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Renormalization constants for scalar, pseudoscalar, and tensor currents*

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We calculate the renormalization constants describing nucleon and pion matrix elements of scalar, pseudoscalar, and tensor (S, P, T) current densities. For certain of the constants, expressions can be obtained using standard SU_3 and chiral $SU_3 \otimes SU_3$ methods. To get the remaining constants, we employ the quark model with spherically symmetric quark wave functions to relate the S, P, T renormalization constants to known parameters of the usual vector and axial-vector (V, A) currents. We also evaluate the renormalization constants using the MIT "bag" model quark wave functions. We summarize our results in tabular form, compare the results of the various calculational methods used, and attempt to estimate the accuracy of our predictions.

I. INTRODUCTION

A number of recent papers have examined the possibility that neutral currents may involve scalar, pseudoscalar, and tensor (S,P,T) weak couplings in addition to or in place of the usually assumed vector and axial-vector (V,A) Lorentz structures. In particular, expressions have been given for deep inelastic neutrino nucleon scattering[1,2] (using the quark parton model) and for various low-energy nuclear correlations,[1] assuming a completely general Lorentz structure for the weak neutral current. In order to make phenomenological studies of S,P,T weak neutral couplings which simultaneously use deep-inelastic information on the one hand, and exclusive channel or low-energy nuclear results on the other, it is essential to know the renormalization constants describing the nucleon and pion matrix elements of the S,P,T current densities. The purpose of this paper is to estimate these renormalization constants by using various dynamical models of hadron structure. Our results will be applied in a subsequent publication to a detailed analysis, using current-algebra techniques, of soft-pion production by a weak neutral current of arbitrary Lorentz structure.

Within a general quark-model framework, the currents which we study have the form (for S,P,V,A,T structures, respectively)

$$\begin{aligned}
&\mathfrak{F}_j=\bar{\psi}\,\tfrac{1}{2}\lambda_j\,\psi\,,\\
&\mathfrak{F}_j^5=\bar{\psi}\gamma_5\,\tfrac{1}{2}\lambda_j\,\psi\,,\\
&\mathfrak{F}_j^\lambda=\bar{\psi}\gamma^\lambda\,\tfrac{1}{2}\lambda_j\,\psi\,,\\
&\mathfrak{F}_j^{5\lambda}=\bar{\psi}\gamma^\lambda\gamma_5\,\tfrac{1}{2}\lambda_j\,\psi\,,\\
&\mathfrak{F}_j^{\lambda\eta}=\bar{\psi}\sigma^{\lambda\eta}\,\tfrac{1}{2}\lambda_j\,\psi\,,\\
&\qquad j=0,\ldots,8
\end{aligned}\tag{1}$$

with ψ being the quark field, $\sigma^{\lambda\eta}=(\frac{1}{2}i)[\gamma^\lambda,\gamma^\eta]$, $\lambda_0=(\frac{2}{3})^{1/2}$, and with $\lambda_{1,\ldots,8}$ being the usual SU_3 matrices. For describing $\Delta S=0$ neutral current effects, only the $j=0,3,8$ components of the above nonets are relevant. We write the nucleon matrix elements of these components as[3]

$$\begin{aligned}
&\langle N(p_2)|\,\mathfrak{F}_j\,|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)\,F_S^{(j)}(k^2)t_j u(p_1)\,,\\
&\langle N(p_2)|\,\mathfrak{F}_j^5\,|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)F_P^{(j)}(k^2)\gamma_5 t_j u(p_1)\,,\\
&\langle N(p_2)|\,\mathfrak{F}_j^\lambda\,|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)[F_1^{(j)}(k^2)\gamma^\lambda+iF_2^{(j)}(k^2)\sigma^{\lambda\eta}k_\eta]\,t_j u(p_1)\,,\\
&\langle N(p_2)|\,\mathfrak{F}_j^{5\lambda}|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)[g_A^{(j)}(k^2)\gamma^\lambda\gamma_5+h_A^{(j)}(k^2)k^\lambda\gamma_5]\,t_j u(p_1)\,,\\
&\langle N(p_2)|\,\mathfrak{F}_j^{\lambda\sigma}|N(p_1)\rangle=\mathfrak{N}_N\bar{u}(p_2)\left[T_1^{(j)}(k^2)\sigma^{\lambda\sigma}+\frac{iT_2^{(j)}(k^2)}{M_N}(\gamma^\lambda k^\sigma-\gamma^\sigma k^\lambda)+\frac{iT_3^{(j)}(k^2)}{M_N^{\,2}}(P^\lambda k^\sigma-P^\sigma k^\lambda)\right]t_j u(p_1)\\
&\qquad=\mathfrak{N}_N\bar{u}(p_2)\left[T_1^{(j)}(k^2)\sigma^{\lambda\sigma}+\frac{i\hat{T}_2^{(j)}(k^2)}{M_N}(\gamma^\lambda k^\sigma-\gamma^\sigma k^\lambda)+\frac{T_3^{(j)}(k^2)}{M_N^{\,2}}(\sigma^{\lambda\nu}k_\nu k^\sigma-\sigma^{\sigma\nu}k_\nu k^\lambda)\right]t_j u(p_1)\,,\\
&\hat{T}_2^{(j)}(k^2)\equiv T_2^{(j)}(k^2)+2T_3^{(j)}(k^2)\,,\\
&k=p_2-p_1,\quad P=p_2+p_1,\quad \mathfrak{N}_N=\left(\frac{M_N}{p_{20}}\,\frac{M_N}{p_{10}}\right)^{1/2},\\
&t_3=\tfrac{1}{2}\tau_3,\quad t_0=\tfrac{1}{2}(\tfrac{2}{3})^{1/2},\quad t_8=\tfrac{1}{2}(\tfrac{1}{3})^{1/2}\,.
\end{aligned}\tag{2}$$

In the above expression, τ_3 is the nucleon Pauli isospin matrix and the spinors $\bar{u}(p_2)$, $u(p_1)$ are understood to include nucleon isospinors. The vector and axial-vector form factors defined above are related to the standard nucleon form factors $F_{1,2}^{V,S}(k^2), g_A(k^2), h_A(k^2)$ by

$$F_{1,2}^{(3)}(k^2) = F_{1,2}^{V}(k^2), \quad g_A^{(3)}(k^2) = g_A(k^2),$$
$$F_{1,2}^{(8)}(k^2) = 3F_{1,2}^{S}(k^2), \quad h_A^{(3)}(k^2) = h_A(k^2). \tag{3}$$

