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Algebra of causality

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A set of trilinear equal-time commutation relations is proposed as a generalized second-quantization scheme for fields satisfying conventional statistics. The scheme specifies the bilinear equal-time commutation relations between distinct fields (a finite number for each spin for the representations considered) in addition to the commutation relations of fields with themselves. Multiplet schemes for a particular representation of "generalized fields" satisfying the trilinear equal-time commutation relations are studied. It is shown that generalized vacuum expectation values can be defined, and that the S-matrix formalism can be developed for generalized fields. Differences with the conventional S-matrix expansions are examined, in particular in connection with possible applications to renormalization procedures. A novel regularization method, based on the bilinear equal-time commutation relations between distinct fields, is considered.

INTRODUCTION

Because of their relevance to quantum field theory, bilinear equal-time commutation relations between distinct fields have repeatedly been discussed in the literature. The conclusion in both the Hamiltonian[1,2] and axiomatic[3] contexts is that a "regular locality"[3] can be Klein-transformed, by a succession of Klein transformations[4] if necessary, to the "normal locality"[1,3] for which, by definition, distinct fermions anticommute and boson-boson and boson-fermion field variables commute for spacelike separations of the arguments. In a regular locality, be definition, any two field variables either commute or anticommute for spacelike separations of the arguments, but the commutation behavior is not (necessarily) normal. Covariant components of a field are assumed to have the same commutation behavior with respect to other fields.

Klein transformations, though nonunitary, should presumably not affect the physical content of a theory. It has generally been recognized that in a theory with anomalous locality there are selection rules, superselection rules, or conservation laws, "even-oddness conservation laws" in axiomatic language,[3] which conceivably may be of physical interest.[5,6] The possible physical relevance of the selection rules inherent in a regular locality, as distinguished from a normal locality, would be of more compelling interest if from first principles it would be possible to derive a set of fields for which the locality is not normal. In other words, it is of interest to consider second-quantization schemes which specify the bilinear equal-time commutation relations between distinct fields, as well as the commutation relations of fields with themselves. In this connection the "algebra of causality" defined by the following trilinear equal-time commutation relations

$$[\Psi_{\alpha,i}(\vec{x}), [\chi_{\beta,j}(\vec{y}), \chi'_{\gamma,k}(\vec{z})]_-]_- = \delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}M'\chi'_{\gamma,k}(\vec{z}) - \delta(\vec{x}-\vec{z})(\gamma_4)_{\alpha\gamma}M\chi_{\beta,j}(\vec{y}) \tag{1a}$$

and

$$[\Phi_{\mu,l}(\vec{x}), [\chi_{\nu,m}(\vec{y}), \chi'_{\rho,n}(\vec{z})]_+]_- = i\bar{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})N'\chi'_{\rho,n}(\vec{z}) + i\bar{\delta}_{\mu\rho}\delta(\vec{x}-\vec{z})N\chi_{\nu,m}(\vec{y}) \tag{1b}$$

has been studied.[7-9] The following representations of "generalized fields" (operators satisfying the algebra of causality) have been considered:

$$\Psi_i(x) = A_i \times \psi(x)\,;\quad \bar{\Psi}_i(x) = A_i' \times \bar{\psi}(x) \tag{2a}$$

$$\Phi_j(x) = B_j \times \phi(x)\,;\quad \Pi_j(x) = B_j' \times \Pi(x)\,. \tag{2b}$$

For any representation the bilinear equal-time commutation relations between the various "component fields" ψ and ϕ are determined by the space-time independent matrices A_i or B_j with which they respectively are associated, using Eqs. (1) for the determination and other considerations that may be necessary. The notation employed in Eqs. (1) and (2) and the meaning of the symbols used have repeatedly been discussed.[7-9] For the representation considered, all the "undetermined multipliers" M, M', N, and N', which depend upon the generalized fields in whose trilinear commutation relations they occur, commute with each other, but these multipliers do not necessarily satisfy bilinear commutation relations with all the generalized fields of the representation. In spite of this the trilinear commutation relations (1) can still be derived from the action principle[10,11] provided the generalized variations, generalized fields, multipliers, and symmetrized and antisymmetrized generators of the infinitesimal transformations $\tilde{G}(\Phi_j)$ and $G(\Psi_j)$ satisfy relations of the form

$$[\Psi_j, G(\Psi_k)]_- = ic_j\delta_{jk}\delta\Psi_k, \quad (3a)$$

$$[\delta\Psi_j, \chi]_- = \frac{c_j}{2}[\delta\Psi_j, M\chi]_-, \quad c_j^2 = I \quad (3b)$$

and

$$[\Phi_j, \tilde{G}(\Phi_k)]_- = ic_j'\delta_{jk}\delta\Phi_k, \quad (3c)$$

$$[\delta\Phi_j, \chi]_+ = \frac{c_j'}{2}[\delta\Phi_j, N\chi]_+, \quad c_j'^2 = I, \quad (3d)$$

where all the numerical matrices c_j and c_j' commute with each other and with all the generalized fields of the representation. The various multipliers M, M', N, and N' are symmetric in the canonically conjugate fields being contracted, and they are assumed to vanish if the fields being contracted are not related by canonical conjugation.[8] Each c_j and c_j' depends on a pair of pertinent canonically conjugate field variables, whereas each M and N depends in addition also on χ. For the representations to be considered the conditions (3) are satisfied.

In Eqs. (1a) and (1b) any two field variables may be kinematically related or unrelated, each field operator independently of the other fields denotes either a field variable or its canonical conjugate, and the "ordered Kronecker delta" $\bar{\delta}_{\mu\nu}$ is antisymmetric in its subscripts, i.e., $\bar{\delta}_{\mu\nu} = \pm\delta_{\mu\nu}$, depending on whether μ refers to a field variable or its canonical conjugate.[7] The Ψ and Φ fields differ in the symmetrization of their generators of the infinitesimal transformations [cf. Eqs. (3a) and (3c)], the generators being antisymmetrized in the former and symmetrized in the latter case. Moreover, the generalized fields χ_i and χ_j' can independently of each other and of the other generalized fields refer either to a Ψ or to a Φ field.

BILINEAR COMMUTATION RELATIONS BETWEEN THE COMPONENT FERMIONS

In connection with possible physical applications it is expedient to consider representations for which at least one Ψ_i equal-time commutes with one Φ_j:

$$[\Psi_{\alpha,i}(\vec{x}), \Phi_{\mu,j}(\vec{y})]_- = 0. \quad (4)$$

As has been shown,[8] Eqs. (1) and (4) imply that for the representations considered all Ψ_i commute for equal times with all Φ_j, and that

$$[\Phi_{\mu,i}(\vec{x}), \Phi'_{\nu,j}(\vec{y})]_- = 0, \quad \Phi_j' \neq \Pi_i, \quad (5)$$

and furthermore that all the Φ_i are "generalized bosons":

$$[\Phi_{\mu,i}(\vec{x}), \Pi_{\nu,i}(\vec{y})]_- = \tfrac{1}{2}i\bar{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})B_iB_i' \times \delta(\Phi_i, \Pi_i), \quad (6a)$$

$$[B_i, B_i']_- = 0. \quad (6b)$$

The bilinear equal-time commutation relations of the component bosons with all component fields associated with generalized fields of any representation of the type considered [i.e., subject to Eq. (4)] are thus specified by Eqs. (4)–(6).

