Crossing in $2 \longrightarrow 3$ partial-wave amplitudes

William H. Klink*
Institute for Nuclear Research, Warsaw, Poland
and Research Institute for Theoretical Physics, Helsinki, Finland

(Received 8 October 1974)

Crossing in $2 \longrightarrow 3$ reactions is investigated, and it is shown that by using canonical variables we can express crossing in a particularly simple form.

The most general five-particle amplitude involving spinless arbitrary-mass particles requires five relativistically invariant variables plus a "sign" invariant. That is, if the two incoming particles are labeled $1'$ and $2'$, and the three outgoing particles 1, 2, and 3, then the reaction $1'+2'\to 1+2+3$ is conventionally described by an amplitude

$$\overline{F}'^{2\to 3}(s_{1'2'},t_{1'3},t_{2'3},\mathrm{sgn}[\epsilon_{\mu\nu\alpha\beta}p_{1'}^{\mu}p_{2'}^{\nu}p_1^{\alpha}p_2^{\beta}],s_{12},s_{13}) \propto \langle 123|T|1'2'\rangle, \tag{1}$$

where $s_{ij}=(p_i+p_j)^2$, $t_{ij}=(p_i-p_j)^2$, and "sgn [ ]" denotes the sign of the quantity in brackets. If parity invariance is assumed, it is possible to eliminate the dependence of $\overline{F}'^{2\to 3}$ on the "sgn" variable. Assuming parity invariance and using center-of-mass variables, one can write

$$\begin{aligned}\overline{F}^{2\to 3}(s_{1'2'},\hat{n},s_{12},s_{13}) &= \overline{F}'^{2\to 3}(s_{1'2'},t_{1'3},t_{2'3},s_{12},s_{13})\\ &= \sum_{J'M} Y_{J'M}(\hat{n})\overline{\mathfrak{a}}^{2\to 3}(s_{1'2'},J',M,s_{12},s_{13}).\end{aligned} \tag{2}$$

The expansion coefficient $\overline{\mathfrak{a}}^{2\to 3}$ is the partial-wave amplitude of the $1'+2'\to 1+2+3$ reaction, with J' the total angular momentum and M the spin projection chosen along the direction of particle 3. $\hat{n}$ is the unit vector relating the direction of particle $1'$ relative to the plane formed by particles 1, 2, and 3 in their c.m. frame; it is defined through the polar angle θ and azimuthal angle φ:

$$\begin{aligned}\cos\theta &\equiv \cos\theta_{1'3(1'2')}, \quad 0\le\theta\le\pi\\ &= \hat{p}_{1'}\cdot\hat{p}_3\\ &= \frac{(s_{1'2'}+M_{1'}^2-M_{2'}^2)(s_{1'2'}+M_3^2-s_{12})+2s_{1'2'}(t_{1'3}-M_{1'}^2-M_3^2)}{\lambda^{1/2}(s_{1'2'},M_{1'}^2,M_{2'}^2)\lambda^{1/2}(s_{1'2'},s_{12},M_3^2)},\\ \cos\varphi &= \frac{(\vec{p}_3\times\vec{p}_1)}{|\vec{p}_3\times\vec{p}_1|}\cdot\frac{(\vec{p}_3\times\vec{p}_{1'})}{|\vec{p}_3\times\vec{p}_{1'}|}, \quad 0\le\varphi\le 2\pi.\end{aligned} \tag{3}$$

The notation $\theta_{ij(kl)}$ means the angle between particles i and j in the frame where $\vec{p}_k+\vec{p}_l=\vec{0}$. If the parentheses (kl) are dropped, it is understood that the angle is evaluated in the c.m. frame. In order to define φ uniquely—rather than just $\cos\varphi$—it is necessary to specify the sign of the invariant

$$\begin{aligned}\epsilon_{\mu\nu\alpha\beta}p_{1'}^{\mu}p_{2'}^{\nu}p_1^{\alpha}p_2^{\beta} &= \epsilon_{\mu\nu\alpha\beta}p_{1'}^{\mu}p^{\nu}p_1^{\alpha}p_2^{\beta}\\ &= -(s_{1'2'})^{1/2}\epsilon_{ijk}p_{1'}^{i}p_1^{j}p_2^{k}\\ &= (s_{1'2'})^{1/2}\vec{p}_{1'}\cdot\vec{p}_2\times\vec{p}_1.\end{aligned} \tag{4}$$

