Covariant propagators from the Singh-Hagen Lagrangian

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Prescriptions for constructing covariant propagators from the Singh-Hagen Lagrangian are presented. Propagators are derived explicitly for spin $J \leq 5$ (J an integer) according to this prescription. They agree with the propagators proposed by Harnad and Snoke in connection with the O(4) propagator. They do not depend on arbitrary parameters contained in the free Lagrangian. Some comments on daughter fields associated with contact-interaction fields are given.

I. INTRODUCTION

Recently Singh and Hagen[1] proposed the most general quadratic second-order Lagrangian of an arbitrary-spin field as a further extension of the Chang Lagrangians for spin $J \leqslant 4$.[2] By the introduction of a Lagrange multiplier into the Lagrangian, all the nonderivative constraints in the subsidiary conditions[3] imposed on a tensor (or tensor-spinor) field are automatically derived. The nonderivative constraint equations are assumed to hold in advance so that only the differential equations of motion follow from the Euler-Lagrange equations. This procedure is accomplished by introducing auxiliary fields into the Lagrangian.[4]

For simplicity, we restrict ourselves to massive spin-J (J an integer) fields. Moreover, we make the usual choice by selecting the representation $D(J/2, J/2)$ of the proper orthochronous Lorentz group, for which case the field is a symmetric and traceless tensor of rank J. Under the subgroup O(3) of spatial rotations, the representation $D(J/2, J/2)$ is reducible. Therefore, all lower-spin states are contained in this representation. In the free-field limit, these redundant components are eliminated by imposing the Lorentz condition on the field.

The nonderivative constraints are expressed by the form[5]

$$\phi^{(J)} = d^{(J)} \phi^{(J)} \tag{1}$$

where $d^{(J)}$ is an orthogonal covariant projection operator. This projection operator is uniquely determined from the symmetry and tracelessness properties of the nonderivative constraints and is called the O(4) projection operator belonging to the irreducible representation $D(J/2, J/2)$ of O(4).[6] When a nonconserved current couples the field, then the Lorentz condition is no longer imposed on the field. Thus lower-spin components appear. We can regard these lower-spin states as daughter fields. This is done by using the projection operators which decompose Eq. (1) into a spin-K field which corresponds to the field transforming according to the representation $D(K)$ of O(3).[7] This observation strongly suggests that the propagator derived from the Singh-Hagen Lagrangian should be the propagator proposed by Harnad[8] and Snoke[9] in connection with the covariant O(4) propagator.[10,11]

Quantization of the free field is given by Chang[2] and Singh and Hagen[1] using the action principle[12]; canonical commutation rules are derived, the equations of motion are brought to the first-order form (thereby facilitating the introduction of minimal electromagnetic coupling), and the positive definiteness of the energy is proved.

A general prescription for constructing propagators from the Singh-Hagen Lagrangian is presented based on the Schwinger formulation[13] of the Green's function. Following this prescription, we construct propagators explicitly for $J \leqslant 5$.[14] The results are found to be in agreement with those proposed by Harnad and Snoke. Although arbitrary parameters are contained in the free Lagrangian, the propagators do not depend on these parameters.

Prescriptions for constructing covariant propagators from the Lagrangian are presented in Sec. II. Propagators are constructed explicitly for $J \leqslant 5$ in Sec. III. The final section is devoted to discussion.

II. PRESCRIPTIONS FOR CONSTRUCTING THE COVARIANT PROPAGATOR

The most general quadratic second-order Lagrangian proposed by Singh and Hagen is

$$\begin{aligned}\mathcal{L}_0 = {} & \tfrac{1}{2}\phi^{(J)}(\partial^2 - m^2)\phi^{(J)} + \tfrac{1}{2}J(\partial\,\phi^{(J)})^2 \\ & + c^{(J)}\Big\{\phi^{(J-2)}(\partial\partial\phi^{(J)}) - \tfrac{1}{2}\phi^{(J-2)}(\partial^2 - a_2^{(J)}m^2)\phi^{(J-2)} + \tfrac{1}{2}b_2^{(J)}(\partial\phi^{(J-2)})^2 - d_2^{(J)}\phi^{(J-4)}(\partial\partial\phi^{(J-2)}) \\ & - \sum_{q=3}^{J}\Big(\prod_{k=2}^{q-1} c_k^{(J)}\Big)\big[\tfrac{1}{2}\phi^{(J-q)}(\partial^2 - a_q^{(J)}m^2)\phi^{(J-q)} - \tfrac{1}{2}b_q^{(J)}(\partial\phi^{(J-q)})^2 \\ & - m\phi^{(J-q)}(\partial\phi^{(J-q+1)}) + d_q^{(J)}\phi^{(J-q-2)}(\partial\partial\phi^{(J-q)})\big]\Big\} \\ & + \Phi^{(J)}(I^{(J)} - d^{(J)})\phi^{(J)} + \sum_{q=2}^{J}\Phi^{(J-q)}(I^{(J-q)} - d^{(J-q)})\phi^{(J-q)}\ ,\end{aligned} \tag{2}$$