Even if the Ψ_i are "generalized fermions,"

$$[\Psi_{\alpha,i}(\vec{x}), \bar{\Psi}_{\beta,i}(\vec{y})]_+ = \tfrac{1}{2}\delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}A_iA_i' \times \delta(\Psi_i, \bar{\Psi}_i), \quad (7a)$$

$$[A_i, A_i']_- = 0, \quad (7b)$$

and it is assumed that distinct component fermion fields satisfy bilinear equal-time commutation relations with each other, Eqs. (1) [and (4)] do not uniquely specify these bilinear commutation relations, unless another reasonable requirement is introduced, e.g., that there should be a one-to-one correspondence between the matrices and the bilinear equal-time commutation relations between the component fields (cf. below). The assumption that the Ψ_i are (generalized) fermions is not as restrictive as might appear, because Eqs. (1) imply that any generalized half-integral-spin field which anticommutes for equal times with at least one other half-integral-spin field belonging to the same representation must be a fermion. The trilinear commutation relations (1) interrelate the commutation relations between distinct generalized fields to the commutation relations of a generalized field with itself.

The operator Kronecker deltas $\delta(\Phi_i, \Pi_i)$ and $\delta(\Psi_i, \bar{\Psi}_i)$ respectively occurring in Eqs. (6a) and (7a) by definition[12] have the same commutation behavior with respect to other component fields as the product of the component fields associated with the generalized fields being contracted.

It is expedient to consider a particular representation based on the generators[11]

$$C_1 = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_y \end{pmatrix}, \quad C_2 = \begin{pmatrix} \sigma_y & 0 \\ 0 & \sigma_x \end{pmatrix}, \quad C_3 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad (8)$$

and to divide the matrices obtained from these generators into four sets:

$$S_1 = (I, iC_1C_3C_1C_3, iC_1C_2, C_1C_3C_2C_3), \quad (9a)$$

$$S_2 = ((C_1C_3 \pm C_3C_1), (C_2C_3 \pm C_3C_2)), \quad (9b)$$

$$S_3 = (C_1, C_2, C_3C_1C_3, C_3C_2C_3), \quad (9c)$$

$$S_4 = (C_3, iC_1C_2C_3, C_1C_3C_1, iC_1C_3C_2). \quad (9d)$$

No matrix in set S_3 satisfies a bilinear commutation relation with a matrix in S_4. However, any other two of the matrices (9) either commute or anticommute. Since distinct component fields are assumed to satisfy bilinear equal-time commutation relations with each other, one can infer that the matrices in sets S_3 and S_4 must either all be associated with generalized fermions or all with generalized bosons. The *a priori* ambiguity of dividing the matrices (9) between fermions and bosons is thus reduced but not eliminated. For the correspondence between fields and particles to be considered the matrices in S_1 and S_2 will be associated with bosons, and those in S_3 and S_4 with fermions.

The bilinear equal-time commutation relations between the component fermions are determined by the following relation:

$$[\Psi_{\alpha,i}(\vec{x}), \Psi'_{\beta,j}(\vec{y})]_+ = 0\,, \quad \Psi'_j \neq \overline{\Psi}_i\,, \tag{10}$$

provided the matrices associated with Ψ_i and Ψ'_j both belong either to S_3 or to S_4. All bilinear commutation relations between component fermions, whose respective matrices do not belong to the same set, are then determined from the requirement that the component fields respectively associated with the generalized fields $C_1 \times \psi$ and $C_3 \times \psi$, for example, anticommute for equal times.

The bilinear equal-time commutation relations between all the component fields of the representation considered are then determined in a manner consistent with Eqs. (1), and the resulting regular locality is summarized in Table I.

The component fields associated with the generalized fields $C_1 \times \psi$ and $C_3 \times \psi$ could equally well be assumed to commute for equal times. Subject to the condition (10) the effect would be that in the fermion-fermion part of Table I all the signs in the upper left- and lower right-hand quadrants are reversed. This effectively implies that in the fermion-fermion part of the table rows (and columns) are interchanged pairwise, e.g., the rows

TABLE I. Bilinear equal-time commutation relations between the component fields for the representation of the algebra of causality considered in this discussion. The commutation relations are obtained from Eqs. (1), (4), (5), (10), and the assumption that the component fields associated with $C_1 \times \psi$ and $C_3 \times \psi$ anticommute for equal times. Other representations, for which the locality may be different, can, of course, be considered. In the shorthand notation employed, 1313 (written vertically or horizontally), for example, refers to the component field associated with the generalized field $C_1C_3C_1C_3 \times \phi$.

3	+	+	−	−	+	−	−	+									
132	+	+	−	−	−	+	+	−									
131	−	−	+	+	−	+	+	−									
123	−	−	+	+	+	−	−	+									
323	+	−	−	+	+	+	−	−									
313	−	+	+	−	+	+	−	−									
2	−	+	+	−	−	−	+	+									
1	+	−	−	+	−	−	+	+									
23 − 32	−	+	−	+	−	+	−	+	−	−	+	+	+	+	−	−
23 + 32	+	−	+	−	+	−	+	−	−	−	+	+	+	+	−	−
13 − 31	+	−	+	−	−	+	−	+	−	−	+	+	−	−	+	+
13 + 31	−	+	−	+	+	−	+	−	−	−	+	+	−	−	+	+
1323	−	−	−	−	+	+	+	+	−	−	−	−	+	+	+	+
12	+	+	+	+	−	−	−	−	−	−	−	−	+	+	+	+
1313	+	+	+	+	+	+	+	+	−	−	−	−	−	−	−	−
I	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−	−
	1	2	313	323	123	131	132	3	*I*	1 3 1 3	12	1 3 2 3	1 3 + 3 1	1 3 − 3 1	2 3 + 3 2	2 3 − 3 2

(and columns) corresponding to C_1 and $C_3C_2C_3$. However, these rows and columns differ in their commutation behavior with respect to the component boson fields. Yet an inspection shows that the symmetry of Table I is of such a nature that effectively the same regular localities are obtained regardless of whether the component fields associated with $C_1\times\psi$ and $C_3\times\psi$ are assumed to commute or anticommute for equal times.

If the bilinear commutation relations between the component fields are determined as outlined, the generalized fermions of the representation considered also satisfy the following equal-time commutation relation:

$$[\Psi_{\alpha,i}(\vec{x}),\Psi'_{\beta,j}(\vec{y})\Psi''_{\gamma,k}(\vec{z})]_-=0\,,\quad \Psi_i\neq\overline{\Psi}'_j,\Psi_i\neq\overline{\Psi}''_k\,, \tag{11}$$

provided the matrices associated with Ψ'_j and Ψ''_k both belong to S_3 or both belong to S_4. Furthermore, for the representation considered, the right-hand sides of all the bilinear commutation relations (6a) and (7a), though not, strictly speaking, c numbers, commute with each other and with all the generalized fields of the representation. This is true even if the two matrices associated with canonically conjugate generalized field variables are not equal, though they are required to commute. Except for a factor $\frac{1}{2}i\overline{\delta}_{\mu\nu}\delta(\vec{x}-\vec{y})$ or $\frac{1}{2}\delta(\vec{x}-\vec{y})(\gamma_4)_{\alpha\beta}$ the right-hand sides of Eqs. (6a) and (7a) are respectively equal to the factors c'_j and c_j occurring in Eqs. (3).

Subject to Eq. (4) and the division of the matrices (9) between the generalized fermions and bosons, the bilinear equal-time commutation relations of the component bosons with all the component fields of the representation considered are uniquely determined by Eqs. (1). However, Eqs. (10) and (11) are consistent with but not implied by Eqs. (1) [and (4)]. The question therefore arises whether the bilinear equal-time commutation relations between the component fermion fields can be determined in some other manner to yield a regular locality essentially different from Table I. If the bilinear equal-time commutation relations between the component fermion fields whose associated matrices are in the same set, S_3 or S_4 [Eqs. (9)], are not determined by Eq. (10), but by some combination of commutation and anticommutation relations [consistent with Eqs. (1)] for distinct generalized fermions whose associated matrices are in the same set, it is, in fact, possible to obtain regular localities which essentially differ from Table I in the fermion-fermion part, and which in some cases are degenerate. In a "degenerate locality" component fields associated with different matrices have the same bilinear equal-time commutation behavior with respect to all the component fields of the representation. As a consequence the table corresponding to such a degenerate locality contains two (or more) identical columns (rows).

