# Comment on a scaling law for pion-nucleon scattering

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A recently proposed scaling law for meson-baryon scattering is shown to be in disagreement with important features of $\pi N$ data.

In two recent papers[1] Blaha *et al.* have proposed a new scaling law for meson-nucleon scattering above 4 GeV/$c$ in the ranges $|t|<3$ and $|u|<1$ [$t$ and $u$ are always given in units of $(\text{GeV}/c)^2$]. They claim that the law is valid "for all available meson-nucleon scattering data within the experimental errors." It is the purpose of this note to point out that there are large systematic deviations far beyond the experimental errors. Therefore it is hard to see the significance of the proposed scaling variable.

The universal expression is given for the differential cross section $\sigma(k,t)$ divided by its forward value,

$$\frac{\sigma(k,t)}{\sigma(k,0)}=f(\tau)\,, \tag{1}$$

where $k$ = pion lab momentum and $\tau$ is the dimensionless variable

$$\tau=-bt(1+t/s)\,. \tag{2}$$

$f(\tau)$ is approximated by two exponentials:

$$\begin{aligned} f(\tau)&=\exp(-\tau)\ \text{ if } \tau\leq 5\\ &=\exp(-0.70\,\tau-1.5)\ \text{ if } \tau\geq 5\,. \end{aligned} \tag{3}$$

The parameter $b(s)$ is the slope at $t=0$. It follows from experimental data.

As mentioned by Blaha *et al.*, the agreement with the data at $|t|<0.6$ is due to the prescription. The interesting questions are whether the scaling law describes correctly (i) the strong $s$ dependence at fixed $t$ in the range $0.8<|t|<3$ and (ii) a surprising relation between the forward and backward peaks. If the law is valid, the differential cross sections at a certain energy are equal at momentum transfer squared $t$ and

$$t'=-s-t\,. \tag{4}$$

In particular the backward cross section ($t_b=-4q^2$, $q$ = c.m. pion momentum) agrees with the cross section at

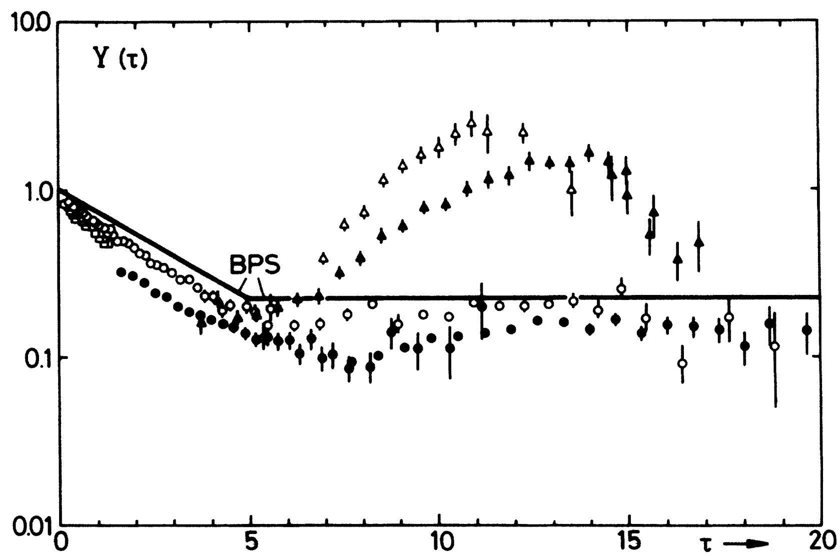


FIG. 1. Comparison of $\pi^-p$ data with the proposed scaling law. According to Blaha, Pardee, and Sukhatme (BPS) (Ref. 1), all data points should lie near the straight lines denoted by "BPS." $Y(\tau)=[\sigma(k,t)/\sigma(k,0)]\exp(0.7\tau)$. Data: 3.92 GeV/$c$: Alitti *et al.* (Ref. 2)△. 5.8 and 5.9 GeV/$c$: Owen *et al.* (Ref. 3) ▲. 14.15 GeV/$c$: Dzierba *et al.* (Ref. 4)○. 13.8 GeV/$c$: Cornillon *et al.* (Ref. 5) ○. 22.15 GeV/$c$: Foley *et al.* (Ref. 6)□. 22.6 GeV/$c$: Cornillon *et al.* (Ref. 5) ●. Arguments favoring a renormalization factor of $N=1.25$ for the 22.6-GeV/$c$ data of Cornillon *et al.* have been given by Höhler and Jakob and by Eichmann and Höhler (see Ref. 9).

$$t_b' = 4q^2 - s = -2(m^2 + \mu^2) + \frac{(m^2 - \mu^2)^2}{s} . \tag{5}$$

This means that in terms of the new variable the backward peak agrees with part of the forward peak, namely $t < -1.7$ at 4 GeV/$c$ and $t < -1.8$ at very large energies.

