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Coordinates as dynamical viariables: A possible description of strong interactions*

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As a possible generalization of dual resonance models, a description of strong interactions is proposed in terms of q-number coordinates. Some basic properties of the latter are conjectured by abstraction from dual resonance models and on the basis of a correspondence principle. Fields on q-number coordinates are defined; interaction between matter fields and coordinate fluctuations is obtained from a covariance argument. Feynman rules are derived for the calculation of Green's functions. Two explicit examples are worked out as an illustration. It is shown in particular that a certain type of dual resonance amplitudes may be recovered as an approximate solution.

I. INTRODUCTION

Dual resonance models[1] (DRM) provide an interesting description of hadronic phenomena. While such models are certainly unphysical in their details, it is a widespread opinion that they correctly reflect some important qualitative aspects of strong interactions.

I suggest that the qualitative succes of DRM is due to the fact that they provide a description of strong interactions which is essentially "kinematical" in its nature. In particular, coordinates (which play an entirely passive role in ordinary field theories) are endowed with a new meaning by being treated as q numbers.

It is therefore this aspect of DRM which has to be abstracted and suitably generalized.

Recently, a significant step in this direction has been taken by Ramond[2] and by Kaku and Kikkawa.[3] These authors construct a second-quantized theory based on the string picture.[4] The basic object in their theory is a field "defined on a string," say $\Phi[Y(\tau)]$. The coordinates $Y^\mu(\tau)$ are those of a submanifold of Minkowski space; τ denotes the set of parameters used to parametrize the submanifold. (The string is, of course, characterized by *two* parameters, τ^0, τ^1.) In this picture, the evolution of $Y^\mu(\tau)$ is governed by equations of motion derived from the Nambu-Chang-Mansouri Lagrangian,[4] whereas, in the author's interpretation, Φ "creates and annihilates strings." Strong interactions are described by adding cubic and quartic terms to the Lagrangian which governs the evolution of Φ.

There are "two levels" of the dynamics described by such a theory: (i) One has to specify the motion of the string; hadronic levels are described in terms of the excitations of its normal modes. (ii) One has to construct a dynamics of the "superfield," $\Phi[Y(\tau)]$. Thus the argument of the second-quantized field Φ is itself a dynamical variable, describing the creation and annihilation of the normal modes of the string.

One can now ask whether it is possible to generalize the string picture to higher-dimensional objects in the hope of avoiding the intrinsic problems[1] of the string theory. In particular, one is led to examine the limiting case when Y^μ is considered to be a function of *four* parameters, say x^μ ($\mu=0, \ldots, 3$). The variables x^μ may be chosen as Cartesian coordinates in Minkowski space. In this case, however, one is no longer dealing with an object moving in space-time, but rather with a mapping of space-time onto itself. [Classically, the coordinates $Y^\mu(x)$ are general, curvilinear coordinates of the points of Minkowski space. It is assumed that the classical mapping is one to one.]

As a next step, the Y^μ may be regarded as dynamical variables ("q numbers"); one attempts to describe strong interactions in terms of the "quantum fluctuations" of the mapping around the identity map.

In order to realize this idea, one introduces "matter fields" (describing, say, quarks) such that their arguments are the q numbers, $Y^\mu(x)$, instead of c-number coordinates x^μ. This is similar to the procedure followed in Refs. 2 and 3; however (as it is evident from the previous discussion), its physical meaning is slightly different. One may postulate (as I do in this paper) that the *classical* evolution of the field is governed by a Lagrangian. In the corresponding quantum theory, one specifies a weight factor as a certain imaginary exponential in the Feynman path integral.

The Lagrangian of the quark field may contain the usual kinetic term and terms describing electromagnetic and weak interactions. However, the Lagrangian *does not* contain, for example, a quark-gluon interaction put in "by hand" to describe strong interactions. The latter are described by the "quantum fluctuations" of Y^μ and by their coupling to the matter field. This coupling may be determined on the basis of a covariance argument

(Sec. III). In this sense, one is led to a "kinematic" theory of strong interactions.

Once this interpretation of strong interactions is adopted, it is very tempting to conjecture that strong interactions are *universal*: The argument of *every* field (including the electromagnetic and lepton fields) is a dynamical coordinate, Y^μ. Strictly speaking, this is not a logical necessity: I indicate briefly in Sec. III that one can construct "mixed" theories, in which "ordinary" fields coexist with ones depending on a dynamical coordinate. Such mixed theories, however, lack much of the aesthetic appeal of a "universal" theory; they are not examined in this paper any further. (It is, of course, the experimental consequences which should ultimately decide between the two types of theories. The result of a simple model calculation reported in Sec. V suggests that adoption of the universality hypothesis need not lead to conflicts with the usual tests of quantum electrodynamics.)

The plan of the paper is the following. In Sec. II some basic properties of a dynamical coordinate are examined; they are partly abstracted from DRM, partly conjectured on the basis of a correspondence principle. Section III deals with the question of constructing Lagrangians for fields which depend on dynamical coordinates. The guiding principles invoked are that the Lagrangians should resemble "classical" Lagrangians as much as possible and that their forms should not change as the mapping $Y^\mu(x)$ fluctuates around the identical map (general covariance). Feynman rules are constructed and examined in Sec. IV; in Sec. V some model calculations are reported, mainly in order to illustrate the use of the techniques developed in the preceding sections. Some concluding remarks are made in Sec. VI.

Notation. Tetrad indices are written to the right of a vertical bar ($|$), unless (as in Sec. V) there do not occur indices referring to a general metric. Exceptions are a classical, Minkowskian coordinate which is always denoted by x^μ (instead of $x^{|\mu}$) and the Minkowskian metric tensor which is denoted by $\eta^{\mu\nu}$. The signature of the metric is $(+---)$. Differentiation with respect to x^μ is often denoted by a subscript μ, separated by a comma from the functional symbol.

Constant Dirac matrices (referring to a tetrad basis) satisfy

$$\{\gamma^{|\mu}, \gamma^{|\nu}\} = 2\eta^{\mu\nu} ,$$
$$\gamma^{|0}\gamma^{|\mu}\gamma^{|0} = \gamma^{|\mu\dagger}; \;\; \gamma^{|5} = \gamma^{|0}\gamma^{|1}\gamma^{|2}\gamma^{|3} .$$

Almost everywhere the natural system of units is used: $\hbar = c = L = 1$, where L is the scale length of strong interactions introduced in Sec. II. An exception is Sec. II itself, where the "quasiclassical" limit ($L \to 0$) of Y^μ is examined; there one sets $\hbar = c = 1$, but keeps L.

II. COORDINATES AS DYNAMICAL VARIABLES

Classical Minkowski space is a four-dimensional, pseudo-Euclidean manifold. This property has to be carried over in some form into a picture in which coordinates are treated as dynamical variables; otherwise, there is little hope that ordinary field theory (e.g., quantum electrodynamics) can be recovered as some appropriate limiting case of one's theoretical constructs. It is reasonable to demand therefore that a suitable quantum average of the dynamical variable in question should coincide with points of a classical Minkowski space. I assume that the quantum average can be written in the form of a Feynman functional integral. Therefore, the requirement just stated should be written as

$$\langle Y^\mu \rangle \equiv \frac{\int \delta Y^\nu \, e^{iW[Y]} \, Y^\mu}{\int \delta Y^\nu \, e^{iW[Y]}} = x^\mu , \qquad (2.1)$$

where the dynamical variable corresponding to the coordinate has been denoted by Y^μ. The functional $W[Y]$ should be some suitably chosen action. The "classical" coordinate may be used to parametrize the dynamical variable Y^μ: One can write $Y^\mu = Y^\mu(x)$. One can assume further that the action of inhomogeneous Lorentz transformations is defined on the classical coordinates, x^μ. In particular, under translations one should have

$$T(a)\, Y^\mu(x) = Y^\mu(x+a) , \qquad (2.2a)$$

with, of course,

$$\langle Y^\mu(x+a) \rangle = x^\mu + a^\mu . \qquad (2.2b)$$

The last requirement can be satisfied if one writes

$$Y^\mu(x) = x^\mu + \xi^\mu(x) , \qquad (2.3)$$

and demands that $\xi^\mu(x)$ (the "fluctuating part" of the coordinate) have a vanishing expectation value. The field $\xi^\mu(x)$ transforms under translations of the x^μ in the "usual" way: $x^\mu \to x^\mu + a^\mu$ is represented by

$$\xi^\mu(x) \to \xi^\mu(x+a) .$$

Intuitively, one should further demand that the fluctuations of the coordinate have a finite *range*, characterized by a scale length, L. (On the basis of experience with dual models one expects perhaps $L \approx 1\ \mathrm{GeV}^{-1} \approx 10^{-14}$ cm.) If one lets L go to zero artificially, a *correspondence principle* should hold: The quantum fluctuations of the coordinate should disappear.