A regular locality, consistent with Eqs. (1) and (4), essentially different from Table I results if, for example, the bilinear equal-time commutation relations between the component fields are determined as before, except that any two generalized fermions associated with distinct matrices in set S_4 [Eq. (9d)] are assumed to commute for equal times.

If couplings are to be suitably symmetrized, e.g., antisymmetrized in generalized fermion pairs, as is indicated by Eqs. (1), it appears impossible to couple two commuting generalized fermions to a boson, for example. This difficulty may conceivably be circumvented by the introduction of a Hermitian matrix, e.g., $iC_1C_3C_1C_3$ into the generalized interaction.

As an example of a degenerate locality consistent with Eqs. (1) and (4) the commutation relations between component fermions associated with distinct matrices in set S_4 are determined as follows:

$$\begin{aligned}[][C_3\times\psi(\vec{x}),iC_1C_2C_3\times\psi(\vec{y})]_+ &\\ &=[C_1C_3C_1\times\psi(\vec{x}),iC_1C_3C_2\times\psi(\vec{y})]_+\\ &=[C_3\times\psi(\vec{x}),C_1C_3C_1\times\psi(\vec{y})]_-\\ &=[C_3\times\psi(\vec{x}),iC_1C_3C_2\times\psi(\vec{y})]_-\\ &=[iC_1C_2C_3\times\psi(\vec{x}),C_1C_3C_1\times\psi(\vec{y})]_-\\ &=[iC_1C_2C_3\times\psi(\vec{x}),iC_1C_3C_2\times\psi(\vec{y})]_-\\ &=0\,,\end{aligned} \tag{12}$$

all the other bilinear commutation relations being determined as before. The resulting locality differs from Table I in the fermion-fermion part, the columns corresponding to C_3 and $C_1C_3C_1$ being equal, as are the columns corresponding to $C_1C_2C_3$ and $C_1C_3C_2$. This degenerate locality is contained in Table I if only different columns are considered.

It is of interest to consider the following superpositions:

$$\frac{1}{\sqrt{2}}(C_3\times\psi\pm C_1C_3C_1\times\psi')\,, \tag{13a}$$

$$\frac{1}{\sqrt{2}}(C_1C_2C_3\times\psi\pm C_1C_3C_2\times\psi')\,. \tag{13b}$$

For the degenerate locality under discussion ψ and ψ' in each case anticommute with each other and have the same bilinear equal-time commutation relations with respect to all the other component fields of the representation. Hence, in each case one may set

$$\psi = \psi' , \tag{14}$$

and consider the pairwise-degenerate generalized fields

$$\frac{1}{\sqrt{2}}(C_3 \pm C_1C_3C_1) \times \psi , \tag{15a}$$

$$\frac{1}{\sqrt{2}}(C_1C_2C_3 \pm C_1C_3C_2) \times \psi . \tag{15b}$$

The degeneracy is removed if the bilinear equal-time commutation relations are again determined by Eq. (10), and Table I is recovered if the following correspondence is made:

$$\begin{aligned} C_3 &\to \frac{1}{\sqrt{2}}(C_3 + C_1C_3C_1) , \\ iC_1C_2C_3 &\to \frac{i}{\sqrt{2}}(C_1C_2C_3 + C_1C_3C_2) , \\ C_1C_3C_1 &\to \frac{1}{\sqrt{2}}(C_3 - C_1C_3C_1) , \\ iC_1C_3C_2 &\to \frac{i}{\sqrt{2}}(C_1C_2C_3 - C_1C_3C_2) , \end{aligned} \tag{16}$$

and

$$S_4 \to S_4' = \left(\frac{1}{\sqrt{2}}(C_3 \pm C_1C_3C_1), \frac{i}{\sqrt{2}}(C_1C_2C_3 \pm C_1C_3C_2)\right) . \tag{17}$$

The factors $1/\sqrt{2}$ in Eqs. (13) and (15)–(17) have been introduced for normalization purposes.

Instead of the matrices in sets S_1, S_2, S_3, and S_4, Eqs. (9), it is expedient to consider the sixteen linearly independent matrices in sets S_1, S_2, S_3, and S_4', since in the latter four sets any two matrices either commute or anticommute. Table I is recovered if condition (10) is applied to any two distinct generalized fermions with associated matrices in the union of S_3 and S_4'. The anomalous regular locality of Table I can therefore be obtained from a normal locality for the generalized fields [Eqs. (4), (5), (6a), (7a), and (10)]. If physically relevant expressions are suitably symmetrized the implications of using the matrices in S_3 and S_4 or in S_3 and S_4' can be expected to be identical.

The locality of Table I can also be obtained in a manner consistent with Eqs. (1) if instead of Eq. (4) it is required that one Ψ_i anticommutes for equal times with one Φ_j, assuming that Eqs. (5) and (10) remain valid.

Although there is some latitude in choosing the commutation relations of the generalized fields consistent with Eqs. (1) and Table I, it is apparently not possible to make simultaneously a Klein transformation of both the generalized and the component fields to the normal case if only conventional Klein transformations as applied to the component fields are considered. Moreover, the Klein-transformed fields in general will no longer satisfy the algebra of causality, which has physical content, e.g., it contains statements about the commutativity of physically relevant expressions for spacelike separations of the arguments. Since in any case Klein transformations should not affect the physical content of a theory,[6] the above outlined procedure of determining the bilinear equal-time commutation relations between the component fields from the normal case for the generalized fields appears to be not only consistent with but also indicated by Eqs. (1), the supplementary condition (4) being postulated for physical applications.

Additional justification for this procedure derives from a possibly significant mathematical distinction between the locality of Table I and any other essentially different locality consistent with Eqs. (1) and (4) for the representation under consideration. Since a minus sign in Table I means commutation and a plus sign means anticommutation relations, the product of two columns (rows) can be defined if the product of two equal signs equals a minus, and the product of two unequal signs equals a plus sign. Using this convention, the square of each column or row is obviously proportional to the unit column (row), and Table I can be generated by suitable multiplication of, for example, the four columns in the fermion-fermion part of the table corresponding to C_1, C_2, $C_3C_1C_3$, and C_3 [$\to (1/\sqrt{2})(C_3 + C_1C_3C_1)$]. Alternatively, Table I can be generated from the array of Table II, for example, by repeatedly multiplying rows and columns in this table until the resulting product table is closed under multiplication. Four column (row) generators are needed instead of three matrix generators, Eq. (8), because column

TABLE II. Repeated multiplication (as defined in the text) of rows and columns in this table, and subsequent rearrangement of the product, yields the array of Table I, which is closed under multiplication.

23 − 32	−	+	+	−
13 + 31	−	+	−	+
2	−	+	+	+
1	+	−	−	−
	1	2	1 3 + 3 1	2 3 − 3 2

(row) multiplication is, by definition, commutative. For any locality of the representation considered [subject to Eq. (4)] essentially different from Table I this one-to-one correspondence between matrices and columns (and rows) of signs under multiplication can apparently not be established. Therefore, in this sense, only the locality of Table I is a faithful representation of the matrices.

As will be discussed in the next section, there is a possible phenomenological way of deriving the locality of Table I for the matrix representation (9) being considered.