First we shall test the scaling law in the range $|t| < 3$. Since even large deviations are difficult to see if the ordinate varies by a factor of $10^7$ (Ref. 1), we have plotted the quantity

$$Y(\tau) \equiv f(\tau)\exp(0.7\tau) , \tag{6}$$

which is expected to become constant beyond $\tau = 5$ [Eq. (3)].

Figure 1 shows clearly that the data points (Refs. 2–6) are *not* lying on a universal curve. There is a strong systematic momentum dependence which, at $\tau = 11$, amounts to a factor of 10 between 4 and 14 GeV/$c$. This variation is much larger than the statistical errors. It is confirmed by other accurate $\pi^+ p$ scattering data which are not shown in our Fig. 1.[7] The discrepancy is somewhat smaller for the 5-GeV/$c$ data of Eide *et al.*,[8] which differ from those of other authors at $t < -1$.

Above 10 GeV/$c$ the new scaling law differs only slightly from scaling in the variable $bt$ in the $t$ range of interest, since $|t/s| < 0.1$ at $t = -2$. If one wants to check whether the data confirm the additional term $t/s$, one has to consider the quadratic $t$ dependence of the quantity

$$\ln\left[\frac{\sigma(k_1,t)}{\sigma(k_1,0)}\,\frac{\sigma(k_2,0)}{\sigma(k_2,t)}\right] = 0.7\,bt^2(s_1^{-1} - s_2^{-1}) , \tag{7}$$

where $k$ = pion lab momentum. The data at $k_1 = 13.8$ GeV/$c$ and $k_2 = 22.6$ GeV/$c$ (Ref. 5) do not favor the $t/s$ term in (2), but they are not accurate enough for a final conclusion.

The failure of the scaling law can also be expressed in terms of an "effective trajectory" $\alpha_{\text{eff}}(t)$, which is defined by

$$k^2 \frac{d\sigma}{dt} = F(t) k^{2\alpha_{\text{eff}}(t)} . \tag{8}$$

It is well known that in $\pi^\pm p$ scattering determinations of $\alpha_{\text{eff}}(t)$ in the range 4–14 GeV/$c$ lead to negative values, for instance $\alpha_{\text{eff}}(-2) = -0.7$. However, the scaling law predicts positive values,[9] for instance $\alpha_{\text{eff}}(-2) = +0.2$.

As far as the proposed relation between the forward and backward peaks is concerned, it is surprising that the authors claim universality, since it is known that the $\pi^\pm p$ and $K^\pm p$ backward peaks differ considerably from each other. These differences can be understood as consequences of the different $u$-channel baryon exchanges.[10] Therefore it can be expected that even asymptotically there is no universal law for meson-baryon backward scattering.

Figure 2(a) shows the forward and backward peaks in $\pi^+ p$ scattering at 10 GeV/$c$, together with the proposed fit. The shapes of the $\tau$ dependences