The function

$$\lambda(x,y,z)=x^2+y^2+z^2-2(xy+yz+zx)$$

is the usual triangle function.[1]

Now to implement crosssing from the $1'+2'\to 1+2+3$ channel to the $1'+2'+\overline{3}\to 1+2$ channel in the most convenient manner, we choose another variable in place of the s_{13} subenergy, namely $\cos\theta_{13(12)}$, that is, the angle between particles 1 and 3 in the frame where particles 1 and 2 have equal and opposite momenta—the so-called helicity angle. This angle is readily expressed in terms of relativistic invariants to be

$$\cos\theta_{13(12)}=\frac{(s_{12}+M_1^2-M_2^2)(s_{1'2'}-s_{12}-M_3^2)-2s_{12}(s_{13}-M_1^2-M_3^2)}{\lambda^{1/2}(s_{12},M_1^2,M_2^2)\lambda^{1/2}(s_{1'2'},s_{12},M_3^2)}, \tag{5}$$

and the amplitude in this new variable becomes

$$F^{2\to 3}(s_{1'2'}, \hat{n}, s_{12}, \cos\theta_{13(12)}) = \overline{F}^{2\to 3}(s_{1'2'}, \hat{n}, s_{12}, s_{13})$$
$$= \sum_{J'M} Y_{J'M}(\hat{n})\mathcal{A}^{2\to 3}(s_{1'2'}, J', M, s_{12}, \cos\theta_{13(12)})$$
$$= \sum_{J'M} P_{J'M}(\cos\theta_{1'3(1'2')})e^{-iM\varphi}\mathcal{A}^{2\to 3}(s_{1'2'}, J', M, s_{12}, \cos\theta_{13(12)})\,. \tag{6}$$

It is to be noted that the azimuthal angle φ enters in Eq. (6) only as an exponential $(e^{i\varphi})^{-M}$. In trying to find simple crossing properties for the $2\to 3$ partial-wave amplitude, we find it convenient to write out $e^{i\varphi}$—rather than φ—as a relativistic invariant:

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$
$$= \frac{(\vec{p}_3\times\vec{p}_1)\cdot(\vec{p}_3\times\vec{p}_{1'})}{|\vec{p}_3\times\vec{p}_1||\vec{p}_3\times\vec{p}_{1'}|} + i\,\frac{(\vec{p}_3\times\vec{p}_1)\cdot\vec{p}_{1'}}{|\vec{p}_3\times\vec{p}_1||\vec{p}_{1'}|\sin\theta}$$
$$= \frac{1}{|\vec{p}_3\times\vec{p}_1||\vec{p}_3\times\vec{p}_{1'}|}[(\vec{p}_3\times\vec{p}_1)\cdot(\vec{p}_3\times\vec{p}_{1'}) + i\,|\vec{p}_3|(\vec{p}_3\times\vec{p}_1)\cdot\vec{p}_{1'}]$$
$$= \frac{\epsilon_{\rho\delta\alpha\beta}p_2^\delta p_3^\alpha p_1^\beta \epsilon^\rho{}_{\gamma\mu\nu}p_{2'}^\gamma p_3^\mu p_{1'}^\nu + \tfrac{1}{2}i\lambda^{1/2}(s_{1'2'}, s_{12}, M_3{}^2)\epsilon_{\mu\nu\alpha\beta}p_{2'}^\mu p_{1'}^\nu p_3^\alpha p_1^\beta}{[(\epsilon_{\mu\nu\alpha\beta}p_2^\nu p_3^\alpha p_1^\beta)^2]^{1/2}[(\epsilon_{\mu\nu\alpha\beta}p_{2'}^\nu p_3^\alpha p_{1'}^\beta)^2]^{1/2}}\,. \tag{7}$$

