{\gamma_p^\omega} \approx \frac{5.9}{4.9} = 1.2. \tag{2.11}$$

Such a 20% breaking is not unreasonable. However, in order to avoid the detection of a falling component in σ_{K^+p}, the exchange-degeneracy relation $\gamma_K^f = \gamma_K^\omega$ would have to be reasonably well satisfied (or perhaps even $\gamma_K^f < \gamma_K^\omega$).

In the next section we describe our determination of

$$\beta_{\pi p}^f = \gamma_\pi^f \gamma_p^f = 16.8 \pm 0.8 \text{ mb}. \tag{2.12}$$

The fit successfully assumes $\alpha_f = \alpha_\omega$.

If one assumes f-ω exchange degeneracy, then one expects $\gamma_p^f = \gamma_p^\omega$ and $\gamma_K^f = \gamma_K^\omega$. Furthermore, SU(3) (see Refs. 8 and 15–17) predicts $\gamma_\pi^f = 2\gamma_K^f$ (here there is no scale ambiguity of the type found in comparing $\frac{1}{2}\gamma_\pi^\rho$ with γ_K^ω). Hence, the theoretical prediction (which also assumes factorization and $\beta_{\pi p}^{f'} = 0$) is

$$\beta_{\pi p}^f \underset{\rm SU(3)}{=} 2\beta_{Kp}^f \underset{f-\omega}{=} 2\beta_{Kp}^\omega = 15.9 \pm 0.3 \text{ mb}, \tag{2.13}$$

in excellent agreement with (2.12).

On the other hand, the ρ-f exchange-degeneracy relation[10–14] (B5) (which requires a theoretical extension of duality ideas to unmeasured meson-meson amplitudes) predicts

$$\beta_{\pi p}^f \underset{\rho-f;\, f-\omega}{=} \gamma_\pi^\rho \gamma_p^\omega = 11.1 \pm 0.1 \text{ mb}. \tag{2.14}$$

Observe that the ρ-f relation suffers from the same scale ambiguity that was encountered in (2.7).

Of course, both (2.13) and (2.14) would have to be modified if (2.11) were true. However, (2.13) depends only on the product $\gamma_K^f \gamma_p^f$, which cannot differ very much from β_{Kp}^ω because of the behavior of σ_{K^+p}.

Some possible tests of ρ-A_2 exchange degeneracy are described in Sec. II B.

B. Charge- and isospin-exchange cross sections

In this section we predict the values of the forward differential cross sections for various nondiffractive reactions in terms of the Regge parameters determined from $\Delta(\pi p)$ and $\Delta(Kp)$. Our motivation is to test various aspects of Regge theory, such as the Regge phase, ρ-A_2 exchange degeneracy, and SU(3), as well as to further test the parameters of our fits. The predictions are compared with experiment.

1. $\pi^- p \to \pi^0 n$

The amplitude for $\pi^- p$ charge exchange is

$$T_{\pi^- p \to \pi^0 n} = \frac{1}{\sqrt{2}}(T_{\pi^+ p \to \pi^+ p} - T_{\pi^- p \to \pi^- p}) = -\sqrt{2}\,\beta_{\pi p}^\rho \left(\tan\frac{\pi\alpha_\rho}{2} + i\right) p_{\rm lab}{}^{\alpha_\rho}. \tag{2.15}$$

If we use Eq. (A2) the forward differential cross section is

$$\left(\frac{d\sigma}{dt}\right)_0 = \frac{2}{16\pi}(\beta_{\pi p}^\rho)^2 \left[1 + \tan^2\left(\frac{\pi\alpha_\rho}{2}\right)\right] p_{\rm lab}{}^{2\alpha_\rho - 2} = 1.83 p_{\rm lab}{}^{-0.85} \text{ mb (GeV)}^{-2}, \tag{2.16}$$

where we have used the ρ parameters from Table II. Comparison with experiment is complicated by the fact that the ρ coupling to nucleons is primarily of the helicity-flip type,[3] so the differential cross section dips in the forward direction. The Regge pole prediction for charge exchange is in good agreement with previous charge-exchange data, but is in serious disagreement with the new Caltech-LBL data of Barnes *et al.*[2] (see Fig. 4). It should be noted that there is a discrepancy between the data of Barnes *et al.* and the earlier data of Bolotov *et al.*[21] in the overlapping energy range. Nevertheless, if both the new charge-exchange data and the Fermilab cross-section data are correct, the simple Regge-pole picture fails at high energies. The ratio of real to imaginary parts of the πp isospin-one amplitude falls with energy below the constant value predicted by the Regge-pole picture at high energies.[22] A dispersion relation calculation of the real part of the πp isospin-one amplitude shows that the simple power law for $\Delta(\pi^+ p)$ cannot remain valid at higher energies. In order to estimate the necessary change in $\Delta(\pi^+ p)$ we have assumed

$$\text{Im} T_{\pi^- p \to \pi^0 n} = \begin{cases} -\sqrt{2}\,\beta_{\pi p}^\rho p_{\rm lab}{}^{\alpha_\rho}, & p_{\rm lab} < 200 \text{ GeV}/c \\ -\sqrt{2}\,\bar\beta_{\pi p}^\rho p_{\rm lab}{}^{\bar\alpha_\rho}, & p_{\rm lab} > 200 \text{ GeV}/c \end{cases}$$

where $\beta^{\rho}_{\pi p}$ and α_ρ are the parameters of Table II, and we have determined $\bar{\beta}^{\rho}_{\pi p}$ and $\bar{\alpha}_\rho$ such that $\mathrm{Im}T_{\pi^- p\to\pi^0 n}$ is continuous at 200 GeV/c and $\mathrm{Re}T_{\pi^- p\to\pi^0 n}$ as computed by dispersion relations gives the Caltech-LBL value for $(d\sigma/dt)_0$ at 100 GeV/c. We obtain $\bar{\alpha}_\rho = 0.4$. This exercise indicates that a change in the behavior of $\Delta(\pi^+ p)$ is expected, at high energies, though not necessarily a drastic one.[23,24] Our value for α_ρ agrees with the value 0.56 ± 0.10 recently determined[25] from the reaction $\pi^+ p \to \omega\Delta^{++}$ below 10 GeV/c.

2. $K^0_L p \to K^0_S p$

The regeneration amplitude is given by

$$T_{K^0_L p\to K^0_S p} = \tfrac{1}{2}(T_{K^0 p\to K^0 p} - T_{\bar{K}^0 p\to \bar{K}^0 p})$$
$$= \beta^{\rho}_{K\rho}\left(\tan\frac{\pi\alpha_\rho}{2} + i\right) p_{\mathrm{lab}}{}^{\alpha_\rho}$$
$$- \beta^{\omega}_{Kp}\left(\tan\frac{\pi\alpha_\omega}{2} + i\right) p_{\mathrm{lab}}{}^{\alpha_\omega}. \qquad (2.17)$$

Combining Eqs. (A2) and (2.17) with the Regge-fit parameters in Table II, we predict both the forward differential cross section and the phase ϕ of the forward amplitude $T_{K^0_L p\to K^0_S p}$. The results are compared[26] with experiment in Figs. 5 and 6. The agreement is excellent. *This verifies the parameters and Regge phase relation for the* ω, which strongly dominates the amplitude $(\beta^{\omega}_{Kp}/\beta^{\rho}_{Kp} = 6.1)$. Incidentally, the ω is largely helicity nonflip,[3] so the cross section peaks in the forward direction.

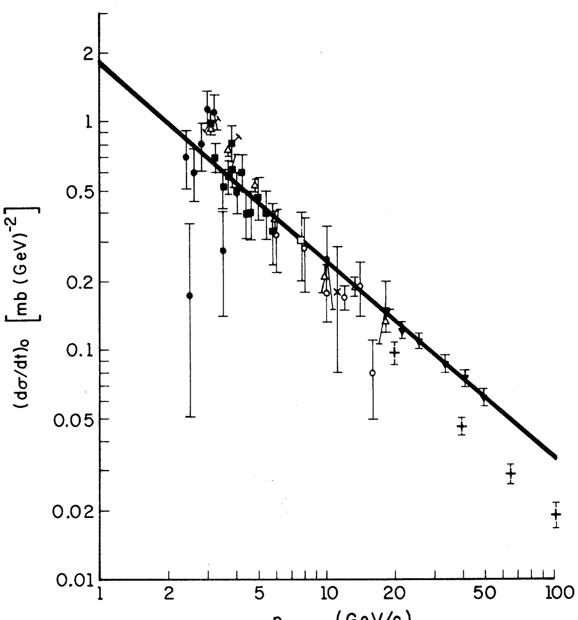

FIG. 4. $(d\sigma/dt)_0$ for $\pi^- p \to \pi^0 n$. The line is the prediction (2.16). The data are from the following references: ● M. A. Wahlig *et al.*, 1968; ■ M. Yvert, 1968; ○ I. Mannelli *et al.*, 1965; □ O. Guisan *et al.*, 1971; ▲ O. Guisan *et al.*, 1968; × P. Bonamy *et al.*, 1970; complete references in Particle Data Group Summary, LBL Report No. LBL-63, 1973 (unpublished); △ P. Sonderegger *et al.*, Phys. Lett. 20, 75 (1966); A. V. Stirling *et al.*, Phys. Rev. Lett. 14, 763 (1965); ▼ V. N. Bolotov *et al.*, Phys. Lett. 38B, 120 (1971); Nucl. Phys. B73, 365 (1974); + A. V. Barnes *et al.*, Ref. 2.