MULTIPLET STRUCTURE

Since the generalized fields of the representation under consideration and the associated locality (Table I) are obtained from first principles,[10] it seems natural to try to exploit the selection rules inherent in the locality in connection with the observed internal symmetries of elementary particles.

The commutator or anticommutator of two matrices, one from set S_3 [Eq. (9c)] and the other from set S_4 [Eq. (9d)] or S_4' [Eq. (17)], is equal (proportional) to a matrix in S_2 [Eq. (9b)]. Since up to a possible numerical factor the square of each matrix in sets (9) and (17) is equal to the unit matrix, the generalized bosons with associated matrices in S_2 can mediate (trilinear) interactions between two generalized fermions, one with matrix in S_3 and the other with matrix in S_4 or S_4', in such a manner that the matrix obtained by multiplying together all the matrices associated with the generalized fields entering any one of these interactions ("interaction matrix") is proportional to the unit matrix. Hence, for the locality of Table I the product of the component fields entering any such interaction commutes for spacelike separations with all the component fields of the representation. These observations heuristically suggest that in any attempt to establish a correspondence between generalized fields of the representation considered and physical particles, the matrices in S_3 should be associated with generalized fermions (baryons) of even strangeness, and those in $S_4(S_4')$ with baryons of odd strangeness or vice versa. Furthermore, the matrices in S_2 should be associated with generalized bosons that mediate transitions between baryon multiplets which differ in strangeness by one unit, i.e., with kaons, for example.

Making a judicious choice for the generalized fields to represent one baryon singlet and the bosons heuristically labeled $K^{\pm}$, K^0, $\bar{K}^0$, $W^{\pm}$, W^0, and $\bar{W}^0$ (Fig. 1), it is possible, using the considerations outlined above, to obtain the supermultiplet scheme of Fig. 1 by repeated (symmetrized) matrix multiplication, as indicated in the figure. The Yukawa-type couplings are supposed to be antisymmetrized in the generalized fermions, and the locality of Table I is used to obtain the interaction matrices. Not only are the interaction matrices, by construction, proportional to the unit matrix, but the couplings are also Hermitian in the matrices and in the component fields, and local in the generalized as well as in the component fields, since both the coupled generalized and component fields have equal-time commutation behavior consistent with locality.[5] Starting from the singlet at each stage only two baryon multiplets with different matrix structure are obtained (Fig. 1). Any two distinct commuting matrices in S_3 or $S_4(S_4')$ can be selected for the baryon singlet. Different choices lead to physically equivalent schemes. There similarly is some latitude in the choice of the matrices for the generalized bosons of Fig. 1. It is to be expected that the theory can be developed in such a manner that this latitude in the choice of the matrices will be physically inconsequential.

If a suitable selection of generalized fields from Fig. 1 is made, it is heuristically possible to obtain baryon octet and decimet structures (Table III), though such problems as the $\frac{3}{2}$ spin of the decimet fields or the presumed spin 1 of the intermediate bosons are left unsolved. Table III is an expanded version of Fig. 1 and contains a tentative phenomenological correspondence between generalized fields and particles.[9] For each field the canonically conjugate momentum can be obtained from a suitably constructed generalized (matrix) Lagrangian with suitable interactions, each field being uniquely characterized by its spin (parity), charge, and two commuting matrices (A_i, A_i') or (B_i, B_i'). In Fig. 1 there are some hypothetical generalized fermions for which there appears to be no phenomenological correspondence in the scheme considered. Such fields do not occur in Table III, and as a consequence the generalized field labeled Λ, for example, is a singlet and not a member of a triplet. Furthermore, there are no allowed minimal strangeness-nonconserving electromagnetic transitions between the fields of Table III.

Though Table III and Fig. 1 have been phenomenologically constructed assuming trilinear baryonic couplings to bosons, it is possible to regard the fields in Table III as given *a priori* and subject them to different interactions. If it is required that all suitably symmetrized trilinear interactions involving generalized fermions be Hermitian and local, some such couplings will be allowed and

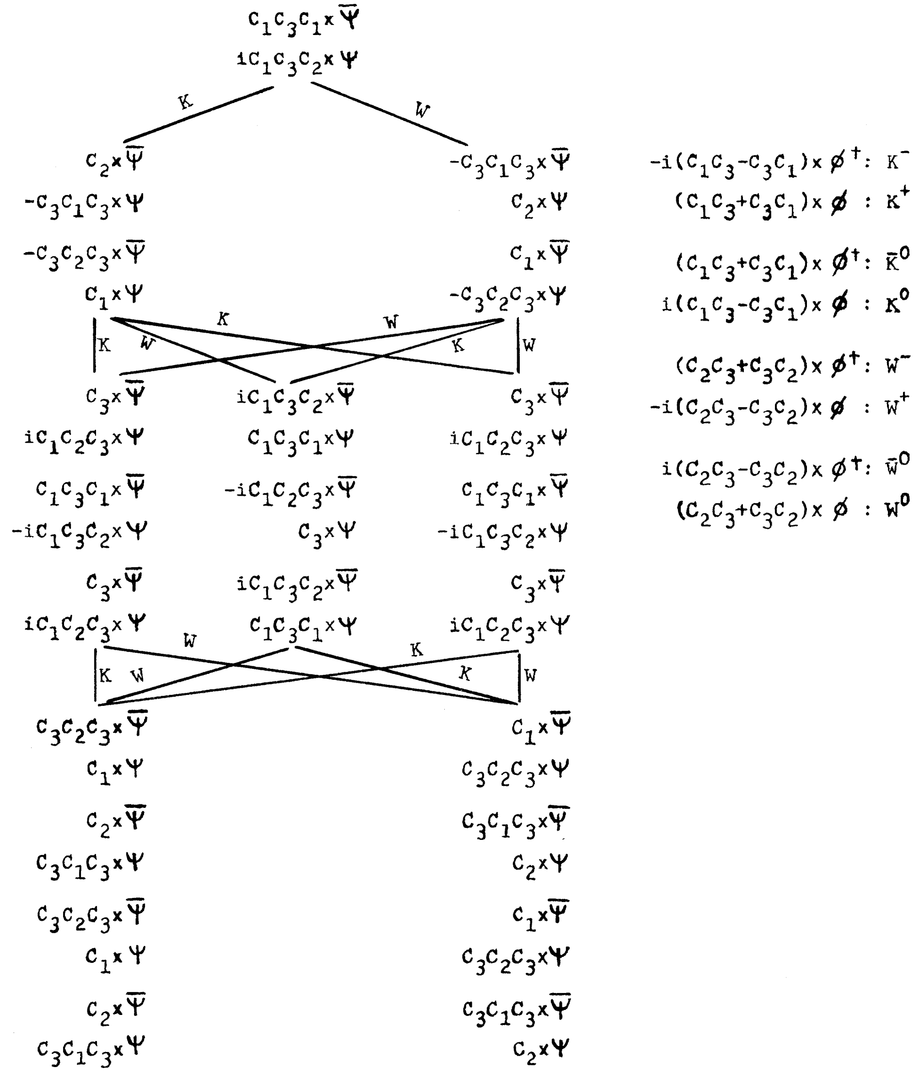

FIG. 1. A hypothetical supermultiplet scheme for generalized fields of the representation being considered. The multiplets are generated from a generalized fermion singlet by generalized, Hermitian, local, properly symmetrized Yukawa couplings, mediated by generalized integral-spin fields heuristically labeled K and W. All the interaction matrices are proportional to the unit matrix. It is understood that the generalized fermions within the same isomultiplet are distinguished by their charges.

others will obviously be forbidden.

