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Comments on gauge invariance in Compton scattering

D. L. Weaver
Department of Physics, Tufts University, Medford, Massachusetts 02155
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The scattering of neutral vector mesons from a pseudoscalar-meson target is considered. It is shown that using the SL(2, C) description for the vector particles, one obtains a clear understanding of gauge invariance in the limit of vanishing vector-meson mass, i.e., there are no gauge-invariance conditions on the invariant amplitudes. Some related properties of the description are also given.

The formulation and enforcement of gauge-invariance requirements is usually considered an important part of the theoretical discussion of photonic reactions.[1] In fact, if one uses the appropriate description for the vector particles, the gauge-invariance conditions on the invariant amplitudes are trivial; there are none. The purpose of this note is to demonstrate the above statement. Following Ref. 1, the reaction studied is the scattering of neutral vector mesons from a pseudoscalar-meson target, because the ideas can be made clear without the complication of a target with spin. Nevertheless, the validity of the points to be made is more general than the example chosen. The description of particles with spin which is most useful for such a discussion is the SL$(2, C)$ or symmetric spinor[2] description. Such a description allows construction of the most general scattering amplitude in a simple way, shows the dependence of the invariants on the mass of the vector meson explicitly, makes it clear that no relations are required among the invariant amplitudes in the limit that the neutral-vector-meson mass goes to zero (becomes the photon), and shows that gauge invariance imposes no additional constraints on a correctly formed scattering amplitude. The SL$(2, C)$ description also allows a simple generalization from photons to any massless-particle "Compton scattering" as indicated below.

The first step is to construct the most general Lorentz-invariant $\mathcal{R}$ operator, defined in terms of the scattering operator S by

$$S = 1 + i\int d^4x\, \mathcal{R}(x)$$

for the reaction

$$V_1(K_1) + \pi(Q_1) \to V_2(K_2) + \pi(Q_2)$$

and all the reactions related by crossing and time reversal. The $\mathcal{R}$ operator in general contains nine terms (plus charge conjugates) for $V_2 \neq V_1$. This number is reduced to five by space-inversion invariance, and finally to four if $V_2 = V_1$. The result for the latter case may be written

$$\mathcal{R}(x) = \sum_{i=1}^{4} A_i M_i\, \phi\phi^\dagger ,$$

where[3]

$$M_1 = \{\chi_1\varphi_2^\dagger + \varphi_1\chi_2^\dagger\} ,$$
$$M_2 = \{\chi_1 QQ\chi_2^\dagger + \varphi_1 QQ\varphi_2^\dagger\} ,$$

and M_3 and M_4 are the same as M_2, but with $Q \equiv \frac{1}{2} \times (Q_1 + Q_2)$ replaced by K_1K_2 and $(K_1 + K_2)Q$, respectively. The matrix elements of $\mathcal{R}$ integrated over all space-time are proportional to the usual T-matrix elements, and one finds, for example, the crossing properties of the A_i by the standard techniques, i.e., if the vector meson is self-conjugate, then comparison of the matrix elements

$$\int d^4x \langle \pi(Q_2)V(K_2) | \mathcal{R} | \pi(Q_1)V(K_1)\rangle$$

and

$$\int d^4x \langle \pi(Q_2)V(-K_1) | \mathcal{R} | \pi(Q_1)V(-K_2)\rangle$$

yields the crossing properties.

The next step is to study the $\mathcal{R}$ operator as a function of the vector-meson mass m, and to find the limit as m goes to zero. There are two points to make. The first is that the spinor operators themselves, χ and φ, go smoothly to the massless limit appropriate for the photon, i.e., the helicity-zero parts of the spinors vanish. The second point is that the derivatives of the vector-meson field operators are proportional to m and so go smoothly to zero in the massless limit, i.e., $\lim_{m\to 0} K_1\chi_1 = 0$, etc. So, as the vector-meson mass goes to zero, M_3 vanishes as m^2 and M_4 vanishes as m. Note that there are no relations among the A_i in this limit, in contrast to the usual treatment.[1] Note also that gauge invariance is an automatic consequence of the SL$(2, C)$ description. In the $m \to 0$ limit the $\mathcal{R}$ operator thus contains only the M_1 and M_2 terms and describes Compton scattering. To convert to tensor notation for Compton scattering[4] one uses the relations[5]

$$\chi \sim i\sigma_\mu^\dagger \sigma_\nu \sigma_2 F_{\mu\nu} ,$$

$$\varphi \sim -i\sigma_\mu \sigma_\nu^\dagger \sigma_2 F_{\mu\nu} ,$$

where $F_{\mu\nu}$ is the antisymmetric electromagnetic field tensor operator. In fact, χ and φ are projections of $F_{\mu\nu}$ that only annihilate a particular helicity component. Thus, both kinds of spinor are required for parity-conserving reactions.

It is easy to generalize the $\mathcal{R}$ operator for Compton scattering to the elastic scattering of spin-s massless particles from the same target. The result is

$$\mathcal{R}(x) = A_1\{\chi_1\varphi_2^\dagger + \varphi_1\chi_2^\dagger\}\phi\phi^\dagger$$
$$+A_2\{\chi_1 Q\cdots Q\chi_2^\dagger + \varphi_1 Q\cdots Q\varphi_2^\dagger\}\phi\phi^\dagger ,$$

where Q appears $2s$ times in the A_2 term, and χ, φ are now symmetric spinors with $2s$ indices.

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[1]See, for example, H. J. W. Muller and N. Vahedi-Faridi, Phys. Rev. D 7, 1254 (1973) and references contained therein.

[2]See, for example, D. L. Weaver and D. M. Fradkin, Nuovo Cimento 37, 440 (1965).

[3]In the usual spinor notation (see Ref. 2) a vector meson is described by the symmetric spinor pair $\chi_{\dot\alpha\dot\beta}, \varphi^{\gamma\delta}$ coupled by a second-order wave equation. The Greek indices run from 1 to 2 and Lorentz scalars are made by summing over an upper and a lower spinor index of the same type (dotted or undotted). The symmetry of the spinors makes the number of independent components three, appropriate for a spin-one particle. The four-momentum operator $-i\partial/\partial x_\mu$ (in units $\hbar = c = 1$) is written as K_1 when it operates on V_1, as K_2 when it operates on V_2, etc., so that the order of derivative factors may be disregarded. In writing invariants the notation followed is, for example, $\chi_1 QQ\chi_2^\dagger \equiv \chi^{(1)}_{\dot\alpha_1\dot\alpha_2} Q^{\gamma_1\dot\alpha_1} Q^{\gamma_2\dot\alpha_2} \chi^{(2)}_{\gamma_1\gamma_2}$. The { } brackets indicate symmetrization in the usual way to avoid infinities, and † indicates Hermitian conjugation of field operators, i.e., $(\chi_{\dot\alpha\dot\beta})^\dagger = \chi_{\alpha\beta}$, and the complex conjugate, transpose of matrices.

[4]See, for example, W. A. Bardeen and W. Tung, Phys. Rev. 173, 1423 (1968).

[5]Here the Greek indices μ, ν, run from 1 to 4, $\vec\sigma$ are the usual Pauli matrices, and σ_4 is i times the 2×2 identity.