This can be achieved if (a) $W[Y]$ is proportional to a negative power of L (say, L^{-2}) and (b) the classical coordinate, x^{μ}, is a solution of the Euler-Lagrange equations

$$\frac{\delta W}{\delta Y^{\mu}}=0 \ . \tag{2.4}$$

These conditions for the fulfilment of a correspondence principle are well known; they have been investigated by Morette DeWitt in connection with the quasiclassical limit ($\hbar \to 0$) of Feynman path integrals.[6] In fact, if the two conditions just stated are met, then as L approaches zero the phase factor $\exp(iW[Y])$ undergoes rapid oscillations as one integrates over Y. The main contribution to the integral comes from the region where the phase is stationary; the condition for that is given by (2.4). By assumption, the solution of those equations is just the classical coordinate. On writing W as the integral of a local Lagrangian, say,

$$W[Y]=L^{-2}\int d^4x\, \mathfrak{L}(Y(x)) \ , \tag{2.5}$$

one can introduce intrinsic variables in functional integrals of the type (2.1). [The intrinsic variables are $Y^{\mu}(x)-Y^{\mu}_{\rm cl}(x)$, where $Y^{\mu}_{\rm cl}$ is the appropriate solution of (2.4). Thus the intrinsic variables turn out to be just the $\xi^{\mu}(x)$ from (2.3).] Translation invariance of the action requires $\partial\mathfrak{L}/\partial Y^{\mu}=0$; cf. Eq. (2.3). On expanding the action around its stationary value, one gets

$$W[Y]=L^{-2}W[x] + \tfrac{1}{2}L^{-2}\int d^4x \frac{\partial^2\mathfrak{L}}{\partial Y^{\mu}{}_{,\alpha}\,\partial Y^{\nu}{}_{,\beta}}\,\xi^{\mu}{}_{,\alpha}\,\xi^{\nu}{}_{,\beta}+\cdots \ . \tag{2.6}$$

Thus, indeed, $\langle\xi^{\mu}\rangle=0$ at least to *leading order* in L.

Can one achieve that $\langle\xi^{\mu}\rangle=0$ (i.e., $\langle Y^{\mu}\rangle=x^{\mu}$) *identically*? Clearly, a sufficient condition for that is the invariance of the Lagrangian under the transformation $Y^{\mu}\to\xi^{\mu}$ in the functional integral, for evidently $\mathfrak{L}(Y)$ has to be *even* in Y^{μ}; therefore, if $\mathfrak{L}$ is invariant under $Y^{\mu}\to\xi^{\mu}$, it is also even in ξ^{μ}. Hence, by symmetry, one has $\langle\xi^{\mu}\rangle=0$. Evidently $\mathfrak{L}(Y)$ is invariant under $Y^{\mu}\to\xi^{\mu}$ if it depends on Y^{μ} only through its antisymmetrized derivative, $u_{\mu\nu}=Y_{\mu,\nu}-Y_{\nu,\mu}$. (Notice that $Y_{\mu,\nu}-Y_{\nu,\mu}=\xi_{\mu,\nu}-\xi_{\nu,\mu}$.) However, this is not the only way of constructing Lagrangians which are invariant under the transformation $Y^{\mu}\to\xi^{\mu}$; the model Lagrangian used in Sec. V is an obvious counterexample. By means of similar considerations, one can conjecture some other important properties of the coordinate fluctuations. Consider, in particular, the correlation function

$$\langle Y^{\mu}(x_1)Y^{\nu}(x_2)\rangle \equiv x_1^{\mu}\,x_2^{\nu}+\langle\xi^{\mu}(x_1)\,\xi^{\nu}(x_2)\rangle \ .$$

Clearly, one expects that as $L\to 0$, the fluctuation correlations disappear, $\langle\xi^{\mu}(x_1)\,\xi^{\nu}(x_2)\rangle\to 0$. By using dimensional and invariance considerations, one may write

$$\langle\xi^{\mu}(x)\,\xi^{\nu}(0)\rangle = L^2\left(\eta^{\mu\nu}g_0(x^2)+\frac{\partial^2 g_1(x^2)}{\partial x_{\mu}\partial x_{\nu}}\right) \ . \tag{2.7}$$

The tensor in angular brackets has to be dimensionless. One expects further that the behavior of g_0 and g_1 near the light cone is governed by the high-momentum behavior of their Fourier transforms. Thus,

$$g_0(x^2)\sim\int\frac{d^4p}{(P^2)^2}e^{ipx}\sim\ln x^2 \ ,$$

$$g_1(x^2)\sim\int\frac{d^4p}{(p^2)^3}e^{ipx}\sim x^2 \ .$$

(More precisely, near the light cone g_0 and g_1 have to behave as *associated* homogeneous functions of x^2 of degree 0 and 1, respectively.) Hence one infers that (barring some anomalies) the fluctuation correlations must have a rather mild singularity on the light cone. For instance, if the Lagrangian in (2.5) is assumed to be quadratic in Y^{μ}, then its leading term near the light cone (i.e., the term containing the highest derivative) must be proportional to the *second* derivative of Y^{μ}, e.g., in the form $\partial_{\alpha}\partial_{\beta}Y_{\mu}\partial^{\alpha}\partial^{\beta}Y^{\mu}$. Such terms are automatically invariant under $Y^{\mu}\to\xi^{\mu}$ (since $\partial_{\alpha}\partial_{\beta}x^{\mu}=0$); therefore, there is no need to antisymmetrize the derivatives in them.

From now on, the question of the scale length, L, approaching zero will not be discussed again in this paper. Hence the natural system of units, $\hbar=c=L=1$, will be used throughout.

III. FIELDS AND LAGRANGIANS

In order to proceed, one has to give a meaning to a *field* (the electron field, perhaps a quark field, etc.) defined on the coordinates $Y^{\mu}(x)$. To this end, one notices that in the functional formalism used here, $\xi^{\mu}(x)$ is just a c-number field and, hence, so is $Y^{\mu}(x)=x^{\mu}+\xi^{\mu}(x)$. ("Quantizing" the coordinate amounts to averaging over the various functional forms of ξ^{μ}.) Therefore, $Y^{\mu}(x)$ with a *fixed* functional form of $\xi^{\mu}(x)$ may be looked upon as a curvilinear system of coordinates introduced in Minkowski space. Hence, a "field defined on the Y^{μ}" [again, for a *given* form of $\xi^{\mu}(x)$] is just a field defined on an arbitrary curvilinear system of coordinates.

In constructing a Lagrangian density for a field or a system of fields, it is physically reasonable to require that the form of that Lagrangian density remain unchanged as $\xi^{\mu}(x)$ runs through its various functional forms, in other words, that the Lagrangian be a scalar density under general coordinate transformations. The procedure for such a construction is well known in the general theory of relativity and it may be taken over without any change.

The basic tool in constructing generally covariant Lagrangians for fields of arbitrary spin is the tetrad formalism.[7]

On using (2.3), one can immediately see that the tetrad vectors are given by

$$e^{\mu}{}_{|\nu} = Y^{\mu}{}_{,\nu} = \delta^{\mu}{}_{\nu} + \xi^{\mu}{}_{,\nu} \ . \tag{3.1}$$

In terms of the tetrad vectors, the contravariant metric tensor is given by

$$g^{\mu\nu} = e^{\mu}{}_{|\rho} e^{\nu}{}_{|\sigma} \eta^{\rho\sigma} = \eta^{\mu\nu} + \xi^{\mu,\nu} + \xi^{\nu,\mu} + \eta^{\rho\sigma} \xi^{\mu}{}_{,\rho} \xi^{\nu}{}_{,\sigma} \ . \tag{3.2}$$

It is important to keep in mind that the tetrad vectors (3.1) satisfy the symmetry relation

$$\frac{\partial e^{\mu}{}_{|\alpha}}{\partial x^{\beta}} = \frac{\partial e^{\mu}{}_{|\beta}}{\partial x^{\alpha}} \ . \tag{3.3}$$

This relation is a consequence of the fact that *the space is flat* and hence Y^{μ} is obtained from the *globally* Minkowskian system of classical coordinates x^{μ} by means of a coordinate transformation.

After these preliminary remarks, Lagrangian densities are constructed in a straightforward way. Suffice it to quote a few relevant examples of Lagrangians of *"free"* fields.

(a) Neutral scalar field.

$$\mathfrak{L} = \frac{1}{2}\left(g^{\mu\nu} \frac{\partial \Phi}{\partial Y^{\mu}} \frac{\partial \Phi}{\partial Y^{\nu}} - m^2 \Phi^2 \right), \tag{3.4}$$

where $\Phi = \Phi(Y(x))$ and $g^{\mu\nu}$ is given by (3.2). [It would be perhaps more appropriate to write $\delta\Phi/\delta Y^{\mu}(x)$ in (3.4) instead of $\partial\Phi/\partial Y^{\mu}$. However, the simpler notation may be used as long as it does not give rise to confusion.]

(b) Maxwell field.

$$\mathfrak{L} = -\tfrac{1}{4} F_{\alpha\beta} F_{\gamma\delta} g^{\alpha\gamma} g^{\beta\delta} \ , \quad F_{\alpha\beta} \equiv \frac{\partial A_{\alpha}}{\partial Y^{\beta}} - \frac{\partial A_{\beta}}{\partial Y^{\alpha}} \ . \tag{3.5}$$

(One remembers that the Christoffel symbols drop out of the expression of the curl of a vector.)

