PHYSICAL REVIEW D VOLUME 11, NUMBER 11 1 JUNE 1975

Models for weak currents with Han-Nambu three-triplets*

C. Saclioglu
The Enrico Fermi Institute and the Department of Physics, The University of Chicago, Chicago, Illinois 60637
(Received 5 November 1974)

A mechanism introduced by Stech to suppress $\Delta S = 1$ neutral currents is adapted to the Han-Nambu model. Consequences of different schemes arising from the freedom in baryon number assignment are investigated. Though first order processes are eliminated in all of these models, only two of them give sufficiently small second-order rates for $K_L^0 \to \mu^+\mu^-$. In these cases, it is possible to obtain $\sin\theta_C = 1/3\sqrt{2} \cong 0.235$ in a rather natural manner. These two models and one of the others can be extended to SU(2)⊗U(1) gauge theories of weak and electromagnetic interactions which are anomaly-free in the quark sector and thus require heavy leptons to cancel leptonic anomalies. The rates for neutral events in inclusive neutrino-nucleon reactions predicted by these gauge theories are compatible with experimental results when $\sin^2\theta_W \cong \frac{1}{3}$. The models giving acceptable $K_L^0 \to \mu^+\mu^-$ rates do not predict any enhancement of neutrino-nucleon total cross sections associated with the production of SU(3)″ nonsinglet states. None of the above schemes helps explain the $\Delta T = \frac{1}{2}$ rule.

I. INTRODUCTION

Although the Han-Nambu model[1] has become increasingly more popular as a theory of hadronic constituents since its introduction, it has so far not been possible to determine (with any appearance of uniqueness) the forms of weak interaction currents in terms of the quark fields of the theory. Specifically, the question is how to choose the transformation properties of the weak currents in the SU(3)′ and SU(3)″ spaces. An attractive form for the charge operator is obtained by the requirement of preserving the Gell-Mann–Nishijima relation, but one still has a great deal of freedom of choice for the other currents. This arises from the fact that observed hadrons are classified as SU(3)″ singlet states, allowing the addition of arbitrary SU(3)″ nonsinglet terms to the weak operators effecting transitions between them. On the other hand, there are long-standing puzzles not explained by the simple quark model, such as the absence of $\Delta S = 1$ neutral processes, the $\Delta T = \frac{1}{2}$ rule, and the Cabibbo angle. It is therefore a possibility worth investigating that at least some of these can be understood in the Han-Nambu model by employing the freedom in the SU(3)″ space in a judicious manner. One recent example in this direction is an idea put forward by Stech[2]: by taking I-spin and V-spin changing currents with transformation properties (8′, 1″), (1′, 8″), respectively, the $\Delta S = 1$ neutral currents which are proportional to the commutators of the former two are eliminated. The observed $\Delta S = 1$ transitions and the Cabibbo angle are then assumed to be related to the mixing of a set of current quarks which have the same quantum numbers, such as the three quarks at the center of the Han-Nambu diagram shown in Fig. 1. As the mixing can be expressed as a unitary transformation, commutators and selection rules remain unchanged. Naturally, only the 1″ part of the resulting currents is effective between observed hadron states. But, to be more accurate, the basic fields in this reference are not Han-Nambu quarks, as each carries a positive unit of baryon number and baryons are constructed from three particle fields and two antiparticle fields.

In this paper we first investigate the possibility of using this mechanism in the Han-Nambu model for various baryon number assignments. Then it must be seen if the second-order effects are sufficiently small. Barring the possibility of very fortuitous leptonic cancellations, we find that only two models give acceptably small rates for $K_I^0 \to \mu^+\mu^-$. In these models it is possible to derive $\sin\theta_C = 1/3\sqrt{2}$ in a somewhat natural way, but we are also forced to assume that the radiative corrections to the vector coupling constant G_v must be approximately four times as large as those for similar theories. From a theoretical

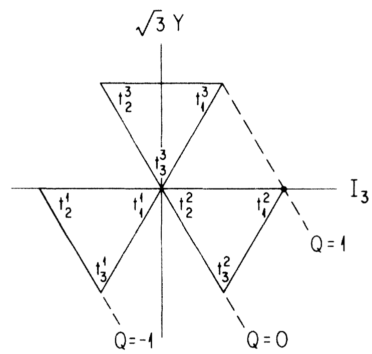

FIG. 1. Positions of the Han-Nambu three triplets on the I_3-Y diagram.