where $c^{(J)}$, $a_q^{(J)}, b_q^{(J)}$, $c_q^{(J)}$, and $d_q^{(J)}$ $(q=2,\ldots,J)$ are real parameters and m denotes the mass of a spin-J particle. Here $\phi^{(J)}$ and $\phi^{(J-q)}$ $(q=2,\ldots,J)$ are real fields, $\phi^{(J-q)}$ $(q=2,\ldots,J)$ is an auxiliary field, and $\Phi^{(J)}$ and $\Phi^{(J-q)}$ $(q=2,\ldots,J)$ are Lagrange multipliers.[15] Moreover, $I^{(P)}$ and $d^{(P)}$ $(P=0,1,2,\ldots,J-2,J)$ are defined by

$$I^{(P)}_{\mu_1\cdots\mu_P;\nu_1\cdots\nu_P} = g_{\mu_1\nu_1}\cdots g_{\mu_P\nu_P}\ , \tag{3}$$

$$d^{(P)}_{\mu_1\cdots\mu_P;\nu_1\cdots\nu_P} = \sum_{r=0}^{[P/2]} \alpha_r^{(P)} \sum c\,(g_{\mu_1\mu_2}g_{\nu_1\nu_2}\cdots g_{\mu_{2r-1}\mu_{2r}}g_{\nu_{2r-1}\nu_{2r}})\prod_{i=2r+1}^{P} g_{\mu_i\nu_i}\ , \tag{4}$$

with

$$\alpha_r^{(P)} = (-)^r (P-r)!\,r!/(P!)^2\ , \tag{5}$$

where $d^{(P)}$ is called the O(4) projection operator.[16,6]

A. Euler-Lagrange equations

The Euler-Lagrange equations follow from Eq. (2). For later convenience, one contracts these equations with $\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}$. The results are found to be

$$(\partial^2 - m^2)\phi^{(J,\lambda)} = J\,T_1^{(J;0,\lambda)} - c^{(J)}T_3^{(J;0,\lambda)}\ , \tag{6, 0}$$

$$\phi^{(J,\lambda)} = (\partial^2 - a_2^{(J)}m^2)\phi^{(J-2,\lambda)} + b_2^{(J)}T_1^{(J;2,\lambda)} + mc_2^{(J)}T_2^{(J;2,\lambda)} + d_2^{(J)}T_3^{(J;2,\lambda)}\ , \tag{6, 2}$$

$$m\phi^{(J-2,\lambda)} = (\partial^2 - a_3^{(J)}m^2)\phi^{(J-3,\lambda)} + b_3^{(J)}T_1^{(J;3,\lambda)} + mc_3^{(J)}T_2^{(J;3,\lambda)} + d_3^{(J)}T_3^{(J;3,\lambda)}\ , \tag{6, 3}$$

and

$$\begin{aligned} c_{q-1}^{(J)}c_{q-2}^{(J)}m\phi^{(J-q+1,\lambda)} - d_{q-2}^{(J)}\phi^{(J-q+2,\lambda)} = {} & c_{q-1}^{(J)}c_{q-2}^{(J)}[(\partial^2 - a_q^{(J)}m^2)\phi^{(J-q,\lambda)} + b_q^{(J)}T_1^{(J;q,\lambda)} \\ & + mc_q^{(J)}T_2^{(J;q,\lambda)} + d_q^{(J)}T_3^{(J;q,\lambda)}] \quad \text{for } q=4,\ldots,J\ ,\end{aligned} \tag{6, q}$$

where $\phi^{(J-q,\lambda)}$ is a symmetric and traceless tensor of rank $J-\lambda$;

$$\phi^{(J-q,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}} = \partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}\phi^{(J-q)}_{\mu_1\cdots\mu_{J-q}}\ . \tag{7}$$

We derive Eqs. (6, q) in detail in the Appendix. Also, $T_i^{(J;q,\lambda)}$ $(i=1,2,3)$ are defined in the Appendix [Eqs. (A1), (A2), and (A3)]. By setting $\lambda=q$ in Eqs. (6, q) Euler-Lagrange equations are obtained.

B. Determination of coefficients in the Lagrangian

Since the coefficients ($c^{(J)}$, $a_q^{(J)}$, $b_q^{(J)}$, $c_q^{(J)}$, and $d_q^{(J)}$) are independent of λ, we can easily determine them by setting $\lambda = J$ in Eqs. $(6, q)$. Then, Eqs. $(6, q)$ reduce to

$$(\partial^2 - m^2)\phi^{(J,J)} = J f_{0,J}^{(J)} \partial^2 \phi^{(J,J)} - c^{(J)} f_{0,J}^{(J)} f_{1,J}^{(J)} \partial^4 \phi^{(J-2,J)} , \tag{8, 0}$$

$$\phi^{(J,J)} = (\partial^2 - a_2^{(J)} m^2)\phi^{(J-2,J)} + b_2^{(J)} f_{2,J}^{(J)} \partial^2 \phi^{(J-2,J)} + m c_2^{(J)} f_{2,J}^{(J)} \partial^2 \phi^{(J-3,J)} + d_2^{(J)} f_{2,J}^{(J)} f_{3,J}^{(J)} \partial^4 \phi^{(J-4,J)} , \tag{8, 2}$$

