Deep-inelastic eN and νN scattering: A unified description via dual Regge poles and SU(3)

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The deep-inelastic scattering of electrons and neutrinos off nucleons is studied phenomenologically by means of a model that describes the scattering of weak and electromagnetic currents off nucleons. It is found that one may impose the constraints of Regge behavior, duality, SU(3) symmetry, and current algebra in a model for the structure functions and obtain a very accurate and economical description of the data. This approach demonstrates a remarkable resemblance between the strong interactions of hadrons and their weak and electromagnetic interactions.

I. INTRODUCTION

The ideas of Regge behavior at high energy, SU(3) symmetry for amplitudes, and duality have in the last few years been widely accepted as offering a strong phenomenological foundation for studying the interactions of hadrons. The success of these ideas is particularly striking in the medium-energy region (3–70 GeV) where highly quantitative agreement with a wide variety of strong-interaction data has been found.[1]

Many investigations[2] have suggested that the three ideas mentioned above, having worked so well in describing the strong interactions of hadrons, might be applicable to cases where hadrons are involved in weak and electromagnetic (EM) interactions. Usually these cases also involve leptons, but methods are known for separating the part of the amplitude involving the leptons from the part of the amplitude involving only the weak or EM current and hadrons.

The deep-inelastic scattering of electrons[3] and neutrinos[4] off nucleons provides a means of testing whether EM and weak current-hadron scattering can be described by the same ideas that seem to govern hadron-hadron scattering (Fig. 1).

We will demonstrate below highly quantitative evidence in favor of the view that the weak and EM current-hadron scattering data are described by the ideas of Regge behavior, SU(3), and duality.

In Sec. II the electron and neutrino structure functions are briefly discussed in order to establish kinematics and notation. There follows then the description of a quantitative model for deep-inelastic current-nucleon scattering which embodies the ideas of Regge behavior, SU(3), and duality. Section III gives the comparison between the model and the existing data, and gives the predictions of the model for expected FNAL measurements. Section IV lists the conclusions.

II. A MODEL FOR DEEP-INELASTIC LEPTON SCATTERING

The experimental data[3,4] for leptons scattering off nucleons in the scaling limit are summarized in the distributions $F_i^{lN}(x)$, $i=1, 2, 3$; l either e^-, ν, or $\bar{\nu}$; N either p or n; and $x=q^2/2M\nu$, where M is the nucleon mass and q^2, ν are the virtual current's mass squared and lab energy, respectively. The variable x is bounded by $0 \leq x \leq 1$, and $\omega = x^{-1}$ is the Bjorken scaling variable.[5]

The number of independent structure functions may be reduced as follows:

(1) $F_3^{eN}(x)=0$ for all x since the electromagnetic current has a definite (natural) parity. The V–A form of the weak current provides an interference term $F_3^{\nu N}(x)$ that contributes in neutrino scattering.

(2) The Callan-Gross[6] relation gives $xF_1(x) = F_2(x)$, which amounts to assuming that the weak and electromagnetic currents interact with spin-$\frac{1}{2}$ objects (quarks, partons, etc.).

(3) Isospin conservation implies $F_2^{\nu p}(x) = F_2^{\bar{\nu} n}(x)$ and $F_2^{\bar{\nu} p}(x) = F_2^{\nu n}(x)$, with similar relations holding for $F_3(x)$.

With the above restrictions, we need only consider the six structure functions $F_2^{ep}(x)$, $F_2^{en}(x)$, $F_2^{\nu p}(x)$, $F_2^{\bar{\nu} p}(x)$, $F_3^{\nu p}(x)$, and $F_3^{\bar{\nu} p}(x)$. Of these six structure functions, only the first two have been measured accurately.[3]

The inelastic structure functions may be interpreted as total cross sections for scattering an electromagnetic or weak current off a nucleon. We may therefore calculate the structure functions via the optical theorem by considering the imaginary part of the appropriate forward elastic scattering amplitudes. These amplitudes naturally depend upon the properties of both the incident currents and the target nucleons and upon the interaction mechanism between the current and target. We consider each of these in turn:

(1) A crucial and powerful assumption is that the electromagnetic current γ and the natural-parity components $W^{\pm}$ of the weak current be treated as members of the same SU(3) octet. In particular, we assume the standard classifications

$$\gamma \sim \rho^0 + (\tfrac{1}{3})^{1/2}\,\omega_8\,, \qquad W^{\pm} \sim \rho^{\pm} - A_1^{\pm}\,, \tag{1}$$

where we use a suggestive notation involving the natural- and unnatural-parity vector mesons. The Cabibbo angle[7] is also neglected, since $\sin^2\theta_C \sim 0.05$. The proton and neutron are members of the standard SU(3) octet.