standpoint, it is desirable that schemes of this type allow an extension to a gauge theory of weak and electromagnetic interactions. It will be shown that for three cases including the two mentioned above, one can construct an $SU(2)_L \otimes U(1)$ theory. If we insist that these theories should be anomaly[3] free, at least two new heavy leptons will be needed. The coupling of our neutral currents to neutrinos is uniquely determined in the framework of a gauge theory, making their applications to inclusive neutrino-nucleon scattering possible. The results agree with the CERN-Gargamelle[4] and Fermi National Accelerator Laboratory (Fermilab)[5] neutral-current data for a Weinberg angle such that $\sin^2\theta_W \cong \frac{1}{3}$. If the SU(3)″ nonsinglet threshold is assumed to be finite, all models except the acceptable ones predict substantial increases in nucleon structure functions[6] for neutrino scattering above that threshold. Considering that there is no evidence of such effects at Fermilab neutrino energies, these models again appear preferable to the rest. Finally, we show that these models do not offer an explanation of the $\Delta T = \frac{1}{2}$ rule. The plan of the sections follows the order of topics mentioned above.

II. THE MECHANISM AND THE MODELS

In this section we shall outline the mechanism for suppressing $\Delta S = 1$ neutral currents proposed in Ref. 2 (hereafter to be called paper II) and show how it can be used with Han-Nambu quarks for various baryon number assignments. Recalling that these processes arise from the nonvanishing of the commutators $[I_+, V_-]$ and $[V_+, I_-]$ in the simple quark model, we can choose the transformation properties of $I_\pm$ and $V_\pm$ so that they commute exactly. For example, the charge raising current of paper II and one of our cases is taken to be

$$j_{+\mu} = I'_{+\mu} + V''_{+\mu} \equiv \bar{\mathrm{t}}\,{}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^{\alpha}_{2} + \bar{\mathrm{t}}\,{}^{3}_{\beta}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^{1}_{\beta}\,, \tag{1}$$

where the upper indices refer to SU(3)″ and the lower to SU(3)′. The two pieces above are of the form (8′, 1″) and (1′, 8″), thus they obviously commute. The fact that the above operators can be interpreted as generators of the two SU(3) symmetries is an appealing feature of this scheme not shared by models which employ (8′, 8″)-type currents.[7]

Clearly, the current (1) also prohibits charged $\Delta S = 1$ transitions between observed SU(3)″ singlet hadrons. In this theory, these are assumed to result from a mixing of current quarks which have the same quantum numbers. In paper II, where the triplets have equal baryon numbers, the mixing has the form

$$\begin{aligned}
\mathrm{t}^1_1 &= \tfrac{1}{2}(1+\cos\theta)t^1_1 - \tfrac{1}{2}(1-\cos\theta)t^2_2 + \frac{\sin\theta}{\sqrt{2}}\,t^3_3\,,\\
\mathrm{t}^2_2 &= -\tfrac{1}{2}(1-\cos\theta)t^1_1 + \tfrac{1}{2}(1+\cos\theta)t^2_2 + \frac{\sin\theta}{\sqrt{2}}\,t^3_3\,,\\
\mathrm{t}^3_3 &= \frac{\sin\theta}{\sqrt{2}}\,(t^1_1 + t^2_2) - \cos\theta\, t^3_3\,,\\
\mathrm{t}^\alpha_\beta &= t^\alpha_\beta \quad \text{for } \alpha \neq \beta\,.
\end{aligned} \tag{2}$$