$$m\phi^{(J-2,J)} = (\partial^2 - a_3^{(J)} m^2)\phi^{(J-3,J)} + b_3^{(J)} f_{3,J}^{(J)} \partial^2 \phi^{(J-3,J)} + m c_3^{(J)} f_{3,J}^{(J)} \partial^2 \phi^{(J-4,J)} + d_3^{(J)} f_{3,J}^{(J)} f_{4,J}^{(J)} \partial^4 \phi^{(J-5,J)} , \tag{8, 3}$$

and

$$c_{q-1}^{(J)} c_{q-2}^{(J)} m \phi^{(J-q+1,J)} - d_{q-2}^{(J)} \phi^{(J-q+2,J)} = c_{q-1}^{(J)} c_{q-2}^{(J)} \big[(\partial^2 - a_q^{(J)} m^2)\phi^{(J-q,J)} + b_q^{(J)} f_{q,J}^{(J)} \partial^2 \phi^{(J-q,J)}$$
$$+ m c_q^{(J)} f_{q,J}^{(J)} \partial^2 \phi^{(J-q-1,J)} + d_q^{(J)} f_{q,J}^{(J)} f_{q+1,J}^{(J)} \partial^4 \phi^{(J-q-2,J)} \big] \quad \text{for } q = 4, \ldots, J. \tag{8, q}$$

Equations $(8, q)$ are exactly the Singh-Hagen expressions, in which the coefficients are determined so as to eliminate the auxiliary fields $\phi^{(J-q)}$ $(q = 2, \ldots, J)$. The results are found to be

$$\begin{aligned} c^{(J)} &= \frac{J(J-1)^2}{2J-1} , \\ a_q^{(J)} &= \frac{q(2J-q+1)(J-q+2)}{2(2J-2q+3)(J-q+1)} , \\ b_q^{(J)} &= \frac{(J-q)^2}{2J-2q+3} , \\ c_q^{(J)} &= \frac{(q-1)(J-q)^2(J-q+2)(2J-q+2)}{2(J-q+1)(2J-2q+1)(2J-2q+3)} , \end{aligned} \tag{9}$$

and

$d_q^{(J)} = 0$ for $q = 2, \ldots, J$.

It is worth noticing that these results are invariant under the transformation

$\phi^{(J-q)} \to \alpha_q \phi^{(J-q)}$ for $q = 1, \ldots, J$, $\alpha_q \neq 0$

so that different choices of the α_q's will yield equivalent Lagrangians. Suitable choices of the α_q's are easily seen to yield the Lagrangians obtained by Chang for $J = 2$, 3, and 4.

C. The covariant propagator

Since a quantization of free fields was performed by Chang[2] and Singh and Hagen,[1] fields are assumed to be quantized from now on. Following the Schwinger formulation of the Green's function,[13] we derive the propagator in this subsection. One adds the free Lagrangian density $\mathcal{L}_0$ to an interaction term $\mathcal{L}_I$ of the form

$$-\mathcal{L}_I = j^{(J)} \phi^{(J)} , \tag{10}$$

where $j^{(J)}$ is an external c-number source obeying no constraints. Then, Eq. (6, 0) is slightly modified as

$$(\partial^2 - m^2)\phi^{(J,\lambda)} - J T_1^{(J;0,\lambda)} + c^{(J)} T_3^{(J;0,\lambda)} = j^{(J,\lambda)} , \tag{6$'$, 0}$$

where $j^{(J,\lambda)}$ is defined by

$$j^{(J,\lambda)}_{\mu_{\lambda+1} \cdots \mu_J} \overset{\cdot}{=} \partial^{\mu_1} \cdots \partial^{\mu_\lambda} d^{(J)}_{\mu_1 \cdots \mu_J ; \nu_1 \cdots \nu_J} j^{(J)\nu_1 \cdots \nu_J} . \tag{11}$$

After a lengthy calculation, the auxiliary fields $\phi^{(J-q,\lambda)}$ are eliminated from Eqs. (6′, 0), (6, 2), (6, 3), and $(6, q)$ and the following equation of motion is obtained by using Eqs. (9) and (A13):