3. $\pi^- p \to \eta n$

The reaction $\pi^- p \to \eta n$ should be dominated by the A_2 trajectory. If we assume ρ-A_2 exchange degeneracy,[9-14]

$$\gamma^{A_2}_{p\bar{n}} = \sqrt{2}\,\gamma^{A_2}_{p} = \sqrt{2}\,\gamma^{\rho}_{p}, \qquad (2.18)$$
$$\alpha_\rho = \alpha_{A_2}.$$

The $\pi^- - \eta - A_2$ coupling is predicted by the SU(3) (see Refs. 8, and 15–17) relation (B23)

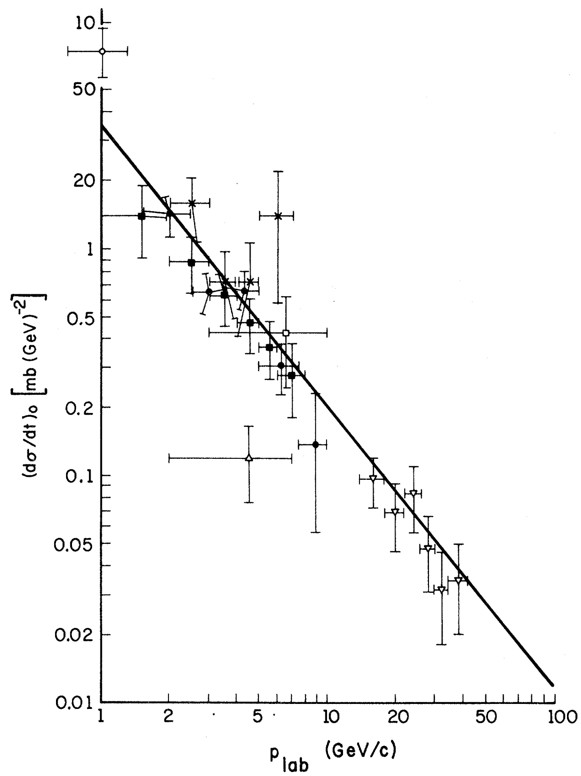

FIG. 5. $(d\sigma/dt_0)$ for $K^0_L p \to K^0_S p$ compared with the prediction from (2.17). The data are from the following references: ✖ P. Darriulat *et al.*, 1970; ○ L. B. Leipuner *et al.*, 1963; □ C. D. Buchanan *et al.*, 1971; △ A. Firestone *et al.*, 1966; ■ A. D. Brody *et al.*, 1971; complete references in Particle Data Group Summary, LBL Report No. LBL-55, 1972 (unpublished); ● G. W. Brandenburg *et al.*, Phys. Rev. D 9, 1939 (1974); ▽ V. K. Birulev *et al.*, Phys. Lett. 38B, 452 (1972); JINR report, 1972 (unpublished). See also the note in Ref. 26.

$$\gamma^{A_2}_{\pi^-\eta} = \frac{2}{\sqrt{3}}\gamma^{A_2}_{K} = \frac{2}{\sqrt{3}}\gamma^{\rho}_{K}\,. \tag{2.19}$$

Since (2.19) relates couplings of the *same* trajectory to different particles, there is no scale ambiguity. Hence (using an *even*-signature factor)

$$T_{\pi^- p\to \eta n} = 2(\tfrac{2}{3})^{1/2}\beta^{\rho}_{Kp}\left(-\cot\frac{\pi\alpha_\rho}{2}+i\right)p_{\rm lab}{}^{\alpha_\rho} \tag{2.20}$$

and

$$\left(\frac{d\sigma}{dt}\right)_0 = 0.381 p_{\rm lab}{}^{-0.852}\ \text{mb (GeV)}^{-2}\,. \tag{2.21}$$

To make contact with experiment (in which the two-photon decay of the η is measured), we multiply (2.21) by $\Gamma_{\eta\to 2\gamma}/\Gamma_{\rm tot} = 0.38$ to obtain

$$\left(\frac{d\sigma}{dt}\right)_{0\ \eta\to 2\gamma} = 145 p_{\rm lab}{}^{-0.852}\ \mu\text{b (GeV)}^{-2}\,. \tag{2.22}$$

Again the experimental data are complicated by a dip in the forward direction, but a comparison to the Regge prediction is shown in Fig. 7. While the prediction agrees well with previous data, it differs substantially from the new LBL-Caltech data of Dahl *et al.*[27] There is some discrepancy in the region of overlap between the data of Dahl *et al.* and those of Bolotov *et al.*[28] Again, if we assume that the new data are correct, the combined assumptions of Regge-pole theory and ρ-A_2 exchange degeneracy fail at high energies. As in the charge-exchange case, the ratio of real to imaginary for isospin-one amplitudes may fall below the value predicted by the Regge-pole picture at high energies, or the assumption of ρ-A_2 degeneracy may be invalid.

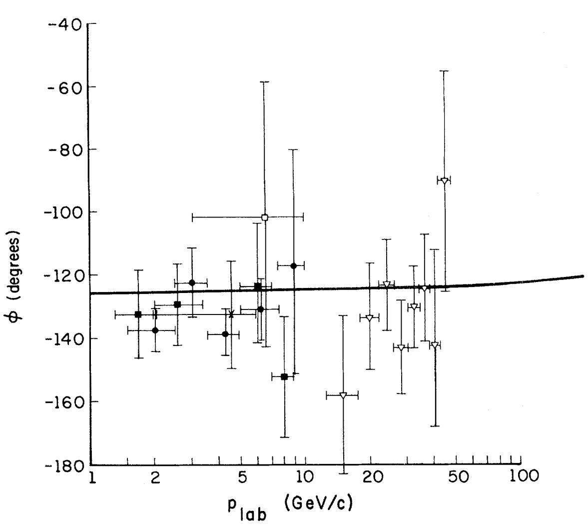

FIG. 6. The phase of the forward amplitude for $K^0_L p \to K^0_S p$, compared with the prediction from (2.17). The data are from Fig. 5.

4. $K^- p\to \bar{K}^0 n$ and $K^+ n \to K^0 p$

These reactions are a stringent test of ρ-A_2 exchange degeneracy as well as the value of β^{ρ}_{Kp}. The amplitudes are

$$\begin{aligned} T_{K^- p\to \bar{K}^0 n} &= T_{K^- n\to K^- n} - T_{K^- p\to K^- p} \\ &= -2(T^{\rho}_{Kp} + T^{A_2}_{Kp}) \end{aligned} \tag{2.23}$$

and

$$\begin{aligned} T_{K^+ n\to K^0 p} &= T_{K^+ p\to K^+ p} - T_{K^+ n\to K^+ n} \\ &= -2(T^{\rho}_{Kp} - T^{A_2}_{Kp})\,, \end{aligned} \tag{2.24}$$

where

$$\begin{aligned} T^{\rho}_{Kp} &= \beta^{\rho}_{Kp}\frac{1-e^{-i\pi\alpha_\rho}}{\sin\pi\alpha_\rho}p_{\rm lab}{}^{\alpha_\rho}\,, \\ T^{A_2}_{Kp} &= \beta^{A_2}_{Kp}\frac{-1-e^{-i\pi\alpha_{A_2}}}{\sin\pi\alpha_{A_2}}p_{\rm lab}{}^{\alpha_{A_2}}\,. \end{aligned} \tag{2.25}$$

If ρ-A_2 exchange degeneracy is valid, the differential cross sections should be equal for all (small) values of t. This seems roughly true experimentally[3,29] for $p_{\rm lab} \gtrsim 5$ GeV/c, but not for smaller $p_{\rm lab}$. Fits to the low-energy data[30] have generally assumed broken ρ-A_2 exchange degeneracy to ac-

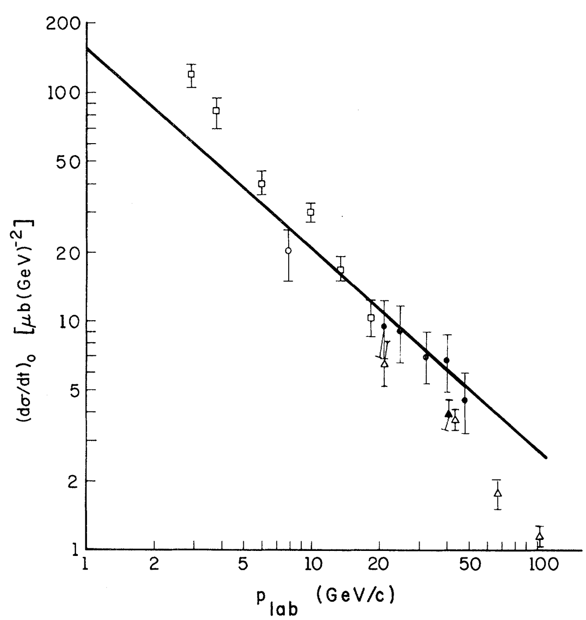

FIG. 7. $(d\sigma/dt)_0$ for $\pi^- p\to \eta n \to 2\gamma + n$. The line is the prediction (2.22). The data are from the following references: ● V. N. Bolotov *et al.*, Nucl. Phys. B73, 387 (1974); □ O. Guisan *et al.*, Phys. Lett. 18, 200 (1965); ○ O. Guisan, 1971 (see Fig. 4); ▲ W. D. Apel *et al.*, in Proceedings of the Conference on High Energy Physics, London, England, 1974 (unpublished); △ O. Dahl *et al.*, *ibid.* 1974 (unpublished).

count for the nonequality of the K^-p and K^0n t distributions. The forward cross sections (which are again hard to measure because of the sharp forward dip) are predicted from ρ-A_2 exchange degeneracy to be

$$\left(\frac{d\sigma}{dt}\right)_0 = \frac{1}{\pi}\left(\frac{\beta^{\rho}_{Kp}}{\sin\pi\alpha_\rho}\right)^2 p_{\rm lab}{}^{2\alpha_\rho - 2}$$
$$= 1.49 p_{\rm lab}{}^{-0.85}\ \text{mb (GeV)}^{-2}\,. \qquad (2.26)$$

Equation (2.26) is compared with experiment in Fig. 8. The agreement is good above 4 GeV/c

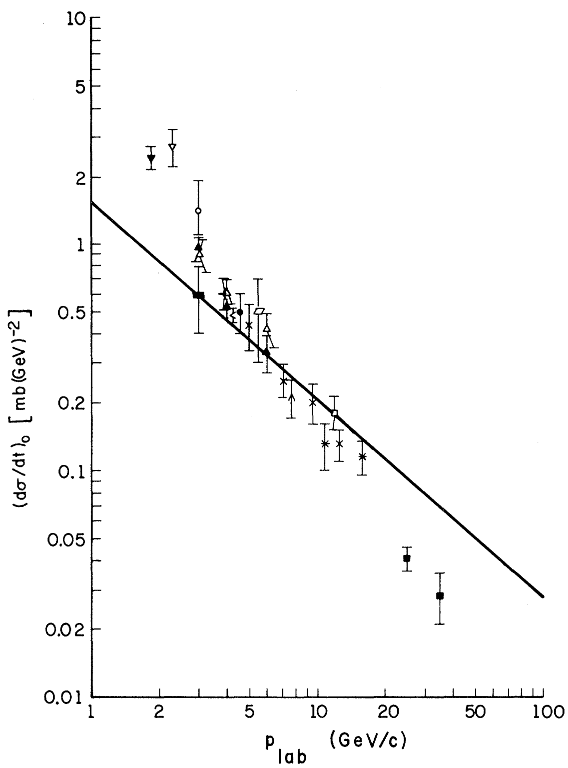

FIG. 8. $(d\sigma/dt)_0$ for $K^-p \to \bar{K}^0n$ (solid points) and $K^+n \to K^0p$ (open points) compared with the prediction (2.26). The data for K^-p are from the following references: ▼ P. J. Litchfield *et al.*, 1971; + L. Moscoso *et al.*, 1970; × P. Astbury *et al.*, 1966; ∧ A. Buffington *et al.*, 1968; complete references in CERN Report No. CERN/HERA 72-2, 1972 (unpublished); ■ V. N. Bolotov *et al.*, Serpukhov Reports Nos. IHEP 73-53, 1973 (unpublished), and 73-58, 1973 (unpublished); ▲ R. Diebold *et al.*, Ref. 29; ● M. Aguilar-Benitez *et al.*, Phys. Rev. D 4, 2583 (1971); – J. Badier *et al.*, Report No. CEA-R-3037, 1966 (unpublished); < R. Blokzijl *et al.*, Nucl. Phys. B51, 535 (1973); * K. J. Foley *et al.*, Phys. Rev. D 9, 42 (1974). The data for K^+n are from the following references: ▽ I. Butterworth *et al.*, 1965; □ A. Firestone *et al.*, 1970; complete references in CERN Report No. CERN/HERA 72-2, 1972 (unpublished); △ R. Diebold *et al.*, Ref. 29; ○ Y. Goldschmidt-Clermont *et al.*, Phys. Lett. 27B, 602 (1968); ◇ D. Cline *et al.*, Nucl. Phys. B22, 247 (1970).

except for the two highest-energy points.[31]

The rapid falloff of the cross section suggested by the last two points would be hard to understand on the basis of broken exchange degeneracy and might require strong absorption effects. Higher-energy experiments on $K^-p \to \bar{K}^0n$ to settle these issued are clearly desirable.

