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# Corrections to the nuclear Goldberger-Treiman relation and pionic form factor. II. Forbidden transition*

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Corrections to the nuclear Goldberger-Treiman relation and the pionic nuclear form factor are investigated using the $\Delta J=0$ transition with parity change as an example of forbidden transitions. It is shown that (1) the corrections are as large as the main term, implying that the original Goldberger-Treiman relation with pion-pole dominance is no longer valid, (2) the pionic form factor $f_{\pi if}(q^2)$ is a very rapidly varying function of $q^2$ for $|q^2|<m_\pi^2$, and (3) the ratio $[f_{\pi if}(0)/f_{\pi if}(-m_\pi^2)]$ is essentially zero.

## I. INTRODUCTION

It is well known that the assumption of pion-pole dominance in the dispersion relation for the matrix element of the divergence of the axial-vector current holds reasonably well for the nucleon case. In fact, the pion-pole term represents more than 90% of the total contribution at $q^2=0$. In other words, the correction to the original Goldberger-Treiman relation[1] is less than 10% and the pion-nucleon form factor is a smooth function of $q^2$, at least, for $|q^2|\lesssim m^2_\pi$.

In a recent paper[2] it has been demonstrated that a similar but slightly larger deviation from pion-pole dominance is present in nuclear cases where transitions are allowed (i.e., $\Delta J=0,1$ and no parity change) than is present in the nucleons. The difference arises from the contribution of anomalous thresholds to the dispersion integral. This deviation can also be characterized by the departure of the pion-nucleus form factor, $K_{\pi if}(0)=f_{\pi if}(0)/f_{\pi if}(-m_\pi^2)$, from unity, where $f_{\pi if}(0)$ and $f_{\pi if}(-m_\pi^2)$ are, respectively, the $\pi N_i N_f$ nuclear form factors evaluated at $q^2=0$ and $-m_\pi^2$, $f_{\pi if}(-m_\pi^2)$ being the physical coupling constant. For example, it was found[2] that $K_{\pi if}(0)\cong 0.8$ for $^3\mathrm{He}\to{}^3\mathrm{H}$ and $K_{\pi if}(0)\cong 0.6$ for $^{12}\mathrm{C}\to{}^{12}\mathrm{B}$, whereas $K_{\pi np}(0)=0.92$. This shows that for very light nuclei, the pion may still be treated as soft, as is done in the nucleon case; i.e., pion-pole dominance is still reasonably good. On the other hand, even in the region of $A=12$, one expects a large deviation from pion-pole dominance.

In this paper, we extend the work of Ref. 2 to the case of forbidden transitions, using the $0^-\to 0^+$ transition as an example. We present an answer to the question as to what extent pion-pole dominance (or the nuclear Goldberger-Treiman relation) is valid when the transitions are forbidden ones. In this connection we obtain the value of $K_{\pi if}(0)$ and the $q^2$ dependence of $f_{\pi if}(q^2)$ explicitly for the $0^-\to 0^+$ transition.

## II. FORMULATION

From general invariance arguments, the most general matrix elements for the $0^-\to 0^+$ transition are given by

$$\langle f\,|\,V_\alpha^{(\pm)}(0)|\,i\,\rangle=0\,,$$
$$\langle f|A_\alpha^{(\pm)}(0)|\,i\,\rangle=\frac{1}{2M}\left[F_A(q^2)Q_\alpha+\frac{2M\Delta M}{m_\pi^2}F_P(q^2)q_\alpha\right], \tag{1}$$
$$q_\alpha=(p_f-p_i)_\alpha,\ \ Q_\alpha=(p_i+p_f)_\alpha$$

where $M$ is the nuclear mass $[M=\frac{1}{2}(M_i+M_f)]$ and $\Delta M=M_i-M_f$; $p_i$ and $p_f$ are, respectively, the momenta for the initial and final nuclei. Also $F_A(q^2)$ and $F_P(q^2)$ are the axial-vector and induced pseudoscalar form factors.