The nonvanishing pion matrix elements of the scalar, pseudoscalar, and tensor currents are

$$\langle \pi^a(p_2)|\,\mathfrak{F}_j\,|\,\pi^b(p_1)\rangle = \mathfrak{N}_\pi F_{S\pi}^{(j)}(k^2)\delta^{ab}t_j, \quad j = 0, 8$$
$$\langle \pi^a(p_2)|\,\mathfrak{F}_3^{\lambda\sigma}\,|\,\pi^b(p_1)\rangle = \mathfrak{N}_\pi \frac{T_\pi^{(3)}(k^2)}{M_N}\epsilon^{ab3}(P^\lambda k^\sigma - P^\sigma k^\lambda), \tag{4}$$
$$\mathfrak{N}_\pi = \frac{1}{(2p_{10}2p_{20})^{1/2}}, \quad \epsilon^{123} = 1.$$

Our analysis will give values at $k^2 = 0$ (and, in certain cases, first derivatives at $k^2 = 0$) for the various form factors which appear in the above expressions. Effectively, the $k^2 = 0$ values are the strong interaction renormalization constants describing scalar, pseudoscalar, and tensor density couplings to nucleons and pions.

Two principal calculational methods are used in what follows. First, values for certain of the renormalization constants can be obtained by using standard SU_3 and chiral $SU_3 \otimes SU_3$ methods. For the remaining constants, we use the quark model with spherically symmetric quark wave functions to relate the S, P, T renormalization constants (and certain first derivatives at $k^2 = 0$) to known parameters of the usual V, A currents. We also give a direct calculation in the quark model using the specific quark wave functions obtained in the MIT "bag" model. Our calculational procedures are further briefly described in Sec. II below. Results of the computations are tabulated in Sec. III, while in Sec. IV we compare results obtained by the various calculational methods used and attempt to estimate the accuracy of our predictions.

II. CALCULATIONAL METHODS

A. SU_3 and chiral $SU_3 \otimes SU_3$ predictions

We begin by discussing those renormalization constants which can be determined within the framework of the Gell-Mann–Oakes–Renner (GMOR) model[4] for SU_3 and chiral $SU_3 \otimes SU_3$ breakdown. In this model, the strong interaction Hamiltonian has the form

$$\mathcal{H} = \mathcal{H}_0 + \kappa(\mathfrak{F}_0 + c\mathfrak{F}_8), \tag{5}$$

with $\mathcal{H}_0$ chiral $SU_3 \otimes SU_3$ symmetric and with $\kappa(\mathfrak{F}_0 + c\mathfrak{F}_8)$ a symmetry-breaking term.[4] The parameter κ has the dimension of mass, while the parameter c is determined by the pseudoscalar meson masses to have the value $c \approx -1.25$. Since κ is not fixed in the GMOR model,[5] we can only determine values of the scalar and pseudoscalar renormalization constants relative to any one of them, say, relative to $F_S^{(8)}(0)$.

We begin by getting relations for the scalar density renormalization constants. Within the scalar octet, SU_3 symmetry relates $F_S^{(3)}(0)/F_S^{(8)}(0)$ to $\alpha_{SS} \approx -0.44$, the $D/(D+F)$ value of the baryon octet semistrong mass splitting, giving

$$F_S^{(3)}(0)/F_S^{(8)}(0) = 1/(3 - 4\alpha_{SS}). \tag{6}$$

The ninth scalar renormalization constant $F_S^{(0)}(0)$ cannot be calculated by SU_3 symmetry, but can be related to the experimentally measurable pion-nucleon "σ term" parameter[6] $\sigma_{\pi NN}$ and the nucleon SU_3 mass splitting parameter Δm defined respectively by

$$\sigma_{\pi NN} = \tfrac{1}{3}(\sqrt{2} + c)\langle N|\sqrt{2}\,\kappa\,\mathfrak{F}_0 + \kappa\mathfrak{F}_8|N\rangle$$
$$\approx 45 \pm 20 \text{ MeV},$$
$$\Delta m = \langle N|\kappa\mathfrak{F}_8|N\rangle \tag{7}$$
$$= \frac{1}{c}\left[M_N - \tfrac{1}{2}(M_\Lambda + M_\Sigma)\right]$$
$$\approx 173 \text{ MeV},$$

giving

$$\frac{F_S^{(0)}(0)}{F_S^{(8)}(0)} = \frac{1}{2}\left[\frac{3\sigma_{\pi NN}/\Delta m}{\sqrt{2} + c} - 1\right]. \tag{8a}$$

We remark that if $F_S^{(0)}(0)$ and $F_S^{(8)}(0)$ were equal, as is predicted in the quark model, then Eq. (8a) would fix $\sigma_{\pi NN}$ to have the value

$$\sigma_{\pi NN} = \Delta m(\sqrt{2} + c)$$
$$\approx 28 \text{ MeV}. \tag{8b}$$

Finally, we consider the pion scalar coupling $F_{S\pi}^{(8)}(0)$, which can be evaluated relative to $F_S^{(8)}(0)$ by noting that

$$\frac{F_{S\pi}^{(8)}(0)}{F_S^{(8)}(0)} = \frac{2M_\pi\langle\pi|\kappa c\mathfrak{F}_8|\pi\rangle}{\langle N|\kappa c\mathfrak{F}_8|N\rangle}$$
$$= \frac{\tfrac{1}{2}(M_\eta{}^2 + M_\pi{}^2) - M_\pi{}^2}{\tfrac{1}{2}(M_\Lambda + M_\Sigma) - M_N}$$
$$= \frac{M_\pi{}^2}{\sqrt{2} + c}\,\frac{1}{\Delta m}, \tag{9}$$

where the final equality is obtained by using Eq. (7) and the GMOR relation $M_\eta{}^2/M_\pi{}^2 = (\sqrt{2} - c)/(\sqrt{2} + c)$.