$$
\begin{aligned}
(\partial^2-m^2)\phi^{(J)}_{\mu_1\cdots\mu_J} = {}& j^{(J,0)}_{\mu_1\cdots\mu_J} - \frac{1}{m^2}\sum_C \partial_{\mu_1} j^{(J,1)}_{\mu_2\cdots\mu_J} \\
& - \frac{1}{m^4}\left[\frac{J-1}{J(2J-1)}\,\partial^2 - \frac{1}{J}\,m^2\right]\sum_C g_{\mu_1\mu_2} j^{(J,2)}_{\mu_3\cdots\mu_J} + \frac{1}{m^4}\,\frac{2(J-1)}{2J-1}\sum_C \partial_{\mu_1}\partial_{\mu_2} j^{(J,2)}_{\mu_3\cdots\mu_J} \\
& + \frac{1}{m^6}\left[\frac{J-2}{J(2J-1)}\,\partial^2 - \frac{2(J-1)}{J(2J-1)}\,m^2\right]\sum_C g_{\mu_1\mu_2}\partial_{\mu_3} j^{(J,3)}_{\mu_4\cdots\mu_J} - \frac{1}{m^6}\,\frac{2(J-2)}{2J-1}\sum_C \partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3} j^{(J,3)}_{\mu_4\cdots\mu_J} \\
& + \frac{1}{m^8}\left[\frac{(J-2)(J-3)}{J(J-1)(2J-1)(2J-3)}\,\partial^4 - \frac{2(J-2)}{J(J-1)(2J-1)}\,\partial^2 m^2 + \frac{2}{J(2J-1)}\,m^4\right]\sum_C g_{\mu_1\mu_2} g_{\mu_3\mu_4} j^{(J,4)}_{\mu_5\cdots\mu_J} \\
& - \frac{1}{m^8}\left[\frac{2(J-2)(J-3)}{J(2J-1)(2J-3)}\,\partial^2 - \frac{2(J-2)}{J(2J-1)}\,m^2\right]\sum_C g_{\mu_1\mu_2}\partial_{\mu_3}\partial_{\mu_4} j^{(J,4)}_{\mu_5\cdots\mu_J} \\
& + \frac{1}{m^8}\,\frac{4(J-2)(J-3)}{(2J-1)(2J-3)}\sum_C \partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4} j^{(J,4)}_{\mu_5\cdots\mu_J} \\
& - \frac{1}{m^{10}}\left[\frac{(J-3)(J-4)}{J(J-1)(2J-1)(2J-3)}\,\partial^4 - \frac{4(J-2)(J-3)}{J(J-1)(2J-1)(2J-3)}\,\partial^2 m^2 + \frac{2(J-2)}{J(J-1)(2J-1)}\,m^4\right] \\
& \times \sum_C g_{\mu_1\mu_2} g_{\mu_3\mu_4}\partial_{\mu_5} j^{(J,5)}_{\mu_6\cdots\mu_J} \\
& + \frac{1}{m^{10}}\left[\frac{2(J-3)(J-4)}{J(2J-1)(2J-3)}\,\partial^2 - \frac{4(J-2)(J-3)}{J(2J-1)(2J-3)}\,m^2\right]\sum_C g_{\mu_1\mu_2}\partial_{\mu_3}\partial_{\mu_4}\partial_{\mu_5} j^{(J,5)}_{\mu_6\cdots\mu_J} \\
& - \frac{1}{m^{10}}\,\frac{4(J-3)(J-4)}{(2J-1)(2J-3)}\sum_C \partial_{\mu_1}\partial_{\mu_2}\partial_{\mu_3}\partial_{\mu_4}\partial_{\mu_5} j^{(J,5)}_{\mu_6\cdots\mu_J} + \cdots \,.
\end{aligned} \tag{12}
$$

In general, Eq. (12) can be rewritten in the form

$$(\partial^2-m^2)\phi^{(J)} = \hat{\Theta}^{(J)} j^{(J)} \,. \tag{13}$$

Schwinger's expression for the Green's function[17] is given by

$$G^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(x,y) = \left.\frac{\delta\langle 0|\phi^{(J)}_{\mu_1\cdots\mu_J}(x)|0\rangle}{\delta j^{(J)\nu_1\cdots\nu_J}(y)}\right|_{j^{(J)}=0} . \tag{14}$$

It follows from the form of the right-hand side of Eq. (14) that $G^{(J)}$ can be written in the form

$$G^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(x,y) = \Theta^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(\partial_x)\Delta_+(x-y,m^2) \,, \tag{15}$$

where

$$\Delta_+(x-y,m^2) = \lim_{\epsilon\to 0}\int \frac{d^4p}{(2\pi)^4}\,\frac{\exp[i(x-y)\cdot p]}{p^2+m^2-i\epsilon} \,. \tag{16}$$

Operating on both sides of Eq. (14) with a Klein-Gordon operator and using Eq. (13) along with $(\partial_x{}^2-m^2)\Delta_+(x-y,m^2)=\delta^{(4)}(x-y)$, we find the prescription for determining $\Theta^{(J)}$:

$$\Theta^{(J)} = \hat{\Theta}^{(J)} \,. \tag{17}$$

Once $\hat{\Theta}^{(J)}$ is obtained explicitly, the covariant propagator is easily derived as the Fourier transform of Eq. (15). Thus

$$\mathcal{P}^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J} = \hat{\Theta}^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(ip)/(p^2+m^2-i\epsilon) \,. \tag{18}$$