(2) At large values of the invariant mass squared $W^2 = M^2 + 2M\nu - q^2$ of the current-hadron system, $x = q^2/2M\nu \to 0$ and by analogy with strong-interaction physics we expect[8] the structure functions to be dominated by the exchange of the single Regge trajectories[9] ρ-A_2-ω-f and the vacuum trajectory P. As shown in Ref. 1 in a highly detailed study of pseudoscalar meson-baryon scattering, the constraints of duality and no-exotics provide a very clean and highly constrained description of the ρ-A_2-ω-f amplitudes. We therefore expect the Regge-exchange amplitudes to be of the form

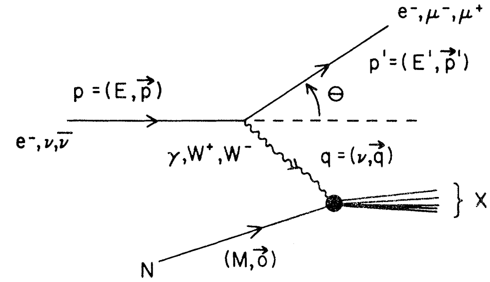

FIG. 1. A diagram of the process $l + N \to l' + X$.

$$\mathrm{Im}\,\langle cd|\,T^{(s)}\,|ab\rangle_{t=0} = [\langle \tilde{c}ab\tilde{d}\rangle\,(d+f) + (\langle \tilde{c}a\tilde{d}b\rangle - \langle a\tilde{c}\rangle\langle b\tilde{d}\rangle)(d-f)]\,x^{1-\alpha(0)}\,, \tag{2}$$

where $d(0)$, $f(0)$ are independent residue functions evaluated at $t=0$ and $\alpha(0) \simeq 0.55$ for ρ-A_2-ω-f exchange. The matrices a, b, $\tilde{c}$, $\tilde{d}$ ($\tilde{c}$ is the transpose of c), and the traces are defined in Ref. 1, and the trace factors in Eq. (2) correspond to the familiar planar duality diagrams.[10]

(3) It is natural to assume that the vector and axial-vector couplings of the components of $W^{\pm}$ to the Regge trajectories do not depend upon the parity (chiral symmetry). Then the amplitude for $W^+p \to W^+p$, for example, is [in the notation of Eq. (1)]

$$\begin{aligned} A(W^+p \to W^+p) &= A(\rho^+p \to \rho^+p) + A(A_1^+p \to A_1^+p) \\ &\quad - 2A(\rho^+p \to A_1^+p) \\ &= 2A(\rho^+p \to \rho^+p) - 2A(\rho^+p \to A_1^+p)\,. \end{aligned} \tag{3}$$

The term $2A(\rho^+p \to \rho^+p)$ is the $F_2^{\nu p}(x)$ contribution and $-2A(\rho^+p \to A_1^+p)$ is the $xF_3^{\nu p}(x)$ contribution to $A(W^+p \to W^+p)$. Note that $F_2^{\nu p}(x) = -xF_3^{\nu p}(x)$ follows from isosymmetry whenever the Pomeron does not contribute to $F_2^{\nu p}(x)$. [The Pomeron never contributes to $xF_3^{\nu p}(x)$.] Similar relations hold for $\bar{\nu}p$ scattering.