This relativistically invariant expression for $e^{i\varphi}$ is obtained by writing all the cross products as invariants. For example,

$$(\vec{p}_3\times\vec{p}_1)\cdot\vec{p}_{1'} = \epsilon_{ijk}p_{1'}^i p_3^j p_1^k$$
$$= \frac{1}{(s_{1'2'})^{1/2}}\epsilon_{\mu\nu\alpha\beta}(p_{1'}+p_{2'})^\mu p_{1'}^\nu p_3^\alpha p_1^\beta$$
$$= \frac{1}{(s_{1'2'})^{1/2}}\epsilon_{\mu\nu\alpha\beta}p_{2'}^\mu p_{1'}^\nu p_3^\alpha p_1^\beta\,. \tag{8}$$

$[p_\mu]^2$ is shorthand notation for $p_\mu p^\mu$. The striking thing about the relativistically invariant expression for $e^{i\varphi}$ is that, under the interchange $p_3\to -p_{\overline{3}}$, $e^{i\varphi}$ remains unchanged. The denominator is symmetric with respect to the incoming particles $1'$, $2'$ and outgoing particles 1, 2. In the numerator the same symmetry holds, for in the triangle function, $s_{1'2'}$ and s_{12} interchange their roles as total energy and subenergy. Thus, if a partial-wave analysis is made of the reaction $1'+2'+\overline{3}\to 1+2$, $e^{i\varphi}$ will be unchanged and gives the azimuthal angle between the planes formed by $\hat{p}_{\overline{3}}-\hat{p}_{1'}$ and $\hat{p}_{\overline{3}}-\hat{p}_1$ in the $1', 2'$ or 1, 2 c.m. frame.

Further, the variables needed for this $3\to 2$ amplitude are s_{12} ~total energy, $t_{1\overline{3}}$ ~scattering angle in the 1, 2 c.m. frame and two subenergies, chosen to be $s_{1'2'}$ and $\cos\theta_{1\overline{3}(1'2')}$. Thus,

$$F^{3\to 2}(s_{12}, \cos\theta_{1\overline{3}(12)}, \varphi, s_{1'2'}, \cos\theta_{1'\overline{3}(1'2')}) = \overline{F}'^{3\to 2}(s_{12}, t_{1\overline{3}}, t_{2\overline{3}}, s_{1'2'}, s_{1'\overline{3}})\,. \tag{9}$$

But if the expression for $\cos\theta_{1'3(1'2')}$ in Eq. (3) is examined and compared with the form of the helicity angle, Eq. (5), it is seen that under crossing, because $t_{1'3}\to s_{1'\overline{3}}$ and $s_{13}\to t_{1\overline{3}}$,

$$\cos\theta_{1'3(1'2')}\to -\cos\theta_{1'\overline{3}(1'2')}$$

and

$$\cos\theta_{13(12)}\to -\cos\theta_{1\overline{3}(12)}\,;$$

that is, the roles of scattering angle and helicity angle are interchanged in the two channels. Therefore,

$$F^{3\to 2}(s_{12}, \cos\theta_{1\overline{3}(12)}, \varphi, s_{1'2'}, \cos\theta_{1'\overline{3}(1'2')}) = F_{\text{cont}}^{2\to 3}(s_{1'2'}, -\cos\theta_{1'3(1'2')}, \varphi, s_{12}, -\cos\theta_{13(12)})\,, \tag{10}$$

where $F_{\text{cont}}(\;)$ means the $2\to 3$ amplitude analytically continued in the variables $s_{1'2'}$ and s_{12} into the physical domain of the $3\to 2$ reaction.[2]