The distinction indicated above between the fields t and t is similar to the distinction in Cabibbo theory between the $\mathfrak{N}$ quark and $\mathfrak{N}^C \equiv \mathfrak{N}\cos\theta_C + \lambda\sin\theta_C$. As this amounts to a unitary transformation on the quarks, the commutators remain unchanged and $\Delta S = 1$ neutral processes are still suppressed. But applying (2) on (1), and taking the SU(3)″ singlet part of the resulting expression as an effective current between observed hadrons, we have

$$j_{+\mu}(\mathrm{eff}) = \left(\frac{2+\cos\theta}{3}\right)\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_2 + \frac{\sqrt{2}\,\sin\theta}{3}\,\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_3\,, \tag{3}$$

which, for $\sin\theta \cong \frac{1}{2}$ gives the suppression factors $0.955 \cong \cos\theta_C$ and $0.236 \cong \sin\theta_C$. As the usual $\cos\theta_C$ can only be extracted from the data through model-dependent radiative corrections,[8] one can regard the current (3) in agreement with experiment for $\Delta S = \Delta Q = 1$ transitions. The neutral component of the SU(2) triplet of which (1) is a member is

$$\begin{aligned}
j_{3\mu} &= I'_{3\mu} + V''_{3\mu}\\
&\equiv \tfrac{1}{2}\bar{\mathrm{t}}\,{}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^\alpha_1 - \tfrac{1}{2}\bar{\mathrm{t}}\,{}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^\alpha_2\\
&\quad + \tfrac{1}{2}\bar{\mathrm{t}}\,{}^{3}_{\alpha}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^3_\alpha - \tfrac{1}{2}\bar{\mathrm{t}}\,{}^{1}_{\alpha}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathrm{t}^1_\alpha.
\end{aligned} \tag{4}$$

The effective neutral current below is obtained from this by the method leading to Eq. (3):

$$\begin{aligned}
j_{3\mu}(\mathrm{eff}) &= \frac{(5+\cos\theta)}{6}\left[\tfrac{1}{2}\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_1 - \tfrac{1}{2}\bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_2\right]\\
&\quad + \frac{\sin^2\theta}{8}\left[\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_1 + \bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_2 - 2\bar{t}^{\alpha}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t^\alpha_3\right].
\end{aligned} \tag{5}$$

This obviously does not include a neutral $\Delta S = 1$ piece.

Clearly, the manner in which the above scheme can be adapted to the Han-Nambu model depends crucially on the baryon number assignment in that model. For example, calling the triplets t^1, t^2, t^3 after the components of the basic SU(3)″ antitriplet, it can be immediately seen that the assignment $(1, 1, -1)$ does not even allow a V'' changing current. It may therefore be a worthwhile digression here if we discuss various possible assignments. A more detailed account of this question can be found in the last paper of Ref. 1. Of course, one possibility is using fractional baryon numbers $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. This is completely compatible with the above summarized scheme of paper II, but is inconsistent with one attractive feature of the Han-Nambu model, that of using fields with integral quantum numbers. We will consider it only in Sec. III while studying induced $\Delta S = 1$ neutral processes in the theory that was outlined above. With integral baryon numbers we can have the cases

$$B = (1, 0, 0), \tag{6a}$$

$$(0, 1, 0), \tag{6b}$$

$$(0, 0, 1), \tag{6c}$$

$$(1, 1, -1), \tag{6d}$$

$$(1, -1, 1), \tag{6e}$$

and

$$(-1, 1, 1). \tag{6f}$$

If the zero baryon number quarks in cases (6a), (6b), and (6c) can decay, they must do so by producing an odd number of leptons,[9] showing that they possess nonzero lepton number. Ignoring these peculiar leptons with strong interactions, we will consider (6d), (6e), (6f). In these cases, quarks with positive baryon numbers can decay into baryons, and quarks with negative baryon numbers can decay into antibaryons. Case (6d) allows the viewpoint of identifying the symmetry group of observed strong interactions with the diagonal isospin SU(2) subgroup of SU(3)′×SU(3)″. The usual large symmetry breaking along the hypercharge axis can also be thought of as being related to the baryon number assignment. The decay of quarks would then proceed with characteristically the same strength as this mass splitting, i.e., quarks would decay through strong interactions. In (6e) $I''_{\pm}$ do not commute with baryon number. Thus in this case we would have to assume that the exact SU(2) symmetry of strong interactions is generated by $\vec{I}'$ alone. This assignment is the most suitable for the purposes of constructing a model similar to that of paper II, but does not indicate the strength of quark decay into hadrons as was possible in (6d). A simple Hamiltonian describing such decays could be an expression such as Eq. (24) of the third paper of Ref. 1, except now the third and second triplets would have to be interchanged. Case (6f) does not admit $\vec{I}''$ and $\vec{V}''$ currents, and thus is not very useful for building models of this type.