Using the Gell-Mann–Lévy PCAC (partial conservation of axial-vector current) relation[3]

$$\partial_\alpha A_\alpha^{(\pm)}(x)=a_\pi m_\pi^3\,\phi_\pi^{(\pm)}(x)\,, \tag{2}$$

where $a_\pi=0.94\pm0.01$ and $\phi_\pi^{(\pm)}(x)$ is the pion field, we obtain from Eq. (1)

$$F_A(q^2)+\frac{q^2}{m_\pi^2}F_P(q^2)=\frac{a_\pi m_\pi^2}{m_\pi^2+q^2}f_{\pi if}(q^2)\,. \tag{3}$$

In deriving Eq. (3), we have used

$$(\Box^2-m_\pi^2)\phi_\pi^{(\pm)}(x)=-j_\pi^{(\pm)}(x)\,, \tag{4}$$

$$\langle f\,|\,j_\pi^{(\pm)}(0)|\,i\,\rangle\equiv\frac{\Delta M}{m_\pi}f_{\pi if}(q^2)\,. \tag{5}$$

Rewriting Eq. (3) for $f_{\pi if}(q^2)$, we have

$$f_{\pi if}(x)=\frac{F_A(x)}{a_\pi}(1+x)\left\{1+x\left[\frac{F_P(x)}{F_A(x)}\right]\right\}, \tag{6}$$

where

$$x\equiv\frac{q^2}{m_\pi{}^2}.$$

Setting $x=0$ in Eq. (6) gives the modified Goldberger-Treiman relation

$$\begin{aligned}F_A(0)&=a_\pi f_{\pi if}(0)\\&=a_\pi f_{\pi if}(-m_\pi{}^2)\left[\frac{f_{\pi if}(0)}{f_{\pi if}(-m_\pi{}^2)}\right].\end{aligned}\tag{7}$$

We note that Eqs. (3), (6), and (7) hold for nuclear states of arbitrary spin and parity in the nonrelativistic limit of nuclei. That is, they take the same forms for both allowed and forbidden transitions so long as $F_A$ and $F_P$ are properly defined.

In the case of allowed transitions, we have, to a very good approximation,[2]

$$\frac{F_P(x)}{F_A(x)}\cong-\frac{1}{1+x}, \tag{8}$$

so that, when Eq. (8) is substituted into Eq. (6),

$$f_{\pi if}(x)\cong\frac{1}{a_\pi}F_A(x)\quad\text{for } |x|\lesssim 1, \tag{9}$$

implying that the $q^2$ dependence of $f_{\pi if}(q^2)$ and $F_A(q^2)$ is very similar for $|q^2|\lesssim m_\pi{}^2$. Also, as shown in Ref. 2, this approximate equality of the $q^2$ dependence of $f_{\pi if}(q^2)$ and $F_A(q^2)$ leads to the result that $K_{\pi if}(0)$ is not very different from unity. This implies that the correction to the original Goldberger-Treiman relation due to the non-pion-pole contribution is small. Moreover, the correction to Eq. (8) due to the modification of the pion propagator in nuclear matter does not change Eq. (9) to any significant extent.[2] In summary, for allowed transitions, both $f_{\pi if}(q^2)$ and $F_A(q^2)$ are given by

$$\begin{aligned}f_{\pi if}(q^2)&=f_{\pi if}(0)(1-\tfrac{1}{6}aR^2q^2+\cdots),\\F_A(q^2)&=F_A(0)(1-\tfrac{1}{6}bR^2q^2+\cdots),\end{aligned}\tag{10}$$

where $R\cong(1/m_\pi)A^{1/3}$ is the nuclear radius and $a\cong b$ are of order of unity. The approximate equality $a\cong b$ implies that when pions interact with nuclei, the pions actually "see" the size of the nuclei as high-energy electrons would do and the form factors fall off accordingly as $q^2$ increases. Furthermore, if the transition radii in the form factors are not very different from the radius of the stable nucleus, then $a$ and $b$ are of order of unity.

For the natural-parity transitions such as $1^-\to0^+$, $2^+\to0^+$, ..., the ratio $(F_P/F_A)$ is, as in the case of allowed transition, of order of unity[4] for $|q^2|\ll m_\pi{}^2$. Thus, $f_{\pi if}(q^2)$ and $F_A(q^2)$ will have a similar $q^2$ dependence.

In the case of the unnatural-parity transitions such as $0^-\to0^+$, $\frac{1}{2}^-\to\frac{1}{2}^+$, ..., as we shall see below, the ratio $(F_P/F_A)$ is no longer of order of unity for $|q^2|\ll m_\pi{}^2$, but instead is significantly larger than unity. This leads to a drastic change of $q^2$ dependence of $f_{\pi if}(q^2)$ from that of $F_A(q^2)$.

