Statistical scaling in inclusive γp, $e^- p$, and $e^\pm e^-$ reactions

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(Received 24 June 1974)

High-energy inclusive γp, $e^- p$, and $e^\pm e^-$ reactions are discussed in connection with the modified Fermi model. Evidence for statistical scaling in these processes is presented.

I. INTRODUCTION

Recently, much interest has been focused on inclusive experiments in photoproduction, electroproduction, and electron-positron-annihilation processes. Violations of Bjorken and/or Feynman scaling have been observed[1-13] in hadron (h) production in

$$\gamma + p \to h + \text{anything}, \tag{1}$$

in

$$e^- + p \to e^- + h + \text{anything} \tag{2}$$

(which in the one-photon-exchange approximation can be written as

$$\gamma_v + p \to h + \text{anything}, \tag{3}$$

where γ_v is the virtual photon with negative mass squared q^2), and in inclusive $e^- e^+$ reactions.

In photoproduction ($q^2 = 0$) of charged pions and kaons the following characteristics are seen[1,2,4-7] [as in the experimental analyses, we use for reactions (1) and (3) the center-of-mass system of γ (γ_v) and p, where both the transverse momentum p_T and the longitudinal momentum $p_L = xp_{\max}$ of h are measured with respect to the direction of the photon]:

(a) The invariant cross section $Ed^3\sigma/d^3p$ *increases* with increasing total energy of the outgoing hadron system, $\sqrt{s}$, for *large* transverse momentum ($p_T \geq 0.5$ GeV/c, say).

(b) At fixed x values, the above-mentioned energy dependence becomes *stronger* as p_T increases.

(c) The p_T distribution is x-dependent. $Ed^3\sigma/d^3p$ is *not* factorizable into independent functions of x and p_T.

(d) In the central region ($-0.1 \leq x \leq 0.1$, say) the partially integrated cross section

$$F(x, s) = \frac{1}{\pi} \int dp_T^2 \frac{E}{p_{\max}} \frac{d^2\sigma}{dx\, dp_T^2} \tag{4}$$

increases with increasing energy $\sqrt{s}$.

In connection with electroproduction processes, the following facts are known[2,8-12]:

(e) The distribution *drops* with q^2 for $x \geq 0.3$ and *small* p_T. At the same energy, the difference between $q^2 = 2.0$ and $q^2 = 1.2$ (GeV/c)^2 is *much smaller* than that between $q^2 = 1.2$ and q^2 equal or near 0 [e.g., 0.4 (GeV/c)^2]. (Here, we adopt the usual convention that positive x corresponds to the "photon-fragmentation region".)

(f) The average transverse momentum $\langle p_T \rangle$ *increases* with *increasing* q^2 for $x > 0$. (Cf. Fig. 19 in Ref. 2.)

(g) Near $x = 0$, $F(x, s)$ and $\langle p_T \rangle$ *increase* with increasing total energy $\sqrt{s}$.

Furthermore, the following observations have also been made[2,12,13]:

(h) The average multiplicity $\langle n \rangle$ for fixed $\sqrt{s}$ *decreases* with increasing q^2 in the interval 0 and 1 (GeV/c)^2 but it remains *constant* for $1 \leq q^2 \leq 8$ (GeV/c)^2.

(i) For fixed q^2 (either $= 0$ or $\neq 0$), $\langle n \rangle$ *increases* with increasing $\sqrt{s}$.

(j) $\langle n \rangle$ does *not* depend only on the Bjorken variable

$$\omega = \frac{s - M^2}{q^2}, \tag{5}$$

where M is the mass of the proton.

From experiments on high-energy electron-positron annihilation processes,

$$e^+ + e^- \to \text{hadrons} \tag{6}$$

and

$$e^+ + e^- \to h + \text{anything} \tag{7}$$

the following results have been obtained[3,14,15]:

(k) The ratio

$$R = \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)}, \tag{8}$$

where $\mu^\pm$ are the muons, is *strongly increasing* with increasing total energy of the incoming $e^+ e^-$ system (we shall also call it $\sqrt{s}$).

(l) The inclusive single-hadron distribution as a function of the Feynman variable depends on the total energy $\sqrt{s}$.

(m) The distribution mentioned in (l) is *isotropic* in the interval $90° \pm 41°$ of the c.m. angle θ.