(4) The Pomeron contributes to F_2^{eN} and $F_2^{\nu N}$. Neglecting any octet component for the Pomeron (an approximation that will be justified later), the SU(3) amplitude takes the form for $x \to 0$

$$\mathrm{Im}\langle cd|\,T_P^{(s)}\,|ab\rangle_{t=0} = \langle a\tilde{c}\rangle\langle b\tilde{d}\rangle\,P x^{1-\alpha_P(0)}\,, \tag{4}$$

where P is the Pomeron residue at $t=0$ and $\alpha_P(0) = 1$. Using Eqs. (1)–(4) one can now evaluate the six structure functions for $x \to 0$,

$$\begin{aligned} F_2^{ep}(x) &= T_P + \tfrac{2}{9}T_1 + \tfrac{1}{3}T_2\,, \\ F_2^{en}(x) &= T_P + \tfrac{1}{18}T_1 + \tfrac{1}{2}T_2\,, \\ F_2^{\nu p}(x) &= 3T_P + 2T_2\,, \\ F_2^{\bar{\nu}p}(x) &= 3T_P + T_1 + T_2\,, \\ xF_3^{\nu p}(x) &= -2T_2\,, \\ xF_3^{\bar{\nu}p}(x) &= -T_1 - T_2\,, \end{aligned} \tag{5}$$

where $T_P = P$ [see Eq. (4)] and

$$T_1 = (3f + d)x^{1-\alpha(0)}\,,$$

$$T_2 = (f - d)x^{1-\alpha(0)}\,.$$

The particular linear combinations of f and d used in T_1 and T_2 will be explained below.

(5) The duality relations between t-channel exchanges and s-channel resonances are already contained in Eq. (2). According to Bloom and Gilman[11] the structure functions near $x=1$ are built up by a sum over individual resonances. The threshold behavior of the structure functions is thus controlled by the asymptotic behavior of the resonance form factors,

$$F_2(x) \sim (1-x)^{n-1} \text{ as } x \to 1\,,$$

if the form factors $G(q^2) \sim (1/q^2)^{n/2}$ as $q^2 \to \infty$. Following Rosner[12] and Chaichian and Kitakado,[13] we observe that the normal J^p s-channel resonances [e.g., p, $N(1520)$] are dual to the t-channel combination $(3f+d)$, while the abnormal J^p s-channel resonances [e.g., $\Delta(1236)$, $N(1670)$] are dual to the t-channel combination $(f-d)$. The nucleon and $\Delta(1236)$ form factors are known experimentally to behave[14,15] as $(1/q^2)^2$ and $(1/q^2)^3$, respectively. We therefore parametrize the terms in Eqs. (5) as

$$T_1 \sim (1-x)^3\,,\qquad T_2 \sim (1-x)^5,\quad x\to 1\,. \tag{6}$$

The amplitudes T_1 and T_2 may be understood as being built up via duality from separate sets of exchange-degenerate baryon resonances.[16] The Pomeron is presumably dual to the nonresonant background beneath the resonances[17,18] and we found it sufficient to parametrize T_P as

$$T_P \sim (1-x)^9,\quad x\to 1\,. \tag{7}$$

The exponent in Eq. (7) is not determined well phenomenologically but rather is chosen to ensure that T_P vanishes faster than the resonances whenever x is not near zero.

There is of course no guarantee that nonleading terms in Eqs. (5)–(6) are not needed, and we find in fact (see Sec. III) that such terms are necessary in a detailed fit to the electron-nucleon structure functions.

(6) The model as presented in Eqs. (5)–(7) is already quite restrictive, but some arbitrariness may be further removed by enforcing the Adler sum rule[19]

$$\int_0^1 \frac{dx}{x}\,[F_2^{\bar\nu p}(x)-F_2^{\nu p}(x)]=2\,,$$

and the Gross–Llewellyn Smith sum rule[20]

$$\int_0^1 dx[F_3^{\bar\nu p}(x)+F_3^{\nu p}(x)]=-6\,.$$

Although the validity of the Adler sum rule has been challenged,[21] we find no need in this investigation to abandon the sum rule.

(7) We find that the parton distribution functions[22] may be expressed in our model as

$$\begin{aligned} xu(x) &= \tfrac{3}{4}T_P+\tfrac{1}{2}T_1+\tfrac{1}{2}T_2\,,\\ xd(x) &= \tfrac{3}{4}T_P+T_2\,,\\ x\bar u(x) &= x\bar d(x)\\ &= \tfrac{1}{2}x[s(x)+\bar s(x)]\\ &= \tfrac{3}{4}T_P\,, \end{aligned} \tag{8}$$

so our model is similar to the valence-sea models.[23] Equations (5) do not permit a model in which the Pomeron couples only to $\bar u$, $\bar d$, s, $\bar s$. From Eqs. (6)–(8) we see that near threshold $(x\to 1)$, the proton quark dominates the parton distributions, in agreement with an analysis by Feynman.[22]