In the following we will present two schemes each for assignments (6d) and (6e). We will refer to them as 1(a), 1(b) and 2(a), 2(b), respectively. In schemes 1(a) and 1(b) baryon conservation rules out a V'' current between triplets; thus our models will not be very closely analogous to that of paper II.

Model 1(a). Here we construct commuting $I_{\pm}$ and $V_{\pm}$ currents by using the antitriplets together with the triplets. Specifically, we will consider

$$\begin{aligned} j_{+\mu} = {} & \bar{t}_1^{\alpha} \gamma_\mu \frac{(1+\gamma_5)}{2} t_2^{\alpha} \\ & + \left[\bar{t}_1^{2} \gamma_\mu \left(\frac{1+\gamma_5}{2}\right) t_2^{c3} - \bar{t}_2^{2} \gamma_\mu \left(\frac{1+\gamma_5}{2}\right) t_1^{c3} \right] \\ & - \bar{t}_3^{c3} \gamma_\mu \left(\frac{1+\gamma_5}{2}\right) t_3^{1}, \end{aligned} \tag{7}$$

where t^c denotes the charge conjugate field to t, given by $t^c = \gamma_2 t^*$ in the Dirac-Pauli representation of the γ matrices. These currents are shown in Fig. 2, in which it can be easily seen that the last three terms raise V spin. As the first term is an SU(3)″ singlet and an I'_3-raising operator, it commutes with the combination of the next two terms which behave as an invariant determinant under SU(2) transformations generated by $\vec{I}'$. It is clear from the lower indices of the last term that it also commutes with I'_+. Now we need a mixing between quarks and antiquarks to recover a current from (7) that can be considered effectively

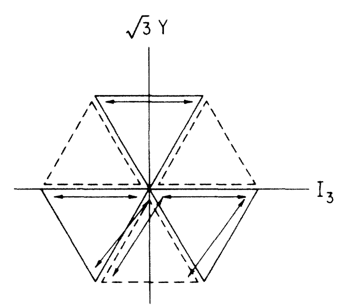

FIG. 2. Charge-changing currents in model 1(a) are represented by arrows. Dashed-line triangles denote the antitriplets.

equivalent to the Cabibbo current. As can be observed in Fig. 2, this is a natural possibility in the Han-Nambu model, where every quark has the same quantum numbers (with the possible exception of baryon number) as at least one antiquark. One well-known transformation of the required type is the Pauli-Gürsey transformation.[10] In particular, we will take the following set of Pauli-Gürsey mixings:

$$\begin{aligned}
\mathbf{t}_1^3 &= (\tfrac{1}{2}\theta^2)t_1^3 + \theta\gamma_5 t_3^{c1} ,\\
\mathbf{t}_2^3 &= (1-\tfrac{1}{2}\theta^2)t_2^3 + \theta\gamma_5 t_3^{c2} ,\\
\mathbf{t}_3^3 &= (1-\theta^2)t_3^3 - \theta\gamma_5(t_1^{1c}+t_2^{2c}) ,\\
\mathbf{t}_1^1 &= (1-\tfrac{1}{2}\theta^2)t_1^1 - \theta\gamma_5 t_3^{c3} ,\\
\mathbf{t}_2^2 &= (1-\tfrac{1}{2}\theta^2)t_2^2 - \theta\gamma_5 t_3^{c3} ,
\end{aligned} \tag{8}$$

where terms of order θ^3 and higher will be ignored hereafter. We have chosen the above form by demanding that the transformations generate a model Hamiltonian for allowing the decay of single quarks into hadrons. For example, when (8) is applied on a common quark mass term of the form $m\bar{\mathbf{t}}_\beta^\alpha \mathbf{t}_\beta^\alpha$, the result is

$$2m\theta(\bar{t}_2^3\gamma_5 t_3^{c2} + \bar{t}_3^1\gamma_5 t_1^{c3} - \bar{t}_3^3\gamma_5 t_1^{c1} - \bar{t}_3^3\gamma_5 t_2^{c2}) + \text{H.c.},$$