III. THE TOTAL CROSS SECTIONS

The experimental results of Ref. 1 show that, with the exception of antiprotons, all total cross sections of hadrons on protons eventually grow with energy. For the first time a clear increase has been observed in $\pi^\pm p$, $\pi^\pm D$, K^-p, and K^-D total cross sections. In addition, the accuracy of measurements in Ref. 1 is sufficient to begin a phenomenological examination of the way in which the cross sections increase.

To facilitate discussion, we will deal here with the total cross sections for pp, π^+p, and K^+p interactions. From relatively low-energy experiments it is clear that, beyond the resonance region, the pp and π^+p cross sections fall substantially, while the K^+p cross section does not. Regge theory attributes the falloff of the π^+p cross section to the ρ and f trajectories, while the absence of a sharp falloff in the K^+p cross section comes from the

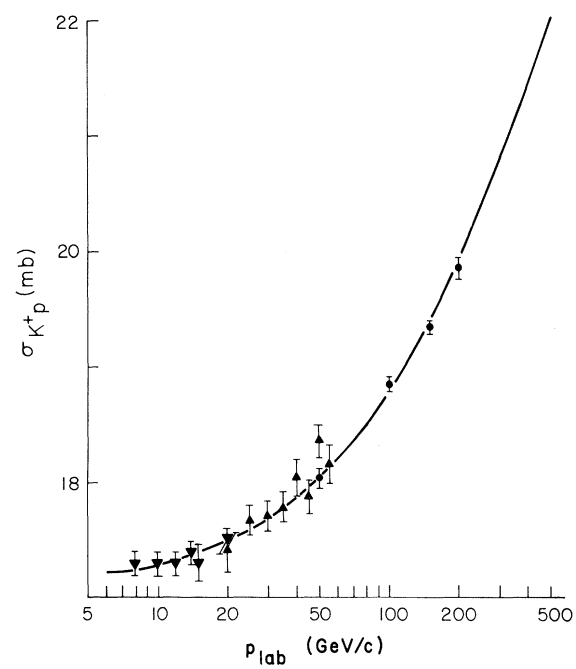

FIG. 9. σ_{K^+p}. The curve is the result of the fit (3.3) and the data are from the following references: ▼ W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965); ▲ S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); ● A. S. Carroll *et al.*, Ref. 1.

exchange degeneracy of the f and ω, ρ and A_2 trajectories [see Eqs. (A5)]. The low-energy falloff of the pp cross section is not well understood, and may result from a breaking of exchange degeneracy, a lower-lying singularity, or some other mechanism. Above 50 GeV/c, all three total cross sections begin to grow with energy, with the K^+p cross section increasing sharply at lower energies, as shown in Figs. 9–11.

Following the scheme outlined above, we parametrize the total cross sections with the general form

$$\begin{aligned}\sigma_{K^+p} &= D_{Kp}\,,\\ \sigma_{\pi^+p} &= D_{\pi p} + \beta^f_{\pi p}\, p_{\rm lab}{}^{\alpha_f - 1} - \beta^\rho_{\pi p} p_{\rm lab}{}^{\alpha_\rho - 1}\,, \qquad (3.1)\\ \sigma_{pp} &= D_{pp} + \beta_{pp} p_{\rm lab}{}^{-n}\,.\end{aligned}$$

The values of $\beta^\rho_{\pi p}$ and α_ρ are taken directly from the difference measurements of $\Delta(\pi p)$, and the intercept α_f is taken to be the ω intercept, determined from the $\Delta(Kp)$ measurements as described in Sec. II. In each reaction, D_{ip} is the diffractive component of the total cross section,[32] the object of primary interest, which contains the increasing part of the cross section for each reaction. The additional term included in σ_{pp} to account for the low-energy falloff has β_{pp} and n as free parameters in the fit. *One can successfully fit the pp data without such a term* using, for example, $\sigma_{pp} = a_i + b_i[\ln(p_{\rm lab}/c_i)]^{\alpha_i}$ with the low-energy falloff of σ_{pp} coming from this diffractive term (see Table III). However, we will also investigate strictly increasing diffractive terms, for which an additional nondiffractive term is necessary for σ_{pp}. Thus, we choose a strategy which maintains exchange degeneracy in the K^+p reaction, and we include the additional nondiffractive term in σ_{pp}.

The results of the fitting analysis show the total cross-section data to be compatible with a number of functional forms for the diffractive components. Diffractive components such as[33] $D_{ip} = a_i + b_i \ln p_{\rm lab} + c_i(\ln p_{\rm lab})^2$, $D_{ip} = a_i + b_i(\ln p_{\rm lab})^{c_i}$, and $D_{ip} = C_i \ln[(p_{\rm lab} + m_i)/b_i]$ all give reliable fits to the cross-section data (see Table III). It is clear from this analysis that the experimental data are not measured to high enough energies or sufficient accuracy to single out one parametrization over all others.

We investigate further the parametrization[34]

$$D_{ip} = C_i \ln\left(\frac{p_{\rm lab} + m_i}{b_i}\right). \qquad (3.2)$$

Using this parametrization along with Eqs. (3.1) involves making a three-parameter fit to the K^+p cross section, a four-parameter fit to the π^+p cross section, and a five-parameter fit to the pp cross section. This parametrization is fairly sensitive to the coefficients C_i for each reaction, and it is found that a reliable fit to the data can be made with $C_K : C_\pi : C_p$ in the ratio $1:1:\frac{3}{2}$. The

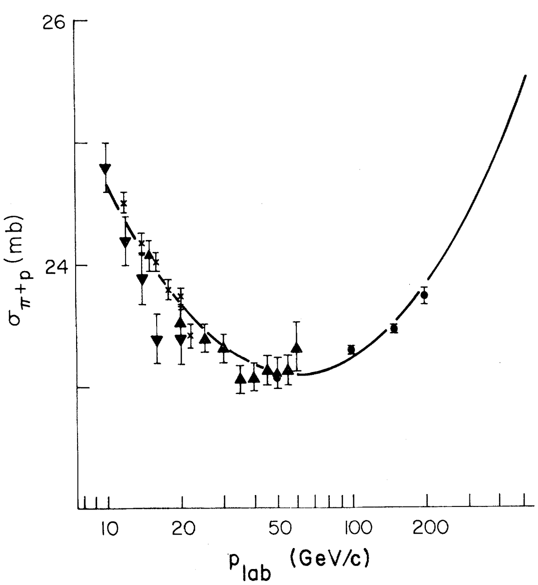

FIG. 10. σ_{π^+p}. The curve is the fit (3.3) and the data are from the following references: × K. J. Foley *et al.*, Phys. Rev. Lett. 19, 330 (1967); the other symbols are as in Fig. 9.

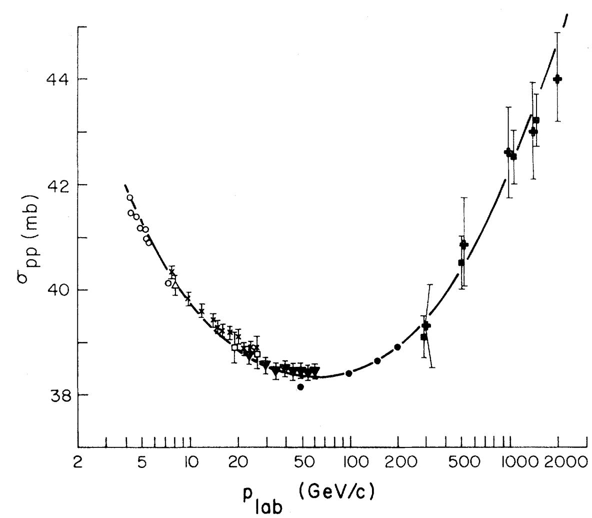

FIG. 11. σ_{pp}. The curve is the fit (3.3) and the data are from the following references: + S. R. Amendolia *et al.*, Phys. Lett. 44B, 119 (1973); ■ U. Amaldi *et al.*, *ibid.* 44B, 112 (1973); ▼ S. P. Denisov *et al.*, Phys. Lett. 36B, 415 (1971); ● A. S. Carroll *et al.*, Ref. 1; × K. J. Foley *et al.*, Phys. Rev. Lett. 19, 330 (1967); □ G. Bellettini *et al.*, Phys. Lett. 14, 164 (1965); ○ D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1968); △ J. Ginestet *et al.*, Nucl. Phys. B13, 283 (1969).

TABLE III. Parametrizations of σ_{K^+p}, σ_{π^+p}, and σ_{pp} using various diffractive forms. In cases (1)–(3), the fits are made as described below Eqs. (3.1). In case (4), σ_{pp} is parametrized without a term $\beta_{pp} p_{\rm lab}^{-n}$. χ^2/ν given for each fit is the total χ^2 divided by the number of degrees of freedom.