To get relations for the pseudoscalar density renormalization constants, we consider axial-vector current divergences in the GMOR model. Taking first the divergence of $\mathfrak{F}_3^{5\lambda}$, we find

$$\partial_\lambda \mathfrak{F}_3^{5\lambda} = -i[F_3^5, \kappa(\mathfrak{F}_0 + c\mathfrak{F}_8)]$$
$$= i\,\frac{\sqrt{2}+c}{\sqrt{3}}\,\kappa\mathfrak{F}_3^5, \tag{10}$$

which when sandwiched between nucleon states implies that

$$2M_N g_A = \frac{\sqrt{2}+c}{\sqrt{3}}\,\kappa F_P^{(3)}(0), \tag{11}$$

with $g_A = g_A^{(3)}(0)$. Rewriting Eq. (7) for Δm as

$$\Delta m = \frac{1}{2}\frac{\kappa}{\sqrt{3}} F_S^{(8)}(0) \tag{12}$$

and dividing Eq. (11) by Eq. (12) we get

$$\frac{F_P^{(3)}(0)}{F_S^{(8)}(0)} = \frac{g_A}{\sqrt{2}+c}\,\frac{M_N}{\Delta m}\,. \tag{13}$$

Next we take the divergence of $\mathfrak{F}_8^{5\lambda}$, giving

$$\partial_\lambda \mathfrak{F}_8^{5\lambda} = -i[F_8^5, \kappa(\mathfrak{F}_0 + c\mathfrak{F}_8)]$$
$$= i\kappa\left[\frac{\sqrt{2}-c}{\sqrt{3}}\,\mathfrak{F}_8^5 + \frac{\sqrt{2}\,c}{\sqrt{3}}\,\mathfrak{F}_0^5\right], \tag{14}$$

which when sandwiched between nucleon states gives

$$2M_N g_A^{(8)}(0) = \frac{\sqrt{2}-c}{\sqrt{3}}\,\kappa F_P^{(8)}(0) + \frac{2c}{\sqrt{3}}\,\kappa F_P^{(0)}(0)\,. \tag{15}$$

Using

$$g_A^{(8)}(0) = g_A(3-4\alpha_A) \tag{16}$$

[where $\alpha_A \approx 0.66$ is the $D/(D+F)$ value of the baryon octet axial-vector vertex] and dividing by Eq. (12) gives the second relation

$$(\sqrt{2}-c)F_P^{(8)}(0) + 2cF_P^{(0)}(0) = (3-4\alpha_A)\frac{M_N g_A}{\Delta m}\,F_S^{(8)}(0)\,. \tag{17}$$

A second independent relation for $F_P^{(8)}(0)$ and $F_P^{(0)}(0)$ cannot be obtained in the GMOR model. We note, however, that if $F_P^{(8)}(0)$ and $F_P^{(0)}(0)$ were equal, as in the quark model, then Eq. (17) would reduce to

$$F_P^{(8)}(0) = (3-4\alpha_A)\,\frac{g_A}{\sqrt{2}+c}\,\frac{M_N}{\Delta m}\,F_S^{(8)}(0)$$
$$= (3-4\alpha_A)F_P^{(3)}(0)\,, \tag{18}$$

an analog of the SU_3 relation of Eq. (16). We remark finally that standard pion pole dominance arguments give for the induced pseudoscalar form factor $h_A^{(3)}(k^2)$ the expression

$$h_A^{(3)}(k^2) = \frac{2M_N g_A}{M_\pi^2 - k^2}\,, \tag{19}$$

from which we get

$$h_A^{(3)}(0) = \frac{2M_N g_A}{M_\pi^2}\,. \tag{20}$$

The formulas obtained in this section are listed in column 1 of Tables I and II.

B. Quark model predictions

We next turn to the quark model, within which we can calculate expressions for all of the scalar, pseudoscalar, and tensor renormalization constants, and for certain of the form-factor first derivatives as well. We use for the nucleon the standard spin–internal-symmetry wave functions of the nonrelativistic quark model,[7]

$$\begin{aligned}|p, s_z = \tfrac{1}{2}\rangle_{QM} = (\tfrac{1}{18})^{1/2}[&2|\mathcal{P}\uparrow\mathfrak{N}\downarrow\mathcal{P}\uparrow\rangle + 2|\mathcal{P}\uparrow\mathcal{P}\uparrow\mathfrak{N}\downarrow\rangle + 2|\mathfrak{N}\downarrow\mathcal{P}\uparrow\mathcal{P}\uparrow\rangle - |\mathcal{P}\uparrow\mathcal{P}\downarrow\mathfrak{N}\uparrow\rangle - |\mathcal{P}\uparrow\mathfrak{N}\uparrow\mathcal{P}\downarrow\rangle \\ &- |\mathcal{P}\downarrow\mathfrak{N}\uparrow\mathcal{P}\uparrow\rangle - |\mathfrak{N}\uparrow\mathcal{P}\downarrow\mathcal{P}\uparrow\rangle - |\mathfrak{N}\uparrow\mathcal{P}\uparrow\mathcal{P}\downarrow\rangle - |\mathcal{P}\downarrow\mathcal{P}\uparrow\mathfrak{N}\uparrow\rangle],\ \text{etc.},\end{aligned} \tag{21}$$

where p denotes the proton and $\mathcal{P}$, $\mathfrak{N}$ denote quarks. In treating the nucleon spatial wave function, we assume three colored quark triplets to be present, with the physical nucleon constructed as a color singlet.[8] The nucleon states are then completely antisymmetric in the color index, and so satisfy Fermi statistics with completely symmetric spatial wave functions, which we form from one-particle quark orbitals. For the quark orbitals in a nucleon we assume a spherically symmetric Dirac wave-function form:

$$\psi(\vec{r}) = \frac{\mathfrak{N}_q}{(4\pi)^{1/2}}\begin{pmatrix} iJ_0(r) \\ -\vec{\sigma}\cdot\hat{r}\,J_1(r)\end{pmatrix}\chi\,, \tag{22}$$

with J_0 and J_1 arbitrary functions of r, with χ being the quark Pauli spinor, and with the normalization constant $\mathfrak{N}_q$ fixed by the condition

$$1 = \int d^3r\,\psi^\dagger(\vec{r})\psi(\vec{r})$$
$$= \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}[J_0^2(r) + J_1^2(r)]\,. \tag{23}$$

TABLE I. Nucleon parameters.