If we examine the partial-wave amplitudes associated with these reactions, it is seen that J', the total angular momentum, is associated with the scattering angle $\theta_{1'3(1'2')}$. Since the helicity angle becomes the scattering angle in the crossed channel, this suggests making an expansion in the helicity angle also using Legendre functions; as shown in Ref. 3, P_{JM} (helicity angle) is the transform of the helicity angle to a variable J which can be thought of as the spin of the 12 system. We thus have partial-wave amplitudes $\mathcal{A}^{2\to 3}(s'J'MsJ)$ and $\mathcal{A}^{3\to 2}(sJMs'J')$, where (sJ) is the energy and angular momentum of the 1, 2 system while $(s'J')$ is the energy and angular momentum of the $1', 2'$ system; M is the common spin projection. The crossing relations of Eq. (10) look particularly simple in these variables:

$$\mathcal{A}^{3\to 2}(sJ, M, s'J') = (-1)^{J+J'}\mathcal{A}_{\text{cont}}^{2\to 3}(s'J', M, sJ)\,. \tag{11}$$

When coupled with time-reversal invariance, which

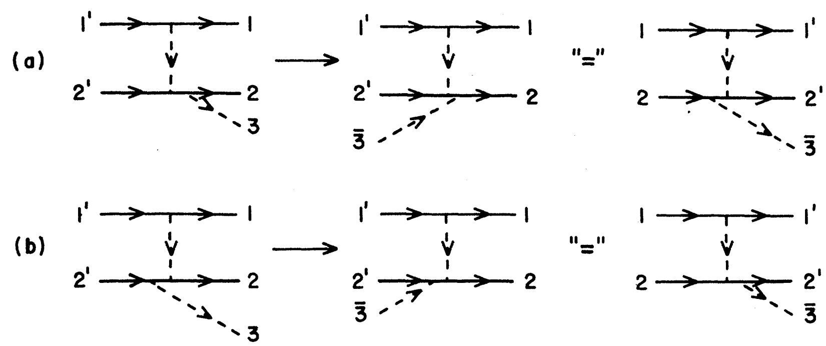

FIG. 1. Simple production Feynman diagrams.

also relates 2 → 3 and 3 → 2 partial-wave amplitudes, a general relationship involving the interchange of (sJ) and $(s'J')$ in 2 → 3 partial-wave amplitudes follows. It should be emphasized that the crossing result, Eq. (11), holds for any of the final particles; after one has decided which particle is to be crossed the choice of appropriate variables is fixed. To illustrate how these variables may be used, we consider the simplest production Feynman diagrams, as seen in Fig. 1. For simplicity, we assume that the mass of particles $1'$, $2'$, 1, and 2 is M, while the mass of particle 3 is m. The arrows in Fig. 1 mean "crossing," while "=" means the time-reversed diagram, the amplitude of which is the same as the crossed diagram. The amplitude for the direct channel diagrams of Fig. 1(a) and Fig. 1(b) is given by

$$
\begin{aligned}
F^{2\to3}_{(a)}+F^{2\to3}_{(b)} &= \frac{1}{t_{11'}-m^2}\left(\frac{1}{s_{23}-M^2}+\frac{1}{t_{2'3}-M^2}\right)\\
&=[A+B(s',s)\cos\theta_{1'3(1'2')}-B(s,s')\cos\theta_{13(12)}\\
&\quad +C\cos\theta_{1'3(1'2')}\cos\theta_{13(12)}+D\sin\theta_{1'3(1'2')}\sin\theta_{13(12)}\cos\varphi]^{-1}\\
&\quad\times\left[\frac{1}{E(s',s)+F(s,s')\cos\theta_{13(12)}}+\frac{1}{E(s,s')-F(s',s)\cos\theta_{1'3(1'2')}}\right],
\end{aligned}
\tag{12}
$$