In this note we attempt to show that the above-mentioned experimental findings can be understood in the framework of Fermi's statistical approach.[16,17]

II. APPLICATION OF THE MODIFIED FERMI MODEL TO INCLUSIVE PHOTOPRODUCTION PROCESSES

In an earlier paper[18] a modified Fermi model has been proposed to describe the large-transverse-momentum phenomena observed in high-energy proton-proton collisions. The physical picture which underlies that model is as follows: Secondary hadrons with large transverse momentum ($p_T \geqslant 2$ GeV/c, say) found in the neighborhood of center-of-mass angle (θ) equal to 90° are produced in *violent collision processes* of the two initial protons. In such processes, the average momentum transfer is larger, the average impact parameter smaller, and the average multiplicity higher than their counterparts in the normal collisions. (For the sake of simplicity, here we shall only consider the central collision. For discussions on the more general case see Refs. 17 and 18.) When a violent collision process between two high-energy protons take place, the energy available in their center-of-mass system will be first deposited inside a conglomerate. By a succession of hadronic reactions, this energy can give rise to states representing a certain number of particles. Eventually statistical equilibrium is reached in this conglomerate among all states that are compatible with the conservation laws. The probability that the collision may result in the formation of one of the possible final states is proportional to the probability that the state in question will have all of its particles contained at the same time inside the conglomerate. The conglomerate at its initial stage, the volume of which is that of two Lorentz-contracted protons, must expand before it decays. (This is necessary in order to obtain a self-consistent statistical or hydrodynamical description of such processes. Cf. Ref. 18 and papers 11 and 12 cited therein.) Because of the lack of direct information on the detailed expansion and decay mechanism, the following simple ansatz has been made: *The conglomerate decays at a critical volume V, the magnitude and shape of which is not specified; it is only assumed that V does not depend on the total energy of the incoming protons*. In the high-energy limit (where the mass of the observed particle is neglected) this picture leads us to the extremely simple result for the single-particle inclusive cross section

$$\frac{d^3\sigma}{d^3p}(s,\vec{p}) \sim e^{-p/kT} , \tag{9}$$

$$T \sim s^{1/8} , \tag{10}$$

where T is the temperature of the conglomerate at the moment of decay, k is the Boltzmann constant, s the total c.m. energy squared, and p ($p = |\vec{p}|$) the momentum of the observed particle. (Here we have considered the simplest case where only one kind of particle is produced. The generalization to more realistic cases is straightforward.) It follows from Eqs. (9) and (10) that

$$\ln\frac{d^3\sigma}{d^3p}(s,\vec{p}) = \text{const}\cdot z , \tag{11}$$

where

$$z = ps^{-1/8} \tag{12}$$

plays the role of a scaling variable (hereafter referred to as "the statistical scaling variable"). Evidently, the variable used in the previous paper[18] is a special case of (12). Other characteristic features of the above-mentioned model [cf. Eq. (24) and Eqs. (15) and (19) of Ref. 18] are as follows: The energy dependence of the total averaged multiplicity is $s^{3/8}$. The angular dependence of the p_T distribution near $\theta = 90°$ is extremely weak (isotropic for central collisions).

Since photons at high energies behave like hadrons,[19-21] it is quite natural to expect that this model (originally proposed to describe hadron collisions) can *also* be applied to high-energy γp processes. We studied the available data[1,2,4-7,22] and found that *the experimental findings, especially those mentioned in (a), (b), (c), and (d) of Sec. I, can indeed be readily understood in terms of this model*. That is to say, also in high-energy photon-proton collisions there are statistical processes in which the multiplicity is higher, and/or in which hadrons with larger (compared with the average over-all processes) transverse momentum are produced. Similarly to the proton-proton case, we expect that products of such statistical processes will dominate in the large-p_T, large-θ ($p_T \gg \langle p_T\rangle$, $\theta \approx 90°$, i.e., $p_T \gg p_L$) region. As a quantitative test of this model Eq. (11) is confronted with all available large-p_T data in or near the central region (all data with $|x| \lesssim 0.2$ and $p_T/p_L > 1$ are included). The results are shown in Figs. 1–4. The slight difference in slope for pions and kaons is not unexpected. This is because Eq. (11) stands for the high-energy limit where the mass (m) of the observed particle is neglected. For cases where the mass is comparable with the momentum (p), $(p^2+m^2)^{1/2}s^{-1/8}$ would be a more suitable variable. Although the available experimental results[1,2,4-7] (cf. also (a), (b), (c), and (d) of Sec I) already *exclude* models which predict *s independence* in the kinematical region $p_T \gg p_L$, the $s^{1/8}$ behavior proposed in this model still needs further experimental tests. For the lack of suffi-