Renormalization constant	SU_3 or chiral $SU_3 \otimes SU_3$ prediction	Quark-model prediction, in terms of $I_{1,\ldots,5}$	Quark-model phenomenological relation	Numerical value from column 3 in MIT model	Numerical value from column 4
$F_S^{(8)}(0)$	$\cdots$	$3(1-2I_2)$	$3(-\frac{1}{2}+\frac{9}{10}g_A)$	1.44	1.86
$F_S^{(0)}(0)$	$F_S^{(8)}(0)\frac{1}{2}\left(\frac{3\sigma_{\pi NN}/\Delta m}{\sqrt{2}+c}-1\right)$	$3(1-2I_2)$	$3(-\frac{1}{2}+\frac{9}{10}g_A)$	1.44	1.86
$F_S^{(3)}(0)$	$F_S^{(8)}(0)/(3-4\alpha_{SS})$	$1-2I_2$	$-\frac{1}{2}+\frac{9}{10}g_A$	0.48	0.62
$F_P^{(8)}(0)$	$\left\{(\sqrt{2}-c)F_P^{(8)}(0)+2cF_P^{(0)}(0)\right.$	$\frac{4}{3}M_N I_3$	μ_p	2.64	2.79
$F_P^{(0)}(0)$	$\left.=(3-4\alpha_A)\frac{M_N g_A}{\Delta m}F_S^{(8)}(0)\right\}$	$\frac{4}{3}M_N I_3$	μ_p	2.64	2.79
$F_P^{(3)}(0)$	$F_S^{(8)}(0)\frac{g_A}{\sqrt{2}+c}\frac{M_N}{\Delta m}$	$\frac{20}{9}M_N I_3$	$\frac{5}{3}\mu_p$	4.41	4.65
$F_1^{(8)}(0),\ F_1^{(0)}(0)$		3	3	3	3
$F_1^{(3)}(0)$		1	1	1	1
$F_2^{(8)}(0),\ F_2^{(0)}(0)$		$\frac{2}{3}I_3-\frac{3}{2M_N}$	$(\mu_p-3)/(2M_N)$	$-0.36/(2M_N)$	$-0.21/(2M_N)$
$F_2^{(3)}(0)$		$\frac{10}{9}I_3-\frac{1}{2M_N}$	$(\frac{5}{3}\mu_p-1)/(2M_N)$	$3.40/(2M_N)$	$3.65/(2M_N)$
$F_1^{(8)\prime}(0),\ F_1^{(0)\prime}(0)$		$\frac{1}{2}(I_1+I_5)-\frac{1}{3M_N}I_3+\frac{3}{4M_N^2}$	$\frac{1}{2}r_p^2-\frac{\mu_p}{4M_N^2}+\frac{3}{4M_N^2}$	$\frac{0.25}{M_\pi^2}$	$\frac{0.17}{M_\pi^2}$ [a]
$F_1^{(3)\prime}(0)$		$\frac{1}{6}(I_1+I_5)-\frac{5}{9M_N}I_3+\frac{1}{4M_N^2}$	$\frac{1}{6}r_p^2-\frac{5\mu_p}{12M_N^2}+\frac{1}{4M_N^2}$	$\frac{0.065}{M_\pi^2}$	$\frac{0.035}{M_\pi^2}$
$\mu_p \equiv M_N[F_2^{(3)}(0)+\frac{1}{3}F_2^{(8)}(0)]+1$		$\frac{4}{3}M_N I_3$	μ_p	2.64	Expt: 2.79
$r_p^2 \equiv 3F_1^{(3)\prime}(0)+\frac{3F_2^{(3)}(0)}{2M_N}+F_1^{(8)\prime}(0)+\frac{F_2^{(8)}(0)}{2M_N}$		I_4+I_5	r_p^2	$\frac{0.50}{M_\pi^2}$	Expt: $\frac{0.33}{M_\pi^2}=0.66\ \mathrm{F}^2$

(*Continued on following page*)

TABLE I (continued)

Renormalization constant	SU_3 or chiral $SU_3 \times SU_3$ prediction	Quark-model prediction, in terms of $I_{1,\ldots,5}$	Quark-model phenomenological relation	Numerical value from column 3 in MIT model	Numerical value from column 4
$g_A^{(8)}(0)$	$(3-4\alpha_A)g_A$	$1-\frac{4}{3}I_2$	$\frac{3}{5}g_A$	0.65	0.74
$g_A^{(0)}(0)$		$1-\frac{4}{3}I_2$	$\frac{3}{5}g_A$	0.65	0.74
$g_A^{(3)}(0)$		$\frac{5}{3}(1-\frac{4}{3}I_2)$	g_A	1.09	Expt: 1.24
$h_A^{(3)}(0)$	$\frac{2M_N}{M_\pi^2}g_A$	$\frac{4}{9}M_N I_5$	$\frac{5}{3}M_N(\frac{1}{6}r_p^2-\frac{3}{5}g_A')$	$\frac{2M_N}{M_\pi^2}\,0.037$	$\frac{2M_N}{M_\pi^2}\,0.017$
$g_A^{(3)\prime}(0)$		$\frac{5}{3}(\frac{1}{6}I_4-\frac{1}{10}I_5)$	g_A'	$\frac{0.066}{M_\pi^2}$	Expt: $\frac{2.48}{(0.90\text{ GeV})^2}=\frac{0.060}{M_\pi^2}$
$g_A^{(8)\prime}(0)/g_A^{(8)}(0)$, $g_A^{(0)\prime}(0)/g_A^{(0)}(0)$			g_A'/g_A		
$T_1^{(8)}(0)$, $T_1^{(0)}(0)$		$1-\frac{2}{3}I_2$	$\frac{1}{2}+\frac{3}{10}g_A$	0.83	0.87
$T_1^{(3)}(0)$		$\frac{5}{3}(1-\frac{2}{3}I_2)$	$\frac{5}{3}(\frac{1}{2}+\frac{3}{10}g_A)$	1.38	1.45
$T_2^{(8)}(0)$, $T_2^{(0)}(0)$		$-\frac{4}{15}M_N^2 I_5$	$M_N^2[-\frac{1}{6}r_p^2+\frac{3}{5}g_A']$	-1.98	-0.88
$T_2^{(3)}(0)$		$-\frac{4}{9}M_N^2 I_5$	$\frac{5}{3}M_N^2[-\frac{1}{6}r_p^2+\frac{3}{5}g_A']$	-3.30	-1.48
$T_3^{(8)}(0)$, $T_3^{(0)}(0)$		$\frac{1}{2}[-2M_N I_3+\frac{4}{15}M_N^2 I_5+\frac{1}{2}-\frac{1}{3}I_2]$	$\frac{1}{2}[-\frac{3}{2}\mu_p-T_2^{(8)}(0)+\frac{1}{2}T_1^{(8)}(0)]$	-0.78	-1.44
$T_3^{(3)}(0)$		$\frac{1}{2}[-\frac{2}{3}M_N I_3+\frac{4}{9}M_N^2 I_5+\frac{5}{6}-\frac{5}{9}I_2]$	$\frac{1}{2}[-\frac{1}{2}\mu_p-T_2^{(3)}(0)+\frac{1}{2}T_1^{(3)}(0)]$	1.34	0.41
$\hat{T}_2^{(8)}(0)$, $\hat{T}_2^{(0)}(0)$		$-2M_N I_3+\frac{1}{2}-\frac{1}{3}I_2$	$-\frac{3}{2}\mu_p+\frac{1}{2}T_1^{(8)}(0)$	-3.54	-3.75
$\hat{T}_2^{(3)}(0)$		$-\frac{2}{3}M_N I_3+\frac{5}{6}-\frac{5}{9}I_2$	$-\frac{1}{2}\mu_p+\frac{1}{2}T_1^{(3)}(0)$	-0.62	-0.66
$T_1^{(3)\prime}(0)$		$\frac{5}{3}(\frac{1}{6}I_4-\frac{1}{30}I_5)$	$\frac{5}{3}[\frac{1}{24}r_p^2+\frac{9}{20}g_A']$	$0.085/M_\pi^2$	$0.068/M_\pi^2$ [b]
$T_1^{(8)\prime}(0)/T_1^{(8)}(0)$, $T_1^{(0)\prime}(0)/T_1^{(0)}(0)$			$T_1^{(3)\prime}(0)/T_1^{(3)}(0)$		