## III. $0^-\to0^+$ TRANSITION

In this section we generalize the previous analysis carried out for allowed transitions to the case of the unnatural-parity forbidden transitions using the $0^-\to0^+$ transition as an example. We wish to investigate the $q^2$ dependence of $f_{\pi if}(q^2)$ and to estimate $K_{\pi if}(0)$. To this end we start by estimating the ratio $(F_P/F_A)$ in the impulse approximation.

The matrix element of the axial-vector current in the impulse approximation is

$$\begin{aligned}\langle f|A_\alpha^{(\pm)}(0)|i\rangle&=g_A(-\langle\vec\sigma e^{i\vec q\cdot\vec r}\rangle,\ i\langle\gamma_5e^{i\vec q\cdot\vec r}\rangle)\\&\quad-\frac{2m_pg_P'}{m_\pi{}^2+q^2}\langle\gamma_4\gamma_5e^{i\vec q\cdot\vec r}\rangle(\vec q,\ iq_0),\end{aligned}\tag{11}$$

where we have used the definitions

$$A_\alpha\equiv(\vec A,\ iA_0), \tag{12}$$

$$\langle\mathcal{O}e^{i\vec q\cdot\vec r}\rangle=\left\langle\psi_f\left|\sum_{a=1}^{A}\mathcal{O}^{(a)}\tau_\pm^{(a)}e^{i\vec q\cdot\vec r^{(a)}}\right|\psi_i\right\rangle, \tag{13}$$

and $g_P'$ is the nucleon-induced pseudoscalar form factor with the pion pole taken out, and $g_A\cong-g_P'$ for small $q^2$. For the $0^-\to0^+$ transition, we have

$$\begin{aligned}\langle\vec\sigma e^{i\vec q\cdot\vec r}\rangle&=\langle\vec\sigma\rangle+\langle\vec\sigma(i\vec q\cdot\vec r)\rangle+\cdots\\&=\tfrac{1}{3}\vec q\langle i\vec\sigma\cdot\vec r\rangle+\cdots,\\\langle\gamma_5e^{i\vec q\cdot\vec r}\rangle&=\langle\gamma_5\rangle+\cdots,\\\langle\gamma_4\gamma_5e^{i\vec q\cdot\vec r}\rangle&=\langle\gamma_4\gamma_5\rangle+\cdots,\end{aligned}\tag{14}$$

and hence, Eq. (11) becomes

$$\begin{aligned}\langle f|A_\alpha^{(\pm)}(0)|i\rangle&\cong g_A(-\tfrac{1}{3}\langle i\vec\sigma\cdot\vec r\rangle\vec q,\ i\langle\gamma_5\rangle)\\&\quad-\frac{2m_p}{m_\pi{}^2+q^2}g_P'\langle\gamma_4\gamma_5\rangle(\vec q,\ iq_0).\end{aligned}\tag{15}$$

On the other hand, Eq. (1) can be rewritten as, neglecting $|\vec q|/M$ terms,

$$\langle f | A_\alpha^{(\pm)}(0) | i \rangle \cong F_A(q^2)(0, i) + \frac{\Delta M}{m_\pi^2} F_P(q^2)(\vec{q}, iq_0) = \left( \frac{\Delta M F_P(q^2)}{m_\pi^2} \vec{q}, i \left[ F_A(q^2) - \left( \frac{\Delta M}{m_\pi} \right)^2 F_P(q^2) \right] \right), \tag{16}$$

where we have set $q_0 \cong -\Delta M$. Comparing Eqs. (15) and (16), we obtain the following relations:

$$F_P(q^2)\left(\frac{\Delta M}{m_\pi^2}\right) \cong -\tfrac{1}{3} g_A \langle i\vec{\sigma}\cdot\vec{r}\rangle - \frac{2m_p}{m_\pi^2 + q^2} g_P' \langle \gamma_4\gamma_5\rangle ,$$

$$F_A(q^2) - \left(\frac{\Delta M}{m_\pi}\right)^2 F_P(q^2) \cong g_A\langle\gamma_5\rangle + \frac{2m_p}{m_\pi^2+q^2} g_P' \Delta M \langle\gamma_4\gamma_5\rangle . \tag{17}$$