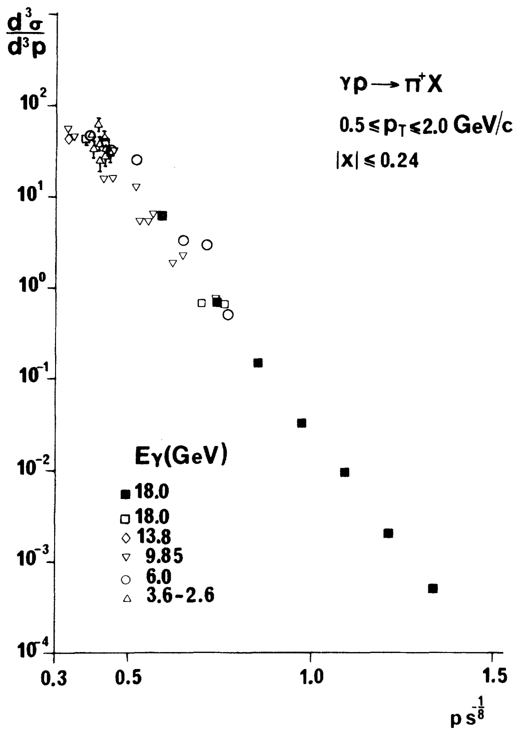

FIG. 1. Cross section for inclusive photoproduction of π^+ in the center-of-mass system versus $z = p s^{-1/8}$, where $p = |\vec{p}|$ is the momentum of the π^+ and $\sqrt{s}$ is the total c.m. energy. $d^3\sigma/d^3p$ is given in μb/GeV^3. □ Boyarski *et al.*, Ref. 7 (data taken from Ref. 1); □ Boyarski *et al.*, Ref. 7 (data taken from Ref. 4); ○ Burfeindt *et al.*, Ref. 4; △ Struczinski *et al.*, Ref. 6; ▽, ◇ Kaune *et al.*, Ref. 22.

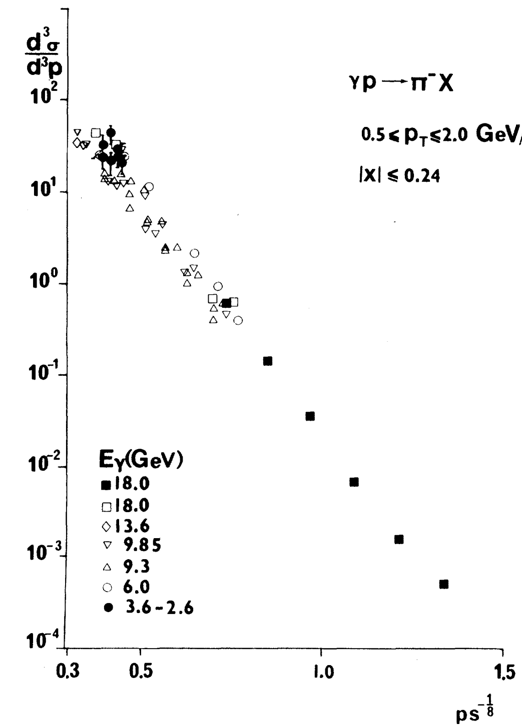

FIG. 2. Same as Fig. 1 for π^-. ■ Boyarski *et al.*, Ref. 7 (data taken from Ref. 1); □ Boyarski *et al.*, Ref. 7 (data taken from Ref. 4); ○ Burfeindt *et al.*, Ref. 4; △ Moffeit *et al.*, Ref. 5; ○ Struczinski *et al.*, Ref. 6; ▽, ◇ Kaune *et al.*, Ref. 22.

cient large-p_T data near $x=0$, the $s^{1/8}$ dependence shown in the figures above is not as obvious as in the proton-proton case.[18] Future data in this kinematical range will be very helpful.

Furthermore, according to the physical picture of this model, products of statistical processes can also be found in the fragmentation regions ($0.2<|x|<1$, say). That is to say, for sufficiently large transverse momentum p_T (cf. Sec. III and Ref. 13) the single-particle distribution can be written as

$$\frac{d^3\sigma}{d^3p}(p_T, x; s) \approx g(p_T, x) + Ae^{-Bz}, \qquad (13)$$

where $g(p_T, x)$ denotes the limiting distribution of the fragmentation products.[23] A and B are positive real constants. To compare Eq. (13) with experiments, we consider *sets of data points with the same value for the statistical variable z*. Plotting in each group $d^3\sigma/d^3p$ $(p_T, x; s)$ with *fixed x* vs. p_T, we should find that data points taken at *different s* values lie on *one* curve. We note that the second term on the right-hand side of Eq. (13) is only significant for sufficiently large p_T values ($p_T > p_L$, say). At the present stage, there are unfortunately not enough high-p_T data for a critical test of this formula.