which happens to be an expression considered for this purpose in Sec. III of the last paper of Ref. 1. Returning to our original concern, we can subject (7) to the transformations (8) and isolate the SU(3)″ singlet part of the result to obtain

$$j_{+\mu}(\text{eff}) = \left(1 - \frac{2\theta^2}{3}\right)\bar{t}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_2^\alpha + \frac{2\theta}{3}\,\bar{t}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_3^\alpha , \tag{9}$$

which, for $\theta = 0.333$ gives the factors 0.926 and 0.222 in front of the I'_+ and V'_+ currents. Identification of the former quantity with $\cos\theta_C$ is possible if the radiative corrections[8] for G_v/G_μ in this theory are approximately 10% of the leading term. The neutral current given by the commutator of (7) and its charge-lowering counterpart is

$$\begin{aligned}
j_{3\mu} = {}& \tfrac{1}{2}\bar{\mathbf{t}}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_1^\alpha - \tfrac{1}{2}\bar{\mathbf{t}}_2^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_2^\alpha \\
& + \tfrac{1}{2}\left[\bar{\mathbf{t}}_1^2\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_1^2 + \bar{\mathbf{t}}_2^2\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_2^2 - \bar{\mathbf{t}}_3^1\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_3^1 + \bar{\mathbf{t}}_2^3\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\mathbf{t}_2^3 + \bar{\mathbf{t}}_1^3\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\mathbf{t}_1^3 - \bar{\mathbf{t}}_3^3\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)\mathbf{t}_3^3\right] .
\end{aligned} \tag{10}$$

Carrying out the steps analogous to those in obtaining (9) from (7), we derive the effective neutral current

$$\begin{aligned}
j_{3\mu}(\text{eff}) = \tfrac{1}{12}[& (5-\theta^2)\bar{t}_1^\alpha\gamma_\mu t_1^\alpha - (1-\theta^2)\bar{t}_2^\alpha\gamma_\mu t_2^\alpha - (2+\theta^2)\bar{t}_3^\alpha\gamma_\mu t_3^\alpha + (3-\theta^2)\bar{t}_1^\alpha\gamma_\mu\gamma_5 t_1^\alpha - 3(1-\theta^2)\bar{t}_3^\alpha\gamma_\mu\gamma_5 t_3^\alpha \\
& - \theta^2\bar{t}_3^\alpha\gamma_\mu\gamma_5 t_3^\alpha] .
\end{aligned} \tag{11}$$

Again we see that neutral $\Delta S = 1$ terms are absent in both (10) and (11).

Model 1(b). This model employs a mechanism similar to that of Glashow, Iliopoulos, and Maiani[11] while retaining some properties of paper II. It is based on the isospin-changing current

$$j_{+\mu} = \bar{\mathbf{t}}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_2^\alpha + \bar{\mathbf{t}}_3^2\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)\mathbf{t}_3^1 \tag{12}$$

shown in Fig. 3. The fields $\mathbf{t}_2^1$ and $\mathbf{t}_3^1$ are assumed to be mixed through the rotation

$$\begin{aligned}
\mathbf{t}_2^1 &= \cos\phi\, t_2^1 + \sin\phi\, t_3^1 ,\\
\mathbf{t}_3^1 &= -\sin\phi\, t_2^1 + \cos\phi\, t_3^1 ,
\end{aligned} \tag{13}$$

so that orthogonal combinations of t_2^1 and t_3^1 are carried separately to t_1^1 and t_3^2. Thus t_1^1 and t_3^2 behave like the $\mathcal{P}$ and the $\mathcal{P}'$ quarks of Glashow, Iliopoulos, and Maiani. Substituting (13) in (12) and picking out the SU(3)″ singlet part we find

$$j_{+\mu}(\text{eff}) = \left(\frac{2+\cos\phi}{3}\right)\bar{t}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_2^\alpha + \frac{\sin\phi}{3}\,\bar{t}_1^\alpha\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_3^\alpha . \tag{14}$$

For $\phi \cong 41°$, the two coefficients above become 0.920 and 0.220. Again the smallness of the fac-

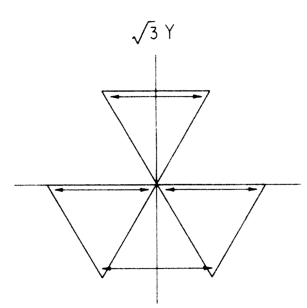

FIG. 3. Currents in model 1(b).