III. COMPARISON OF THE MODEL WITH DATA

The data to be fitted by the model are the structure function $F_2^{ep}(x)$, the ratio $F_2^{en}(x)/F_2^{ep}(x)$, and the difference $F_2^{ep}(x)-F_2^{en}(x)$. Within the model framework discussed in Sec. II, a very good description of the data was found to be given by

$$\begin{aligned} T_P &= 0.23(1-x)^9\,,\\ T_1 &= [7.1(1-x)^3-7.4(1-x)^5+4.4(1-x)^7\\ &\quad +10.3(1-x)^{13}/\ln x]\,x^{0.45}\,,\\ T_2 &= [5.9(1-x)^5-5.5(1-x)^7]\,x^{0.45}\,. \end{aligned} \tag{9}$$

Since the Pomeron contribution is practically negligible in the region of measured x values, we have neglected any octet component in the Pomeron. The $\ln x$ term in T_1 may be interpreted as a possible absorptive correction, although its origin is purely phenomenological.[24]

There are seven parameters in Eq. (9), but two of them are determined by the Adler sum rule and the Gross–Llewellyn Smith sum rule, and another is determined by the experimental constraint[4]

$$B=-\int_0^1 xF_3^{\nu N}(x)dx\Big/\int_0^1 F_2^{\nu N}(x)dx \simeq 0.86\,. \tag{10}$$

The remaining four parameters were determined by minimizing χ^2. The χ^2's for the fits are 0.69/pt. for $F_2^{ep}(x)$ (Fig. 2), 0.86/pt. for $F_2^{en}(x)/F_2^{ep}(x)$ (Fig.

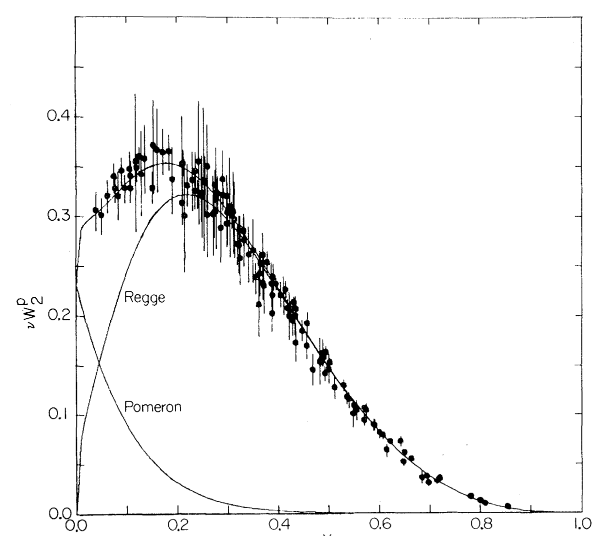

FIG. 2. Experimental data and model results for the ep structure function $\nu W_2^p(x)$.

3), and 0.91/pt. for $F_2^{ep}(x) - F_2^{en}(x)$ (Fig. 4). The total χ^2 per degree of freedom is 0.76 for 209 data points and 4 free parameters.

Besides being able to describe the electron scattering data, the model also predicts $F_2^{\nu,\bar{\nu}}(x)$ and $F_3^{\nu,\bar{\nu}}(x)$. Unfortunately, experimental data on these distributions are not yet available. However, several quantities related to integrals of the distributions have been measured and can be compared to the predictions of the model.

(1) Defining the isospin-averaged structure functions

$$F^{\nu N}_{2,3} = \tfrac{1}{2}(F^{\nu p}_{2,3} + F^{\nu n}_{2,3}),$$

the total cross sections σ^{ν}, $\sigma^{\bar{\nu}}$ are given by

$$\sigma^{\nu,\bar{\nu}} = \frac{G^2ME}{\pi}\left[\tfrac{2}{3}\int F_2^{\nu N}(x)\,dx \mp \tfrac{1}{3}\int xF_3^{\nu N}(x)\,dx\right] = \alpha^{\nu,\bar{\nu}}E\,,$$

showing the famous linear rise of the cross sections with incident neutrino energy E. The model predicts

$$\alpha^{\nu} = 0.72\,,$$
$$\alpha^{\bar{\nu}} = 0.29\,,$$

to be compared with the experimental values[25] $\alpha^{\nu} = 0.69 \pm 0.14$ and $\alpha^{\bar{\nu}} = 0.27 \pm 0.05$, respectively. These results depend upon the Pomeron contribution to F_2 being small. Although the quantity B in Eq. (10) was constrained at the beginning, allowing B to vary does not result in any improvement in the fit and in fact B does not change appreciably.