III. THE EXPLICIT FORM OF PROPAGATORS FOR $J \leq 5$

Information on propagators for $J \leq 5$ is provided in Eq. (12). From Eqs. (12) and (13), $\hat{\Theta}^{(J)}$ is obtained[18]:

$$\hat{\Theta}^{(J)}=d^{(J)}-\frac{1}{m^2}\sum_C\{\partial(\partial d^{(J)})\}$$
$$-\frac{1}{m^4}\left[\frac{J-1}{J(2J-1)}\partial^2-\frac{1}{J}m^2\right]\sum_C\{g(\partial\partial d^{(J)})\}+\frac{1}{m^4}\frac{2(J-1)}{2J-1}\sum_C\{\partial\partial(\partial\partial d^{(J)})\}$$
$$+\frac{1}{m^6}\left[\frac{J-2}{J(2J-1)}\partial^2-\frac{2(J-1)}{J(2J-1)}m^2\right]\sum_C\{g\partial(\partial\partial\partial d^{(J)})\}-\frac{1}{m^6}\frac{2(J-2)}{2J-1}\sum_C\{\partial\partial\partial(\partial\partial\partial d^{(J)})\}$$
$$+\frac{1}{m^8}\left[\frac{(J-2)(J-3)}{J(J-1)(2J-1)(2J-3)}\partial^4-\frac{2(J-2)}{J(J-1)(2J-1)}\partial^2m^2+\frac{2}{J(2J-1)}m^4\right]\sum_C\{gg(\partial\partial\partial\partial d^{(J)})\}$$
$$-\frac{1}{m^8}\left[\frac{2(J-2)(J-3)}{J(2J-1)(2J-3)}\partial^2-\frac{2(J-2)}{J(2J-1)}m^2\right]\sum_C\{g\partial\partial(\partial\partial\partial\partial d^{(J)})\}$$
$$+\frac{1}{m^8}\frac{4(J-2)(J-3)}{(2J-1)(2J-3)}\sum_C\{\partial\partial\partial\partial(\partial\partial\partial\partial d^{(J)})\}$$
$$-\frac{1}{m^{10}}\left[\frac{(J-3)(J-4)}{J(J-1)(2J-1)(2J-3)}\partial^4-\frac{4(J-2)(J-3)}{J(J-1)(2J-1)(2J-3)}\partial^2m^2+\frac{2(J-2)}{J(J-1)(2J-1)}m^4\right]$$
$$\times\sum_C\{gg\partial(\partial\partial\partial\partial\partial d^{(J)})\}$$
$$+\frac{1}{m^{10}}\left[\frac{2(J-3)(J-4)}{J(2J-1)(2J-3)}\partial^2-\frac{4(J-2)(J-3)}{J(2J-1)(2J-3)}m^2\right]\sum_C\{g\partial\partial\partial(\partial\partial\partial\partial\partial d^{(J)})\}$$
$$-\frac{1}{m^{10}}\frac{4(J-3)(J-4)}{(2J-1)(2J-3)}\sum_C\{\partial\partial\partial\partial\partial(\partial\partial\partial\partial\partial d^{(J)})\}\ . \tag{19}$$

A tedious but straightforward calculation shows that $\hat{\Theta}^{(J)}$ results in the following elegant form[19] for each J ($J\leq 5$):

$$\hat{\Theta}^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(\partial)=(d^{(J)}\tilde{\Theta}^{(J)}d^{(J)})_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}\ . \tag{20}$$

Here $\tilde{\Theta}^{(J)}(\partial)$ is the usual O(3) projection operators obtained by Fierz[20] and Fronsdal.[21] The $\tilde{\Theta}^{(J)}(\partial)$ is expressed by

$$\tilde{\Theta}^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}(\partial)=\sum_{r=0}^{[J/2]}a_r^{(J)}\sum_C(\tilde{g}_{\mu_1\mu_2}\tilde{g}_{\nu_1\nu_2}\cdots\tilde{g}_{\mu_{2r-1}\mu_{2r}}\tilde{g}_{\nu_{2r-1}\nu_{2r}})\times\prod_{i=2r+1}^{J}\tilde{g}_{\mu_i\nu_i}\ , \tag{21}$$

with

$$a_r^{(J)}=\frac{(-)^r(2J-2r)!\,r!\,2^{2r}}{(2J)!\,(J-r)!} \tag{22}$$

and

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{m^2}\ . \tag{23}$$

It follows from Eqs. (17), (18), and (20) that the propagators for $J\leq 5$ are

$$\mathcal{P}^{(J)}_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}=\frac{[d^{(J)}\tilde{\Theta}^{(J)}(ip)d^{(J)}]_{\mu_1\cdots\mu_J;\nu_1\cdots\nu_J}}{p^2+m^2-i\epsilon}\ , \tag{24}$$

which are exactly as expected, i.e., the propagators proposed by Harnad and Snoke. These propagators reduce to the conventional propagator on the mass shell ($p^2=-m^2$) and to the O(4) propagator at $p_\mu=0$. Note that Eq. (24) is independent of α_q's which are arbitrary parameters in $\mathcal{L}_0$. Therefore the propagators for $J\leq 4$ are those derived from Chang's Lagrangian.