III. STATISTICAL PROCESSES IN DEEP-INELASTIC e^-p COLLISIONS

We shall discuss high-energy deep-inelastic e^-p collisions (in particular the Bjorken limit) in the framework of the Chou-Yang picture.[24] We recall that one of the main features of this picture is that *the virtual photon in process (3)* with lab energy ν and mass squared $-q^2$ *does not fragment* (because it has an energy deficiency compared with its momentum). This readily explains the observed q^2 dependence of single-particle distributions in the photon fragmentation region [cf. (e) of Sec. I].

Comparison of the reactions (1) and (3) leads us also to the following question: Can statistical processes take place between the virtual photon and the proton in reaction (3)? We think the answer is *yes* (cf. Appendix A of Ref. 25). Since in statistical processes the quantum numbers, especially the masses of incoming particles, do *not*

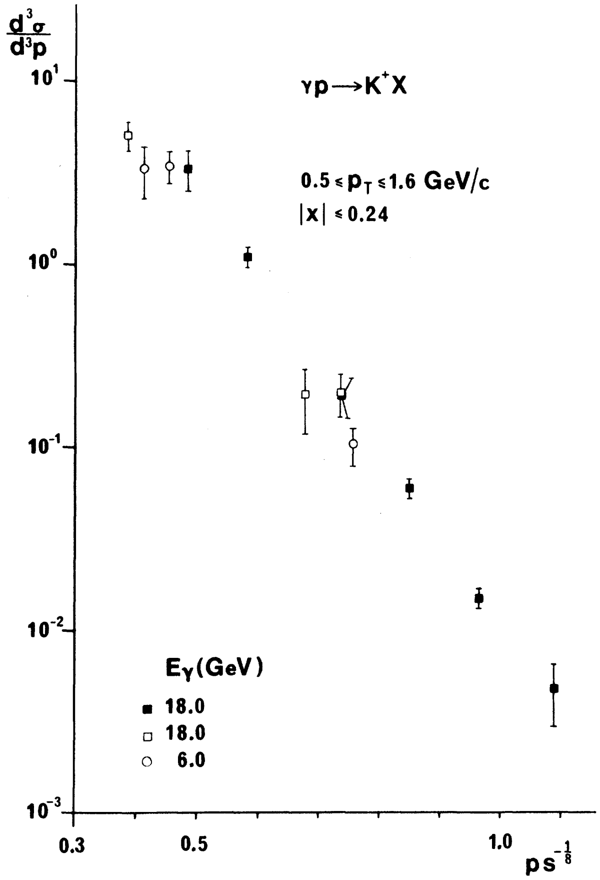

FIG. 3. Same as Fig. 1 for K^+. ■ Boyarski *et al.*, Ref. 7 (data taken from Ref. 1); □ Boyarski *et al.*, Ref. 7 (data taken from Ref. 4); ○ Burfeindt *et al.*, Ref. 4.

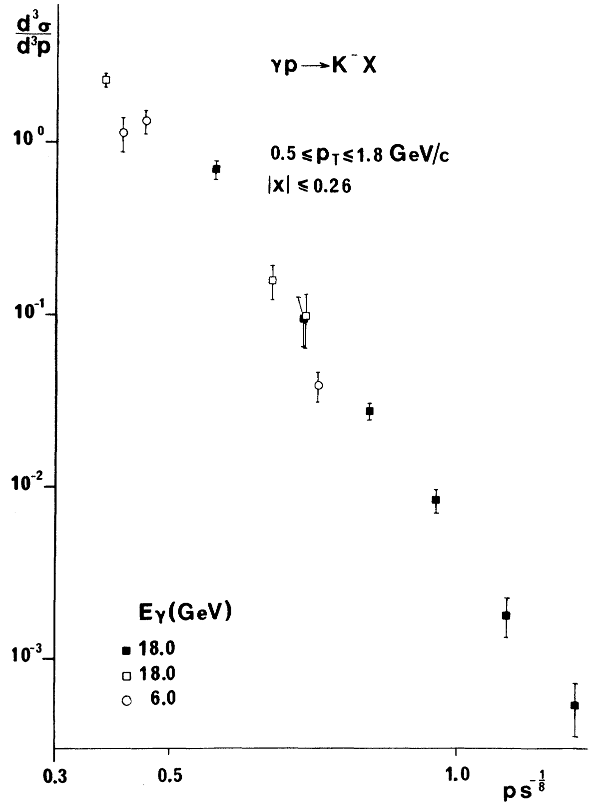

FIG. 4. Same as Fig. 3 for K^-.