Reaction	K^+p	π^+p	pp
No. of data pts.	19	25	66
$p_{\rm lab}$ range (GeV/c)	8–200	10–200	4.5–2000
(1) Diffractive term: $D_{ip}=C_i\ln[(p_{\rm lab}+m_i)/b_i]$ ($C_i, \beta^f_{\pi p}, \beta_{pp}$ in millibarns; $p_{\rm lab}, m_i, b_i$ in GeV/c)			
	$C_i=3.27\pm0.07$	$C_i=3.27\pm0.07$	$C_i=4.91\pm0.11$
	$m_i=149\pm7$	$m_i=206\pm23$	$m_i=541\pm54$
	$b_i=0.80\pm0.03$	$b_i=0.33\pm0.02$	$b_i=0.30\pm0.06$
	$\chi^2/\nu=0.56$	$\beta^f_{\pi p}=16.8\pm0.8$	$\beta_{pp}=11.1\pm0.3$
		$\chi^2/\nu=1.27$	$n=0.58\pm0.05$
			$\chi^2/\nu=0.99$
(2) Diffractive term: $D_{ip}=a_i+b_i\ln p_{\rm lab}+c_i(\ln p_{\rm lab})^2$ ($a_i, b_i, C_i, \beta^f_{\pi p}, \beta_{pp}$ in millibarns; $p_{\rm lab}$ in GeV/c)			
	$a_i=18.3\pm0.3$	$a_i=14.4\pm3.3$	$a_i=22.6\pm0.9$
	$b_i=-0.98\pm0.20$	$b_i=1.56\pm1.08$	$b_i=-2.19\pm0.34$
	$c_i=0.24\pm0.03$	$c_i=0.0\pm0.1$	$c_i=0.41\pm0.02$
	$\chi^2/\nu=0.45$	$\beta^f_{\pi p}=29.4\pm5.1$	$\beta_{pp}=23.5\pm0.9$
		$\chi^2/\nu=1.07$	$n=0.07\pm0.02$
			$\chi^2/\nu=1.16$
(3) Diffractive term: $D_{ip}=a_i+b_i(\ln p_{\rm lab})^{c_i}$ ($a_i, b_i, \beta^f_{\pi p}, \beta_{pp}$ in millibarns; $p_{\rm lab}$ in GeV/c)			
	$a_i=17.1\pm0.1$	$a_i=11.2\pm3.2$	$a_i=7.65\pm0.98$
	$b_i=0.01\pm0.006$	$b_i=3.14\pm1.81$	$b_i=1.62\pm0.26$
	$c_i=3.35\pm0.29$	$c_i=0.76\pm0.18$	$c_i=1.31\pm0.06$
	$\chi^2/\nu=0.48$	$\beta^f_{\pi p}=32.2\pm2.8$	$\beta_{pp}=39.7\pm1.0$
		$\chi^2/\nu=1.07$	$n=0.16\pm0.10$
			$\chi^2/\nu=1.70$
(4) Diffractive term[a]: $D_{ip}=a_i+b_i[\ln(p_{\rm lab}/c_i)]^{d_i}$ ($a_i, b_i, \beta^f_{\pi p}, \beta_{pp}$ in millibarns; $p_{\rm lab}, c_i$ in GeV/c)			
	$a_i=17.3\pm0.1$	$a_i=21.8\pm0.3$	$a_i=38.3\pm0.1$
	$b_i=0.23\pm0.28$	$b_i=0.49\pm0.09$	$b_i=0.44\pm0.05$
	$c_i=7.6\pm6.5$	$c_i=27.7\pm5.7$	$c_i=63.2\pm1.5$
	$d_i=2.02\pm0.64$	$d_i=1.53\pm0.25$	$d_i=2.02\pm0.09$
	$\chi^2/\nu=0.48$	$\beta^f_{\pi p}=5.8\pm2.1$	$\chi^2/\nu=1.13$
		$\chi^2/\nu=1.09$	

[a] σ_{pp} is fitted without a term $\beta_{pp} p_{\rm lab}^{-n}$ in this case.

ratio $C_K/C_\pi=1$ suggests that SU(3) arguments may hold for the coefficients C_i, while the ratio $C_K:C_\pi:C_p$ of $1:1:\frac{3}{2}$ would be expected from extending the naive quark model[35] to the coefficients. Taking the over-all normalization of the C_i from the pp cross section gives the parametrization[36]

$$\sigma_{K^+p}=3.27\ln\left(\frac{p_{\rm lab}+149}{0.80}\right),$$

$$\sigma_{\pi^+p}=3.27\ln\left(\frac{p_{\rm lab}+206}{0.33}\right)+\frac{16.8}{p_{\rm lab}^{0.57}}-\frac{2.62}{p_{\rm lab}^{0.43}},\tag{3.3}$$

$$\sigma_{pp}=4.91\ln\left(\frac{p_{\rm lab}+541}{0.30}\right)+\frac{11.1}{p_{\rm lab}^{0.58}},$$

where σ_{ip} are in millibarns and $p_{\rm lab}$ is in GeV/c. These fits are illustrated by the solid curves in Figs. 9–11. The parametrization of the data is not particularly sensitive to the values of m_i and b_i. By a small adjustment of C_i in each reaction, reliable fits may be made with m_i and b_i varying considerably. It can be concluded, however, that the scale b_i corresponds to a scale (in units of s) of $s_0\simeq1$–2 GeV^2, which is a reasonable value for the energy scale. In addition, the fits indicate that the threshold parameter m_i increases as one goes from K^+p to π^+p to pp reactions. This gives a phenomenological mechanism for the faster growth of the K^+p cross section at low energies. *If such a parametrization holds true, all the total cross sections will grow asymptotically as a single power*[34] *of* $\ln(p_{\rm lab})$, *with* σ_{K^+p} *and* σ_{π^+p} *approaching each other, and both approaching* $\frac{2}{3}\sigma_{pp}$ *asymptotically, as predicted by the naive quark model*[35] [Eq. (B30)].

An interesting result of the parametrization of σ_{π^+p} is the extraction of $\beta^f_{\pi p}$, a free parameter in the fit. While the value of the parameter depends on the functional form chosen for the diffractive component, the form of Eq. (3.2) results in the value $\beta^f_{\pi p}=16.8\pm0.8$ mb. This is in close agreement with the value $2\beta^\omega_{Kp}=15.9\pm0.25$ mb extracted from $\Delta(\pi p)$ and $\Delta(Kp)$ as described in Sec. II.

The extraction of the nondiffractive component in σ_{pp} is again dependent on the form chosen for the pp diffractive term, as can be seen from Table III. For the diffractive form of Eq. (3.2), we find $n=0.58$, a value quite close to that of the f-ω intercept. However, even with the particular diffractive form we have chosen, reliable fits can be found with n ranging from 0.5 to 0.7, by changing β_{pp}, C_p, m_p, and b_p appropriately.

In conclusion, it should be stressed that the total cross-section data admit to a number of possible parametrizations with different diffractive components. While the parametrization discussed here appears to represent the diffractive component of total cross sections in a unified and theoretically appealing way, it is by no means unique. At this time, our parametrization should be considered a reasonable interpolation of the data.

IV. DEUTERON SCREENING AND TOTAL CROSS SECTIONS ON DEUTERIUM TARGETS

Until now we have considered only proton-target cross sections. The reason for this is that the conventional elastic Glauber correction[37] needed to extract neutron cross sections from the deuterium data is not sufficiently reliable at high energies; the inelastic rescattering corrections become important.[38-40]

In this section we will calculate the screening corrections, which constitute a sensitive test of the small-t behavior of the data for the inclusive reaction $pp\to p+X$ and of the nonvanishing of the triple-Pomeranchukon coupling.

Using the calculated screening corrections and the neutron-target cross sections (which are predicted from exchange degeneracy), we then predict the deuteron-target cross sections and compare them with the data. Our goal is to test the inelastic Glauber theory, as the uncertainties in this are larger than the effects of a small breaking of exchange degeneracy. It should be noted that the relations $\sigma_{pn}=\sigma_{pp}$ and $\sigma_{K^+n}=\sigma_{K^+p}$ follow from ρ-A_2 exchange degeneracy alone. Since the ρ and A_2 couple relatively weakly (Table II), any breaking of ρ-A_2 exchange degeneracy is irrelevant for our present purposes in the Fermilab energy range. Similarly, σ_{K^-n} and $\sigma_{\bar{p}n}$ can be computed from $\sigma_{\bar{p}p}$ and σ_{K^-p}, the ρ parameters from Table II, and the assumption of ρ-A_2 exchange degeneracy (again, a small breaking would be irrelevant here). The πn cross sections are related by isospin to the πp cross sections in Eq. (A.5).

The total cross section for particle i on deuterium is

$$\sigma_{id}=\sigma_{ip}+\sigma_{in}-\delta_i\ , \tag{4.1}$$

where δ_i is the Glauber screening correction. We will assume that the amplitudes are purely imaginary, which is an excellent approximation at Fermilab energies.[41]

The Glauber correction can be written

$$\delta_i=\delta_i^{\text{el}}+\delta_i^{\text{inel}}\ , \tag{4.2}$$

where δ_i^{el} is the elastic correction and δ_i^{inel} is the contribution from diffractively produced higher-mass intermediate states.[39] The elastic contribution is given by

$$\delta_i^{\text{el}}=\frac{\sigma_{ip}\sigma_{in}}{8\pi(R^2+b_i)}. \tag{4.3}$$

We have chosen a Gaussian form for the deuteron form factor, $\exp(-R^2\vec{q}^{\,2})$ with $R^2=37\ \text{GeV}^{-2}$ (see Ref. 39); b_i is the elastic slope.

The inelastic correction can be computed from a knowledge of the diffractive part of the inclusive cross section $i+p\to p+X$ and is very sensitive to the small t behavior of this cross section. In the triple-Regge model, a nonvanishing triple-Pomeranchukon vertex is expected to give important contributions to δ_i^{inel}. Two recent high-precision experiments[42,43] have been performed at Fermilab in the triple-Regge region. The $p+p\to p+X$ experiment[42] has x in the range 0.98 to 0.999 and $-t$ in the range 0.01 to 0.05 GeV^2, while the $p+d\to d+X$ experiment[43] measures down to $-t=0.03$ GeV^2. Neither experiment shows any indication of the triple-Pomeranchukon vertex starting to vanish. (An older experiment[44] at the ISR, led to some controversy concerning the triple-Pomeranchukon vertex contribution,[45] but measured down only to $-t=0.3$ GeV^2.) From the new $p+d\to d+X$ data,[43] Goulianos[46] has extracted and parametrized the $p+p\to p+X$ cross section (which is in excellent agreement with the subsequent direct measurement[42]). The inclusive cross section drops off sharply for M^2 below 1.7 GeV^2 (M is the missing mass). Above $M^2=1.7$ GeV^2, the data are represented very accurately[46] by

$$\frac{d\sigma_i}{dt\,dM^2}=\frac{A_i}{M^2}Be^{Bt}\ , \tag{4.4}$$

where

$$B=6\left[1+\frac{0.06}{(M-1.35)^2+0.02}\right]\text{GeV}^{-2} \tag{4.5}$$

and $A_{i=p}=0.7$ mb. One can easily recognize that the parametrization (4.4) becomes the triple-Pomeranchukon expression for large values of the missing mass M and has the property of taking properly into account the correct t dependence for small missing masses.