where $A, B, \ldots, F$ are functions of s', s, and masses given by

$$
\begin{aligned}
A &= M^2-\tfrac{1}{4}[4s's+\lambda(s',s,m^2)]^{1/2},\\
B(s',s) &= \frac{1}{4s'}\lambda^{1/2}(s',M^2,M^2)\lambda^{1/2}(s',s,m^2),\\
C &= \frac{1}{4s's}\lambda^{1/2}(s',M^2,M^2)\lambda^{1/2}(s,M^2,M^2)\\
&\quad\times[4s's+\lambda(s',s,m^2)]^{1/2},\\
D &= \frac{1}{2s's}\lambda^{1/2}(s',M^2,M^2)\lambda^{1/2}(s,M^2,M^2),\\
E(s',s) &= \tfrac{1}{2}(s'-s+m^2),\\
F(s',s) &= \frac{1}{2s'}\lambda^{1/2}(s',s,m^2)\lambda^{1/2}(s',M^2,M^2),
\end{aligned}
\tag{13}
$$

and $\sqrt{s'}\geq\sqrt{s}+m$. Notice that the functions A, C, and D have arguments symmetric in s' and s, so these arguments are not explicitly indicated.

The amplitude describing the reaction resulting from crossing particle 3 is now obtained by letting $\cos\theta_{1'3(1'2')}\to-\cos\theta_{1'\bar{3}(1'2')}$, $\cos\theta_{13(12)}\to-\cos\theta_{1\bar{3}(12)}$, $\varphi\to\varphi$, and analytically continuing the functions $A, B, \ldots, F$ to the region where $\sqrt{s}\geq\sqrt{s'}+m$. When this is done the amplitude $F^{3\to2}_{(a)+(b)}$ results, which, by time reversal, is $F^{2\to3}_{(b)+(a)}$, the same as the original amplitude in Eq. (12). That is, all the functions $A, B, \ldots, F$ either remain unchanged under crossing, or the energies s' and s are interchanged. The minus signs of the polar angles result in the opposite time-reversed diagram of Fig. 1. Such a result is quite general, namely, it is always possible to find pairs of multiparticle Feynman diagrams with the property that when one particle is crossed, the form of the Feynman diagram in canonical variables is unchanged. In a subsequent paper it will be shown that such a feature holds for all multiparticle amplitudes, independent of a Feynman diagram setting, and leads one to consider multiparticle amplitudes as self-reciprocal functions.[4]

To conclude, we discuss whether our crossing result is also compatible with exact three-body unitarity (below the four-body threshold) in two simple examples. In Ref. 3 it was shown that the simplest three-body unitarity equations involve a $3\to3$ partial-wave amplitude which can be viewed as an operator on a Hilbert space defined by

$$f\in\mathcal{H}:\|f\|^2=\sum_M\int ds_q\,|f(M,s_q)|^2<\infty,\ q=1,2.$$

The $2\to3$ and $3\to2$ partial-wave amplitudes are vectors in this Hilbert space obtained from the unitarity equations; by changing variables, from $s_q=(s_{12},s_{13})\to(s_{12},\cos\theta_{13(12)})\to(s,J)$, the Hilbert space becomes

$$f\in\mathcal{H}:\|f\|^2=\sum_{M,J}\int\mathcal{J}(s',s)ds\,|f(M,s,J)|^2<\infty,\tag{14}$$

where $\mathcal{J}(s,s')$ is the Jacobian of the transformation (5), in which the Dalitz boundary becomes a rectangle. Since the $3\to3$ partial-wave amplitude completely determines the three-body unitarity equations and since not much is known about such partial-wave amplitudes we give two examples.