tor 0.920 necessitates large radiative corrections for agreement with experiment to be possible. An interesting point about this model and that of scheme 2(b) is the closeness of the mixing angle to "natural" mixing, $\phi = 45°$. In this case one gets $0.235 \cong \sin\theta_C$, a value in excellent agreement with a recent fit to octet baryon decays.[12] However, radiative corrections in this case would have to be as large as 20%, which is rather unlikely. In the subsequent discussions of this model we will take $\phi = 45°$ for calculational simplicity wherever accuracy in the value of $\cos\theta_C$ will not be needed.

As the two terms in (12) commute, we obtain the strangeness-conserving neutral current

$$\begin{aligned} j_{3\mu} = {} & \tfrac{1}{2}\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{1} - \tfrac{1}{2}\bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2} \\ & + \tfrac{1}{2}\bar{t}^{2}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{2}_{3} - \tfrac{1}{2}\bar{t}^{1}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{1}_{3}\,. \end{aligned} \tag{15}$$

With the by now familiar procedure, the effective neutral current turns out to be

$$j_{3\mu}(\text{eff}) = I'_{3\mu} \equiv \tfrac{1}{2}\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{1} - \tfrac{1}{2}\bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2}\,. \tag{16}$$

Model 2(a). This is very similar to the model of paper II, except now a mixing of t^2_2 with t^1_1 and t^3_3 is inadmissible because of baryon number conservation. Thus we use the current (1) shown in Fig. 4 and the simpler transformation of triplet fields:

$$\begin{aligned} t^1_1 &= \cos\beta\, t^1_1 + \sin\beta\, t^3_3\,, \\ t^3_3 &= \sin\beta\, t^1_1 - \cos\beta\, t^3_3\,. \end{aligned} \tag{17}$$

The charged effective current is now

$$\begin{aligned} j_{+\mu}(\text{eff}) = {} & \left(\frac{\cos\beta+2}{3}\right)\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2} \\ & + \frac{2\sin\beta}{3}\,\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{3}\,. \end{aligned} \tag{18}$$

Taking $\sin\theta_C \cong 0.244$ corresponds to setting $\beta \cong 21.4°$. Then $\frac{1}{3}(\cos\beta + 2) = 0.976$, which is very close to $\cos\theta_C \cong 0.972$. The complete and the effective neutral currents are respectively the following:

$$j_{3\mu} = I'_{3\mu} + V''_{3\mu} \equiv \tfrac{1}{2}\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{1} - \tfrac{1}{2}\bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2} + \tfrac{1}{2}\bar{t}^{3}_{\alpha}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{3}_{\alpha} - \tfrac{1}{2}\bar{t}^{1}_{\alpha}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{1}_{\alpha}\,, \tag{19}$$

$$j_{3\mu}(\text{eff}) = \tfrac{1}{2}\left[\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{1} - \bar{t}^{\alpha}_{2}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2}\right] + \frac{\sin^2\beta}{6}\left[\bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{1} - \bar{t}^{\alpha}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{3}\right]. \tag{20}$$

Model 2(b). In this model we take the charged current as indicated in Fig. 5:

$$\begin{aligned} j_{+\mu} &= I'_{+\mu} + \bar{t}^{3}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{1}_{3} \\ &\equiv \bar{t}^{\alpha}_{1}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{\alpha}_{2} + \bar{t}^{3}_{3}\gamma_\mu\left(\frac{1+\gamma_5}{2}\right) t^{1}_{3}\,. \end{aligned} \tag{21}$$

The V''-changing part of the current is now $(1', 8'') + (8', 8'')$, but the two pieces still obviously commute, eliminating first-order neutral $\Delta S = 1$ transitions. The mixing is of the form

$$\begin{aligned} t^1_1 &= \cos\phi\, t^1_1 - \sin\phi\, t^3_3\,, \\ t^3_3 &= \sin\phi\, t^1_1 + \cos\phi\, t^3_3\,, \end{aligned} \tag{22}$$

with $\phi \cong 41°$ as in case 1(b). Thus the effective charged current is the same as that in (14). Also, we will again set $\phi = 45°$ for simplicity in the rest of the discussion on this model. The two neutral

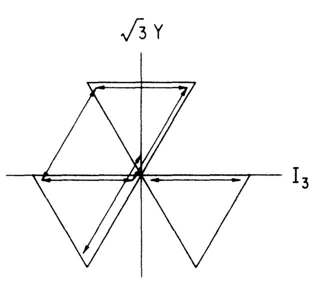

FIG. 4. Currents in model 2(a).