(c) Dirac field. The covariant spinor derivative is given formally by

$$\nabla_{\mu}\Psi = \frac{\partial \Psi}{\partial Y^{\mu}} - \tfrac{1}{4} i \sigma^{|\gamma\delta} t_{\mu|\gamma\delta} \Psi \ ,$$

with (3.6)

$$t_{\mu|\gamma\delta} = \frac{\partial e^{\nu}{}_{|\gamma}}{\partial Y^{\mu}} e^{\rho}{}_{|\delta} g_{\nu\rho} \ ,$$

where, of course, $g_{\nu\rho}$ is the inverse of the tensor (3.2),

$$g^{\mu\nu} g_{\nu\rho} = \delta^{\mu}{}_{\rho} \ .$$

In tetrad components

$$t_{|\mu\gamma\delta} = e_{|\mu}{}^{\rho} t_{\rho|\gamma\delta} \ ,$$

and since

$$\frac{\partial e^{\nu}{}_{|\gamma}}{\partial Y^{\mu}} = \frac{\partial e^{\nu}{}_{|\gamma}}{\partial x^{\alpha}} \frac{\partial x^{\alpha}}{\partial Y^{\mu}} = \frac{\partial e^{\nu}{}_{|\gamma}}{\partial x^{\alpha}} e^{|\alpha}{}_{\mu} \ ,$$

one gets

$$t_{|\alpha\gamma\delta} = \frac{\partial e^{\nu}{}_{|\gamma}}{\partial x^{\alpha}} e_{\nu|\delta} \ , \tag{3.7}$$

where, evidently, $e_{\nu|\delta} = g_{\nu\rho} e^{\rho}{}_{|\delta}$.

Here,

$$\sigma^{|\alpha\beta} = \frac{1}{2i} [\gamma^{|\alpha}, \gamma^{|\beta}] \ .$$

The covariant spinor derivative in tetrad components reads

$$\nabla_{|\mu}\Psi = e^{\alpha}{}_{|\mu} \frac{\partial \Psi}{\partial Y^{\alpha}} - \tfrac{1}{4} i \sigma^{|\gamma\delta} t_{|\mu\gamma\delta} \Psi = \frac{\partial \Psi}{\partial x^{\mu}} - \tfrac{1}{4} i \sigma^{|\gamma\delta} t_{|\mu\gamma\delta} \Psi \ ,$$

in view of Eq. (3.1). The Lagrangian density is written as

$$\mathfrak{L} = \tfrac{1}{2} i [\Psi^{\dagger} \gamma_{|0} \gamma^{|\mu} \nabla_{|\mu} \Psi - (\nabla_{|\mu}\Psi)^{\dagger} \gamma_{|0} \gamma^{|\mu} \Psi] - m \Psi^{\dagger} \gamma_{|0} \Psi \quad ,$$

which, upon introducing $\bar{\Psi} \equiv \Psi^{\dagger} \gamma_{|0}$, gives

$$\mathfrak{L} = \tfrac{1}{2} i \left(\bar{\Psi} \gamma^{|\mu} \frac{\partial \Psi}{\partial x^{\mu}} - \frac{\partial \bar{\Psi}}{\partial x^{\mu}} \gamma^{|\mu} \Psi \right) - m \bar{\Psi} \Psi - \tfrac{1}{4} i \bar{\Psi} \{\gamma^{|\alpha}, \sigma^{|\gamma\delta}\} \Psi t_{|\alpha\gamma\delta} \ . \tag{3.8}$$

On using the relation

$$\{\gamma^{|\alpha}, \sigma^{|\gamma\delta}\} = 2i \epsilon^{|\alpha\gamma\delta\rho} \gamma^{|5} \gamma_{|\rho} \ ,$$

one observes that the last term in (3.8) drops out, since

$$\epsilon^{|\alpha\gamma\delta\rho} t_{|\alpha\gamma\delta} = 0 \ ,$$

in view of (3.7) and of the symmetry relation (3.3). Hence, finally, the Dirac Lagrangian can be rewritten as

$$\mathfrak{L} = \tfrac{1}{2} i \left(\bar{\Psi} \gamma^{\mu} \frac{\partial \Psi}{\partial Y^{\mu}} - \frac{\partial \bar{\Psi}}{\partial Y^{\mu}} \gamma^{\mu} \Psi \right) - m \bar{\Psi} \Psi \ , \tag{3.9}$$

where

$$\gamma^{\mu} = e^{\mu}{}_{|\alpha} \gamma^{|\alpha} \ .$$

The action functional is naturally constructed from each of these Lagrangians (or their sums) by taking

$$W=\int d^4x\,\mathcal{L}\ . \tag{3.10}$$

This expression is generally covariant. In fact, following the usual procedure, for a *given* functional form of $\xi^\mu(x)$, one would write

$$W=\int d^4Y\sqrt{-g}\,\mathcal{L}\ ,$$

where

$$g=\det g_{\mu\nu}=(\det g^{\mu\nu})^{-1}\ .$$

On writing $\det g^{\mu\nu}=-(\det e^\mu{}_{|\alpha})^2$ and using the fact that $\det e^\mu{}_{|\alpha}=(\det e_\mu{}^{|\alpha})^{-1}$, one immediately arrives at (3.10).

Evidently, the form (3.10) of the action is made possible by the fact that the coordinates x^μ furnish a global coordinate system over the entire classical Minkowski space.

Minimal coupling of charged fields to the Maxwell field may be generated by means of a local gauge transformation.

Take the Dirac field as an example. The natural choice for a local gauge transformation is

$$\Psi(Y)\to e^{ie\Lambda(Y)}\Psi(Y)\ , \tag{3.11}$$

where e is the charge carried by the field Ψ. In the usual way, (3.11) leads to the gauge-covariant derivative

$$D_\mu\Psi=\frac{\partial\Psi(Y)}{\partial Y^\mu}-ieA_\mu(Y) \tag{3.12}$$

and to the corresponding expression of the current

$$\begin{aligned}J^\mu&=e\bar\Psi\gamma^\mu\Psi\ ,\\ &=e\bar\Psi\gamma^{|\rho}\Psi e^\mu{}_{|\rho}\ .\end{aligned} \tag{3.13}$$

The transformation (3.11) of the charged field is accompanied by the gauge transformation of the Maxwell field,

$$A_\mu(Y)\to A_\mu(Y)-\frac{\partial\Lambda(Y)}{\partial Y^\mu}\ , \tag{3.14}$$

which is a manifestly covariant relation, of course. It has to be borne in mind, however, that there is an alternative way of introducing electromagnetic interactions.

In fact, on rewriting the Dirac Lagrangian in terms of tetrad components,

$$\mathcal{L}=\tfrac{1}{2}i\left(\bar\Psi\gamma^{|\mu}\frac{\partial\Psi}{\partial x^\mu}-\frac{\partial\bar\Psi}{\partial x^\mu}\gamma^{|\mu}\Psi\right)-m\bar\Psi\Psi\ ,$$

one observes that it can be made invariant under a local gauge transformation of the form

$$\Psi(Y)\to e^{ie\lambda(x)}\Psi(Y)\ ,$$

where the gauge function $\lambda(x)$ depends on the classical coordinate only. Correspondingly, one is forced to introduce the Maxwell field in the form $A_\mu(x)$, and the gauge-covariant derivative of Ψ becomes

$$\begin{aligned}D'_{|\mu}\Psi&=\frac{\partial\Psi}{\partial x^\mu}-ieA_\mu(x)\\ &\equiv e^\nu{}_{|\mu}\frac{\partial\Psi}{\partial Y^\nu}-ieA_{|\mu}(x)\ .\end{aligned}$$

(This is *not* a generally covariant relation, since Ψ and $A_{|\mu}$ depend on two different types of coordinates. Nevertheless, Lorentz invariance of the theory may be saved.) As a matter of fact, one can construct theories in which fields depending on $Y^\mu(x)$ (hadrons?) coexist with others which are defined on the coordinates x^μ (leptons?).

Ultimately, one may be forced into such a choice by the experimental data. However, at present I find this alternative a rather unattractive one from the theoretical point of view.

IV. THE FEYNMAN RULES

In ordinary field theories one can generate the Feynman rules as follows. Suppose that the field variable is $\varphi(x)$, with Lagrangian $\mathcal{L}=Q(\varphi)+gV(\varphi)$ where $Q(\varphi)$ is quadratic in φ, $V(\varphi)$ is the "interaction," with coupling constant g. The generating functional of the Green's functions is constructed by adding a source term, $\int d^4x\,j(x)\,\varphi(x)$, to the action

$$Z=\int\delta\varphi\exp\left\{i\int d^4x\left[Q(\varphi)+gV(\varphi)+j(x)\,\varphi(x)\right]\right\}\ . \tag{4.1}$$

The perturbative expressions of the Green's functions are obtained by expanding the functional derivatives of Z into a power series in g. The resulting functional integrals are of the type

$$\int\delta\varphi\exp\left[i\int d^4x\,Q(\varphi)\right]\left[\int d^4x_1\,V(\varphi(x_1))\cdots\right]\times[\varphi(y_1)\cdots]\ .$$

They are Gaussian and hence readily calculable. How can one generalize this procedure for the case when the fields depend on the coordinates $Y^\mu(x)$?