Since we have

$$g_A \cong -g_P'$$

and (18)

$$\langle\gamma_4\gamma_5\rangle \cong \frac{1}{2m_p}\langle\vec{\sigma}\rangle\cdot\vec{q} + \cdots ,$$

the $g_P'$ terms in Eq. (17) are always negligible compared to the first terms for $\Delta M \ll m_\pi$, and thus, from Eq. (17), we have

$$F_P(q^2) \cong -\tfrac{1}{3} g_A \left(\frac{m_\pi^2}{\Delta M}\right)\langle i\vec{\sigma}\cdot\vec{r}\rangle + \text{small } \vec{q}^2 \text{ terms},$$

$$F_A(q^2) \cong g_A\langle\gamma_5\rangle - \tfrac{1}{3} g_A \Delta M \langle i\vec{\sigma}\cdot\vec{r}\rangle + \text{small } \vec{q}^2 \text{ terms}. \tag{19}$$

Taking the ratio of $F_P(q^2)$ and $F_A(q^2)$ in Eq. (19), we finally obtain

$$\frac{F_P(q^2)}{F_A(q^2)} \cong \left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{1-(3\langle\gamma_5\rangle/\Delta M\langle i\vec{\sigma}\cdot\vec{r}\rangle)} . \tag{20}$$

The ratio $\langle\gamma_5\rangle/\langle i\vec{\sigma}\cdot\vec{r}\rangle$ can be estimated using the well-known Ahrens-Feenberg approximation.[5] It is given by[6]

$$\frac{\langle\gamma_5\rangle}{\langle i\vec{\sigma}\cdot\vec{r}\rangle} = \mp \frac{\Lambda\alpha Z}{2R} \text{ for } \beta^\mp \text{ decay}, \tag{21}$$

where $\Lambda = 1$ to 2 depending on the detailed models used. When the energy difference between states differing by a neutron-proton substitution is dominated by the Coulomb energy difference (Pursey's estimate[7]), $\Lambda \cong 2$ is expected. On the other hand, a partial cancellation of the electrostatic force by the nuclear force effects based on the semiempirical formula for the stable nuclear masses (Ahrens-Feenberg estimate[5]) yields $\Lambda \cong 1$.

Substituting Eq. (21) into Eq. (20), we obtain

$$\frac{F_P(q^2)}{F_A(q^2)} = \left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{1\pm\lambda} ,$$

$$\lambda \equiv \frac{3}{2}\frac{\Lambda\alpha Z}{R(\Delta M)} . \tag{22}$$

All the observed $0^- \to 0^+$ transitions[8] occur for the nuclear systems with $A = 144-206$ (with a range of $\Delta M = 0.83-3.5$ MeV), with an exception of the $0^- \to 0^+$ transition in the $A = 16$ system[9] [$^{16}N^*(0^-) \to {}^{16}O(0^+)$], and they are all $\beta^-$ decay. For heavy-nuclei cases, the values of $\lambda$ range from 5.8 to 48, using $\Lambda = 1$ for the minimum value and $\Lambda = 2$ for the maximum value. Then, the values of the ratio $(F_P/F_A)$ range from 127 to 1200. In the case of the $0^- \to 0^+$ transition in the $A = 16$ system, $(F_P/F_A) = 90-120$, depending on $\Lambda = 1$ or 2, using $\Delta M \cong 10$ MeV. It is clear then that the ratio $(F_P/F_A)$ for the $0^- \to 0^+$ transition is considerably larger than the value for the allowed and natural-parity forbidden ones, which is of order of unity.

Substitution of Eq. (22) into Eq. (6) gives

$$f_{\pi if}(q^2) = \frac{1}{a_\pi} F_A(q^2)\left(1 + \frac{q^2}{m_\pi^2}\right) \times \left[1 + \left(\frac{q^2}{m_\pi^2}\right)\left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{1\pm\lambda}\right]$$

$$\cong \frac{F_A(0)}{a_\pi}\left[1 + \frac{1}{1\pm\lambda}\left(\frac{q}{\Delta M}\right)^2\right] \text{ for } |q^2| \ll m_\pi^2 , \tag{23}$$

where we have assumed that $F_A(q^2)$ has the behavior given by Eq. (10), which is quite reasonable and is also supported by the impulse approximation. As shown in Eq. (23), the form factor $f_{\pi if}(q^2)$ can no longer be treated as a constant even for $\beta$ decay, where $|q^2| \sim m_e^2$, while it is always a good approximation to assume $F_A(q^2) \cong F_A(0)$ for $\beta$ decay; it is then clear that $f_{\pi if}(q^2)$ is a rapidly varying function of $q^2$ even for $|q^2| \ll m_\pi^2$ in contrast to $F_A(q^2)$.