Not all strangeness- or isospin-nonconserving Yukawa couplings for the baryons and bosons of Table III can be ruled out on the basis of lack of Hermiticity or locality. However, phenomenologically a strangeness quantum number can be assigned to all fields of Table III in a natural manner. With such an assignment all $\Delta S = \pm 1$ Yukawa-type transitions between the fields of Table III are ruled out either for lack of Hermiticity or lack of locality or both. Some other considerations, such as group-theoretical arguments, for example, must be invoked to rule out the $\Delta S = \pm 2$ and also some isospin-nonconserving transitions (e.g., from Λ to Δ), which are consistent with both Hermiticity and locality and presumably are not realized physically. In a similar scheme presented previously[8] an attempt was made to rule out all strangeness-nonconserving Yukawa couplings on the basis of non-Hermiticity or lack of locality. However, the scheme of Table III appears to be phenomenologically superior.

All strangeness-conserving trilinear, local, and Hermitian transitions between the baryon multiplets of Table III are schematically represented in Fig. 2. The relative magnitudes and phases of

TABLE III. One of several *a priori* possible phenomenological correspondences between generalized fields of the representation considered and particles. Although, with the exception of the division of generalized fields into two sets, one containing integral and the other half-integral spin fields, questions of spin have been disregarded, it is assumed that the same matrices are associated with a field and all its recurrences. Spinor and vector indices and normalization factors $1/\sqrt{2}$ have for simplicity been omitted.

Octet				Decimet		Leptons	
				$C_1C_3C_1\times\bar\psi$:	$\bar\Omega^+$	$C_1\times\bar\psi$:	$\bar l_1$
				$iC_1C_3C_2\times\psi$:	Ω^-	$C_1\times\psi$:	l_1
$C_2\times\bar\psi$:	$\bar\Xi^+$			$-C_3C_1C_3\times\bar\psi$:	$\bar\Xi^{*+}$	$-C_3C_1C_3\times\bar\psi$:	$\bar l_2$
$-C_3C_1C_3\times\psi$:	Ξ^-			$C_2\times\psi$:	Ξ^{*-}	$C_3C_1C_3\times\psi$:	l_2
$-C_3C_2C_3\times\bar\psi$:	$\bar\Xi^0$			$C_1\times\bar\psi$:	$\bar\Xi^{*0}$	$C_2\times\bar\psi$:	$\bar l_3$
$C_1\times\psi$:	Ξ^0			$-C_3C_2C_3\times\psi$:	Ξ^{*0}	$C_2\times\psi$:	l_3
$C_3\times\bar\psi$:	$\bar\Sigma^+$			$C_3\times\bar\psi$:	$\bar\Sigma^{*+}$	$-C_3C_2C_3\times\bar\psi$:	$\bar l_4$
$iC_1C_2C_3\times\psi$:	Σ^-			$iC_1C_2C_3\times\psi$:	Σ^{*-}	$C_3C_2C_3\times\psi$:	l_4
$C_1C_3C_1\times\bar\psi$:	$\bar\Sigma^0$	$-iC_1C_2C_3\times\bar\psi$:	$\bar\Lambda$	$C_1C_3C_1\times\bar\psi$:	$\bar\Sigma^{*0}$	Bosons	
$-iC_1C_3C_2\times\psi$:	Σ^0	$C_3\times\psi$:	Λ	$-iC_1C_3C_2\times\psi$:	Σ^{*0}	$-i(C_1C_3-C_3C_1)\times\phi^\dagger$:	K^-
$C_3\times\bar\psi$:	$\bar\Sigma^-$			$C_3\times\bar\psi$:	$\bar\Sigma^{*-}$	$(C_1C_3+C_3C_1)\times\phi$:	K^+
$iC_1C_2C_3\times\psi$:	Σ^+			$iC_1C_2C_3\times\psi$:	Σ^{*+}	$(C_1C_3+C_3C_1)\times\phi^\dagger$:	$\bar K^0$
				$C_1\times\bar\psi$:	$\bar\Delta^+$	$i(C_1C_3-C_3C_1)\times\phi$:	K^0
				$C_3C_2C_3\times\psi$:	Δ^-	$(C_2C_3+C_3C_2)\times\phi^\dagger$:	W^-
				$C_3C_1C_3\times\bar\psi$:	$\bar\Delta^0$	$-i(C_2C_3-C_3C_2)\times\phi$:	W^+
$C_2\times\bar\psi$:	$\bar n$			$C_2\times\psi$:	Δ^0	$i(C_2C_3-C_3C_2)\times\phi^\dagger$:	$\bar W^0$
$C_3C_1C_3\times\psi$:	n			$C_1\times\bar\psi$:	$\bar\Delta^-$	$(C_2C_3+C_3C_2)\times\phi$:	W^0
$C_3C_2C_3\times\bar\psi$:	$\bar p$			$C_3C_2C_3\times\psi$:	Δ^+	$iC_1C_2\times\phi$:	η'
$C_1\times\psi$:	p			$C_3C_1C_3\times\bar\psi$:	$\bar\Delta^{--}$	$I\times\phi$:	η
				$C_2\times\psi$:	Δ^{++}	$C_1C_2\times\phi^{(\dagger)}$:	$\pi^\pm$
						$C_1C_3C_2C_3\times\phi$:	π^0
						$C_1C_3C_1C_3\times\phi^{(\dagger)}$:	$W'^\pm$
						$I\times\phi$:	γ

the coupling constants remain to be determined. Tables III and Fig. 2 are of interest also if it should turn out that the intermediate bosons W and W' are not physically realized.

In addition to the generalized fields listed in Table III there are four more (Hermitian) integral spin fields:

$$B_1=(C_1C_3+C_3C_1)\times\phi/\sqrt{2}\,, \tag{18a}$$

$$B_2=i(C_1C_3-C_3C_1)\times\phi/\sqrt{2}\,, \tag{18b}$$

$$B_3=(C_2C_3+C_3C_2)\times\phi/\sqrt{2}\,, \tag{18c}$$

$$B_4=i(C_2C_3-C_3C_2)\times\phi/\sqrt{2}\,. \tag{18d}$$

These fields can be directly (selectively) trilinearly coupled to the generalized fields associated in Table III with leptons, but not to baryons. The generalized boson B_1, Eq. (18a), for example, can be trilinearly and locally coupled to the generalized fermion labeled l_1 (Table III) or to l_2. Such couplings would, however, give rise to neutral currents.

All the free-field and Yukawa-type interaction matrices for allowed couplings of the generalized fields listed in Table III turn out to be diagonal, though they are not all proportional to the unit matrix. Allowed trilinear couplings of the generalized bosons (18) to the generalized fermions associated in Table III with leptons would, for the representation considered, give rise to nondiagonal interaction matrices. However, this may not be objectionable as long as for each field the free field and interaction matrices, associated with allowed interactions into which the field enters, can be diagonalized simultaneously.[8] If this condition is satisfied each component field commutes for spacelike separations with the component Hamiltonian density into which it enters. This is presumably sufficient to insure the locality of the equations of motion of the component field variables. In addition, by virtue of Eqs. (1), all the generalized fields of the representation considered satisfy the necessary causality requirements.

If the generalized fields associated with the generalized fermion singlet and the generalized bosons are chosen as in Fig. 1, the bilinear equal-time commutation relations between all the component fermions of the representation considered can phenomenologically be determined from the requirement that certain observed transitions between particles occur locally in the generalized and the component fields and are of the Yukawa type, and that the corresponding interaction matrices be proportional to the unit matrix. In this manner the fermion-fermion part of the locality of Table I can be obtained phenomenologically.

The method of construction of Fig. 1 can, in principle, be used to extend Table III to include positive-strangeness baryons. However, if this is done the generalized fields begin to repeat themselves, i.e., in some cases generalized fermions are obtained which are associated with matrices (A_i, A_i') that have already occurred in the table in this or reversed order (A_i', A_i) in connection with fields of the same charge. As a consequence minimal strangeness-nonconserving electromagnetic transitions could possibly occur if positive-strangeness baryons were included in Table III and Fig. 1.