First consider the case when the $3\to3$ partial-wave amplitude is normal; then taking the difference of Eqs. (2.5e) and (2.4f) of Ref. 3, we see that $\mathcal{A}^{3\to2*}(s',J',M,s,J)=\mathcal{A}^{2\to3}(s',J',M,s,J)$. Collecting the equations expressing unitarity (when the $3\to3$ partial-wave amplitude is normal), crossing [Eq. (11)], and time-reversal invariance then gives

$$\mathcal{A}^{3\to2*}(s',J',M,s,J)=\mathcal{A}^{2\to3}(s',J',M,s,J)\ \text{(unitarity)},$$
$$(-1)^{J'}\mathcal{A}^{3\to2}(s',J',M,s,J)=(-1)^{J}\mathcal{A}^{2\to3}_{\text{cont}}(s,J,M,s',J')\ \text{(crossing)},\tag{15}$$
$$\mathcal{A}^{3\to2}(s',J',M,s,J)=\mathcal{A}^{2\to3}(s',J',M,s,J)\ \text{(time reversal)}.$$

It can be seen that these equations imply that $\mathcal{A}^{2\to3}$ and $\mathcal{A}^{3\to2}$ are real and that $(-1)^J\mathcal{A}^{2\to3}(s',J',M,s,J)$ is symmetric under the interchange of variables $s,J\leftrightarrow s',J'$. As an example of how these constraints work, set $M=0$ and consider a simple resonance amplitude of the form

$$\mathcal{A}^{2\to3}(s',J',s,J)=\sum_{\alpha=-J_R}^{+J_R}\mathcal{A}_\alpha(s',J')c_\alpha^{J_R}(J)B_\lambda(s),\tag{16}$$

where $\mathcal{A}_\alpha(s',J')$ is the $2\to2$ partial-wave amplitude for the reaction $1'+2'\to3+R$, and R is a resonance of (mass $+\,i\times$ width)$=\lambda$, spin J_R, and spin projection α. $c_\alpha^{J_R}(J)=(P_J,Y_{J_R\alpha})$ and $B_\lambda(s)$ is a Breit-Wigner amplitude. The requirement of $\mathcal{A}^{2\to3}$ real already forces the Breit-Wigner amplitudes to be replaced by, for example, Gaussian amplitudes. Ignoring the time-reversal constraint for the moment, we see that crossing gives the functional equation

$$\sum_\alpha\mathcal{A}_\alpha^*(s',J')(-1)^Jc_\alpha^{J_R}(J)B_\lambda^*(s)=\sum_\alpha\mathcal{A}_\alpha(s,J)(-1)^{J'}c_\alpha^{J_R}(J')B_\lambda(s'),\tag{17}$$

which has the general solution

$$\mathcal{A}_\alpha(s,J)=\sum_\beta K_{\alpha\beta}(-1)^Jc_\alpha^{J_R}(J)B_\lambda^*(s),\tag{18}$$

where $K_{\alpha\beta}$ is a symmetric matrix not depending on any of the variables s',J' or s,J. Thus it is seen that in a production amplitude resulting from the resonant decay of outgoing particles 1 and 2, crossing forces the incoming particles $1'$ and $2'$ to also resonate; the undetermined coefficients $K_{\alpha\beta}$, appropriately squared, specify the probability that the αth spin projection of the incoming resonance will produce the βth spin projection of the outgoing resonance. The amplitude finally reads

$$\mathcal{A}^{2\to3}(s',J',s,J)=B_\lambda^*(s')B_\lambda(s)\sum_{\alpha,\beta}(-1)^{J+J'}c_\alpha^{J_R}(J')K_{\alpha\beta}c_\beta^{J_R}(J),\tag{19}$$
$$F^{2\to3}(s_{1'2'},\cos\theta_{1'3(1'2')},s_{12},\cos\theta_{13(12)})=B_\lambda^*(s_{1'2'})B_\lambda(s_{12})\sum_{\alpha,\beta}P_{J_R\alpha}(-\cos\theta_{1'3(1'2')})\times K_{\alpha\beta}P_{J_R\beta}(\cos\theta_{13(12)}),$$