[a] If $F_1^{(8)}(k^2)$ is parameterized in the dipole form $F_1^{(8)}(k^2)/F_1^{(8)}(0)=(1-k^2/M_S^2)^{-2}$, then this value for $F_1^{(8)\prime}(0)$ corresponds to $M_S=0.84$ GeV.

[b] If $T_1^{(3)}(k^2)$ is parameterized in the dipole form $T_1^{(3)}(k^2)/T_1^{(3)}(0)=(1-k^2/M_T^2)^{-2}$, then this value for $T_1^{(3)\prime}(0)$ corresponds to $M_T=0.91$ GeV.

TABLE II. Pion parameters.

Renormalization constant	SU_3 or chiral $SU_3 \otimes SU_3$ prediction	Quark-model prediction, in terms of $I_{1,\ldots,5}$	Quark-model phenomenological relation	Comment
$F_{S\pi}^{(8)}(0)$	$\frac{M_\pi^2}{\sqrt{2}+c}\frac{1}{\Delta m}F_S^{(8)}(0)$	$\frac{4}{3}M_M 3(1-2I_2)$	$\frac{4}{3}M_M F_S^{(8)}(0)$	Equating columns 2 and 4 $\Rightarrow$ $M_M = 0.53$ GeV
$F_{S\pi}^{(0)}(0)$		$\frac{4}{3}M_M 3(1-2I_2)$	$\frac{4}{3}M_M F_S^{(8)}(0)$	
$T_\pi^{(3)}(0)$		$-\frac{2M_N}{3}fI_3$	$-\frac{1}{2}f\mu_p$	For $f=(\frac{2}{3})^{1/4}$, columns 3 and 4 give -1.19 and -1.26, respectively.

The procedure for calculating nucleon renormalization constants is now completely straightforward.[9] We consider the general quark-model current $\mathfrak{F}_\Gamma = \bar{\psi}\Gamma\psi$ (Γ is a combination of γ and λ matrices) with one-nucleon matrix element

$$\langle N(p_2)|\mathfrak{F}_\Gamma|N(p_1)\rangle = \mathfrak{N}_N \bar{u}(p_2)K_\Gamma(p_2,p_1)u(p_1)\,. \tag{24}$$

Working in the brick-wall frame with

$$\vec{p}_1 = -\tfrac{1}{2}\vec{k},\quad \vec{p}_2 = \tfrac{1}{2}\vec{k}$$
$$p_{10} = p_{20} = [M_N^2 + \tfrac{1}{4}\vec{k}^2]^{1/2}, \tag{25}$$

and using our independent-orbital construction of the nucleon wave function, we get the relation

$$\mathfrak{N}_N \bar{u}(p_2)K_\Gamma(p_2,p_1)u(p_1) = {}_{QM}\Big\langle N\Big|\sum_{\text{quarks}}\mathfrak{M}_\Gamma(\vec{k})\Big|N\Big\rangle_{QM}, \tag{26}$$

with $\mathfrak{M}_\Gamma$ a matrix in the quark spin–internal-symmetry space given by

$$\mathfrak{M}_\Gamma(\vec{k}) = \int d^3r\, e^{i\vec{k}\cdot\vec{r}}\,\frac{\mathfrak{N}_q^2}{4\pi}\,(-\,iJ_0(r),\ \vec{\sigma}\cdot\hat{r}J_1(r)) \times\Gamma\begin{pmatrix} iJ_0(r) \\ -\vec{\sigma}\cdot\hat{r}J_1(r)\end{pmatrix}. \tag{27}$$

Taylor-expanding $e^{i\vec{k}\cdot\vec{r}}$ and equating terms of zeroth, first, and second order in $\vec{k}$ on the left- and right-hand sides of Eq. (26), we get formulas at zero momentum transfer for the form factors appearing in $K_\Gamma(p_2,p_1)$, expressed in terms of integrals over the quark wave function. [In evaluating the order $\vec{k}^2$ relations we drop nucleon recoil terms of order $\vec{k}^2/(8M_N^2)$ on the left-hand side of Eq. (26); these terms are relatively small and do not represent a well-defined correction since a description of nucleon recoil has not been built into the quark-model wave functions.] The quark wave-function integrals which appear are linear combinations of the five basic integrals

$$I_1 = \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}J_0^2(r),$$
$$I_2 = \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}J_1^2(r),$$
$$I_3 = \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}rJ_0(r)J_1(r), \tag{28}$$
$$I_4 = \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}r^2J_0^2(r),$$
$$I_5 = \int d^3r\,\frac{\mathfrak{N}_q^2}{4\pi}r^2J_1^2(r).$$

Expressions for the nucleon scalar, pseudoscalar, vector, axial-vector, and tensor renormalization constants and certain form factor derivatives, in terms of $I_1, \ldots, I_5$, are given in column 2 of Table I. Eliminating the integrals $I_{1,\ldots,5}$ in terms of the normalization condition of Eq. (23) and four experimentally measured parameters of the vector and axial-vector currents [we take these as g_A, $g_A' \equiv g_A'(0)$, r_p^2 = proton squared charge radius,[10] $\mu_p/(2M_N)$ = proton magnetic moment] gives the phenomenological relations listed in column 3 of Table I. These relations are valid in any quark model with a spherically symmetric wave function of the form of Eq. (22); for example, they are valid in both the MIT[9] and the SLAC[11] bag models and in the Bogoliubov model,[12] even though these assign the quarks very different looking wave functions.