In order to interpret the result of Eq. (23), we look at the once-subtracted dispersion relation for $f_{\pi if}(q^2)$:

$$f_{\pi if}(q^2) = f_{\pi if}(-m_\pi^2) + \frac{m_\pi^2}{\pi}\left(1 + \frac{q^2}{m_\pi^2}\right) \times \int_{m_{\rm an}^2}^{\infty} \frac{\mathrm{Im} f_{\pi if}(-m^2)\,dm^2}{m^2(m^2 - m_\pi^2)(1+q^2/m^2)} , \tag{24}$$

where $m_{\rm an}^2$ is the anomalous threshold due to nucleon break-up mechanism. For the single nucleon-nucleus break-up mechanism, the value of $m_{\rm an}^2$ is given by

$$m_{an}{}^2 \cong \frac{8A}{A-1} m_p \epsilon$$
$$\cong (1.7 m_\pi)^2, \tag{25}$$

where $A$ is the mass number and $\epsilon \cong 8$ MeV is the binding energy of a nucleon to the nucleus. Since $m_{an}{}^2$ is in general greater than $m_\pi{}^2$, the integral in Eq. (24) cannot be a rapidly varying function of $q^2$. As shown in Ref. (2), Eq. (24) may be rewritten as

$$f_{\pi if}(q^2) \cong f_{\pi if}(-m_\pi{}^2) + f_{\pi if}(-m_\pi{}^2)[K_{\pi if}(0) - 1] \times \left(1 + \frac{q^2}{m_\pi{}^2}\right)\left(1 - \beta \frac{q^2}{m_\pi{}^2}\right) \quad \text{for } |q^2| \lesssim \tfrac{1}{2} m_\pi{}^2, \tag{26}$$

where $K_{\pi if}(q^2)$ is, as previously mentioned in Sec. I,

$$K_{\pi if}(q^2) = \frac{f_{\pi if}(q^2)}{f_{\pi if}(-m_\pi{}^2)},$$

and

$$0 \lesssim \beta \lesssim \tfrac{1}{3}. \tag{27}$$

The value of $K_{\pi if}(0)$ may be used as a measure of the deviation of the soft-pion value of the coupling constant from the physical value. Deviation of $K_{\pi if}(0)$ from unity is also a direct measure of the extent of variation from the pion-pole dominance.

A direct comparison of Eqs. (23) and (26) (i.e., coefficients of the $q^2$ terms) gives

$$\frac{1}{1 \pm \lambda}\left(\frac{m_\pi}{\Delta M}\right)^2 = \left[\frac{K_{\pi if}(0) - 1}{K_{\pi if}(0)}\right](1 - \beta)$$

or

$$K_{\pi if}(0) = \left[1 - \frac{(m_\pi/\Delta M)^2}{(1 \pm \lambda)(1 - \beta)}\right]^{-1} \tag{28}$$

Since the magnitude of the second term in the denominator is, as shown previously, considerably large compared to unity, we have

$$|K_{\pi if}(0)| \cong \left(\frac{\Delta M}{m_\pi}\right)^2 |1 \pm \lambda|(1 - \beta) \ll 1, \tag{29}$$

which is, of course, consistent with the result that $f_{\pi if}(q^2)$ is a very rapidly varying function of $q^2$. In fact, $f_{\pi if}(0)$ is now very small compared to the physical value $f_{\pi if}(-m_\pi{}^2)$, smaller by one or two orders of magnitude. More explicitly, we find, from Eqs. (7) and (19), an order-of-magnitude estimate of $f_{\pi if}(0)$:

$$|f_{\pi if}(0)| = \frac{1}{a_\pi}|F_A(0)| \cong \frac{g_A}{a_\pi}|\langle\gamma_5\rangle| \sim \frac{g_A}{a_\pi}\left(\frac{m_\pi}{m_p}\right) \sim 10^{-1}. \tag{30}$$