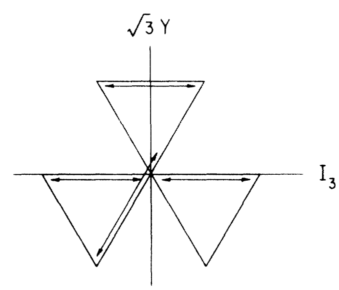

FIG. 5. Currents in model 2(b).

currents are

$$j_{3\mu} = I'_{3\mu} + \tfrac{1}{2}\bar{t}_3^3\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_3^3 - \tfrac{1}{2}\bar{t}_3^1\gamma_\mu\left(\frac{1+\gamma_5}{2}\right)t_3^1 \quad (23)$$

and

$$j_{3\mu}(\mathrm{eff}) = I'_{3\mu} \ . \quad (24)$$

III. $K_L^0 \to \mu^+\mu^-$

One very critical test of all theoretical mechanisms designed to suppress $\Delta S = 1$ neutral currents is whether they also suppress second-order induced effects in agreement with experiment for this process. Here we will consider only paper II, identifying the basic fields with Han-Nambu quarks, and models 1(b), 2(a) and 2(b). We will not discuss model 1(a), as this was mainly intended as an illustration of how the Stech mechanism[2] might be applied with the baryon number assignment (1, 1, −1). Though the calculation of this effect can be theoretically well defined only in a gauge theory (thus also requiring a knowledge of the leptonic spectrum), it would be unrealistic to expect the necessary cancellations to take place anywhere but between the quark triangle parts of the relevant diagrams.[13] We will show in the next section that all the schemes in this section can be extended to gauge theories, therefore all the diagrams are finite and well defined. Thus a sufficient condition for having second-order rates compatible with experiment is that the quark triangle diagrams vanish or cancel each other. Mesons are SU(3)″ singlets, thus we are constrained to have the same upper index on the outgoing down quark and the incoming strange quark. Another point worth mentioning is that we must now keep SU(3)″ nonsinglet terms as $8'' \otimes 8''$ contains an SU(3)″ singlet. Hence we must use the full charge-changing currents after the mixings have been applied. In paper II, these currents are

$$\begin{aligned}&\tfrac{1}{2}(1+\cos\theta)\bar{t}_1^1 t_2^1 - \tfrac{1}{2}(1-\cos\theta)\bar{t}_2^2 t_2^1 + \frac{\sin\theta}{\sqrt{2}}\bar{t}_3^3 t_2^1 + \tfrac{1}{2}(-1+\cos\theta)\bar{t}_1^2 t_1^1 + \tfrac{1}{2}(1+\cos\theta)\bar{t}_1^2 t_2^2 + \frac{\sin\theta}{\sqrt{2}}\bar{t}_1^2 t_3^3 \\ &\quad + \bar{t}_1^3 t_2^3 + \tfrac{1}{2}(1+\cos\theta)\bar{t}_1^3 t_1^1 + \tfrac{1}{2}(-1+\cos\theta)\bar{t}_1^3 t_2^2 + \frac{\sin\theta}{\sqrt{2}}\bar{t}_1^3 t_3^3 + \frac{\sin\theta}{\sqrt{2}}\bar{t}_1^1 t_3^1 + \frac{\sin\theta}{\sqrt{2}}\bar{t}_2^2 t_3^1 - \cos\theta\,\bar{t}_3^3 t_3^1 \ , \quad (25)\end{aligned}$$

and its charge-lowering partner. The Dirac matrices between triplet fields have been left out for conciseness. This pair of currents gives rise to the nonvanishing diagrams of Fig. 6. Clearly, the contribution of diagrams 6(a)–6(c) can be made arbitrarily small by assuming the masses of t_1^1, t_2^2 t_3^3 are close enough. But diagrams 6(d) and 6(e) add up, and it is well known[13] that such terms yield results approximately four orders of magnitude bigger than experimental rates. Hence this model cannot be acceptable.