The action is a functional of $\Phi(Y(x))$ (and of Y^μ itself), say,

$$W[\Phi(Y)]=\int d^4x\,Q[\Phi(Y)]+g\int d^4x\,V[\Phi(Y)]\ .$$

In view of the results of the preceding sections, one tries an expression for the generating functional:

$$Z=\int \delta Y^{\mu}\exp\left[i\int d^4x\,\mathfrak{L}(Y)\right]$$
$$\times\int \tilde{\delta}\Phi\exp\left[i\left(W[\Phi(Y)]+\int j(Y)\Phi(Y)\,d^4x\right)\right] \quad . \tag{4.2}$$

This expression has the right appearance; in particular, one hopes that in the limit as the scale length, L, approaches zero, the rapid oscillations of the factor $\exp[i\int d^4x\,\mathfrak{L}(Y)]$ suppress the coordinate fluctuations, and one recovers the generating functional (4.1). The problem presented by the formal expression (4.2) (as indicated by the tilde in $\tilde{\delta}\Phi$) is that one does not know how to perform the integration over Φ. In fact, in the integral over Φ, one should average over the degree of freedom represented by the field, while keeping those represented by the coordinate fluctuations "frozen in." Thus, in order to give a meaning to a functional integral of the type (4.2), one has to "disentangle" the field degrees of freedom from the coordinate fluctuations.

It is evident that this problem arises already in the case of integrals which are Gaussian in Φ. (Such integrals *do not* represent free fields: Φ interacts with the coordinate fluctuations.) In fact, once one learns how to handle Gaussian integrals, others arising in the perturbation expansion of any Green's function can be treated in the same way. (The latter can be obtained from a fundamental Gaussian integral by means of repeated functional differentiation.) In what follows, I describe the solution of the problem just outlined for the cases of the Dirac and Maxwell fields. (By the end of this section it should become obvious how to generalize the procedure for other fields.)

A. The Maxwell field

One starts from the Lagrangian (3.5) with a source term added to it:

$$W_M=-\int d^4x\left[\frac{1}{2}\,\frac{\partial A_\mu}{\partial Y^\nu}\,\frac{\partial A_\rho}{\partial Y^\sigma}(g^{\mu\rho}g^{\nu\sigma}-g^{\nu\rho}g^{\mu\sigma})\right.$$
$$\left.-g^{\mu\rho}J_\mu(Y)A_\rho(Y)\right] \quad , \tag{4.3}$$

where $g^{\mu\nu}$ is given by (3.2).

In order to separate the field degrees of freedom from those represented by the coordinate fluctuations, one introduces an auxiliary field $a_\mu(x)$ and an *auxiliary source*, $j_\mu(x)$, by means of the equations

$$A_\mu(Y(x))=\int d^4y\,a_\mu(y)\,\delta^{(4)}(Y(x)-y) \quad ,$$
$$J_\mu(Y(x))=\int d^4y\,j_\mu(y)\,\delta^{(4)}(Y(x)-y) \quad . \tag{4.4}$$

One has further

$$\frac{\partial A_\mu}{\partial Y^\nu}=\int d^4y\,a_\mu(y)\,\frac{\partial}{\partial Y^\nu}\,\delta^{(4)}(Y-y)$$
$$=\int d^4y\,\frac{\partial a_\mu(y)}{\partial y^\nu}\,\delta^{(4)}(Y-y) \quad . \tag{4.5}$$

Next, one defines the following two kernels:

$$K^{\mu\nu}(y,y')=\int d^4x\,g^{\mu\nu}(x)\,\delta^{(4)}(Y(x)-y)\,\delta^{(4)}(Y(x)-y')$$
$$\equiv\delta^{(4)}(y-y')\,K^{\mu\nu}(y) \quad , \tag{4.6}$$

where

$$K^{\mu\nu}(y)\equiv\int d^4x\,g^{\mu\nu}(x)\,\delta^{(4)}(Y(x)-y)$$

and

$$K^{\mu\nu;\rho\sigma}(y,y')=\int d^4x\,g^{\mu\nu}(x)\,g^{\rho\sigma}(x)\,\delta^{(4)}(Y(x)-y)$$
$$\times\delta^{(4)}(Y(x)-y')$$
$$\equiv\delta^{(4)}(y-y')\,K^{\mu\nu;\rho\sigma}(y) \quad , \tag{4.7}$$

where

$$K^{\mu\nu;\rho\sigma}(y)\equiv\int d^4x\,g^{\mu\nu}(x)\,g^{\rho\sigma}(x)\,\delta^{(4)}(Y(x)-y) \quad .$$

[These kernels are, of course, functionals of $\xi^\mu(x)$.] In terms of these quantities, the action (4.3) can be rewritten as follows:

$$W_M=-\int d^4y\left\{\frac{1}{2}\,\frac{\partial a_\mu}{\partial y^\nu}\,\frac{\partial a_\rho}{\partial y^\sigma}\,[K^{\mu\rho;\nu\sigma}(y)-K^{\nu\rho;\mu\sigma}(y)]\right.$$
$$\left.-j_\mu(y)\,a_\nu(y)\,K^{\mu\nu}(y)\right\} \quad . \tag{4.8}$$

Now the functional integration over the field degrees of freedom can be performed: One has to take

$$\tilde{\delta}A_\mu\equiv\delta a_\mu(y) \quad .$$

First of all, one observes that the quadratic form in (4.8) is degenerate. It is constant on the orbits of the gauge group:

$$a_\mu(y)\to a_\mu(y)-\frac{\partial\lambda(y)}{\partial y^\mu} \quad .$$

These gauge transformations on the auxiliary fields are induced by the transformations (3.14) of the fields $A_\mu(Y)$; in order to prove this, one introduces $\lambda(y)$ by

$$\Lambda(Y(x))=\int d^4y\,\lambda(y)\,\delta^{(4)}(Y(x)-y) \quad ,$$

so that

$$\frac{\partial \Lambda}{\partial Y^\mu} = \int d^4y \, \frac{\partial \lambda(y)}{\partial y^\mu} \, \delta^{(4)}(Y(x) - y) \; ,$$

and uses the definition (4.4) of the auxiliary fields.

The integral over the orbits of the gauge group can be factored out with the help of the Faddeev-Popov (FP) method.[8]

A convenient choice of the gauge-fixing term is

$$\Delta W[a] = -\tfrac{1}{2}\int d^4y \, \frac{\partial a_\mu}{\partial y^\nu} \, \frac{\partial a_\rho}{\partial y^\sigma} \, K^{\nu\rho\,;\mu\sigma}(y) \tag{4.9}$$

(Feynman gauge).

Next, one has to calculate the FP factor F, defined by

$$F^{-1} \approx \int \delta\lambda \, e^{i\Delta W[a+\partial\lambda]} \; . \tag{4.10}$$

In Eq. (4.10) and throughout the remainder of this section, the wavy equality sign ($\approx$) stands as a reminder that some infinite constants which are independent of *all* the functional arguments [*including* $\xi^\mu(x)$] have to be absorbed into the definition of the functional measures.

A straightforward integration in (4.10) gives

$$\begin{aligned} F[Y] &= [\mathrm{Det}(iK^{\mu\sigma;\nu\rho}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma)]^{1/2} \\ &= \exp[\tfrac{1}{2}i \, \mathrm{Tr} \ln(K^{\mu\sigma;\nu\rho}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma)] \; . \end{aligned} \tag{4.11}$$

The trace is to be understood in the functional sense. As expected, the FP factor is independent of the functional argument $a_\mu(y)$, since the gauge transformations form an Abelian group. However, unlike in ordinary quantum electrodynamics, *one is not allowed to set* $F \approx 1$. In fact, $F[Y]$ is a highly nonlinear functional of Y^μ, which still has to be integrated over.

Having factored out the integral over the gauge group, we now find it easy to compute the generating functional for the Maxwell field. The remaining integral,

$$\int \delta a_\mu \exp\left\{\tfrac{1}{2}i \int d^4y \left[\frac{\partial a_\mu}{\partial y^\nu}\,\frac{\partial a_\rho}{\partial y^\sigma}\, K^{\mu\rho;\nu\sigma}(y) - j_\mu(y)\, a_\nu(y)\, K^{\mu\nu}(y)\right]\right\} \; ,$$

can be calculated by means of the usual trick of shifting variables in a Gaussian integral. As a result, one finds

$$Z_M \approx \int \delta Y^\mu \exp\left[i \int d^4x \, \mathcal{L}(Y)\right] F[Y] \exp\left\{\tfrac{1}{2}i \int d^4y\, d^4y' [j_\rho(y) K^{\rho\mu}(y) D_{\mu\nu}(y, y') K^{\nu\sigma}(y') j_\sigma(y')]\right\} \; , \tag{4.12}$$

where the Green's function $D_{\mu\nu}(y, y')$ satisfies the equation

$$\frac{\partial}{\partial y^\mu}\left(K^{\mu\rho;\nu\sigma}(y) \, \frac{\partial D_{\sigma\tau}(y, y')}{\partial y^\rho}\right) = -\delta^\nu_\tau \, \delta^{(4)}(y - y') \; , \tag{4.13}$$

together with the usual causal boundary conditions.