VACUUM EXPECTATION VALUES AND S MATRIX

Since for the representation considered the generalized fields are 4×4 matrices, state vectors are (4×1) columns or (1×4) rows. With the exception of the unit matrix, all the matrices (9) and (17) are traceless. Hence, to obtain meaningful generalized vacuum expectation values, it is necessary to introduce a Hermitian operator into the definition of vacuum expectation values of generalized fields.[9] *A priori* this can be done in a number of different ways, e.g., the relevant Hermitian operator can be defined as

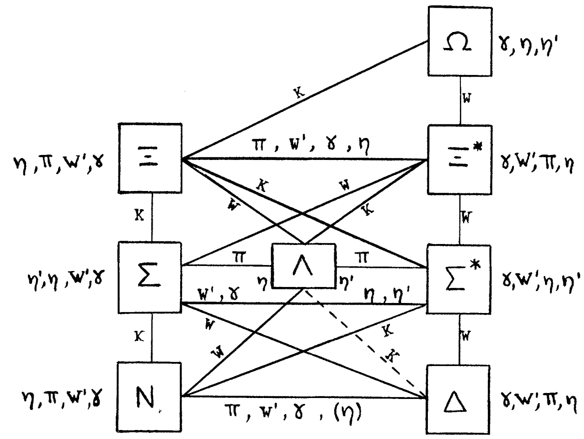

FIG. 2. Schematic representation of the allowed transitions between different isomultiplets obtained when the generalized fields of Table III are coupled by Hermitian, local, strangeness-conserving Yukawa couplings. The Λ-Δ transitions (dashed line) are not consistent with isospin conservation. The allowed transitions within each multiplet can be ascertained in an analogous manner. The symbols next to the rectangles indicate the generalized bosons that can mediate trilinear interactions within the same baryon isomultiplet.

$$R_1 = \tfrac{1}{4}(-1)^Q(I + iC_1C_3C_1C_3 - C_1C_3C_2C_3 - iC_1C_2) = \tfrac{1}{2}(-1)^Q \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (19a)$$

or

$$R_2 = \tfrac{1}{4}(-1)^Q[I + iC_1C_3C_1C_3 \times \delta(\Phi, \Phi^+) - C_1C_3C_2C_3 \times \delta(\Psi, \bar{\Psi}) - iC_1C_2 \times \delta(\Psi', \bar{\Psi}')], \quad (19b)$$

where Q is half the number of singly charged field variables in the expectation value, and the operator Kronecker δ's are those properly associated with the respective matrices. Both R_1 and R_2 are diagonal, and R_2 commutes with all the generalized fields in any generalized vacuum expectation value.

For physically relevant expressions T of the generalized fields of Table III, e.g., for time-ordered products of generalized fields occurring in the S-matrix expansion for allowed generalized Yukawa interactions, the vacuum expectation values are then defined by

$$\langle 0 | R_1 T | 0 \rangle \text{ or } \langle 0 | R_2 T | 0 \rangle, \quad (20)$$

which, for diagonal T (as is assumed), have to be evaluated by taking the trace of the matrix obtained by multiplying together all the matrices associated with the generalized fields and with R in any diagonal generalized vacuum expectation value. Hence,

$$\langle 0|R_1T|0\rangle = \text{numerical factor} \times \langle 0|\text{component fields}|0\rangle, \quad (21a)$$

$$\langle 0|R_2T|0\rangle = \text{numerical factor} \times \text{operator Kronecker } \delta \times \langle 0|\text{component fields }|0\rangle. \quad (21b)$$

While the operators R_1 and R_2 are formally different, they can in connection with diagonal expressions T presumably be used interchangeably and with the same consequences.

For more general manipulations it is expedient to define vacuum expectation values also for expressions for which the resulting matrix is not diagonal. Univalence requires that there must be an even number of half-integral-spin field operators in a nonvanishing vacuum expectation value. Hence, a nondiagonal product matrix of a monomial T in generalized fields of the representation considered, containing an even number of generalized half-integral-spin field operators, must be proportional to one of the off-diagonal matrices in set S_2 [Eq. (9b)].

In analogy to the operators R_1 and R_2 of Eqs. (19) it is therefore possible to define two operators R_3 and R_4, in which the off diagonal matrices of set S_2 [Eq. (9b)] are substituted for the diagonal matrices of S_1 [Eq. (9a)], with a corresponding change in the relevant operator Kronecker δ's.

Alternatively, the definition

$$\langle 0|R_2T|0\rangle \quad (22)$$

for generalized vacuum expectation values of the fields of Table III and Eq. (18) can be generally applicable to any monomial T in generalized fields, regardless of whether the product matrix associated with T is diagonal or off diagonal.

Overall consistency demands that vacuum expectation values of monomials in component fields, such as occur on the right-hand side of Eq. (21), be proportional to the appropriate operator Kronecker δ. The conventional equality of vacuum expectation values to simply c-number functions or distributions actually presupposes the normal case.

In this connection it is expedient to generalize the notion of the operator Kronecker δ to the case where the arguments are two distinct generalized fields χ_i and χ_j' such that the product of the matrices associated with the two fields is even in the generators (8), i.e., $\delta(\chi_i, \chi_j')$, whose square is proportional to unity, has the same commutation behavior with respect to other component fields as the product of the component fields associated with χ_i and χ_j'. Two distinct operator Kronecker δ's either commute or anticommute.

For the above defined operators R_1 and R_2

$$\langle 0|R_1T|0\rangle = \langle 0|R_2T|0\rangle = \langle 0|0\rangle \quad (23)$$

and

$$R_1^2 = R_2^2 = \tfrac{1}{4}I. \quad (24)$$

The factor of $(-1)^Q$ in the definitions (19a) and (19b) is necessary in order to preserve a positive metric for the component fields. This factor occurs because in the construction of Fig. 1 the free-field matrices $-iC_1C_3C_1C_3 = -iC_2C_3C_2C_3$ associated with the charged bosons are 180° out of phase with the free-field matrices associated with the neutral bosons. The generalized fields in Fig. 1 representing the bosons could equally well have been chosen in such a manner that their free-field matrices would all be equal, including phase. Then the factor $(-1)^Q$ would not have been necessary, and it is introduced in Eqs. (19) in order to render the physical implications of the formalism independent of this arbitrariness in the selection of the generalized bosons of Fig. 1.

The above definition of vacuum expectation values permits the development of the S-matrix formalism for the generalized fields of Table III along conventional lines.

Since for the generalized allowed interactions considered

$$[R_2, S]_- = 0, \quad (25)$$

the unitarity of the S matrix appears not to be affected by the matrix structure of the generalized fields, and by the definition (22) of generalized vacuum expectation values.

Formally the usual relation between the chronological and normal products in the interaction representation is unchanged:

$$U^{\cdot}V^{\cdot} = T(UV) - N(UV), \quad (26)$$

where U and V are generalized fields. The generalized contraction $U^{\cdot}V^{\cdot}$, if it does not vanish, is defined by

$$U^{\cdot}V^{\cdot} = \begin{cases} A_iA_i' \times \delta(UV) \times \text{invariant function} \\ \text{or} \\ B_iB_i' \times \delta(UV) \times \text{invariant function}. \end{cases} \quad (27)$$

Hence, with the above definition of vacuum expectation values for generalized fields,

$$\langle 0|R_2U^{\cdot}V^{\cdot}|0\rangle=\langle 0|R_2T(U(x),V(y))|0\rangle$$
$$=0 \text{ or invariant function.} \quad (28)$$

Since, for the representation considered, the generalized fields can be assumed to have normal bilinear equal-time commutation behavior, the signature obtained from the permutation of generalized fields is the same as in the conventional theory.