IV. DISCUSSION

Following our prescriptions, we have constructed propagators for $J\leq 5$ from the Singh-Hagen Lagrangian. The propagators are found to be in agreement with those proposed by Harnad and Snoke. This fact strongly demonstrates that the Harnad-Snoke propagator is derived from the Singh-Hagen Lagrangian by using the Schwinger formulation of the Green's function.

A general structure of the Harnad-Snoke propagator is examined in the framework of the Van Hove model.[7] In particular, slopes of daughter trajectories at $p^2=0$ are given as a closed form. The result depends on a model for the self-energy parts, but it is unlikely that daughter trajectories are parallel to the parent trajectory.[22]

The field $\phi^{(P)}$ ($P=0,1,\ldots,J-2,J$) is decomposed into the following form by using orthogonal projection operators proposed by the authors:

$$\phi^{(P)}=\sum_{L=0}^{[P/2]}\sum_{j=0}^{1}\Phi^{(P,L;P-2L-j)}\ . \tag{25}$$

Here $\Phi^{(P,L;P-2L-j)}$ is defined as

$$\Phi^{(P,L;P-2L-j)}=Q^{(P,L;P-2L-j)}\phi^{(P)}\ , \tag{26}$$

where $Q^{(P,L;P-2L-j)}$ is the projection operator which projects out the spin-$(P-2L-j)$ state corresponding to the representation $D(P-2L-j)$ of O(3) from the representation $D(P/2,P/2)$ of O(4).[23] Equation (25) follows from the fact that

$$d^{(P)}=\sum_{L=0}^{[P/2]}\sum_{j=0}^{1}Q^{(P,L;P-2L-j)}\ . \tag{27}$$

Note that $Q^{(P,L;P-2L-j)}$ contains the orthogonal spin-$(P-2L-j)$ projection operator which is obtained by replacing $\tilde{g}_{\mu\nu}$ in $\tilde{\Theta}^{(P-2L-j)}$ with $\bar{g}_{\mu\nu}=g_{\mu\nu}-\partial_\mu\partial_\nu/\partial^2$.

The field $\Phi^{(J,0;J)}$ belongs to the representation $D(J)$ of O(3) and describes a spin-J state. In the free limit, $j^{(J)}=0$, only the field $\Phi^{(J,0;J)}$ survives. Also, $\Phi^{(J,0;J)}$ satisfies the Klein-Gordon equation and all the subsidiary conditions. Therefore, other fields correspond to "daughter" fields[24] describing lower spin-K ($K=J-2L-j$; $L=0$, $j=1$ and $L=1,\ldots,[J/2]$, $j=0,1$) states.

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APPENDIX

First of all, we define the tensors $T_i^{(J;q,\lambda)}$ $(i=1,\ldots,5)$ of rank $J-\lambda$ as

$$T^{(J;q,\lambda)}_{1;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}d^{(J-q)}_{\mu_1\cdots\mu_{J-q};\alpha_1\cdots\alpha_{J-q}}\partial^{\alpha_1}(\partial_{\beta_1}\phi^{(J-q)\beta_1\alpha_2\cdots\alpha_{J-q}})\ , \tag{A1}$$

$$T^{(J;q,\lambda)}_{2;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}d^{(J-q)}_{\mu_1\cdots\mu_{J-q};\alpha_1\cdots\alpha_{J-q}}\partial^{\alpha_1}\phi^{(J-q-1)\alpha_2\cdots\alpha_{J-q}}\ , \tag{A2}$$

$$T^{(J;q,\lambda)}_{3;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}d^{(J-q)}_{\mu_1\cdots\mu_{J-q};\alpha_1\cdots\alpha_{J-q}}\partial^{\alpha_1}\partial^{\alpha_2}\phi^{(J-q-2)\alpha_3\cdots\alpha_{J-q}}\ , \tag{A3}$$

$$T^{(J;q,\lambda)}_{4;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}d^{(J-q)}_{\mu_1\cdots\mu_{J-q};\alpha_1\cdots\alpha_{J-q}}(\partial_{\beta_1}\phi^{(J-q+1)\beta_1\alpha_1\cdots\alpha_{J-q}})\ , \tag{A4}$$

and

$$T^{(J;q,\lambda)}_{5;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}d^{(J-q)}_{\mu_1\cdots\mu_{J-q};\alpha_1\cdots\alpha_{J-q}}(\partial_{\beta_1}\partial_{\beta_2}\phi^{(J-q+2)\beta_1\beta_2\alpha_1\cdots\alpha_{J-q}})\ , \tag{A5}$$

where $\phi^{(P)}$ is a symmetric and traceless tensor of rank P. By using the formula