play an important role in formation and decay of the conglomerate, there is in fact *a priori no* reason why this should *not* be the case. Furthermore, we speculate that the pulverization process,[24] which is also a violent one, takes place according to the modified Fermi model as well.

It is clear that the characteristic features of the recent experimental results, especially those given in (f), (g), (i), and (j) of Sec. I, can be understood in terms of this model. A direct, quantitative comparison with the data in this case is, however, not so simple as in those for hadron-hadron or photon-hadron processes. The reason is that the pulverization process mentioned above is equivalent to a statistical process at a *lower* total energy ($\sqrt{s'} \approx \sqrt{s}/2$ in the Bjorken limit; cf. Fig. 1 of Ref. 24). Hence, in general, the energy dependence of the observed cross sections is expected to be more complicated than the simple relation given in Eq. (11). As a first-order approximation we make the following simple ansatz: Statistical processes with the whole proton dominate over those with only part of it.

The following particular features of deep-inelastic e^-p processes should be pointed out: Since virtual photons do not fragment, statistical processes can already be significant at small p_T's in the kinematical region $x > 0$. Furthermore, because of the smaller spatial extension of virtual (compared with real) photons, the probability for statistical processes to occur should be larger in deep-inelastic e^-p collisions than that in γp processes.[25] But, since Eq. (11) is only valid for central collisions and for $\theta \approx 90°$ in the case of noncentral collisions (cf. Sec. III of Ref. 18), we expect Eq. (11) to agree in general with data in the neighborhood of $\theta = 90°$. Comparison with experiments[2,9,12,26,27] is given in Figs. 5 and 6. It is interesting to see that the slopes of these plots are precisely the same as those in Figs. 1 and 2. For the reasons stated above, only data points with $p_T/p_L > 1$ [for data where p_T is not sharply given: $(p_T)_{max}/p_L > 1$] are included. It should also be mentioned that the present physical picture provides a natural explanation for the experimental result given in (h) of Sec. I: While the

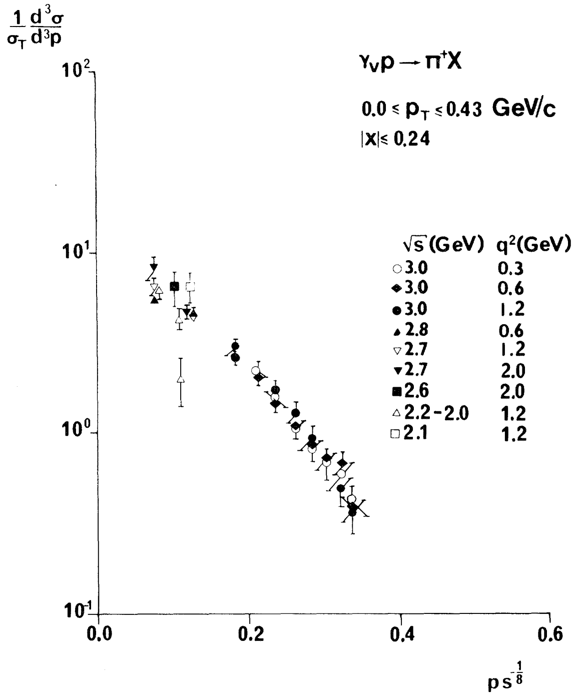

FIG. 5. Cross section for inclusive electroproduction of π^+ in the center-of-mass system of the virtual photon and the proton vs. $z = ps^{-1/8}$. $(1/\sigma_T)d^3\sigma/d^3p$ is given in GeV^{-3}. ●, ◇, ○ Lazarus *et al.*, Ref. 26; ■, □ Bebek *et al.*, Ref. 9; ▲, △, ▼, ▽ Ahrens *et al.*, Ref. 27.