(2) Perkins has discussed how to predict the integral

$$\int_0^1 F_2^{\nu N}(x)\,dx = I^{\nu N}$$

in terms of the electron data and several additional assumptions.[25] Since the model given here agrees with the electron data and with Perkins's assumptions, it is no surprise that we get $I^{\nu N} = 0.49$, to be compared with the experimental value 0.49 $\pm$ 0.07.

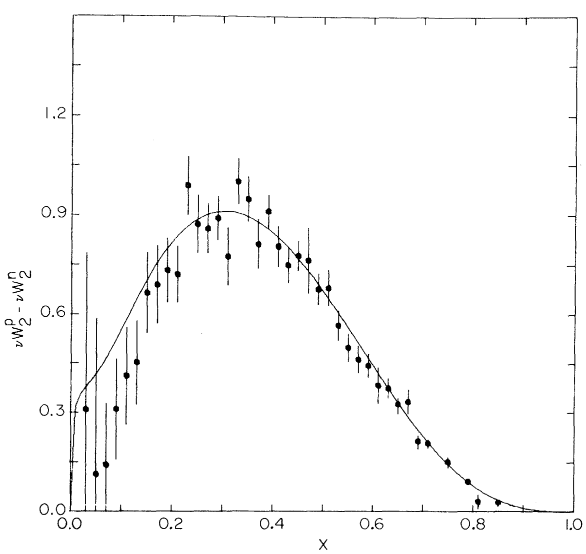

FIG. 3. Experimental data and model results for the difference $\nu W_2^p - \nu W_2^n$.

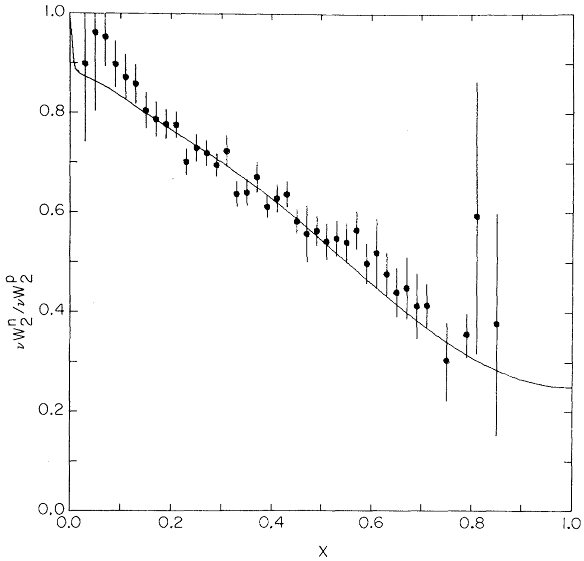

FIG. 4. Experimental data and model results for the ratio $\nu W_2^n/\nu W_2^p$.

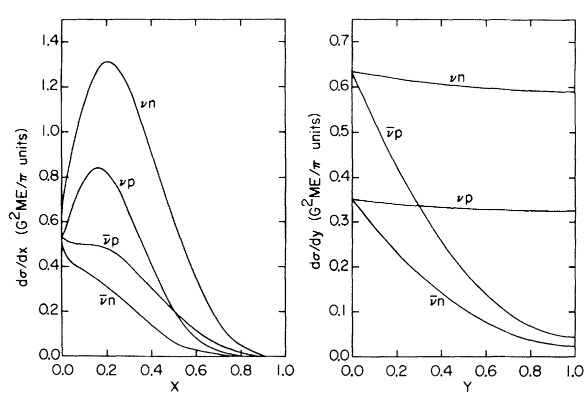

FIG. 5. Model predictions for the neutrino differential cross sections $d\sigma/dx$ and $d\sigma/dy$.