and if time-reversal invariance is imposed, the Breit-Wigner amplitudes must be replaced by some appropriate real functions such as Gaussians. Unfortunately, it seems difficult to translate the meaning of S normal into a physical restriction; one restriction is that if S is normal and the production amplitude is not zero (as is assumed here) then there must be a three-body force present.[3] Equation (19) states that if there is a resonance in two outgoing particles, then the incoming particles must resonate with the same mass, spin, and width. This result is by no means unexpected and can be derived from other principles. However, the example is meant to illustrate the general result that emerges from crossing in canonical variables, namely that knowledge of the subenergy dependence of a production amplitude enables one to determine the total energy dependence of the crossed amplitude.

As a second example consider the exactly soluble model studied in Ref. 5. Here S, the $3\to 3$ partial-wave amplitude, is given by

$$S=S^d+\sum_{i,j=1}^{N} v_i\lambda_{ij}\otimes w_j^\dagger\,,$$

where $\{v_i\}$ are a set of N vectors in $\mathcal{H}$, λ_{ij} is a matrix satisfying an equation given in Ref. 5, and S^d is the disconnected $3\to 3$ partial-wave amplitude, which is a unitary operator on $\mathcal{H}$ in the simple model discussed in Ref. 5 and depends only on the phase shift $\delta_{J'}(s')$ for the $2\to 2$ reaction below the three-body threshold. Finally $w_i=S^{d\dagger}v_i$. Then Ref. 5 shows that S satisfies three-body unitarity and generates the production amplitudes $\mathcal{A}^{2\to 3}$ and $\mathcal{A}^{3\to 2}$ and the inelasticity parameter η. If time-reversal invariance is added, then S is symmetric which implies that $v_i=w_i$, so that these two requirements become

$$w_i=S^{d\dagger}v_i \text{ (unitarity)},$$

$$w_i=v_i^* \text{ (time reversal)},$$

which means that $\arg v_i=\delta_{J'}(s')$, independent of i. If we compare this with the crossing requirement, Eq. (11) shows that these three requirements are incompatible in this simple exactly soluble model.

This result is not surprising, since the simple $3\to 3$ partial-wave amplitude of Ref. 5 arises from a single two-body force plus a superposition of separable three-body forces. It is to be expected that in any realistic situation, where all the particles can interact with one another, the $3\to 3$ partial-wave amplitude is much more complicated and allows crossing to be incorporated. Such a possibility may occur in the Lee model (Ref. 6), but this requires further study.

In conclusion we have shown that crossing in $2\to 3$ partial-wave amplitudes takes on a particularly simple form in the "correct" partial-wave variables and, when coupled with time reversal, puts constraints on the form of allowed production amplitudes. By using group-theoretical arguments, it will be shown in a subsequent paper that such canonical variables exist for all multiparticle amplitudes.

The author wishes to thank Professor C. Cronstrom for the hospitality shown him at the Research Institute for Theoretical Physics and Professor R. Raczka for the hospitality shown him at the Institute for Nuclear Research.

*Permanent address: Dept. of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242.

[1]The triangle function and other kinematic quantities of interest are discussed by E. Byckling and K. Kajantie, *Particle Kinematics* (Wiley, New York, 1973).

[2]In this paper it is simply assumed that a path of analytic continuation exists for multiparticle amplitudes.

[3]W. H. Klink, Phys. Rev. D 7, 2980 (1973); 7, 2989 (1973).

[4]Self-reciprocal functions for Fourier transforms are discussed by E. C. Titchmarsh, *Introduction to the Theory of Fourier Integrals* (Clarendon, Oxford, 1948), 2nd edition, Chap. 9. The use of such functions in crossing multiparticle amplitudes is discussed by W. H. Klink, Nucl. Phys. B (to be published).

[5]W. H. Klink and Lloyd E. Johnson, Phys. Rev. D 9, 1057 (1974).

[6]O. W. Greenberg and H. Yabuki, Ann. Phys. (N.Y.) 79, 146 (1973).