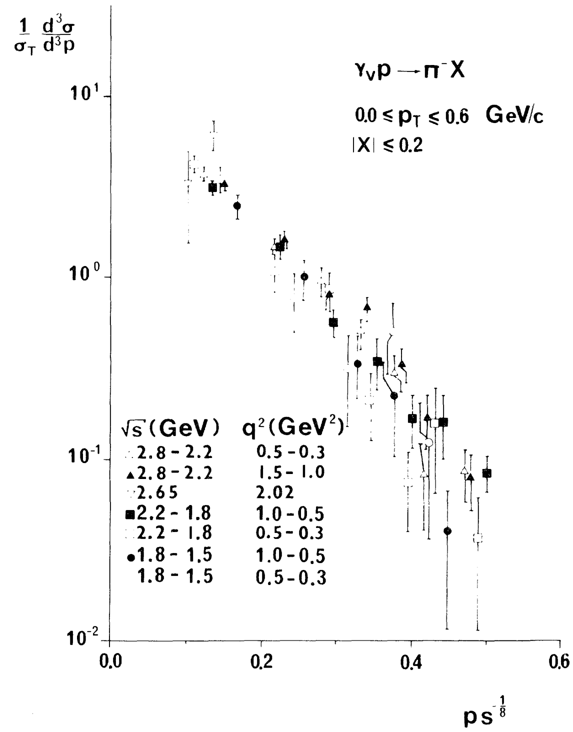

FIG. 6. Same as Fig. 5 for π^-. ●, ○, ■, □, ▲, △ Eckardt *et al.*, Ref. 12; ▽ Bebek *et al.*, Ref. 9.

observed q^2 independence in average multiplicity, $\langle n \rangle$, for $q^2 > 1$ $(GeV/c)^2$ coincides with one of the basic features of the statistical model, the observed decrease in $\langle n \rangle$ when q^2 is moved from zero (real photon) to positive values (virtual photon) is just because virtual photons do not fragment.

IV. e^-e^+ AND e^-e^- PROCESSES

We speculate that statistical processes also take place in high-energy $e^\pm e^-$ hadron-production processes. In fact, because of the condition for statistical processes to take place (small impact parameter) and because of the small spatial extension of the electron (or positron) relative to that of the proton, the relative probability for such a process (compared with other processes) to occur may be higher in $e^\pm e^-$ than in pp reactions. Furthermore, we speculate that fragmentation processes[23] as well as statistical processes can take place between high-energy electrons (or positrons) and photons, and between photons and photons. This means, in particular, that we do *not* confine ourselves to the one-photon approximation while dealing with the reactions (6) and (7) (cf., e.g., Fig. 1 in Ref. 28).

Immediate consequences of the above-mentioned speculations are the following:

(i) At large c.m. angles ($\theta = 90° \pm 30°$, say) and at large transverse momenta ($p_T \geq 0.5$ GeV/c, say) the single-hadron inclusive cross section $d^3\sigma/d^3p(s,p)$ is given by Eq. (11) of Sec. II.

(ii)

$$R \sim s \text{ for } s \to \infty\,, \tag{14}$$

because $e^\pm$ with finite radius led to constant total cross section $\sigma(e^+e^- \to \text{hadrons})$. (cf. Refs. 16 and 23). Here $\sigma(e^+e^- \to \mu^+\mu^-)$ is calculated in the lowest-order QED.

(iii) The total average multiplicity $\langle n \rangle$ has the following asymptotic behavior (this behavior has also been obtained by other authors[29-31]):

$$\langle n \rangle \sim s^{3/8} \text{ for } s \to \infty\,. \tag{15}$$

(iv) The angular distribution is very *flat* near $\theta = 90°$ [cf. Eqs. (15) and (19) of Ref. 18]. In case of *central collisions*, which is certainly not a bad approximation for $e^\pm e^-$ processes (because of the small spatial extension of electrons and positrons), it is *isotropic*.

(v) The typical features mentioned above can *also* be observed in high-energy e^-e^- hadron-production processes.

It seems that (i), (ii), (iii), and (iv) are in good agreement with experiments.[3,14,15] Results from DORIS on e^+e^- inclusive π^0-production experiments as well as those on e^-e^- hadron-production experiments[32] will be extremely exciting.

In connection with the e^+e^- annihilation process, statistical (or thermodynamical) models have been discussed by Bjorken and Brodsky[33,34] and by Engels, Satz, and Schilling.[35] Because of the differences both in spirit and in method, the basic features of their models are qualitatively different from those of ours.

ACKNOWLEDGMENT

We would like to thank Dr. G. Goldhaber, Dr. J. Katz, Dr. P. V. Landshoff, and Dr. W. Theis for helpful discussions.

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