The procedure for calculating pion renormalization constants is analogous to that used for the nucleon, with a few differences which we briefly describe. Just as for the nucleon, we use for the pion the usual spin–internal-symmetry wave functions of the nonrelativistic quark model,[7]

$$|\pi^+\rangle_{QM} = (\tfrac{1}{2})^{1/2}[|\bar{\mathfrak{N}}\uparrow\mathcal{P}\downarrow\rangle - |\bar{\mathfrak{N}}\downarrow\mathcal{P}\uparrow\rangle], \quad \text{etc.} \tag{29}$$

For the quark wave function we use an analog of Eq. (22),

$$\psi(\vec{r}) = \frac{\mathfrak{N}_q f^{-3/2}}{(4\pi)^{1/2}} \begin{pmatrix} iJ_0(r/f) \\ -\vec{\sigma}\cdot\hat{r}J_1(r/f) \end{pmatrix}\chi, \tag{30}$$

with f being a rescaling factor which reflects the fact that quark orbitals in a pion may have a different radius from those in a nucleon. In the MIT bag model[9] f has the value $(\frac{2}{3})^{1/4} \approx 0.90$, not much different from unity. The antiquark wave function is the same as Eq. (30), with the antiquark contribution to a current with even (odd) charge conjugation equal to $+1$ (-1) times the corresponding quark contribution. The pion analog of Eqs. (24)–(27) is evidently

$$\begin{aligned} \langle\pi(p_2)|\mathfrak{F}_\Gamma|\pi(p_1)\rangle &= \mathfrak{N}_\pi K'_\Gamma(p_2,p_1) \\ &= {}_{QM}\Big\langle\pi\Big|\sum_{\text{quarks}}\mathfrak{M}_\Gamma(f\vec{k})\Big|\pi\Big\rangle_{QM}, \\ \mathfrak{N}_\pi &= \frac{1}{2(M_\pi^2+\frac{1}{4}\vec{k}^2)^{1/2}} \end{aligned} \tag{31}$$

with $\mathfrak{M}_\Gamma$ being the same matrix function as defined in Eq. (27). In applying Eq. (31) we only expand out to terms of first order in $\vec{k}$, since neglect of recoil in the case of the pion would be unjustified. To order $\vec{k}$, the normalization factor $\mathfrak{N}_\pi$ is just $1/(2M_\pi)$. In the case of the tensor density coupling to the pion this factor of M_π^{-1} is just cancelled by a corresponding factor of M_π coming from K'_Γ, giving a formula for $T_\pi^{(3)}(0)$ which does not involve the pion mass. On the other hand, in the scalar density case the factor M_π^{-1} survives, giving the relation

$$F_{S\pi}^{(8)}(0) = 4M_\pi(1-2I_2), \tag{32}$$

which explicitly involves the pion mass. Since, however, the quark model leads to a degenerate meson 35-plet, instead of having a nearly massless pion, we reinterpret the factor M_π in Eq. (32) as being M_M, a typical quark-model meson mass, and thus write

$$F_{S\pi}^{(8)}(0) = 4M_M(1-2I_2). \tag{33}$$

As we will see below in Sec. IV, this interpretation of Eq. (32) is in accord with the chiral $SU_3 \otimes SU_3$ formula for $F_{S\pi}^{(8)}(0)$ obtained above. The results of our analysis in the pion case are given in column 2 of Table II (in terms of the integrals $I_{1,\ldots,5}$) and in column 3 of Table II (in terms of vector and axial-vector current parameters).

We conclude this section by giving expressions for the quark orbitals and the integrals $I_{1,\ldots,5}$ in the MIT bag model,[9] which gives a fairly satisfactory account of the measurable parameters of the vector and axial-vector currents. In this model the quarks in a nucleon are confined to a finite spherical region of space of radius R_0, with orbitals

$$\begin{aligned} &J_0(r) = j_0(\omega r/R_0), \quad J_1(r) = j_1(\omega r/R_0), \quad r \le R_0 \\ &J_0(r) = J_1(r) = 0, \quad r \ge R_0, \\ &\quad \omega = 2.04, \quad R_0 = 0.97M_\pi^{-1}, \\ &\quad j_0(z) = \frac{\sin z}{z}, \quad j_1(z) = \frac{\sin z}{z^2} - \frac{\cos z}{z}. \end{aligned} \tag{34}$$

Evaluating the integrals $I_{1,\ldots,5}$ we find in the MIT model

$$\begin{aligned} I_1 &= \frac{2\omega-1}{4(\omega-1)} = 0.740, \\ I_2 &= \frac{2\omega-3}{4(\omega-1)} = 0.260, \\ I_3 &= \frac{R_0}{\omega}\,\frac{4\omega-3}{8(\omega-1)} = 0.304R_0, \\ I_4 &= \frac{R_0^2}{24\omega^2(\omega-1)}(4\omega^3+2\omega^2-4\omega+3) \\ &= 0.357R_0^2, \\ I_5 &= \frac{R_0^2}{24\omega^2(\omega-1)}(4\omega^3-10\omega^2+20\omega-15) \\ &= 0.175R_0^2. \end{aligned} \tag{35}$$

III. TABULATION OF RESULTS

In Tables I and II we tabulate our results for the form factors defined in Eq. (2). To recapitulate, the quantities c, Δm, α_{SS}, α_A, and $\sigma_{\pi NN}$, defined above in Sec. II A, have the values

$$\begin{aligned} &c \approx -1.25, \\ &\Delta m \approx 173 \text{ MeV}, \\ &\alpha_{SS} \approx -0.44, \\ &\alpha_A \approx 0.66, \\ &\sigma_{\pi NN} \approx 45 \pm 20 \text{ MeV}, \end{aligned} \tag{36}$$

while the integrals $I_{1,\ldots,5}$ are defined and evaluated in Eqs. (28) and (35). The mass M_M, a typical quark-model meson mass introduced in Eq. (33), is of order 0.6–0.8 GeV while the scale factor f introduced in Eq. (30) is close to unity, with the value $(\frac{2}{3})^{1/4} \approx 0.90$ in the MIT model.[13]