On the other hand, we have

$$|f_{\pi if}(-m_\pi{}^2)| = \left|\frac{f_{\pi if}(0)}{K_{\pi if}(0)}\right| \cong \frac{g_A}{a_\pi}\left(\frac{m_\pi}{m_p}\right)\left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{|1 \pm \lambda|(1 - \beta)} \cong 10\text{–}100. \tag{31}$$

Since $|K_{\pi if}(0)| \ll 1$, the first and second terms in Eq. (26) have about the same magnitude with *opposite* sign for $|q^2| < m_\pi{}^2$. This is the reason why $f_{\pi if}(q^2)$ is such a rapidly varying function of $q^2$, in spite of the fact that the anomalous integral itself is not a rapidly varying function of $q^2$. Similarly, in the dispersion relation for the matrix element of the divergence of the axial-vector current, the pion-pole term and the remaining integral, whose lower limit is the anomalous threshold, have about the same magnitude with opposite sign for $|q^2| < m_\pi{}^2$. In other words, in the case of the $0^- \to 0^+$ transition, the non-pion-pole term is as large as the pion-pole term and the original Goldberger-Treiman relation, $F_A(0) = a_\pi f_{\pi if}(-m_\pi{}^2)$, is completely inaccurate.

We have also carried out a similar analysis for the case of the $\frac{1}{2}^- \to \frac{1}{2}^+$ transition with the same results. In general, for the $\Delta J^{P_i P_f} = 0^-$ transition (unnatural-parity transitions), when the matrix element of the axial-vector current is written in terms of the nuclear form factors using general invariance argument alone, the kinematical suppression is absent for the leading $F_A$ term as in the case of the $F_V$ term. However, since experiments always indicate a general suppression of the $F_A$ term (large $ft$-values), the value of $F_A$ should be small, which would then lead to a large value of the ratio $F_P/F_A$. Therefore, the various results discussed so far are general ones for the $\Delta J = 0$ with parity change. In other unnatural-parity transitions such as $\Delta J^{P_i P_f} = 2^-, 3^+, \ldots,$ the kinematical suppression always exists and the result is the same as that of natural-parity transitions.

## IV. SUMMARY

Before we summarize our results for the $\Delta J^{P_i P_f} = 0^-$ transition, we first recall that in other transitions such as allowed and natural-parity transitions, $f_{\pi if}(q^2)$ is a very smooth function of $q^2$ for $|q^2| \lesssim m_\pi^2$ and the $q^2$ dependence of $f_{\pi if}(q^2)$ is about the same as that of $F_A(q^2)$. Also, the deviation of $K_{\pi if}(0) = f_{\pi if}(0)/f_{\pi if}(-m_\pi^2)$ from unity is not too large, e.g., $K_{\pi if}(0) = 0.8$ and 0.6 for $^3He \to {}^3H$ and $^{12}C \to {}^{12}B$, respectively.

The results for the $\Delta J^{P_i P_f} = 0^-$ transition are as follows:

(1) The magnitude of $K_{\pi if}(0)$ is considerably smaller than unity,

$$|K_{\pi if}(0)| \ll 1 .$$

(2) As a consequence of (1), $f_{\pi if}(q^2)$ is a very rapidly varying function of $q^2$ for $|q^2| < m_\pi^2$. Even in the region $|q^2| \lesssim m_e^2$,

$$f_{\pi if}(q^2) \cong f_{\pi if}(0)\left[1 + \frac{1}{1 \pm \lambda}\left(\frac{q}{\Delta M}\right)^2\right] ,$$

where $\lambda$ is of order of 10. Recall that $\Delta M \sim O(m_e)$. On the other hand, $F_A(q^2)$ is still a smooth function of $q^2$.

(3) The pion-pole dominance is no longer valid. In fact the pion-pole term and the remaining contribution are about the same in magnitude but opposite in sign, making $\langle f | \partial_\alpha A_\alpha^{(\pm)}(0) | i \rangle$ a rapidly varying function of $q^2$ for $|q^2| < m_\pi^2$. The correction to the original Goldberger-Treiman relation, $F_A(0) = a_\pi f_{\pi if}(-m_\pi^2)$, is as large as the leading term.

In conclusion we remark that the above results are the consequences of the validity of the Gell-Mann–Lévy version of PCAC applied directly to the $\Delta J^{P_i P_f} = 0^-$ transitions.

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