If we use (4.4), the inelastic correction reads[39]

$$\delta_p^{\text{inel}}=2A_p\int_{M_0^2}^{\infty}\frac{Be^{-\gamma^2(M^2-m_p^2)^2}}{R^2+B}\,\frac{dM^2}{M^2}\ , \tag{4.6}$$

where $\gamma=m_dR/s$ ($s\approx 2m_p\times p_{\text{lab}}$), and m_p and m_d are the proton and deuteron masses. δ_p^{inel} has been evaluated numerically taking $M_0^2=1.7$ GeV^2. Typical values for δ_p^{inel} are 0.48 mb at 50 GeV/c and 0.75 mb at 200 GeV/c. As the elastic correction is $\delta_p^{\text{el}}\sim 3$ mb, we see that the inelastic contribution is substantial (as much as 20% of δ_p). We will assume $\delta_p^{\text{inel}}=\delta_{\bar{p}}^{\text{inel}}$ (as is appropriate in the triple-Pomeranchukon region), take[47] $b_p\approx b_{\bar{p}}\approx 11.3$ GeV^{-2}, and use the parametrizations for σ_{pp} and $\sigma_{\bar{p}p}$ obtained in Secs. II and III along with exchange degeneracy to compute $\sigma_{\bar{p}d}$ and σ_{pd}. The resulting cross sections are compared with the experimental data in Fig. 12. We consider the agreement between the theoretical predictions and the experimental data to be remarkable.

Since there are no available data for the inclusive processes $\pi^+ p \to p + X$ or $K^+ p \to p + X$ at small-t values we will make the following approximations in order to compute δ_π^{inel} and δ_K^{inel}.

(a) Start the integration in (4.6) at $M_0^2 = 5\ \text{GeV}^2$ in order to get rid of the small masses, since in this region the triple-Pomeranchukon picture is not expected to be valid (the reader should keep in mind that the calculation is very sensitive to the t dependence). Take

$$A_\pi = \frac{\sigma_{\pi p}}{\sigma_{pp}} A_p \simeq \tfrac{24}{38} A_p, \quad A_K = \frac{\sigma_{Kp}}{\sigma_{pp}} A_p \simeq \tfrac{18}{38} A_p \qquad (4.7)$$

in Eq. (4.4). This approximation has been checked by computing the inclusive distribution in the high-missing-mass region for the $\pi^- p \to p + X$ experiment[48] at 205 GeV/c and good agreement was found.

(b) Use the estimate of the A_1 enhancement of Ref. 40 (one gets $\simeq 0.12$ mb) as representing the contribution of the small missing masses; we have arbitrarily taken 0.08 mb for the small-mass region contribution to the $Kp \to p + X$ reaction. Since the elastic slopes are not known in the Fermilab energy range, we have left b_π and b_K in Eq. (4.3) as free parameters and have determined them through a best fit to the $K^+ d$ and $\pi^\pm d$ cross sections. Therefore, our results for δ_K and δ_π are not entirely predictions; however, the sensitivity of (4.3) to the exact values of b_π and b_K is small. The values obtained are $b_\pi = 6\ (\text{GeV}/c)^{-2}$ and $b_K = 9.5\ (\text{GeV}/c)^{-2}$. The value obtained for b_π is somewhat smaller than would have been guessed from the lower-energy data ($\approx 8.5\ \text{GeV}^{-2}$). The errors in our estimate of δ_π^{inel} and our parametrizations could account for the difference.

The computed cross sections for $\pi^+ d$, $K^- d$ and $K^+ d$ are shown in Figs. 13 and 14 along with the experimental data. The agreement is again excellent, although the prediction for $\sigma_{K^- d}$ is slightly higher than the two highest-energy data points.

We consider the agreement between theory and experiment for the deuteron cross sections to be a *very strong confirmation of the theory of inelastic screening corrections*[38-40] *and of the inclusive data*[42,43] *which indicates a nonvanishing triple-Pomeranchukon vertex.*

Finally, we consider $\Delta(K^+ d)$ and $\Delta(pd)$. From Eqs. (4.1) and (A5) we have

$$\Delta(K^+ d) = 4B_{Kp}^{\omega}\left(1 - \frac{D_{Kp}}{8\pi(R^2 + b_K)}\right) \equiv 4\xi_K B_{Kp}^{\omega} \qquad (4.8)$$

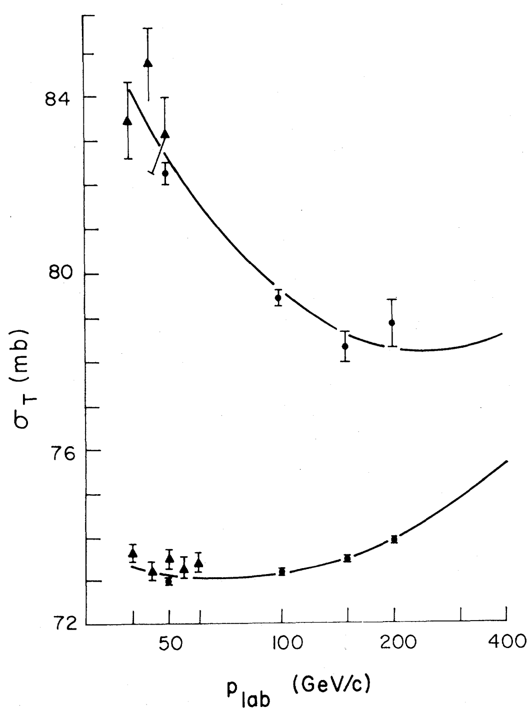

FIG. 12. σ_{pd} upper curve and $\sigma_{\bar{p}d}$ lower curve. The data are from Denisov *et al*. (see Fig. 1) and Carroll *et al*. (Ref. 1). The curves represent the predicted cross sections.

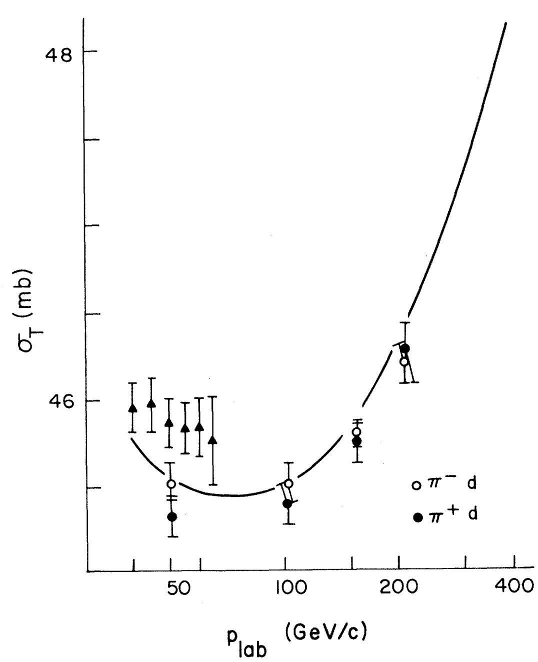

FIG. 13. $\sigma_{\pi^\pm d}$. Same as Fig. 12.

and

$$\Delta(pd) = 4B^{\omega}_{pp}\left(1 - \frac{D_{pp}}{8\pi(R^2 + b_p)}\right)$$
$$\equiv 4\xi_p B^{\omega}_{pp}\,, \qquad (4.9)$$

where we have neglected a very small term proportional to $(B^{\omega})^2$. If we neglect the energy dependence of D_{Kp} and D_{pp} (a second-order effect), the energy dependence of $\Delta(K^+d)$ and $\Delta(pd)$ is predicted to be $p_{\rm lab}{}^{\alpha_\omega - 1}$, which is in complete agreement with the data.

Using the data for σ_{K^+p} and σ_{pp} in the 50–200 GeV/c range[1] we predict from (4.8) and (4.9)

$$\xi_K = 0.96 \pm 0.01\,,$$
$$\xi_p = 0.92 \pm 0.01\,. \qquad (4.10)$$

We have fitted $\Delta(K^+d)$ and $\Delta(pd)$ using the values for α_ω and $\beta^{\omega}_{pp} = 3\beta^{\omega}_{Kp}$ from Table II. For $\Delta(pd)$, $p_{\rm lab}$ was replaced by $p_{\rm lab} - p_0^d$, where p_0^d was fitted in order to describe the very-low-energy behavior correctly. The result is

$$\xi_K = 0.90 \pm 0.01\,,$$
$$\xi_p = 0.87 \pm 0.01\,, \qquad (4.11)$$
$$p_0^d = 0.94 \pm 0.02 \text{ GeV}/c\,.$$

The details of the fit are given in Table I. The agreement between (4.10) and (4.11) is reasonable, especially when one considers that $\Delta(K^+d)$ and $\Delta(pd)$ are small quantities. They are very sensitive to systematic errors in the data and to the approximations made for the screening corrections.

We conclude therefore that ω universality ($\beta^{\omega}_{pp} = 3\beta^{\omega}_{Kp}$) and the screening corrections in Eqs. (4.8) and (4.9) are valid to around 5%.

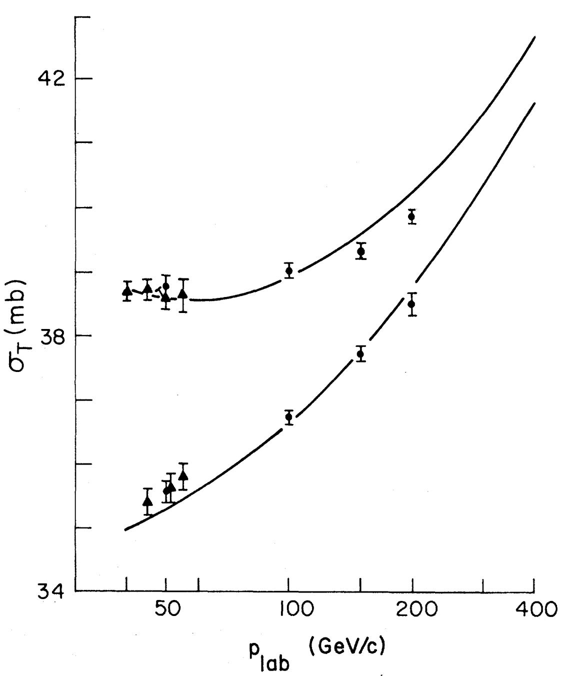

FIG. 14. σ_{K^-d} upper curve and σ_{K^+d} lower curve. Same as Fig. 12.

V. CONCLUSION

We have analyzed and fitted the available data on the total cross sections of $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$ on protons. The resulting parameters were used to predict the forward differential cross sections for several nondiffractive reactions, and the predictions were compared with experiment. The (inelastic) Glauber screening corrections for the scattering of pions, kaons, and nucleons on deuterium were determined; the corrections, when combined with proton-target cross sections, were used to predict the deuteron-target cross sections, which were then compared with experiment.