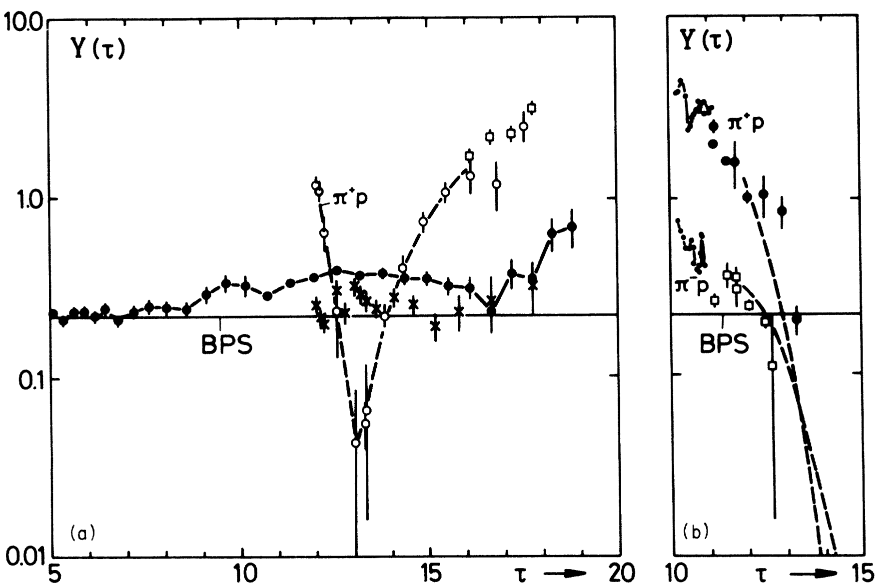


FIG. 2. Comparison of the $\tau$ dependence of the forward and backward peaks, (a) at fixed momentum (10 GeV/$c$) and (b) at fixed angle (180°). Scaling law is indicated by horizontal straight lines labeled BPS. Data in (a): Baglin *et al.* $|t| < 3$ $\pi^+ p$ (Ref. 11) (solid line) ●. Baglin *et al.* $|u| < 1$ $\pi^+ p$ (Ref. 11) □. Owen *et al.* $|u| < 1$ $\pi^+ p$ (Ref. 4) (dashed line) ○. Owen *et al.* $|u| < 1$ $\pi^- p$ (Ref. 4) X. Data in (b): Refs. 4, 13–17; $\pi^+ p$ ●, $\pi^- p$ □. Dashed lines: Regge fit (Ref. 12).

differ considerably, but the order of magnitude is correct.

Figure 2(b) shows the $\tau$ dependence of the $\pi^{\pm}p$ backward cross sections above 4 GeV/$c$.[4,12-17] For some of the $\pi^{\pm}p$ data the deviation from universality amounts to a factor of 10. The agreement with the fit at special energies is accidental.

*Conclusion.* The deviations from the scaling law of Blaha *et al.* are so large that its usefulness is doubtful.

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[1]S. Blaha, W. J. Pardee, and U. P. Sukhatme, Phys. Lett. 42B, 435 (1972); S. Blaha and U. P. Sukhatme, Phys. Rev. D 8, 4221 (1973).
[2]J. Alitti *et al.*, Paper 174 presented to the Second International Conference on Elementary Particles, Aix-en-Provence, 1973 (unpublished).
[3]D. P. Owen *et al.*, Phys. Rev. 181, 1794 (1969).
[4]A. R. Dzierba *et al.*, Phys. Rev. D 7, 725 (1973).
[5]P. Cornillon *et al.*, Phys. Rev. Lett. 30, 403 (1973).
[6]K. J. Foley *et al.*, Phys. Rev. 181, 1775 (1969).
[7]A. Brabson, Phys. Rev. Lett. 25, 553 (1970); H. W. Paik *et al.*, Indiana University Report No. COO-2009-31 (unpublished); I. Ambats *et al.*, Phys. Rev. Lett. 29, 1415 (1972); C. W. Akerlof *et al.*, *ibid.* 27, 219 (1971); D. R. Rust *et al.*, *ibid.* 24, 1361 (1970); see also Ref. 17.
[8]A. Eide *et al.*, Nucl. Phys. B60, 173 (1973).
[9]It was noticed by G. Höhler and H. P. Jakob [Phys. Lett. 49B, 280 (1974)] that at $|t|>0.8$ $\alpha_{\rm eff}(t)$ depends strongly on the momentum range in which the fit was made. *Positive* values of $\alpha_{\rm eff}$ up to $t=-2.8$ have been obtained from data above 10 GeV/$c$. The result is not far from a rather flat Pomeron trajectory ($\alpha_P' < 0.3$) [G. Eichmann and G. Höhler, Karlsruhe report No. TKP 10-74 (unpublished)].
[10]The $K^-p$ cross section is decreasing much faster than the $K^+p$ cross section. It is smaller by two orders of magnitude at 5 GeV/$c$ (Ref. 8).
[11]C. Baglin *et al.*, Phys. Lett. 47B, 85 (1973).
[12]V. Barger and D. Cline, Phys. Lett. 27B, 312 (1968).
[13]W. F. Baker *et al.*, Phys. Rev. Lett. 32, 251 (1974).
[14]A. Babaev *et al.*, Phys. Lett. 38B, 342 (1972).
[15]W. F. Baker *et al.*, Nucl. Phys. B25, 385 (1971).
[16]E. W. Anderson *et al.*, Phys. Rev. Lett. 20, 1529 (1968).
[17]J. Orear *et al.*, Phys. Rev. 152, 1162 (1966).