If the matrices associated with the generalized fields corresponding to the electron and muon, respectively, are $(C_3C_1C_3, -C_3C_1C_3)$ and $(C_3C_2C_3, -C_3C_2C_3)$ (cf. Table III) or vice versa, and only electromagnetic interactions of the electron and muon are considered, quantum electrodynamics is not affected by the matrix structure of these generalized fields. With the above definitions of expectation values, the S-matrix expansion is essentially unchanged in electron and muon electrodynamics. The generalized fields corresponding to the electron and muon differ in their possible interactions with the generalized bosons (18).

CONDITIONAL CANCELLATIONS OF SOME DIVERGENCES

If the fields tabulated in Table III are subjected to the substitution (16), the resulting generalized fields satisfy normal equal-time commutation relations. Hence, with the modifications suggested in the previous section, the conventional S-matrix formalism can be applied to these fields in a straightforward manner. This application of the S-matrix formalism can also be made, with equivalent results, if the substitution (16) is not made, but all relevant expressions are suitably symmetrized.

The matrix structure of the generalized fields or, equivalently, the locality of Table I, has implications which modify some results of the conventional S-matrix formalism in a possibly physically relevant manner. As has been noted previously,[13] the matrix structure of the generalized fields for the correspondence between fields and particles summarized in Table III implies that the lowest-order strong self-energy corrections to, for example, the proton propagator due to emission and reabsorption of pions are 180° out of phase with the loops obtained from emission and absorption of strange particles. It is assumed that the covariant character of the relevant Yukawa couplings is the same in all cases under discussion, e.g., pseudoscalar. A similar phase difference between loops consisting of relatively strange and nonstrange hadrons occurs in connection with the corrections to other baryon propagators.

If the various interactions modifying the propagator add coherently, the phase difference between the loops can be used to eliminate the logarithmic divergences associated with the lowest-order strong self-energy corrections.[13] If other interactions the proton may enter into are disregarded, the coefficients of the logarithmic divergences vanish in the proton case if the masses and coupling constants satisfy the following regularization conditions:

$$G^2(p,p,\pi^0)+G^2(p,n,\pi^+)$$
$$=G^2(p,\Sigma^+,K^0)+G^2(p,\Sigma^0,K^+), \quad (29a)$$

$$G^2(p,p,\pi^0)M_p+G^2(p,n,\pi^+)M_n$$
$$=G^2(p,\Sigma^+,K^0)M_{\Sigma^+}+G^2(p,\Sigma^0,K^+)M_{\Sigma^0}. \quad (29b)$$

The 180° phase difference between the loops, of course, also affects the cutoff-independent contributions to the self-energy corrections.

The conditions (29) insure that the logarithmic divergences vanish regardless of whether the proton line [Fig. 3(a)] is an external or an internal line of a more complicated diagram, i.e., the coefficients of the cutoff-dependent logarithms vanish both for the case of mass and wave-function renormalization.

The origin of the phase difference between the pionic and kaonic loops can be understood from the multiplication (Table IV) of the diagonal matrices contained in set S_1 [Eq. (9a)], the product

TABLE IV. Multiplication table for the Hermitian diagonal matrices contained in S_1 [Eq. (9a)].

$C_1C_3C_2C_3$	$C_1C_3C_2C_3$	$-iC_1C_2$	$-iC_1C_3C_1C_3$	I
iC_1C_2	iC_1C_2	$-C_1C_3C_2C_3$	I	$-iC_1C_3C_1C_3$
$iC_1C_3C_1C_3$	$iC_1C_3C_1C_3$	I	$-C_1C_3C_2C_3$	$-iC_1C_2$
I	I	$iC_1C_3C_1C_3$	iC_1C_2	$C_1C_3C_2C_3$
	I	$iC_1C_3C_1C_3$	iC_1C_2	$C_1C_3C_2C_3$

matrices associated with the various loops being equal to $\pm C_1C_3C_2C_3$. Since the interaction matrices associated with the generalized pionic and kaonic Yukawa vertices are equal to the unit matrix in the case of the proton (as well as for some other baryons) it is to be expected that the relative phase difference between the loops should also be explainable in terms of the bilinear equal-time commutation behavior of the component fields (Table I) in the relevant time-ordered products of the S-matrix expansion. Indeed, if the matrices concerned are disregarded, the occurrence of the 180° phase difference between the pionic and kaonic loops can also be inferred from the permutations of the component fields which arise in the process of contraction of the component fields in the time-ordered products. Whether or not a permutation of component fields gives rise to a factor of -1 is, for the correspondence between fields and particles considered, determined by Tables I and III. It is perhaps a satisfactory aspect of the formalism that the 180° phase difference between the loops can be understood from matrix multiplication of matrices associated with generalized fields satisfying normal bilinear equal-time commutation relations, or from the signatures obtained from the necessary permutations of component fields whose bilinear equal-time commutation behavior is specified by Table I. This phase difference also demonstrates that in the context of the formalism presented the (anomalous) bilinear equal-time commutation relations between distinct (component) fields do affect the S matrix in a possibly physically significant manner.

Although the regularization conditions (29) are suggestive of similar (though not identical) conditions obtained by Pauli and Villars,[14] the cancellation mechanism presented in this discussion is quite different from their method and from that of other authors.[15] The Hamiltonian considered in the present context is not only local and Hermitian, but the coupling constants are real, the relative phase difference of the lowest-order self-energy corrections being obtained from the equal-time commutation behavior of the component fields whose metric is positive definite. Therefore, the fields considered, instead of being "auxiliary" or "shadow" fields, can presumably refer to physical particles with finite (bare) masses.

A similar phase difference occurs between the loops arising from trilinear couplings of baryons to strange (W) and nonstrange (W') "intermediate bosons" (cf. Table III and Fig. 2). The electromagnetic field can heuristically be regarded as the neutral member of a nonstrange (spin 1) W' triplet.

Numerous attempts have been made in different contexts to eliminate the leading divergences associated with the lowest-order weak transition amplitudes and weak self-mass corrections of baryons and leptons due to emission and reabsorption of virtual spin-1 intermediate bosons.[16]

In the present context it is possible to derive regularization conditions for some baryons analogous to Eqs. (29) to eliminate the leading (quadratic) divergence arising from the emission and reabsorption of hypothetical virtual nonstrange (W') and strange (W) intermediate bosons assumed to have spin 1. The phase difference between strange and nonstrange loops necessary for equating the coefficient of the divergence to zero is again obtained from the equal-time commutation behavior of the fields concerned. Considering only trilinear vector couplings of spin-$\frac{1}{2}$ octet fields to intermediate spin-1 bosons in the context of the scheme summarized in Tables I and III and Fig. 2, the conditions for the vanishing of the coefficients of the terms quadratically dependent on the cutoff in the case of the proton propagator, for example, are

$$G_W{}^2(p, n, W'^+) = G_W{}^2(p, \Lambda, W^+)\,, \tag{30a}$$

$$M_n G_W{}^2(p, n, W'^+) = M_\Lambda G_W{}^2(p, \Lambda, W^+)\,. \tag{30b}$$

If the couplings are more generally assumed to be parity nonconserving of the form

$$G_W \bar{\Psi}_a \gamma_\mu (1 + K\gamma_5)\Psi_b W_\mu + \text{H.c.}\,, \tag{31}$$

Eqs. (30) have to be modified by adding terms dependent on the ratios of the relevant axial-vector to vector currents [denoted by K in Eq. (31)].