On remembering the expression (4.6) of $K^{\mu\nu}(y)$, we can write the generating functional Z_M in the form

$$Z_M \approx \int \delta Y^\mu \exp\left[i \int d^4x \, \mathcal{L}(Y)\right] F[Y] D[Y] \exp\left[\tfrac{1}{2}i \int d^4x \, d^4x' J^\mu(Y(x)) D_{\mu\nu}(Y(x), Y(x')) J^\nu(Y(x'))\right] \; . \tag{4.14}$$

The factor $D[Y]$ is given by the expression

$$\begin{aligned} D[Y] &= [\mathrm{Det}(iK^{\mu\rho;\nu\sigma}\partial_\mu\partial_\rho)]^{-1/2} \\ &= \exp[-\tfrac{1}{2}i \, \mathrm{Tr} \ln(K^{\mu\rho;\nu\sigma}\partial_\mu\partial_\rho)] \; , \end{aligned} \tag{4.15}$$

where the trace is to be taken both in the functional sense and with respect to the superscripts ν, σ. As noticed before, the factor $D[Y]$ must not be dropped, due to its dependence on the functional argument Y^μ.

The meaning of an expression of the type $D_{\mu\nu}(Y(x), Y(x'))$ is clear from the preceding discussion; the "recipe" is the following:

(i) First solve Eq. (4.13) for a Green's function $D_{\mu\nu}(y, y')$ depending on the classical coordinates y and y'. [The solution of (4.13), of course, involves the $Y^\mu(x)$, or, equivalently, the fluctuation field, $\xi^\mu(x)$, through the kernel $K^{\mu\nu;\rho\sigma}(y)$.]

(ii) Having gone through step (i), substitute "quantized coordinates," $Y^\mu(x)$, $Y^\mu(x')$ in place of the classical arguments y^μ and y'^μ, respectively. A perturbative construction of such Green's functions is outlined later in this section, using the Green's functions of the Dirac field as an example.

B. The Dirac field

The calculation of the generating functional for a Dirac field is actually simpler than the previous case: No complications due to a gauge group arise.

One starts again from the action derived from (3.9), with an appropriate source term added to it:

$$W_D = \int d^4x \left[\tfrac{1}{2}i\left(\bar\Psi\gamma^\mu \frac{\partial\Psi}{\partial Y^\mu} - \frac{\partial\bar\Psi}{\partial Y^\mu}\gamma^\mu\Psi\right) - m\bar\Psi\Psi + \bar H(Y)\Psi(Y) + \bar\Psi(Y)H(Y)\right] \; . \tag{4.16}$$

Here, as usual, the quantities $\Psi, H,$ are regarded as elements of an infinite-dimensional Grassmann algebra ("anticommuting c numbers"), supported by the manifold $Y^\mu(x)$ of dynamical variables.

Next, one introduces auxiliary fields and sources by means of the equations

$$\Psi(Y(x)) = \int d^4y\,\psi(y)\,\delta^{(4)}(Y(x)-y)\ ,$$
$$H(Y(x)) = \int d^4y\,\eta(y)\,\delta^{(4)}(Y(x)-y)\ , \tag{4.17}$$

thus disentangling the two different kinds of degrees of freedom, as discussed before. In terms of the auxiliary variables, the action of the Dirac field is rewritten as follows:

$$W_D = \int d^4y\Big[\tfrac{1}{2}i\Big(\bar\psi(y)G^\mu(y)\frac{\partial\psi}{\partial y^\mu} - \frac{\partial\bar\psi}{\partial y^\mu}G^\mu(y)\psi(y)\Big) - mK(y)\bar\psi(y)\psi(y) + \bar\eta(y)\psi(y)K(y) + \bar\psi(y)\eta(y)K(y)\Big]\ , \tag{4.18}$$

where

$$K(y) = \int d^4x\,\delta^{(4)}(Y(x)-y)\ ,$$
$$G^\mu(y) = \int d^4x\,\delta^{(4)}(Y(x)-y)\gamma^{\mid\rho}e^\mu{}_{\mid\rho}(x)\ . \tag{4.19}$$

After performing the functional integration over $\psi(y)$, one finds for the generating functional of the Dirac field

$$Z_D \approx \int \delta Y^\mu \exp\Big[i\int d^4x\,\mathcal{L}(Y)\Big]\Delta[Y]\exp\Big[-i\int d^4y\,d^4y'\,\bar\eta(y)K(y)S(y,y')K(y')\eta(y')\Big]\ . \tag{4.20}$$

The Green's function $S(y, y')$ satisfies the following equation:

$$iG^\mu(y)\frac{\partial S(y,y')}{\partial y^\mu} + \Big(i\frac{\partial G^\mu}{\partial y^\mu} - mK(y)\Big)S(y,y') = -\delta^{(4)}(y-y')\ , \tag{4.21}$$

while $\Delta[Y]$ is expressed in terms of a functional determinant:

$$\Delta[Y] = \Big\{\mathrm{Det}\,i\Big[\tfrac{1}{2}i\Big(G^\mu(y)\frac{\overrightarrow{\partial}}{\partial y^\mu} - \frac{\overleftarrow{\partial}}{\partial y^\mu}G^\mu(y)\Big) - mK(y)\Big]\Big\}^{1/2}\ . \tag{4.22}$$

The exponent $+\frac{1}{2}$ (instead of $-\frac{1}{2}$) is a consequence of the fact that the field $\psi(y)$ is treated as an anticommuting variable; cf. Berezin.[9] The generating functional can be reexpressed in terms of the sources $H(Y)$ with the help of (4.17) and (4.19). One finds

$$Z_D \approx \int \delta Y^\mu \exp\Big[i\int d^4x\,\mathcal{L}(Y)\Big]\Delta[Y]\exp\Big[-i\int d^4x\,d^4x'\,\bar H(Y(x))S(Y(x),Y(x'))H(Y(x'))\Big]\ . \tag{4.23}$$

In a similar way, one finds that the generating functional for the combined system of Maxwell and Dirac fields is

$$Z_{\mathrm{MD}} \approx \int \delta Y^\mu \exp\Big[i\int d^4x\,\mathcal{L}(Y)\Big]\Delta[Y]F[Y]D(Y)\mathfrak{z}_M\mathfrak{z}_D\ , \tag{4.24}$$

where

$$\mathfrak{z}_M = \exp\Big[-\tfrac{1}{2}i\int d^4x\,d^4x'\,J^\mu(Y(x))D_{\mu\nu}(Y(x),Y(x'))J^\nu(Y(x'))\Big]\ ,$$
$$\mathfrak{z}_D = \exp\Big[-i\int d^4x\,d^4x'\,\bar H(Y(x))S(Y(x),Y(x'))H(Y(x'))\Big]\ .$$

From here one reads off in the usual way the modified Feynman rules in quantum electrodynamics by adding the interaction term to the action:

$$W_{\mathrm{int}} = e\int d^4x\,\Psi(Y(x))\gamma^{\mid\rho}\Psi(Y(x))e^\mu{}_{\mid\rho}(x) \times A_\mu(Y(x))\ ; \tag{4.25}$$

cf. Eq. (3.13). The terms in the perturbation expansion can be given the usual interpretation in terms of Feynman diagrams, with the following changes in the Feynman rules:

(1) A vertex at a point with classical coordinates x^μ ("classical point x^μ") corresponds to a factor

$$e\gamma^{\mid\rho}e^\mu{}_{\mid\rho}(x)\ .$$

(2) A photon propagating between the classical points x^μ and x'^μ gives a factor

$$D_{\mu\nu}(Y(x),Y(x'))\ .$$

(3) A fermion propagating from the classical point x^μ to the classical point x'^μ gives a factor

$$S(Y(x'),Y(x)).$$

(4) The resulting expression has to be averaged over the coordinate fluctuations with the weight factor

$$\exp\left[i\int d^4x\,\mathcal{L}(Y)\right]\Delta[Y]F[Y]D[Y]$$

as read off from (4.24). Thus, coordinate fluctuations affect the calculation in two different ways.

First, they interact with the matter fields: The coefficients of Eqs. (4.13) and (4.21), and the interaction Lagrangian in (4.25) depend on ξ^μ. This effect is a *dynamical* one: It depends on the expressions of the Lagrangians. In order to take it into account, one has to solve the equations for the Green's functions.

Second, the coordinate fluctuations enter in a way which is independent of the form of the Lagrangian and of the approximations used in solving the equations the Green's functions satisfy. As shown before, the classical arguments, y^μ, y'^μ of the Green's functions have to be replaced by $Y^\mu(x)$ and $Y^\mu(x')$, respectively. This is a purely *kinematical* effect of the coordinate fluctuations.

The Green's functions and the determinantal factors F, D, Δ can be calculated perturbatively.