IV. DISCUSSION

We conclude by comparing the results obtained by the various calculational methods described above and by attempting to estimate the reliability of our predictions for the scalar, pseudoscalar, and tensor current parameters. We turn our attention first to the isovector pseudoscalar renormalization $F_P^{(3)}(0)$ and the isovector induced pseudoscalar amplitude $h_A^{(3)}(0)$, both of which are pion pole dominated. From chiral $SU_3 \otimes SU_3$ and pion pole dominance we find

$$\frac{F_P^{(3)}(0)}{F_S^{(8)}(0)} = \frac{g_A}{\sqrt{2}+c}\,\frac{M_N}{\Delta m} = 41\,,$$
$$h_A^{(3)}(0) = \frac{2M_N g_A}{M_\pi^{\,2}} = \frac{2M_N}{M_\pi^{\,2}}\,1.24\,, \tag{37}$$

while the MIT model gives[14]

$$\frac{F_P^{(3)}(0)}{F_S^{(8)}(0)} = 3.1\,, \quad h_A^{(3)}(0) = \frac{2M_N}{M_\pi^{\,2}}\,0.037\,, \tag{38}$$

both much too small. Evidently, the quark-model predictions for pion pole dominated pseudoscalar quantities behave as if the effective pion mass were

$$\left(\frac{41}{3.1}\right)^{1/2} M_\pi = 0.51 \text{ GeV from } F_P^{(3)}(0)\,,$$
$$\left(\frac{1.24}{0.037}\right)^{1/2} M_\pi = 0.81 \text{ GeV from } h_A^{(3)}(0)\,, \tag{39}$$

not unreasonable values since the quark model does not predict an almost massless pion, but rather gives a pion degenerate with all other pseudoscalar and vector mesons in the 35 representation of SU_6. [In fact, the same MIT model calculation giving the value $f = (\frac{2}{3})^{1/4}$ used in Eq. (30) above leads to a value of the 35 representation central mass of $8\omega/(3fR_0) = 0.87$ GeV, consistent with the above estimates.] Referring to Table II, we see that these values for the effective quark-model pion mass are compatible with the value 0.53 GeV obtained by equating the chiral $SU_3 \otimes SU_3$ with the quark-model predictions for the pion scalar density coupling $F_{S\pi}^{(8)}(0)$.

We consider next the isoscalar pseudoscalar renormalization constants $F_P^{(8)}(0)$ and $F_P^{(0)}(0)$. As we have seen, chiral $SU_3 \otimes SU_3$ gives a single equation [Eq. (17)] relating these two constants to $F_S^{(8)}(0)$, which reduces, when $F_P^{(8)}(0)$ and $F_P^{(0)}(0)$ are equal (as in the quark model), to the simple relation

$$\frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = (3-4\alpha_A)\,\frac{F_P^{(3)}(0)}{F_S^{(8)}(0)} = 15\,. \tag{40}$$

This prediction is evidently in serious disagreement with the quark-model value

$$\frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = 1.5\,. \tag{41}$$

The trouble here is most likely the quark-model prediction that $F_P^{(0)}(0) = F_P^{(8)}(0)$, which leads to near cancellation of the two terms on the left-hand side of Eq. (17) and hence to a large prediction for $F_P^{(8)}(0)$. In actual fact, since there is no light ninth pseudoscalar meson associated with the SU_3-singlet axial-vector current it is likely that $F_P^{(0)}(0) < F_P^{(8)}(0)$. Rewriting Eq. (17) in terms of the ratio

$$r = \frac{F_P^{(0)}(0)}{F_P^{(8)}(0)} \tag{42}$$

we find

$$\frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = \frac{(3-4\alpha_A)M_N g_A}{\Delta m(\sqrt{2} - c + 2cr)}\,, \tag{43}$$

which gives the following predictions for $r = 0.3, 0.5, 0.7$ respectively:

$$r = 0.3{:}\quad \frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = 1.27\,,\quad \frac{F_P^{(0)}(0)}{F_S^{(8)}(0)} = 0.38\,,$$
$$r = 0.5{:}\quad \frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = 1.71\,,\quad \frac{F_P^{(0)}(0)}{F_S^{(8)}(0)} = 0.86\,, \tag{44}$$
$$r = 0.7{:}\quad \frac{F_P^{(8)}(0)}{F_S^{(8)}(0)} = 2.65\,,\quad \frac{F_P^{(0)}(0)}{F_S^{(8)}(0)} = 1.85\,,$$

in reasonable agreement with the quark-model value of Eq. (41).

Continuing our comparison of columns 1 and 2 of Table I, we note that SU_3 predicts

$$F_S^{(8)}(0)/F_S^{(3)}(0) = 3 - 4\alpha_{SS} \approx 4.76\,, \tag{45}$$

with an empirical semistrong $D/(D+F)$ ratio $\alpha_{SS} \approx -0.44$, while the quark model gives

$$F_S^{(8)}(0)/F_S^{(3)}(0) = 3\,. \tag{46}$$

Evidently, Eq. (46) represents pure F-type baryon octet semistrong mass splitting, a feature which is a well-known shortcoming of the quark model. For the corresponding axial-vector coupling ratio SU_3 predicts

$$g_A^{(8)}(0)/g_A^{(3)}(0) = 3 - 4\alpha_A \tag{47}$$

with an empirical value $\alpha_A \approx 0.66$, while the quark model gives

$$g_A^{(8)}(0)/g_A^{(3)}(0) = 3/5\,, \tag{48}$$

corresponding to a value of α_A of 0.6. Although the quark-model value of α_A is quite good in this case, the fact that Eq. (47) vanishes for $\alpha_A = 0.75$ makes the predicted $g_A^{(8)}$ in the quark model differ by more than 60% from the value obtained from SU_3 and the empirical α_A. Obviously, in doing phenomenological calculations the predictions of column 1 of Table II should be used (where they are available) in preference to the quark-model values.

For the value of $F_S^{(8)}(0)$ and for all of the tensor density parameters, we must rely solely on quark-model predictions since no information is furnished by SU_3 or chiral $SU_3 \otimes SU_3$ alone. Hence it is essential to have some *a priori* estimate of the reliability of the quark-model predictions.

The following five considerations would appear to be important in forming such an estimate.