Our conclusions are as follows: (a) The diffractive components of the cross sections are compatible with several different functional forms for the energy dependence. (b) One very compact parametrization predicts that the K^+p, π^+p, and pp cross sections will all rise asymptotically as $\ln s$. The coefficients will be in the ratio $1:1:\frac{3}{2}$ predicted by SU(3) and the naive quark model. (c) The parameters of the falling components of σ_{π^+p} and σ_{pp} depend on the functional form of the diffractive component. (d) ω universality is satisfied extremely well, while ρ universality is compatible with the data. (e) Factorization is successful, the ϕ decouples from nucleons, and the data support $\beta^{f'}_{\pi p} = 0$. (f) The amplitude phase predicted by Regge theory for the ω is correct, while a discrepancy between the charge-exchange and total cross-section data for the ρ indicates a serious problem for the simple Regge-pole picture. (g) SU(3) relations between the residues of nondegenerate trajectories are substantially violated, while SU(3) relations involving the same trajectory are compatible with the data. (h) f-ω exchange degeneracy is supported by the data, although the falling component of σ_{pp} could indicate a small breaking. (i) Recent $\pi^-p \to \eta n$ data and the total cross-section data are inconsistent with the combination of a simple Regge-pole picture and ρ-A_2 exchange degeneracy. (j) ρ-f exchange degeneracy for residues appears violated, but the test is dependent on the parametrization of the diffractive component of σ_{π^+p}. (k) The inelastic contributions to the Glauber screening corrections, computed using inclusive scattering data which suggest a nonvanishing triple-Pomeranchukon coupling, are substantial (up to 20% of the entire correction). (l) The deuteron cross sections which are predicted (assuming ρ-A_2 exchange degeneracy as well as the computed screening corrections) are in excellent agreement with the data.

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APPENDIX A: NOTATION AND CONVENTIONS

Our amplitudes $T(s, t)$ are normalized so that the total cross section is

$$\sigma_{ab}(s) = \frac{1}{p_{\text{lab}}} \operatorname{Im} T_{ab \to ab}(s, 0), \tag{A1}$$

where $p_{\text{lab}} \approx s/2m_N$ is the laboratory momentum in GeV/c. A differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} \frac{|T|^2}{p_{\text{lab}}^2}. \tag{A2}$$

We denote the diffractive component of the total cross section, which we assume is an isosinglet, by $D_{ab}(s)$.

The contribution of a normal Regge *pole* with trajectory $\alpha_i(t)$ to $T_{ab\to ab}$ is given by

$$\beta^i_{ab}(t)\left(\frac{\mp 1 - e^{-i\pi\alpha_i}}{\sin\pi\alpha_i}\right) p_{\text{lab}}{}^{\alpha_i}, \tag{A3}$$

where the minus (plus) sign applies to even- (odd-) signature trajectories. The residue β^i_{ab} factorizes:

$$\beta^i_{ab} = \gamma^i_a \gamma^i_b. \tag{A4}$$

If a or b is a fermion, (A4) is true for each helicity amplitude. At $t=0$, however, only the helicity-nonflip amplitude survives.

We are mainly interested here in the f, f', ρ, ω, ϕ, and A_2 trajectories, all of which have intercepts near $\frac{1}{2}$. Their properties are listed in Table IV. It is generally believed[3] that the ϕ and f' decouple from nucleons.

TABLE IV. Properties of the major Regge trajectories. The last column refers to the principal coupling to nucleons for $t \neq 0$.

Trajectory	I^G	C	Signature	Coupling
f	0^+	+	even	helicity nonflip
f'	0^+	+	even	...
ρ	1^+	−	odd	helicity flip
ω	0^-	−	odd	helicity nonflip
ϕ	0^-	−	odd	...
A_2	1^-	+	even	helicity flip

The total cross sections of interest here are

$$\begin{aligned}
&\sigma_{\pi^-p} = D_{\pi p} + B^f_{\pi p} + B^\rho_{\pi p},\\
&\sigma_{\pi^+p} = D_{\pi p} + B^f_{\pi p} - B^\rho_{\pi p},\\
&\sigma_{\pi^-n} = \sigma_{\pi^+p},\\
&\sigma_{\pi^+n} = \sigma_{\pi^-p},\\
&\sigma_{K^-p} = D_{Kp} + B^f_{Kp} + B^\rho_{Kp} + B^\omega_{Kp} + B^{A_2}_{Kp},\\
&\sigma_{K^+p} = D_{Kp} + B^f_{Kp} - B^\rho_{Kp} - B^\omega_{Kp} + B^{A_2}_{Kp},\\
&\sigma_{K^-n} = D_{Kp} + B^f_{Kp} - B^\rho_{Kp} + B^\omega_{Kp} - B^{A_2}_{Kp},\\
&\sigma_{K^+n} = D_{Kp} + B^f_{Kp} + B^\rho_{Kp} - B^\omega_{Kp} - B^{A_2}_{Kp},\\
&\sigma_{\bar pp} = D_{pp} + B^f_{pp} + B^\rho_{pp} + B^\omega_{pp} + B^{A_2}_{pp},\\
&\sigma_{pp} = D_{pp} + B^f_{pp} - B^\rho_{pp} - B^\omega_{pp} + B^{A_2}_{pp},\\
&\sigma_{\bar pn} = D_{pp} + B^f_{pp} - B^\rho_{pp} + B^\omega_{pp} - B^{A_2}_{pp},\\
&\sigma_{pn} = D_{pp} + B^f_{pp} + B^\rho_{pp} - B^\omega_{pp} - B^{A_2}_{pp},
\end{aligned} \tag{A5}$$

where $B^i_{ab} \equiv \gamma^i_a\gamma^i_b p_{\text{lab}}{}^{\alpha_i - 1}$. Of course, lower-lying trajectories should in principle be added to (A5). The $f'(\phi)$ trajectories, if present, would enter (A5) with the same signs as the $f(\omega)$.

We shall sometimes use the notation

$$\Delta(ab) \equiv \sigma_{\bar a b} - \sigma_{ab}. \tag{A6}$$

APPENDIX B: COMPENDIUM OF THEORETICAL PREDICTIONS FOR REGGE RESIDUES

In this appendix we summarize[49] various theoretical predictions concerning the couplings of secondary Regge poles ($f, f', \rho, \omega, \phi, A_2$) to pions, kaons, and nucleons. We concentrate on three major ideas: (1) exchange degeneracy, which relates the couplings of odd-signature vector trajectories to those of even-signature tensor trajectories; (2) ρ and ω universality, which relates the couplings of vector mesons to pions, to kaons, and to nucleons; (3) SU(3), which relates the couplings of different particles and trajectories within SU(3) multiplets. The different classes of predictions are summarized in Fig. 15.

Several comments are in order: (a) To aid in making reliable tests, predictions are generally expressed in terms of factorized Regge residues rather than as sums or differences of total cross sections. (b) The major predictions of higher symmetries, such as SU(6), are generally equivalent to the union of SU(3) with universality. (c) As the data seem to be in agreement with Regge theory, we do not consider quark-model additivity predictions, except when they are expressible in Regge language. (d) Most of the predictions can be straightforwardly generalized to include hyperon-baryon and strangeness-exchange cross sections.[50]

1. Exchange degeneracy[10-13]

Exchange degeneracy comes in two forms. Weak exchange degeneracy predicts that pairs of opposite-signature trajectories should be equal [e.g., $\alpha_\rho(t)=\alpha_{A_2}(t)$], while strong exchange degeneracy further asserts that the residues are equal [e.g., $\beta^{\rho}_{Kp}(t)=\beta^{A_2}_{Kp}(t)$]. The "modern" theoretical motivation for exchange degeneracy is as follows: (1) Assume the existence of finite-energy sum rules (without wrong-signature fixed poles) for full scattering amplitudes (which are not of definite signature). (2) Adopt the Harari-Freund ansatz[14]: The direct-channel resonances are dual to secondary Regge trajectories while the nonresonant background is dual to the Pomeranchukon singularity (diffraction). (3) Hence, for channels which are exotic (no direct-channel resonances) the secondary Regge trajectories must cancel. This requires the equality of both the trajectory functions and residues of pairs of opposite-signature Regge poles.

The predictions based upon the experimentally accessible meson-baryon and baryon-baryon amplitudes are

$$\begin{aligned}\alpha_\rho &= \alpha_{A_2}\,,\\ \alpha_f &= \alpha_\omega\,,\\ \alpha_{f'} &= \alpha_\varphi\,,\end{aligned}\tag{B1}$$

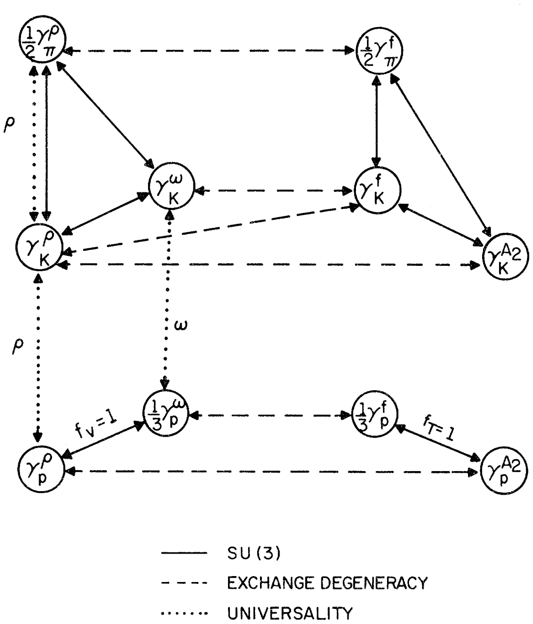

FIG. 15. Theoretical predictions for secondary Regge residues.

and

$$\begin{aligned}\gamma^\rho_p &= s_1\gamma^{A_2}_p\,,\\ \gamma^\rho_K &= s_1\gamma^{A_2}_K\,,\\ \gamma^f_p &= s_2\gamma^\omega_p\,,\\ \gamma^f_K &= s_2\gamma^\omega_K\,,\\ \gamma^{f'}_p &= s_3\gamma^\varphi_p\,,\\ \gamma^{f'}_K &= s_3\gamma^\varphi_K\,,\end{aligned}\tag{B2}$$

where s_1, s_2, and s_3 are ± 1. Equations (B1) and (B2) should also apply for $t\neq 0$ (the predictions for the baryon residues then apply to each spin amplitude separately).