I outline the procedure on the example of the Dirac propagator (4.21) and of the corresponding determinant, $\Delta[Y]$. One writes $Y^\mu(x)=x^\mu+\xi^\mu(x)$ and expands $S(y,y')$ into a functional Taylor series around $\xi^\mu=0$. It is convenient to multiply ξ^μ by a "counting factor" λ, in order to keep track of the powers more easily. At the end of the calculation one sets, of course, $\lambda=1$.

First, one expands the quantities K and G^μ. The main tool used is the power-series expansion of the δ function:

$$\delta^{(4)}(x+\xi(x)-y)=\delta^{(4)}(x-y)+\lambda\xi^\alpha\partial_\alpha\delta^{(4)}(x-y)+\cdots.$$

One obtains

$$\begin{aligned}K(y)&\equiv 1+\lambda k_1(y)+\frac{\lambda^2}{2!}k_2(y)+\cdots\\&=1-\lambda\partial_\alpha\xi^\alpha+\frac{\lambda^2}{2!}\partial_\alpha\partial_\beta(\xi^\alpha\xi^\beta)+\cdots,\\G^\mu(y)&\equiv\gamma^\mu+\lambda g_1^\mu(y)+\frac{\lambda^2}{2!}g_2^\mu(y)\\&=\gamma^\mu+\lambda(\gamma^\nu\partial_\nu\xi^\mu-\gamma^\mu\partial_\alpha\xi^\alpha)\\&\quad+\frac{\lambda^2}{2!}[\gamma^\mu\partial_\alpha\partial_\beta(\xi^\alpha\xi^\beta)-2\gamma^\nu\partial_\alpha(\xi^\alpha\partial_\nu\xi^\mu)]\\&\quad+\cdots.\end{aligned}\tag{4.26}$$

In these and in the subsequent formulas, all indices are contracted with the *Minkowskian* metric tensor. Therefore, the tetrad notation for the *constant* γ matrices entering here is no longer necessary. One now sets

$$S(y,y')=S_0(y,y')+\lambda S_1(y,y')+\frac{\lambda^2}{2!}S_2(y,y')+\cdots,\tag{4.27}$$

inserts (4.26) and (4.27) into (4.21), and collects the coefficients of the powers of λ. In this way, the following infinite system of equations is obtained:

$$\begin{aligned}&(i\gamma^\mu\partial_\mu-m)S_0=-\delta^{(4)}(y-y'),\\&(i\gamma^\mu\partial_\mu-m)S_1+(i\partial_\mu g_1^\mu+ig_1^\mu\partial_\mu-mk_1)S_0=0,\\&(i\gamma^\mu\partial_\mu-m)S_2+2(i\partial_\mu g_1^\mu+ig_1^\mu\partial_\mu-mk_1)S_1\\&\qquad+(i\partial_\mu g_2^\mu+ig_2^\mu\partial_\mu-mk_2)S_0=0,\\&\cdots.\end{aligned}\tag{4.28}$$

This set of equations is readily solved. One finds

$$\begin{aligned}&S_0(y)=\frac{1}{(2\pi)^4}\int d^4k\,e^{iky}(\gamma^\mu k_\mu+m)^{-1},\\&\text{(the free Dirac propagator)}\\&S_1(y_1,y_2)=\int d^4y'\,S_0(y_1,y')v_1(y')S_0(y',y_2),\\&S_2(y_1,y_2)=2\int d^4y'\,d^4y''\,S_0(y_1,y')v_1(y')\\&\qquad\times S_0(y',y'')v_1(y'')S_0(y'',y_2)\\&\qquad+\int d^4y'\,S_0(y,y')v_2(y')S_0(y',y_2),\\&\cdots,\end{aligned}\tag{4.29}$$

where

$$v_l=i\partial_\mu g_l^\mu+ig_l^\mu\partial_\mu-mk_l\quad(l=1,2,\ldots).\tag{4.30}$$

The factor $\Delta[Y]$ can be calculated in the same way by writing

$$\Delta[Y]=\exp[\tfrac{1}{2}i\,\mathrm{Tr}\ln(G^\mu\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu G^\mu-mK)]$$

and using the expansion of K and G^μ. This iterative solution can be represented by Feynman diagrams. On representing the coordinate fluctuation field, ξ^μ, by a dashed line and the propagation of a "bare" fermion (corresponding to S_0) by an oriented continuous one, the solution (4.29) can be represented by the diagrams shown in Fig. 1(a), whereas $-2i\ln\Delta[Y]$ is represented by the diagrams shown in Fig. 1(b).

One notices that to every order in λ there are contact terms ("seagulls") present: The interaction between matter fields and the coordinate fluc-

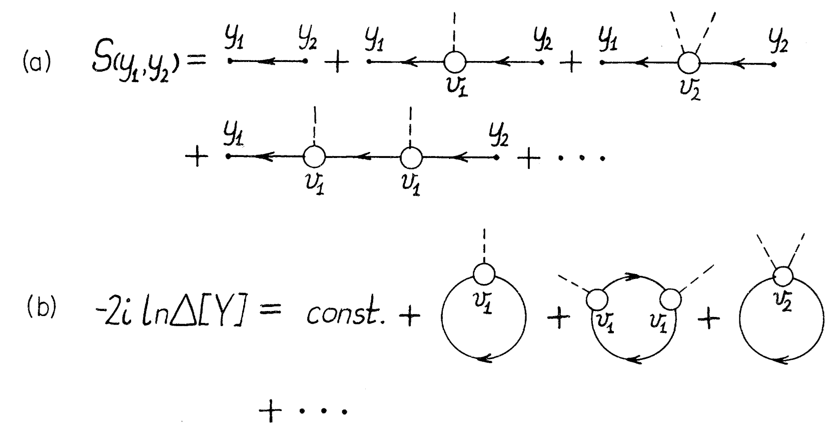

FIG. 1. (a) Feynman diagrams representing the Dirac propagator. (b) Feynman diagrams representing the functional determinant, $\Delta[Y]$. A dashed line represents the fluctuation field ξ; an oriented continuous line represents the propagator of a bare fermion.

tuation cannot be described by a polynomial interaction.

The formal expression of $\ln\Delta[Y]$ as represented by the diagrams in Fig. 1(b) contains ultraviolet divergences and it has to be renormalized. This question will be examined elsewhere.

One understands now the role of the factors F, D, Δ appearing in expression (4.24). They represent the self-interaction of the coordinate fluctuations generated by virtual pairs of quanta of the matter fields. After carrying out the functional integration with respect to ξ^μ, there *do* arise vacuum-to-vacuum diagrams which can be dropped as in usual field theories. However, the effect of the diagrams in Fig. 1(b) on the propagation and mutual interaction of the coordinate fluctuations has to be retained in a consistent treatment.

V. EXAMPLES

These examples of the calculation of on-shell S-matrix elements serve to illustrate the use of the diagram rules.[10] They are designed to shed some light on two questions raised by the theory outlined in the previous sections: (i) What is the influence of coordinate fluctuations on processes calculated in the framework of "well-established" theories (e.g., quantum electrodynamics)? (ii) Can one indeed identify scattering caused by coordinate fluctuations with strong interactions, as was conjectured in the Introduction?

In order to simplify the calculations, I approximate the free Lagrangian for Y^μ by the expression

$$\mathfrak{L}(Y) = -\tfrac{1}{2}\partial_\alpha\partial_\beta Y_\mu \partial^\alpha\partial^\beta Y^\mu\,; \tag{5.1}$$

see Sec. II. The finite range of the coordinate fluctuations is simulated by the introduction of an artificial cutoff into the fluctuation correlations at large distances.

A. First example: "kinematical" influence of coordinate fluctuations on a one-photon process

Consider the scattering of two (distinguishable) leptons through the exchange of one photon. I neglect the dynamical effect of the coordinate fluctuations completely. This means, in particular, that the photon propagator of *classical* arguments is given by the usual expression

$$D_{\mu\nu}(x) = \frac{\eta_{\mu\nu}}{(2\pi)^4}\int \frac{d^4q}{q^2+i\epsilon}e^{iqx}\,. \tag{5.2}$$

Thus, the only effect of the coordinate fluctuations consists of the replacement of the arguments $x_i \to Y(x_i)$ $(i=1,2)$. The matrix element of the process is given by

$$M_{fi} = \frac{e^2}{(2\pi)^4 i}\int d^4x_1\,d^4x_2\,\bar{u}(p_4)\gamma^\mu u(p_1)\bar{u}(p_3)\gamma_\mu u(p_2) \times \int \frac{d^4q}{q^2}\langle e^{iPY(x_1)}e^{iP'Y(x_2)}\rangle\,, \tag{5.3}$$

where

$$P = p_1 - p_4 - q\,,$$
$$P' = p_2 - p_3 + q\,,$$

and the angular brackets stand for the functional average over coordinate fluctuations. The Dirac matrices are coordinate-independent.