$$\begin{aligned}&d^{(P)}_{\mu_1\cdots\mu_P;\alpha_1\cdots\alpha_P}\partial^{\alpha_1}\cdots\partial^{\alpha_i}(\partial_{\beta_1}\cdots\partial_{\beta_j}\phi^{(P-i+j)\beta_1\cdots\beta_j\alpha_{i+1}\cdots\alpha_P})\\&=\sum_{r=0}^{\min(i,[P/2])}\alpha_r^{(P)}\sum_{l=\max(0,i+r-P)}^{\min(r,i-r)}\beta_l^{(P;r,i)}\sum c(g_{\mu_1\mu_2}\cdots g_{\mu_{2r-1}\mu_{2r}})\partial^{2l}\partial_{\mu_{2r+1}}\cdots\partial_{\mu_{i+r-l}}\\&\qquad\times(\partial_{\beta_1}\cdots\partial_{\beta_j}\partial_{\alpha_{2l+1}}\cdots\partial_{\alpha_{r+l}}\phi^{(P-i+j)\beta_1\cdots\beta_j\alpha_{2l+1}\cdots\alpha_{r+l}}{}_{;\mu_{i+r-l+1}\cdots\mu_P})\ ,\end{aligned} \tag{A6}$$

where $\alpha_r^{(J)}$ is given in Eq. (5) and

$$\beta_l^{(P;r,i)}=\frac{(P-i)!\,i!}{2^l l!\,(r-l)!}\ , \tag{A7}$$

the tensors $T_i^{(J;q,\lambda)}$ are calculated in the forms

$$T^{(J;q,\lambda)}_{1;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=f^{(J)}_{q,\lambda}\partial^2\phi^{(J-q,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}}+f^{(J)}_{1;q,\lambda}\sum c\,\partial_{\mu_{\lambda-q+1}}\phi^{(J-q,\lambda+1)}_{\mu_{\lambda-q+2}\cdots\mu_{J-q}}-f^{(J)}_{2;q}\sum c\,g_{\mu_{\lambda-q+1}\mu_{\lambda-q+2}}\phi^{(J-q,\lambda+2)}_{\mu_{\lambda-q+3}\cdots\mu_{J-q}}\ , \tag{A8}$$

$$T^{(J;q,\lambda)}_{2;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=f^{(J)}_{q,\lambda}\partial^2\phi^{(J-q-1,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}}+f^{(J)}_{1;q,\lambda}\sum c\,\partial_{\mu_{\lambda-q+1}}\phi^{(J-q-1,\lambda+1)}_{\mu_{\lambda-q+2}\cdots\mu_{J-q}}-f^{(J)}_{2;q}\sum c\,g_{\mu_{\lambda-q+1}\mu_{\lambda-q+2}}\phi^{(J-q-1,\lambda+2)}_{\mu_{\lambda-q+3}\cdots\mu_{J-q}}\ , \tag{A9}$$

$$\begin{aligned}T^{(J;q,\lambda)}_{3;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=&f^{(J)}_{q,\lambda}f^{(J)}_{q+1,\lambda}\partial^4\phi^{(J-q-2,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}}+g^{(J)}_{1;q,\lambda}\sum c\,\partial_{\mu_{\lambda-q+1}}\partial^2\phi^{(J-q-2,\lambda+1)}_{\mu_{\lambda-q+2}\cdots\mu_{J-q}}\\&+g^{(J)}_{2;q,\lambda}\sum c\,\partial_{\mu_{\lambda-q+1}}\partial_{\mu_{\lambda-q+2}}\phi^{(J-q-2,\lambda+2)}_{\mu_{\lambda-q+3}\cdots\mu_{J-q}}-g'^{(J)}_{2;q,\lambda}\sum c\,g_{\mu_{\lambda-q+1}\mu_{\lambda-q+2}}\partial^2\phi^{(J-q-2,\lambda+2)}_{\mu_{\lambda-q+3}\cdots\mu_{J-q}}\\&-g^{(J)}_{3;q,\lambda}\sum c\,g_{\mu_{\lambda-q+1}\mu_{\lambda-q+2}}\partial_{\mu_{\lambda-q+3}}\phi^{(J-q-2,\lambda+3)}_{\mu_{\lambda-q+4}\cdots\mu_{J-q}}\\&+g^{(J)}_{4;q}\sum c\,g_{\mu_{\lambda-q+1}\mu_{\lambda-q+2}}g_{\mu_{\lambda-q+3}\mu_{\lambda-q+4}}\phi^{(J-q-2,\lambda+4)}_{\mu_{\lambda-q+5}\cdots\mu_{J-q}}\ ,\end{aligned} \tag{A10}$$

$$T^{(J;q,\lambda)}_{4;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\phi^{(J-q+1,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}}\,, \tag{A11}$$

and

$$T^{(J;q,\lambda)}_{5;\mu_{\lambda-q+1}\cdots\mu_{J-q}}=\phi^{(J-q+2,\lambda)}_{\mu_{\lambda-q+1}\cdots\mu_{J-q}}\,, \tag{A12}$$