Equations (B1) and (B2) are equivalent to

$$\begin{aligned}\sigma_{K^+p} &= \sigma_{K^+n} = D_{Kp}\,,\\ \sigma_{pp} &= \sigma_{pn} = D_{pp}\,.\end{aligned}\tag{B3}$$

That is, these cross sections should have no $1/s^{1/2}$ parts.[51]

If one further speculates that the duality arguments apply to meson-meson amplitudes, one can also predict

$$\alpha_\rho = \alpha_f \tag{B4}$$

and

$$\begin{aligned}\gamma^f_\pi &= s_4\gamma^\rho_\pi\,,\\ \gamma^f_K &= s_4\gamma^\rho_K\,,\\ \gamma^{f'}_\pi &= 0\,.\end{aligned}\tag{B5}$$

For such theoretical developments of exchange degeneracy as bootstraps, the prediction of higher symmetries from SU(3) and exchange degeneracy, ideal nonet mixing angles, and the necessary failure of the Harari-Freund ansatz for antibaryon-baryon channels, we refer the reader to Refs. 11–13 and footnotes contained therein.

The exchange-degeneracy predictions (B3) for the total cross sections have generally been thought to be reasonably successful, although σ_{pp} does in fact have a falling component (see Fig. 11). The predictions for nonzero t have met with mixed success.[52]

2. Universality[5-7]

ρ universality is the prediction

$$\tfrac{1}{2}\gamma^\rho_\pi = \gamma^\rho_K = \gamma^\rho_p \tag{B6}$$

at $t=0$, while ω universality states

$$\gamma^\omega_K = \tfrac{1}{3}\gamma^\omega_p\,. \tag{B7}$$

It is implicit in (B7) that

$$\gamma^\varphi_p = 0\,. \tag{B8}$$

Universality can be motivated in two ways: (1) In quark models, universality follows from the assumption that the ρ couples universally to the quark isospin current, and that the ω couples universally to the number of nonstrange quarks in the hadron. (2) Universality also is predicted by the union of vector-meson dominance with SU(3) (with ideal nonet mixing). This requires "smooth extrapolations" among the points (1, 0), $(1, m_\rho^2)$, and $(\alpha_\rho(0), 0)$ in the (J, t) plane.

Equation (B6) implies

$$\Delta(\pi^+p) = \Delta(K^+p) - \Delta(K^+n) \quad \text{(B9a)}$$

$$= \Delta(pp) - \Delta(pn)\,, \quad \text{(B9b)}$$

while (B7) implies

$$\Delta(pp) + \Delta(pn) = 3[\Delta(K^+p) + \Delta(K^+n)]\,. \quad \text{(B10)}$$

Equations (B9) and (B10) can be combined to yield a result independent of neutron targets:

$$\Delta(pp) = 3\Delta(K^+p) - \Delta(\pi^+p)\,. \quad \text{(B11)}$$

Of course, low-lying trajectories ($\alpha \lesssim 0$) should violate (B9)–(B11). Universality, especially for the ω, has been thought to be very successful,[5–7] although tests[5] of the ω universality prediction (B10) have generally ignored the Glauber rescattering corrections, while tests[7] of (B11) have utilized older data (see Fig. 1) for $\Delta(\pi^+p)$ which are systematically too high in the P_{lab} range from 20 to 50 GeV/c.

3. SU(3) (Refs. 8 and 15–17)

Consider the SU(3)-invariant couplings of an octet of pseudoscalar mesons ϕ_l, $l = 1 \ldots 8$, an octet of baryons ψ_l, nine vector mesons v_l, $l = 0 \ldots 8$, and nine tensor mesons t_l. Defining the matrices

$$M \equiv \frac{1}{\sqrt{2}} \sum_{l=1}^{8} \lambda_l \varphi_l\,,$$
$$B \equiv \frac{1}{\sqrt{2}} \sum_{l=1}^{8} \lambda_l \psi_l\,,$$
$$V \equiv \frac{1}{\sqrt{2}} \sum_{l=0}^{8} \lambda_l v_l\,, \quad \text{(B12)}$$
$$T \equiv \frac{1}{\sqrt{2}} \sum_{l=0}^{8} \lambda_l t_l\,,$$

where $\lambda_0 \equiv (\frac{2}{3})^{1/2} I$, the most general invariant couplings are

$$L_{VMM} = \sqrt{2}\, \gamma_{MV}\, \text{Tr}(M[V, M])\,, \quad \text{(B13)}$$

which is pure f type [$(\text{Tr}\,V)(\text{Tr}MM)$ is forbidden by charge conjugation],

$$L_{V\bar{B}B} = \sqrt{2}\, \gamma_{BV}[f_V\, \text{Tr}(\bar{B}[V, B]) + (1 - f_V)\, \text{Tr}(\bar{B}\{V, B\}) + \beta(\text{Tr}\,V)(\text{Tr}\bar{B}B)]\,, \quad \text{(B14)}$$

$$L_{TMM} = \sqrt{2}\, \gamma_{MT}[\text{Tr}(M\{T, M\}) + \epsilon(\text{Tr}\,T)(\text{Tr}MM)]\,, \quad \text{(B15)}$$

which is pure d type, and

$$L_{T\bar{B}B} = \sqrt{2}\, \gamma_{BT}[f_T\, \text{Tr}(\bar{B}[T, B]) + (1 - f_T)\, \text{Tr}(\bar{B}\{T, B\}) + \delta(\text{Tr}\,T)(\text{Tr}\bar{B}B)]\,, \quad \text{(B16)}$$

where γ_{MV}, γ_{BV}, γ_{MT}, γ_{BT}, f_V, f_T, β, ϵ, and δ are arbitrary constants.

The SU(3) predictions can be made in three stages: (1) With no assumptions concerning $\phi - \omega$ and $f - f'$ mixing, the only predictions of interest here are

$$\tfrac{1}{2}\gamma_\pi^\rho = \gamma_K^\rho \quad \text{(B17)}$$

and

$$\gamma_K^{A_2} = \frac{\sqrt{3}}{2}\, \gamma_{\pi^0\eta}^{A_2}\,. \quad \text{(B18a)}$$

Equation (B17), which also follows from ρ universality, leads immediately to the Barger-Rubin relation[8] (B9a), while Eqs. (B17) and (B18a) together imply[17]

$$\frac{d\sigma}{dt}(\pi^-p \to \pi^0 n) + 3\frac{d\sigma}{dt}(\pi^-p \to \eta n) = \frac{d\sigma}{dt}(K^-p \to \bar{K}^0 n) + \frac{d\sigma}{dt}(K^+n \to K^0 p)\,. \quad \text{(B18b)}$$

Equation (B9a) has been considered successful[8, 15, 53]; (B18b) does not work well below 5 GeV/c, but seems successful[54] at 6 GeV/c.

(2) In the second stage one assumes ideal nonet mixing[16]

$$\omega = (\tfrac{1}{3})^{1/2} v_8 + (\tfrac{2}{3})^{1/2} v_0\,,$$
$$\varphi = (\tfrac{2}{3})^{1/2} v_8 - (\tfrac{1}{3})^{1/2} v_0\,, \quad \text{(B19)}$$
$$f = (\tfrac{1}{3})^{1/2} t_8 + (\tfrac{2}{3})^{1/2} t_0\,,$$
$$f' = (\tfrac{2}{3})^{1/2} t_8 - (\tfrac{1}{3})^{1/2} t_0\,.$$

This ideal mixing, which is supported by the Gell-Mann–Okubo formula, is suggested by the quark model; it corresponds to the ω and f being composed of nonstrange quarks only, while the ϕ and f' are composed of strange quarks only. Ideal mixing is also predicted by exchange degeneracy. In addition to (B18a) one now has

$$\tfrac{1}{2}\gamma_\pi^\rho = \gamma_K^\rho = \gamma_K^\omega = (\tfrac{1}{2})^{1/2}\gamma_K^\varphi = \gamma_{MV}\,. \quad \text{(B20)}$$

(3) In the third step one assumes $\gamma_\pi^{f'} = 0$ (which

is supported by the experimental fact that the f' does not decay into two pions and is predicted by exchange degeneracy) and $\gamma_p^{f'} = \gamma_p^{\phi} = 0$ (which is supported by previous Regge fits).[3] Both are predicted by the simple quark model, since the proton and pion are composed of nonstrange quarks only. It is also usual to assume SU(3) breaking in the sense that

$$\alpha_\rho = \alpha_\omega \neq \alpha_\varphi , \quad \alpha_{A_2} = \alpha_f \neq \alpha_{f'} , \tag{B21}$$

which corresponds to giving the strange quark a different mass from the nonstrange quarks.

One now has [in addition to (B20)]

$$\epsilon = 0 , \quad \beta = 2f_V - 1, \quad \delta = 2f_T - 1 , \tag{B22}$$

and

$$\begin{aligned} &\gamma_p^\rho = \frac{1}{4f_V - 1}\gamma_p^\omega = \gamma_{BV} , \\ &\frac{\sqrt{3}}{2}\gamma_{\pi^0\eta}^{A_2} = \gamma_K^{A_2} = \gamma_K^f = -(\tfrac{1}{2})^{1/2}\gamma_K^{f'} = \tfrac{1}{2}\gamma_\pi^f = \gamma_{MT} , \\ &\gamma_p^{A_2} = \frac{1}{4f_T - 1}\gamma_p^f = \gamma_{BT} , \\ &\gamma_p^\varphi = \gamma_p^{f'} = \gamma_\pi^{f'} = 0 . \end{aligned} \tag{B23}$$

Of course, Eqs. (B20), (B21), and (B23) can be extended to $t \neq 0$.

With the exception of the relations (B9a) and (B18b) (which involve the couplings of different particles to the same trajectory), and the Johnson-Treiman and Freund relations (below), there have been few direct tests of Eqs. (B20)–(B23). There have, however, been reasonably successful fits[15,16] to the lower-energy total cross-section data which assumed exact SU(3) as an input constraint.

4. Combinations

By combining any two of exchange degeneracy, universality, and SU(3) much stronger predictions emerge. For example, ρ universality plus SU(3) predicts

$$\gamma_{MV} = \gamma_{BV} , \tag{B24}$$

while the further assumption of ω universality implies

$$f_V = 1 . \tag{B25}$$

From (B20), (B23), and (B25) [(B24) is not required] one finds the Johnson-Treiman formulas[18]

$$\Delta(K^+p) = 2\Delta(K^+n) = 2\Delta(\pi^+p) , \tag{B26}$$

which were originally suggested by $M(12)$ symmetry (which also yields $f_V = 1$). Adding the universality condition (B24), one obtains the Freund relation[19]

$$\Delta(pp) = \tfrac{5}{4}\Delta(pn) = 5\Delta(\pi^+p) . \tag{B27}$$

Relations (B26) and (B27) do not work well at lower energies,[55] but are in reasonable agreement with the new Fermilab data.[1]

The combination of SU(3) with exchange degeneracy implies

$$\begin{aligned} &f_V = f_T , \\ &\beta = \delta , \\ &\gamma_{MV} = \pm\gamma_{MT} , \\ &\gamma_{BV} = \pm\gamma_{BT} , \\ &\alpha_\rho = \alpha_\omega = \alpha_f = \alpha_{A_2} . \end{aligned} \tag{B28}$$

If one combines SU(3), universality, and exchange degeneracy, then all of the couplings can be expressed in terms of $\gamma_{MV} = \gamma_K^\rho$, as illustrated in Fig. 15.