When we use translation invariance, (5.3) becomes

$$M_{fi} = -(2\pi)^4 i\delta^{(4)}(P+P')\bar{u}(p_4)\gamma^\mu u(p_1) \times \bar{u}(p_3)\gamma_\mu u(p_2)M\,, \tag{5.4}$$

where

$$M=\frac{1}{(2\pi)^4}\int d^4x\int\frac{d^4q}{q^2}\langle e^{iPY(x)}e^{-iPY(0)}\rangle$$
$$=\int\frac{d^4q}{q^2}\int\frac{d^4x}{(2\pi)^4}e^{iPx}\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle. \quad (5.5)$$

It is convenient to separate out the "classical" one-photon term by writing

$$M=\frac{1}{t}+K(t),$$

where

$$K(t)=\int\frac{d^4q}{q^2}\int\frac{d^4x}{(2\pi)^4}e^{iPx}[\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle-1],$$
$$t=(p_1-p_4)^2. \quad (5.6)$$

The expression in square brackets has to approach zero at large distances (finite range of the fluctuation correlations). Next, one has to calculate the functional average

$$\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle\approx\int\delta\xi^\mu\exp\left[i\left(\int d^4y\,\mathcal{L}(\xi(y))+P\cdot[\xi(x)-\xi(0)]\right)\right], \quad (5.7)$$

with the Lagrangian (5.1).

One introduces the source function

$$S_\mu(y)=P_\mu[\delta^{(4)}(x-y)-\delta^{(4)}(y)], \quad (5.8)$$

with the help of which the exponent in (5.7) may be written as

$$\int d^4y[\mathcal{L}(\xi(y))+\xi^\mu(y)S_\mu(y)]. \quad (5.9)$$

Thus, the integral (5.7) may be calculated by straightforward Gaussian integration with the result

$$\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle=\exp\{iP^2[g(x)-g(0)]\}, \quad (5.10)$$

where $g(x)$ satisfies the biharmonic equation in four dimensions:

$$\Box^2 g(x)=\delta^{(4)}(x). \quad (5.11)$$

Assumed to be valid in the whole space, the solution of (5.10) with "natural" boundary conditions is given by

$$g(x)=\frac{1}{16\pi^2}\ln x^2. \quad (5.12)$$

Equation (5.9) is meaningless as it stands, since it contains an infinite phase. However, once one specifies the appropriate branch of the logarithm in (5.12) by writing $x^2\to x^2-i\epsilon$, the bracket in (5.10) becomes $g(x^2-i\epsilon)-g(i^{-1}\epsilon)$, which is meaningful as long as $\epsilon\neq 0$. Thus the fluctuation correlations satisfy the boundary condition

$$\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle\to 1\quad (x^2\to+0).$$

One may redefine the Green's function, g, appearing in all these expressions:

$$g(x^2)=\frac{1}{16\pi^2}\ln x^2\quad(x^2\neq 0),$$
$$g(0)=0, \quad (5.13)$$

and simply write

$$\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle=\exp[iP^2g(x)]$$

in place of (5.10).

In order to simulate the finite range of coordinate fluctuations, one may try to insert a step function, $\theta(1-|y^2|)$ under the integral (5.9). This gives

$$\langle e^{iP\xi(x)}e^{-iP\xi(0)}\rangle\equiv 1\text{ for }|x^2|>1.$$

[Due to the fact that the normal derivatives of g are constant on the hyperboloids $|x^2|=1$, there appear no surface terms in (5.10), despite the cutoff.]

In this calculation, one is basically interested in the behavior of $K(t)$ at reasonably low values of t ($-t\lesssim 1$). This is the domain in which the validity of "normal" quantum electrodynamics is established. Hence, the function $K(t)$ should be small if the theory outlined here makes any sense.

On inserting (5.10) into (5.6) together with the arbitrary cutoff just introduced, one gets

$$K(t)=\int\frac{d^4q}{q^2}\int\frac{d^4x}{(2\pi)^4}e^{iPx}[e^{iP^2g(x)}-1]$$
$$\times\Theta(1-|x^2|).$$

The low-t behavior of $K(t)$ is dominated by the first term in the expansion of $\exp[iP^2g(x)]-1$:

$$K(t)\sim i\int\frac{d^4q}{q^2}P^2\int\frac{d^4x}{(2\pi)^4}g(x)e^{iPx}\Theta(1-|x^2|)$$
$$(|t|\lesssim 1),$$

or, by introducing the Fourier transform of $g(x)$,

$$K(t)\sim\frac{i}{(2\pi)^4}\int\frac{d^4q}{q^2}(q-Q)^2g(q-Q), \quad (5.14)$$

where

$$Q_\mu=P_{1\mu}-P_{4\mu}$$

and

$$g(k)=\int d^4x\, g(x)e^{ikx}\Theta(1-|x^2|)\,.$$

It is sufficient to calculate the Fourier transform of g for Euclidean coordinates and momenta; one can then use standard analytic techniques to obtain $g(k)$ in the Minkowskian region.

Assuming then that x^μ, k_μ are Euclidean vectors, one integrates over the angles to obtain

$$g(k)=\frac{4\pi^2 i}{k}\int_0^1 r^2 dr\, J_1(kr)\ln r \tag{5.15}$$

where $k=(k_\mu k_\mu)^{1/2}$. [There is no need to worry about the redefinition (5.13) of the Green's function at this point: The factor r^2 in (5.15) vanishes sufficiently rapidly at $r\to 0$ so as to make such ambiguities irrelevant.]

The integral in (5.15) is calculated by an elementary technique: One expands the Bessel function into its power series, integrates term by term, and resums the resulting series.[11] The result is

$$g(k)=-\tfrac{1}{2}i\left(\frac{1}{k^3}J_1(k)-\frac{2}{k^4}[1-J_0(k)]\right). \tag{5.16}$$

The oscillatory behavior of $g(k)$ for $k\gg 1$ is the price paid for introducing a sharp cutoff in coordinate space (it is sufficient to remember the wave function for the scattering of scalar waves on an impenetrable sphere). For low and high momenta, respectively, $g(k)$ behaves as follows:

$$\begin{aligned} g(k)&\sim -\frac{i}{64}[1+O(k^2)] \quad (k\ll 1),\\ g(k)&\sim -\frac{i}{k^4}(1+\text{oscillating terms})\ (k\gg 1). \end{aligned} \tag{5.17}$$

Thus, apart from the oscillating terms (which are spurious anyway) $g(k)$ can be reasonably well approximated by the expression

$$g(k)\approx -i(k^2+8)^{-2},$$

or, going back to Minkowskian momenta,

$$g(k)\approx -\frac{i}{(k^2-8)^2}\,. \tag{5.18}$$

In terms of "particle" states, one can interpret the expression (5.18) as the propagator of a dipole ghost of *zero* norm[12] located at a mass equal to 8.

[It is amusing to remark in parentheses that the propagator (5.18) in coordinate representation is given by

$$g(x)=\frac{1}{16\pi}H_0^{(2)}((8x^2)^{1/2}) \quad (x^2>0),$$

with the analytic continuation defined by $x^2\to e^{-i\pi}x^2$ for spacelike x. Needless to say, the leading singularity of the last expression on the light cone coincides with that of a Green's function without a cutoff in coordinate space. The fact that the introduction of a cutoff in coordinate space shifts the position of the dipole ghost so far away from $k^2=0$ is a consequence of the very mild behavior of $g(x)$ near the light cone.] If we use the form (5.18) for $g(k)$, the integral in (5.14) can be calculated by means of a standard Feynman diagram technique. The integral (5.14) is logarithmically divergent; thus it has to be subtracted. [One easily convinces oneself that the degree of divergence is the same for all the other integrals which arise from higher-order terms in the expansion of $\exp(iP^2g)-1$; thus altogether one subtraction constant has to be determined.]

One now realizes that the function $K(t)$ in coordinate representation may be written as the vacuum expectation value of a time-ordered product; the value $K(0)$ (the subtraction constant) is related to the value of that product for coincident arguments. The boundary condition (5.13) for the redefined Green's function gives $K(0)=0$. With this, one finds

$$K(t)\sim\frac{1}{16\pi^2}[\ln(1-\tfrac{1}{8}t)+\tfrac{1}{8}t]\,. \tag{5.19}$$

The function $K(t)$ has a branch point at $t=8$; similarly, the integrals arising from higher-order terms in the expansion of $\exp(iP^2g)-1$ have branch points at $t=16$, $t=24,\ldots$, respectively. Thus, indeed, the dominant contribution to $K(t)$ at low values of t comes from the lowest-order term in the expansion. The expansion of (5.19) near $t=0$ reads

$$K(t)=\frac{1}{32\pi^2}\left(\frac{t}{8}\right)^2+O(t^3)\,.$$

Reinserting this into (5.5), one finally obtains

$$M\sim\frac{1}{t}-\frac{1}{32\pi^2}\left(\frac{t}{8}\right)^2 \quad (|t|\lesssim 8),$$

indicating that the *kinematic* effect of coordinate fluctuations on one-photon processes is negligibly small at moderate momentum transfers.