1. Consistency of the quark model with SU_3 and chiral $SU_3 \otimes SU_3$ predictions, where available. This question has just been discussed in detail above. In the case of $F_S^{(8)}(0)$, the 60% discrepancy between Eq. (45) and Eq. (46) suggests an estimate of 60–90% for the possible quark model uncertainty.

2. Comparison of the quark-model predictions for the vector and axial-vector parameters with their known experimental values. Referring to Table I, we see that the MIT-model predictions for g_A, g_A', μ_p, and r_p^2 all agree with experiment[15] to within about 30%, suggesting 30–60% as the general level of reliability for quark-model predictions when other factors (such as pion pole dominance, sensitive cancellations, nucleon recoil corrections, or possible large "glue" contributions) are not involved. In particular, this estimate of the quark-model uncertainty might be expected to apply to the tensor renormalization constant $T_1^{(3)}(0)$.[16]

3. Consistency between the predictions in the final two columns in Table I. Column 5, we recall, gives the predictions of the MIT-model wave functions, while column 6 gives the predictions obtained from the quark-model phenomenological relations of column 4, using as input the empirical values of g_A, g_A', μ_p, and r_p^2. Sensitive cancellations are unlikely to be involved in cases in which the quark-model predictions are relatively large and relatively unvarying from column 5 to column 6, as for example, for $T_1^{(3)}(0)$. On the other hand, when the quark-model predictions are small or strongly varying from column 5 to column 6, as for $T_1^{(8,0)}(0)$, $\hat{T}_2^{(3)}(0)$, and $T_3^{(8,0,3)}(0)$, they may be considerably less reliable than the 30–60% estimated above.

4. Possible importance of neglected nucleon recoil terms. Whereas $F_S^{(8)}(0)$, $T_1^{(8,0,3)}(0)$, and $\hat{T}_2^{(8,0,3)}(0)$ are true static quantities which are insensitive to our neglect of nucleon recoil, expressions for the renormalization constants $T_3^{(8,0,3)}(0)$ are obtained from the second-order term in $\vec{k}$ in Eq. (27) only when nucleon recoil ambiguities are neglected. This introduces an additional source of uncertainty in the quark-model determination of $T_3^{(8,0,3)}(0)$ relative to the uncertainties present in the quark-model determinations of the other renormalization constants.

5. Possible presence of large "glue" contributions. In the quark model only quark contributions to the various current densities are evaluated, while possible contributions from the "glue" which binds the quarks together are ignored. One peculiar feature of tensor densities is the possibility of induced vector meson couplings of the form

$$gF^{\lambda\eta} = g(\partial^\eta A^\lambda - \partial^\lambda A^\eta), \tag{49}$$

with A being a vector meson field. Such couplings can contribute to the induced tensor renormalization constants $T_2(0)$ and $T_3(0)$, while not affecting the value of $T_1(0)$. If all vector gluons carry a color quantum number, then terms like Eq. (49) will be absent in the color-singlet tensor densities of Eq. (1). In this case, the quark-model predictions for $\hat{T}_2^{(0)}(0)$ and $\hat{T}_2^{(8)}(0)$ should, like that for $T_1^{(3)}(0)$, be relatively reliable. On the other hand, if color-singlet–unitary-singlet gluons are present, then the unitary-singlet tensor current $\mathfrak{F}_0^{\lambda\eta}$ could receive important "gluon" contributions from terms of the form of Eq. (49), introducing a possible large uncertainty into the quark-model prediction for $\hat{T}_2^{(0)}(0)$.

Added note. Applying the method of Eqs. (10)–(15) to the ninth axial-vector current $\mathfrak{F}_0^{5\lambda}$ gives the divergence equation

$$\partial_\lambda \mathfrak{F}_0^{5\lambda} = i\kappa(\tfrac{2}{3})^{1/2}(\mathfrak{F}_0^5 + c\mathfrak{F}_8^5), \tag{50}$$

which when sandwiched between nucleon states gives

$$2M_N g_A^{(0)}(0) = \frac{\kappa}{\sqrt{3}}\left[\sqrt{2}\ F_P^{(0)}(0) + cF_P^{(8)}(0)\right]. \tag{51}$$

Dividing Eq. (51) by Eq. (15) then gives the additional chiral $SU_3 \otimes SU_3$ relation

$$\frac{g_A^{(0)}(0)}{g_A^{(8)}(0)} = \frac{\sqrt{2}\,r + c}{\sqrt{2} - c + 2cr}, \tag{52}$$

with $r = F_P^{(0)}(0)/F_P^{(8)}(0)$ being the parameter defined in Eq. (42). For $r = 0.3, 0.5, 0.7$, Eq. (52) gives the respective predictions for $g_A^{(0)}(0)/g_A^{(8)}(0)$,

$$\begin{aligned} r = 0.3:&\quad \frac{g_A^{(0)}(0)}{g_A^{(8)}(0)} = -0.43, \\ r = 0.5:&\quad \frac{g_A^{(0)}(0)}{g_A^{(8)}(0)} = -0.38, \\ r = 0.7:&\quad \frac{g_A^{(0)}(0)}{g_A^{(8)}(0)} = -0.28, \end{aligned} \tag{53}$$

while for the quark-model value $r = 1$, Eq. (52) reduces to the quark-model prediction that $g_A^{(0)}(0)/g_A^{(8)}(0) = 1$. Equations (50)–(53) are valid only when anomalies are not present. When anomalies appear, the above equations apply to the axial-vector renormalization $g_A^{(0)}(0)$ associated with the "symmetry generating" ninth current, but this is no longer the same as the axial-vector renormalization for the physical ninth axial-vector current. (See W. A. Bardeen, Ref. 17).

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[5]In the quark model κ is fixed to have the value $\kappa = (12)^{1/2} \Delta m / F_S^{(8)}(0) = 322$ MeV. [See Eqs. (7) and (12) and the entry for $F_S^{(8)}(0)$ in column 6 of Table I.]

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[14]The quark-model phenomenological relations give similar predictions.

[15]The discrepancies in the case of g'_A and r_p^2 are both in the same direction, suggesting that the MIT wave functions make the nucleon somewhat too large.

[16]In the theoretical framework of the transformation between current and constituent quarks (abstracted from the free quark model), there is a relation between $g_A(0)$ and $T_1(0)$, since the axial charge and certain components of the tensor charge are generators of SU_{6W} currents. However, the relation appears to be sufficiently arbitrary as to be consistent with the results quoted in Table I.

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