where $\phi^{(J-q,\lambda)}$ is given by Eq. (7) and

$$\begin{aligned}
&f^{(J)}_{q,\lambda}=\frac{\lambda-q}{2(J-q)^2}(2J-q-\lambda+1),\quad f^{(J)}_{1;q,\lambda}=\frac{J-\lambda}{(J-q)^2}\,,\quad f^{(J)}_{2;q}=\frac{1}{(J-q)^2}\,,\\
&g^{(J)}_{1;q,\lambda}=\frac{(\lambda-q)(J-\lambda)(2J-q-\lambda)}{(J-q)^2(J-q-1)^2}\,,\quad g^{(J)}_{2;q,\lambda}=\frac{2(J-\lambda)(J-\lambda-1)}{(J-q)^2(J-q-1)^2}\,,\\
&g'^{(J)}_{2;q,\lambda}=\frac{(\lambda-q)(2J-q-\lambda-1)+(J-q-1)}{(J-q)^2(J-q-1)^2}\,,\quad g^{(J)}_{3;q,\lambda}=\frac{2(J-\lambda-1)}{(J-q)^2(J-q-1)^2}\,,
\end{aligned} \tag{A13}$$

and

$$g^{(J)}_{4;q}=\frac{2}{(J-q)^2(J-q-1)^2}\,,$$

with restrictions $q\leq\lambda\leq J$ for $q=0, 2, 3, \ldots, J-1, J$. In understanding the results, it should be noted that tensor indices are arranged in order of subindex size. Under this convention, an arrangement of tensor indices like $\phi_{\mu_q\cdots\mu_p}$ for $p<q$ is forbidden except for the case $q=p+1$, in which we regard this tensor as a scalar. Thus tensors $\phi_{\mu_q\cdots\mu_p}$ vanish for $p<q$.

Euler-Lagrange equations follow from the Singh-Hagen Lagrangian. Moreover, these equations are contracted with $\partial^{\mu_1}\cdots\partial^{\mu_{\lambda-q}}$, then the results turn out to be Eqs. (6, q).

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[1]L. P. S. Singh and C. R. Hagen, Phys. Rev. D 9, 898 (1974); 9, 910 (1974).

[2]S.-J. Chang, Phys. Rev. 161, 1308 (1967).

[3]See, e.g., H. Umezawa, *Quantum Field Theory* (North-Holland, Amsterdam, 1956); Y. Takahashi, *An Introduction to Field Quantization* (Pergamon, New York, 1969).

[4]In the Chang formulation (Ref. 2), auxiliary fields are introduced in order to remove the nonlocality appearing in the field equations [the inverse D'Alembertian operator $(\partial^2)^{-1}$ is contained].

[5]Throughout this work we have omitted tensor indices where they are obvious.

[6]Y. Iwasaki, Phys. Rev. 173, 1608 (1968).

[7]M. Kobayashi and Y. Mori, Prog. Theor. Phys. 52, 275 (1974).

[8]J. P. Harnad, Nucl. Phys. B38, 350 (1972).

[9]J. A. Snoke, Phys. Rev. D 5, 1052 (1972).

[10]This propagator is a further extension of the propagator of massive spin-two particles proposed by Iwasaki (Ref. 11) to that of arbitrary massive spin-J particles.

[11]Y. Iwasaki, Prog. Theor. Phys. 44, 1376 (1970).

[12]J. Schwinger, Phys. Rev. 91, 713 (1953).

[13]J. Schwinger, Proc. Natl. Acad. Sci. U.S.A. 37, 452 (1951).

[14]Unfortunately, it is difficult to derive the propagator as a closed form in our prescription. Since Chang only proposed Lagrangians for $J\leq 4$, construction of the propagator for $J=5$ provides a check for the equivalence of the Chang and Singh-Hagen Lagrangians.

[15]Here $\phi^{(P)}$ carries tensor indices $\mu_1\cdots\mu_P$ and $(\partial\phi^{(P)})$ carries indices $\mu_2\cdots\mu_P$ and stands for $\partial^{\mu_1}\phi^{(P)}_{\mu_1\mu_2\cdots\mu_P}$ and so on. We use natural units, and $g_{\mu\nu}=\mathrm{diag}\,(-1,1,1,1)$. Greek tensor indices run from 0 to 3.

[16]In Eq. (4), Σ_C denotes the sum over all distinct combinations of indices $(\mu_1\cdots\mu_P)$ and $(\nu_1\cdots\nu_P)$.

[17]This definition gives definite predictions of contact interactions. It is a merit of using Schwinger's formulation.

[18]$\{\partial(\partial d^{(J)})\}$ and $\{g\partial(\partial\partial\partial d^{(J)})\}$ stand for

$$\partial_{\mu_1}(\partial^{\alpha_1}d^{(J)}_{\alpha_1\mu_2\cdots\mu_J;\,\nu_1\cdots\nu_J}),$$

and

$$g_{\mu_1\mu_2}\partial_{\mu_3}(\partial^{\alpha_1}\partial^{\alpha_2}\partial^{\alpha_3}d^{(J)}_{\alpha_1\alpha_2\alpha_3\mu_4\cdots\mu_J;\nu_1\cdots\nu_J})$$

and others are defined similarly.

[19]M. Kobayashi and Y. Mori, Gifu University Report No. DPGU-05-74 (unpublished).

[20]M. Fierz, Helv. Phys. Acta 23, 412 (1950).

[21]C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).

[22]The slope of the first daughter trajectory has opposite sign relative to that of the parent trajectory.

[23]See Appendix in Ref. 7.

[24]More precisely, daughter fields are defined only at $p_\mu=0$.