Any logarithmic divergences associated with the lowest-order weak and electromagnetic self-mass corrections can presumably likewise be eliminated if the algebraic sum of the coefficients of all the terms logarithmically dependent on the cutoff is equated to zero. This means that Eqs. (29) have to be modified to also include weak and electromagnetic parameters. Equations (29), thus modified, would then interrelate strong, electromagnetic, and weak coupling constants and particle masses. However, since weak and electromagnetic couplings are presumably much weaker than strong couplings, such a modification of Eqs. (29) can be expected to be small.

In this connection it is of interest to observe that the mass and coupling constant degeneracies implied by Eqs. (29) and (30) can be attributed to the fact that selected interactions have separately been considered in lowest order for only some of the fields of the scheme of Table III and Fig. 2. These degeneracies will presumably be removed if all allowed interactions of all the fields of the scheme are considered simultaneously.

Since only lowest-order effects have been dealt with above, the question naturally arises as to what happens in higher order.

Considering only generalized trilinear couplings of octet baryons with pions and kaons (Table III and Fig. 2) and disregarding other fields and possible allowed interactions, each Feynman diagram of Figs. 3 and 4 actually stands for several graphs, as the internal lines may correspond to strange or nonstrange fields. Table III and Fig. 2 serve as a guide as to which generalized fields of the scheme under consideration can be trilinearly coupled in a local, Hermitian, and strangeness-conserving manner. Abstracting from the considerations outlined above in connection with Fig. 3(a), a diagram is called "subtractive" if some graphs associated with that diagram are, by virtue of the matrix structure of the generalized fields or the bilinear equal-time commutation relations between the component fields, 180° out of phase with respect to other graphs associated with the same diagram. A diagram is "additive" if no such phase difference occurs between the graphs associated with that diagram. Obviously the property of a diagram being "additive" or "subtractive" is relative to the set of fields and interactions being considered, and an additive diagram may become subtractive if the number of fields (and interactions) under consideration is increased, and vice versa. Phase differences between graphs introduced by other arguments, such as SU(2) symmetry, for example, have to be examined separately.

For the representation under consideration the additive or subtractive character of any particular diagram can simply be ascertained by comparing the quantities

$$(-1)^Q \times \text{product of the matrices associated with the propagators of the internal lines} \quad (32)$$

for the various graphs making up the diagram (Q = half the number of singly charged field variables associated with the internal lines of any graph). For the graphs of the diagrams of Fig. 3, i.e., the corrections to the proton propagator due to the (strong) trilinear pionic and kaonic couplings under discussion, the quantities (32) are proportional to the matrix $C_1C_3C_2C_3$. In general the matrices corresponding to the different graphs of a diagram may be different. If the matrices are all diagonal, for example, the additive or subtractive character of the corresponding graphs is determined by comparison of the relevant quantities (32) with the phase relationships between the diagonal matrices implied by the definition of the Hermitian operators R_1 or R_2 [Eqs. (19)].

As is shown in Fig. 3, there may be both additive and subtractive diagrams in the same order of the S-matrix expansion. The diagram of Fig. 3(b) would become subtractive if mesons corresponding to the field labeled η in Table III were also to be included in the discussion. As indicated in Fig. 4, the insertion of a self-energy loop may cause an additive diagram to become subtractive.

Instead of outright subtraction of infinite quantities the subtractive character of the lowest-order self-energy corrections has been used above to equate the algebraic sum of the coefficients of the cutoff-dependent terms to zero, and to yield the relations (29) and (30) between the relevant coupling constants and masses. Whether this procedure can similarly be applied to the elimination of divergences occurring in connection with higher-order diagrams is a question that is presently being studied. Since not all diagrams are subtractive it may be possible to algebraically relate different diagrams of the same order or even diagrams of different orders, e.g., the leading divergence of an additive diagram may be completely or partially canceled by divergences of the same order occurring in connection with diagrams of the same or higher order, or it may be possible to sum all the terms of the same degree of divergence in a perturbative expansion.[17]

It is not known at the present time whether this program can be carried out in a self-consistent manner and in a manner consistent also with unitarity requirements, since with an increasing number of fields and interactions the elimination of divergences associated with higher-order cor-

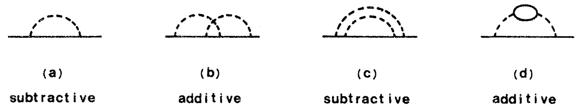

FIG. 3. Lowest-order and fourth-order strong corrections to the proton propagator mediated by pions and kaons. The meaning of the labels "additive" and "subtractive" is explained in the text.

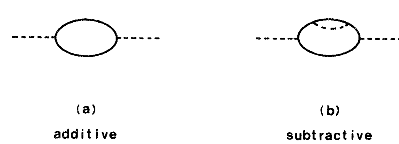

FIG. 4. Lowest-order strong corrections to a meson (pion or kaon) propagator, and a self-energy insertion.

rections may lead to a complicated set of relations between an increasing number of parameters (masses and coupling constants). Conceivably a trend may appear and consistency may possibly be guaranteed only under certain conditions, e.g., that strong interactions possess SU(2) symmetry.

These quantitative questions and related problems are presently being studied.

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[1]G. Lüders, Z. Naturforsch. 13A, 254 (1958).
[2]T. Kinoshita, Phys. Rev. 110, 978 (1958).
[3]H. Araki, J. Math. Phys. 2, 267 (1961). Cf. also N. Burgoyne, in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, New York, 1964), p. 81.
[4]O. Klein, J. Phys. Radium 9, 1 (1938).
[5]K. Nishijima, Prog. Theor. Phys. 5, 187 (1950); S. Oneda and H. Umezawa, *ibid.* 9, 685 (1953); T. Kinoshita, Phys. Rev. 96, 199 (1954); H. Umezawa, J. Podolanski, and S. Oneda, Proc. Phys. Soc. Lond. A68, 503 (1955); H. Umezawa, *Quantum Field Theory* (North-Holland, Amsterdam, 1956), p. 197.
[6]Lecture notes of A. S. Wightman, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963), in particular p. 29; R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and All That* (Benjamin, New York, 1964), p. 146 ff.
[7]H. Scharfstein, Nuovo Cimento 30, 740 (1963); Ph.D. thesis, New York University, 1962 (unpublished).
[8]H. Scharfstein, Phys. Rev. D 5, 403 (1972). This paper contains references to previous publications on trilinear commutation relations.
[9]H. Scharfstein, Nuovo Cimento Lett. 8, 443 (1973).
[10]J. Schwinger, Phys. Rev. 82, 914 (1951).
[11]H. Scharfstein, J. Math. Phys. 7, 1707 (1966).
[12]H. Scharfstein, Phys. Rev. 158, 1254 (1967).
[13]H. Scharfstein, Phys. Rev. 187, 2027 (1969).
[14]W. Pauli and F. Villars, Rev. Mod. Phys. 21, 434 (1949).
[15]Cf., for example, M. E. Arons, M.-Y. Han, and E. C. G. Sudarshan, Phys. Rev. 137, B1085 (1965).
[16]Cf., for example, R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. 28B, 128 (1968); C. Bouchiat, J. Iliopoulos, and J. Prentki, Nuovo Cimento 56A, 1150 (1968); T. D. Lee, *ibid.* 59A, 579 (1969); Y. Y. Lee, Phys. Lett. 28B, 682 (1969); Jean Pestieau and Probir Roy, Phys. Rev. Lett. 23, 349 (1969); H. Terazawa, *ibid.* 22, 254 (1969); Nuovo Cimento Lett. 3, 143 (1970).
[17]Cf. in this connection T. D. Lee, Nuovo Cimento 59A, 579 (1969), in particular the footnote on p. 580, and N. Cabibbo and L. Maiani, Phys. Lett. 28B, 131 (1968).