5. Diffraction

The assumption that the Pomeranchukon singularity is an SU(3) singlet implies

$$D_{\pi p} = D_{Kp} , \tag{B29}$$

while the naive quark additivity assumption[35] implies

$$\tfrac{2}{3}D_{pp} = D_{\pi p} = D_{Kp} . \tag{B30}$$

Equation (B29) has always failed (see Ref. 1 and Figs. 1 and 2) by 20–25%, while $\frac{2}{3}D_{pp} = D_{\pi p}$ is roughly in accord with the data.

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[1]A. S. Carroll *et al.*, Phys. Rev. Lett. 33, 928 (1974); 33, 932 (1974).

[2]A. V. Barnes *et al.*, Proceedings of the Conference on High Energy Physics, London, England, 1974 (unpublished).

[3]Excellent references to previous Regge fits are P. D. B. Collins, Phys. Rep. 1C, 103 (1971); P. D. B. Collins and E. J. Squires, *Regge Poles in Particle Physics* (Springer, New York, 1968).

[4]The errors given on the parameters correspond to the amount by which the parameter must be varied (with the other parameters fixed) in order to increase the total χ^2 of the fit by one unit. Such a determination tends to underestimate the actual uncertainty of the parameter. See, for example, P. R. Bevington, *Data Reduction and Error Analysis for the Physical Sciences*, (McGraw-Hill, New York, 1969), p. 242.

[5]C. A. Levinson, N. S. Wall, and H. J. Lipkin, Phys. Rev. Lett. 17, 1122 (1966).

[6]V. Barger, K. Geer, and F. Halzen, Nucl. Phys. B44, 475 (1972).

[7]H. J. Lipkin, Phys. Rev. D 5, 776 (1972).

[8]V. Barger and M. Rubin, Phys. Rev. 140, B1365 (1965).

[9]One cannot rule out the possibility that the ρ is present and the universality prediction applies to the sum of the ω and ρ contributions.

[10]R. C. Arnold, Phys. Rev. Lett. 14, 657 (1965).

[11]H. J. Lipkin, Nucl. Phys. B9, 349 (1969).

[12]J. Mandula, J. Weyers, and G. Zweig, Ann. Rev. Nucl. Sci. 20, 289 (1970).

[13]M. Kugler, in *Developments in High Energy Physics*, proceedings of the IX Schladming conference on nuclear physics, edited by P. Urban (Springer, Wien, 1970) [Acta Phys. Austriaca Suppl. 7 (1970)], p. 443.

[14]H. Harari, Phys. Rev. Lett. 20, 1395 (1968); P. G. O. Freund, *ibid.* 20, 235 (1968).

[15]V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966).

[16]V. Barger and M. Olsson, and K. V. L. Sarma, Phys. Rev. 147, 1115 (1966).

[17]V. Barger and D. Cline, Phys. Rev. 156, 1522 (1967).

[18]K. Johnson and S. B. Treiman, Phys. Rev. Lett. 14, 189 (1965).

[19]P. G. O. Freund, Phys. Rev. Lett. 15, 929 (1965).

[20]See Ref. 1, 3, 5, and 8, and Figs. 1, 2, and 3.

[21]V. N. Bolotov *et al.*, Phys. Lett. 38B, 120 (1971).

[22]V. Barger and R. J. N. Phillips (unpublished); D. R. O. Morrison, CERN Report No. PHYS 74-31, 1974 (unpublished).

[23]Older data suggested an even larger discrepancy. See R. W. Hanson, Phys. Rev. D 5, 1227 (1972); V. Barger and R. J. N. Phillips, Nucl. Phys. B40, 205 (1972). The latter authors were aware that the discrepancy could be due to a scale error in Ref. 24.

[24]S. P. Denisov *et al.*, Fig. 1.

[25]W. F. Buhl *et al.*, Phys. Lett. 48B, 388 (1974).

[26]We have adjusted the (transmission regeneration) cross sections of Darriulat, Buchanan, and Birulev in Fig. 5 by the factor $|\eta_{+-}^{\rm new}/\eta_{+-}^{\rm old}|$, where $|\eta_{+-}^{\rm new}| = (2.23 \pm 0.05)\times 10^{-3}$; R. Messer *et al.*, Phys. Rev. Lett. 30, 876 (1973); $|\eta_{+-}^{\rm old}|$ is whatever value was used originally to normalize the experiment. This is a 10–15% effect and does not qualitatively alter Fig. 5. We are indebted to H. Sticker for advice on this point.

[27]O. Dahl *et al.*, in Proceedings of the Conference on High Energy Physics, London, England, 1974 (unpublished).

[28]V. N. Bolotov *et al.*, Nucl. Phys. B73, 387 (1974).

[29]R. Diebold *et al.*, Phys. Rev. Lett. 32, 904 (1974).

[30]A. Derem and G. Smadja, Nuovo Cimento A62, 681 (1969); see also the complex Regge-pole model of N. Barik and B. R. Desai, Phys. Rev. D 6, 3192 (1972). Both references assume $\alpha_{A_2} < \alpha_\rho$. A recent study is given by F. Elvekjaer and R. C. Johnson (unpublished).

[31]V. N. Bolotov *et al.*, Fig. 8.

[32]Some relevant review articles and theoretical papers on diffractive components are the following: F. Zachariasen, Phys. Rep. 2C, 1 (1971); D. Horn and F. Zachariasen, *Hadron Physics at Very High Energies* (Benjamin, New York, 1973); P. V. Landshoff and J. C. Polkinghorne, Phys. Rep. 5C, 1 (1972); V. N. Gribov, Zh. Eksp. Teor. Fiz. 53, 654 (1967) [Sov. Phys.—JETP 26, 414 (1968)]; V. N. Gribov and A. A. Migdal, Yad. Fiz. 8, 1002 (1968) [Sov. J. Nucl. Phys. 8, 583 (1969)]; Yad. Fiz. 8, 1213 (1968) [Sov. J. Nucl. Phys. 8, 703 (1969)]; V. Barger and R. J. N. Phillips, Nucl. Phys. B40, 205 (1972); H. Cheng, J. K. Walker, and T. T. Wu, Phys. Lett. 44B, 97 (1973); 44B, 283 (1973).

[33]Parametrizations of the form $D_{ab} = A + B(\ln p_{\rm lab})^2$ are theoretically questionable, since they implicitly set a scale of 1 GeV/c inside the logarithm. In general, such a scale factor should be fitted to the data.

[34]Examples of diffractive components increasing asymptotically as $\ln s$ exist in the literature: J. S. Ball and F. Zachariasen, Phys. Lett. 40B, 411 (1972); V. Barger and R. J. N. Phillips, Ref. 32, and references therein. So far, the only example of an amplitude with σ_t increasing and satisfying Mandelstam analyticity, $\pi\pi$ crossing symmetry, polynomial boundedness, and unitarity for each isospin amplitude has $\sigma_t \sim \ln s$ asymptotically. J. Kupsch, Forschr. Phys. 19, 783 (1971).

[35]E. M. Levin and L. L. Frankfurt, Zh. Eksp. Teor. Fiz. Pis'ma Red. 2, 105 (1965) [JETP Lett. 2, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Lett. 16, 71 (1966); H. J. Lipkin, *ibid.* 16, 1015 (1966); Z. Phys. 202, 414 (1967); Nucl. Phys. B78, 381 (1974). See also J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969), and references therein.

[36]The parametrization of σ_{K^+p}, σ_{π^+p}, and σ_{pp} with $C_K : C_\pi : C_p$ in the ratio $1:1:\frac{3}{2}$ was motivated by a best fit to each reaction separately, which gave C_i values quite close to this ratio. The choice of a single power of $\ln p$ was motivated primarily by simplicity. One can reliably fit the cross section data with

$$D_{ip} = C_i \left[\ln\left(\frac{p_{\rm lab} + m_i}{b_i}\right)\right]^{\alpha_i},$$

with values of α_i ranging roughly from 0.5 to 1.5. The fit to the π^+p cross-section data with (3.3) appears to be the least reliable of the fits, primarily because of the discrepancies among the low-energy data points (see Fig. 10).

[37]R.J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin (Interscience, New York, 1959), Vol. VI, p. 315.

[38]Yu. P. Gorin *et al.*, Yad. Fiz. 15, 953 (1972) [Sov. J. Nucl. Phys. 15, 530 (1972)].

[39]C. Quigg and L.-L. Wang, Phys. Lett. 43B, 314 (1973) and references therein.

[40]J. Kwieciński, L. Lesniak, and K. Zalewski, Cracow report, 1974 (unpublished).

[41]We also ignore two-body charge-exchange corrections [see C. Wilkin, Phys. Rev. Lett. 17, 561 (1966)], which are negligible at Fermilab energies.

[42]V. Bartenev *et al.*, Phys. Lett. B51, 299 (1974).

[43]Y. Akimov *et al.*, Rockefeller University Report No. COO-2232A-1, 1974 (unpublished).

[44]M. G. Albrow *et al.*, Nucl. Phys. B51, 388 (1973); B54, 6 (1973); B73, 40 (1974).

[45]G. C. Fox, in *High Energy Collisions—1973*, edited by C. Quigg (A.I.P., New York, 1973), p. 180; D. P. Roy and R. G. Roberts, Nucl. Phys. B77, 240 (1974); R. D. Field and G. C. Fox, Nucl. Phys. B80, 367 (1974).

[46]K. Goulianos (private communication).

[47]V. Bartenev *et al.*, Phys. Rev. Lett. 31, 1088 (1973).

[48]D. W. G. S. Leith, in *Particles and Fields—1973*, proceedings of the Berkeley meeting of the Division of Particles and Fields of the American Physical Society, edited by H. H. Bingham, M. Davier, and G. R. Lynch (A.I.P., New York, 1973), p. 349.

[49]Our references concentrate on the original papers and on the papers which are *currently* the most useful.

[50]Some predictions may be found in H. J. Lipkin, Phys. Rev. D 7, 846 (1973).

[51]Whether lower-lying trajectories should also cancel in an open question.

[52]See Ref. 3 and Figs. 4, 7, and 8, and V. Barger and R. J. N. Phillips, Ref. 23.

[53]V. Barger, plenary session talk at the Conference on High Energy Physics, London, England, 1974 (unpublished).

[54]See Refs. 53 and 29 and Figs. 4, 7, and 8.

[55]See Refs. 1, 3, 5, and 8 and Figs. 1, 2, and 3.