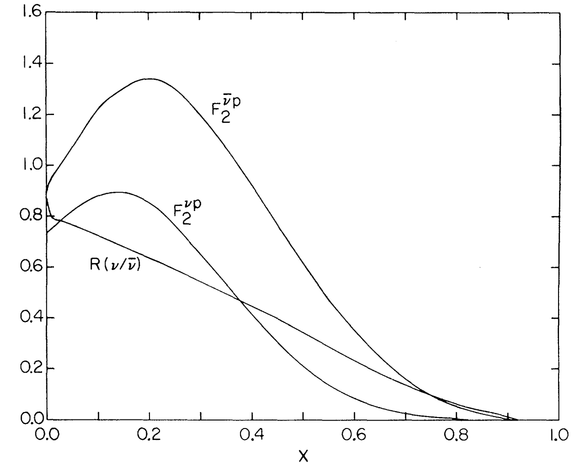

FIG. 6. Model predictions for the neutrino structure functions and their ratio R.

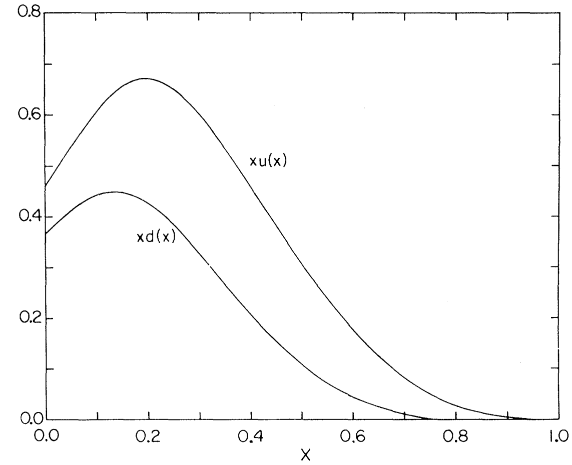

FIG. 7. Model results for the quark-parton distribution functions.

(3) The quantities

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dx}=\frac{G^2ME}{\pi}\left[\tfrac{2}{3}F_2(x)\mp\tfrac{1}{3}xF_3(x)\right],$$

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dy}=\frac{G^2ME}{\pi}\left(\int F_2dx\right)\left[1-(1\mp B)(y-y^2/2)\right],$$

where $B=0.86$ and $y=\nu/E$ ($W^{\pm}$ energy divided by the incident neutrino energy), are shown in Fig. 5. The distributions $F_2^{\nu p}(x)$, $F_2^{\bar{\nu}p}(x)$ and their ratio R are shown in Fig. 6. All of these distributions can eventually be measured experimentally.

(4) For completeness, the quark-parton distribution functions predicted by the model are shown in Fig. 7. We should mention, however, that our model is much more restrictive than the parton model, showing as it does the strong similarity between current-hadron and hadron-hadron scattering. For example, the quark-parton model predicts

$$4\geq F_2^{en}/F_2^{ep}\geq\tfrac{1}{4},$$

while the model given here predicts[13]

$$1\geq F_2^{en}/F_2^{ep}\geq\tfrac{1}{4}.$$

As can be seen in Fig. 3, the data seem naturally bounded by the second set of inequalities rather than by the looser quark-parton set.

(5) The d/f ratio obtained from Eq. (9) is $d/f=1.0$ for $x=1$ and $d/f\sim-0.3$ for $x\sim0.1$. As must be true if the model given here is correct, the d/f ratio in the Regge limit (small x) is about the same as the d/f ratio that was determined in Ref. 1 from meson-baryon scattering data. We remark, as an intriguing aside, that the threshold value $d/f=1$ is just the value required by duality and factorization to overcome the well-known $\bar{B}B$ duality problem.[26]

IV. SUMMARY AND CONCLUSIONS

We have emphasized throughout that one can understand the basic features of the deep-inelastic current-hadron scattering data in terms of well-known ideas from strong interactions. We have expressed the scattering amplitudes in terms of functions which have the correct Regge behavior in the Regge limit ($x\to0$) and the behavior required by duality near threshold ($x\to1$). The model also satisfies the constraints implied by current algebra, i.e., the Adler and Gross–Llewellyn Smith sum rules. The model gives a satisfactory description of all the eN and νN data and also makes predictions for the coming FNAL νN data.

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