B. Second example: scattering caused by coordinate fluctuations

Coordinate fluctuations couple to matter fields; thus they cause scattering. By constructing a simple model, I want to indicate that in a certain (rather crude) approximation, the resulting scattering amplitudes are essentially of the same form as in Mansouri's dual resonance model.[13]

I consider the scattering of scalar mesons for simplicity, assuming that the meson is a bound quark-antiquark ($q\bar{q}$) pair. On going back to Eqs. (4.18) and (4.19) one sees that the coupling between a *scalar* quark pair and the coordinate degrees of freedom is given by a term of the form

$$-m\int d^4y\, K(y)\bar{\psi}(y)\psi(y)\,. \tag{5.20}$$

If the quark pair is bound, one can introduce an effective scalar-meson field, $\varphi(y)$ which (in on-mass-shell amplitudes) replaces $\bar{\psi}\psi$ in the coupling term. Thus, it is assumed that the effective coupling between the meson and the coordinate degrees of freedom is given by the interaction density

$$-m\int d^4y\, \delta^{(4)}(Y(x)-y)\varphi(y)\,. \tag{5.21}$$

In these expressions, m stands for the quark mass.

Using the approximation (5.21) to the interaction, the N-meson amplitude is given by the expression

$$A(k_1,\ldots,k_N) = \frac{g^N}{N!}\sum_P \int \prod_{j=1}^{N} d^4x_j \langle \chi_{k_1}(Y(x_1))\cdots\chi_{k_N}(Y(x_N))\rangle\,. \tag{5.22}$$

(All mesons are considered as incoming ones.)

The meaning of the symbols in Eq. (5.21) is the following. The wave function (depending on the classical argument, y) of an incoming meson of four-momentum may be written as

$$\varphi_k(y) = Z\chi_k(y)\,,$$

where Z is the normalization factor of the bound-state wave function ($0<Z<1$), while $\chi_k(y)$ is a normalized plane wave:

$$\chi_k(y) = \frac{1}{[(2\pi)^3\omega_k]^{1/2}}\, e^{iky}\,. \tag{5.23}$$

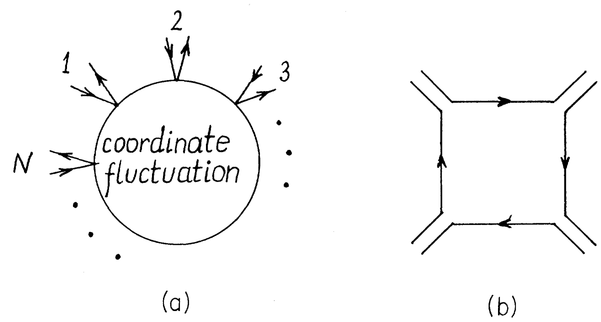

FIG. 2. (a) Quark annihilation diagram of an N-meson amplitude. (b) Quark-exchange diagram, neglected by the approximations made in Sec. V.

The "effective coupling constant," g, is given by $g=-mZ$. The angular bracket means, as before, average over coordinate fluctuations. The quantity in angular brackets turns out to be completely symmetrical with respect to permutations of the external mesons; hence the sum over the $N!$ identical terms in (5.22) cancels the factor $(N!)^{-1}$.

In the language of bound quark pairs, the amplitude (5.22) corresponds to a pure annihilation diagram, as shown in Fig. 2(a). It is to be emphasized that as a consequence of the approximation made in obtaining (5.21) from (5.20), a large class of diagrams [such as the one shown in Fig. 2(b)] is lost. In a more consistent treatment, one should use the original interaction (5.20). External quark pairs may be replaced by effective meson wave functions only in the scattering amplitudes, not in the interaction density.

Next, one has to calculate the average over the coordinate fluctuations. It is given by

$$\langle \chi_{k_1}(Y(x_1))\cdots\chi_{k_N}(Y(x_N))\rangle = \frac{\int \delta Y^\mu \mathfrak{D}[Y]\exp\{i[\int d^4y\, \mathcal{L}(Y) + \sum_{j=1}^N k_{j\mu}Y^\mu(x_j)]\}}{\int \delta Y^\mu \mathfrak{D}[Y]\exp[i\int d^4y\,\mathcal{L}(Y)]}\,. \tag{5.24}$$

The factor $\mathfrak{D}[Y]$ stands for the product of the determinantal factors which generate the self-interaction of coordinate fluctuations through "vacuum loops" of the matter fields; cf. Sec. IV. In what follows, Eq. (5.24) is evaluated in the zero-loop approximation by setting $\mathfrak{D}[Y]=1$.

In order to evaluate the functional integrals in (5.24), it is convenient to go over to a Euclidean metric. I further assume that $\mathcal{L}(Y)$ is given by the simple expression (5.1) with a cutoff in coordinate space at $|(x_i-x_i)^2|=1$, in order to take into account the finite range of the coordinate correlations. The cutoff can be formulated in Euclidean metric in a particularly simple way: All distances are restricted to lie within the unit sphere.

Now the functional integral can be readily evaluated by using the trick of introducing a source function,

$$S_\mu(y) = \sum_{i=1}^N k_{i\mu}\delta^{(4)}(y-x_i)\,,$$

as was done in the previous example. Thus, one is led to evaluate the Euclidean functional integral

$$I = \int \delta Y^\mu \exp\Big[\int d^4y\,\Theta(1-y^2) \times(-\tfrac{1}{2}\partial_\alpha\partial_\beta Y_\mu\partial^\alpha\partial^\beta Y^\mu + Y^\mu S_\mu)\Big]\,. \tag{5.25}$$

This integral is of the same type as the integrals encountered in the functional formulation of dual resonance models; its evaluation is described by Hsue *et al.*[14] in considerable detail. As a result, one finds

$$A(k_1, \ldots, k_N) = \delta^{(4)}(\textstyle\sum k_i)C_N \times \int \prod_{i=1}^{N} d^4x_i\, \Theta(1-x_i^2) \times \exp\Big[-\tfrac{1}{2}\sum_{i,j} k_i k_j\, g(x_i - x_j)\Big], \tag{5.26}$$

where C_N is some constant and the function $g(x)$ satisfies Eq. (5.11), together with the boundary conditions on $x^2 = 1$

$$\partial_n g = \text{const}, \ldots, \quad \partial_n{}^3 g = \text{const},$$

where ∂_n stands for the normal derivative. The solution which satisfies these boundary conditions is given by Eq. (5.12).

After going back to Minkowskian momenta and using (5.12), one obtains finally

$$A(k_1, \ldots, k_N) = \delta^{(4)}(\textstyle\sum k_i)C_N \times \int \prod_{i=1}^{N} d^4x_i\, \Theta(1-|x_i{}^2|) \times \prod_{i\le j} |(x_i - x_j)^2|^{(16\pi^2)^{-1}(k_i\cdot k_j)}. \tag{5.27}$$

This amplitude is evidently of the form studied by Mansouri.[13]

VI. CONCLUDING REMARKS

It was shown in this paper that one can build up a formal theory involving fields which depend on q-number coordinates.

Thus, at least from the theoretical point of view, it appears possible to construct a theory of strong interactions along the lines envisioned in the Introduction.

The role played by the coordinates Y^μ is analogous to the role played by harmonic coordinates in the general theory of relativity. This analogy becomes evident if, in all the previous equations involving the dynamical coordinates, one introduces a representation defined by

$$Y^\mu(x) = x^\mu + \sum_n \left[C_n f_n^\mu(x) + C_n^* f_n^\mu(x)^*\right], \tag{6.1}$$

where the functions f_n^μ form a complete set of solutions of the "wave equation" (2.4); thus $Y^\mu(x)$ itself is a solution; see Sec. II. (The functions f_n^μ have to satisfy the "natural" boundary condition, $f_n^\mu \to 0$ at infinity.) In this representation the set $\{c_n\}$ of complex numbers is the functional argument to be integrated over. Equation (6.1) is the analog of the normal mode expansion used in the string model.[4]

Actually, by the considerations of Sec. II, one is more likely to be led to consider "biharmonic coordinates," as happens in the case of the simple model Lagrangian (5.1).

It would be desirable to have a better guiding principle for the construction of the Lagrangian which governs the coordinate fluctuations; I consider the lack of such a principle the major defect of the theory in its present form. However, it is somewhat reassuring that even in the absence of a "good" Lagrangian, simple models reproduce dual resonance amplitudes in a certain (rather crude) approximation, while the same model assumptions apparently do not lead to disastrous conflicts with quantum electrodynamics in the well-established domain of validity of the latter.

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[10]It should be realized, of course, that the present treatment leaves a gap between the calculation of the Green's function and on-shell S-matrix elements. In a usual theory the on-shell S-matrix elements are factored out of a general Green's function by the following procedure. One takes the coordinates of the sources of initial and final particles into the distant past and future, respectively. In this situation the propagators of the particles before and after scattering are dominated by the stable particle poles; hence the on-shell transition matrix element can be factored out. I do not have a similar proof in the present theory, where the arguments of the Green's functions are q numbers. However, examination of model systems suggests that if the *classical coordinate* of a source is averaged over a space-time cube with sides $\gg L$ ("extended source") then the quantum fluctuations of Y^μ are smeared out. Thus, probably, a factorization procedure similar to the usual theories can be carried out.

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[12]Cf. K. L. Nagy, *State Vector Spaces with Indefinite Metric in Quantum Field Theory* (Nordhoff, Groningen, 1966), Sec. 2.4. The propagator without cutoff corresponds to a massless dipole ghost.

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