1(b). Just as in the Glashow-Iliopoulos-Maiani (GIM)[14] model, the nonvanishing diagrams 7(a) and 7(b) can be made to cancel if the masses of t_1^1 and t_3^3 are within a few GeV of each other. There is nothing in the Han-Nambu model to prevent this possibility.

2(a). Here diagrams 8(a) and 8(b) can cancel, while 8(c) and 8(d) violate the experimental bound.

2(b). In this model only the currents leading to 8(a) and 8(b) of the above case are used, hence the desired cancellation is possible.

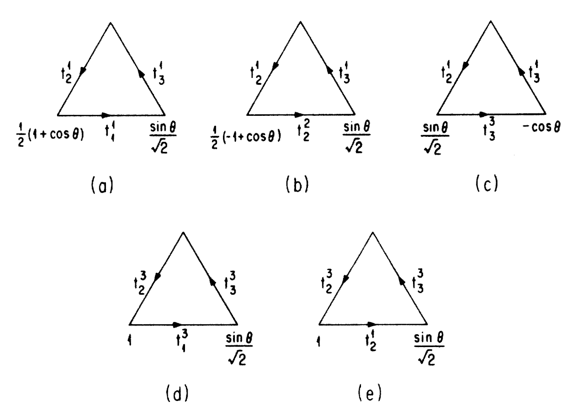

FIG. 6. Diagrams contributing to $K_L^0 \to \mu^+\mu^-$ for Ref. 2. K_L^0 is connected to the upper vertex and charged intermediate bosons couple to lower vertices.

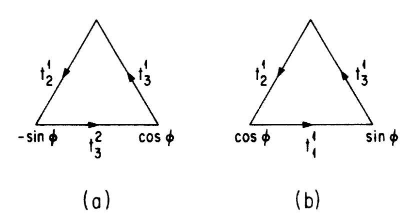

FIG. 7. Diagrams contributing to $K_L^0 \to \mu^+\mu^-$ for model 1(b).

IV. EXTENSION TO A GAUGE THEORY

Unless a model of the preceding variety allows the construction of a gauge theory of weak interactions, it must be considered fundamentally unsatisfactory. We will show below that cases 1(b), 2(a), and 2(b) can be extended to $SU(2)_L \otimes U(1)$ gauge theories[15] of weak and electromagnetic interactions. The $SU(2)_L$ generators are naturally taken as the charges of the triplet of weak currents in each case. Calling these $\tilde{\tau}_i$ $(i=1,2,3)$, we only have to establish that for every set of these charges, there is a U(1) generator $\tilde{Y}$ satisfying the obvious conditions

$$Q=\tilde{\tau}_3+\tilde{Y}\ , \tag{26}$$

$$[\tilde{\tau}_i,\tilde{Y}]=0\ , \tag{27}$$

where Q stands for the electric charge operator, given in the Han-Nambu model by

$$Q=I_3'+\tfrac{1}{2}Y'+I_3''+\tfrac{1}{2}Y''\ . \tag{28}$$

We will also write down the form of the quark multiplets for each model. However, we shall not go into discussions of the choices of Higgs[16] scalars and spontaneous symmetry breaking, as this would involve making additional assumptions about the mass spectrum of the three triplets.

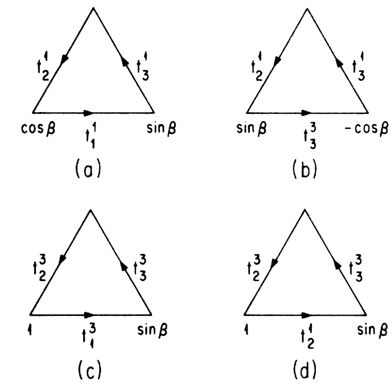

FIG. 8. Diagrams contributing to $K_L^0\to\mu^+\mu^-$ for model 2(a).

Model 1(b). With $\tilde{\tau}_3$ given by (15), we find $\tilde{Y}$, using (26) and (28), as

$$\begin{aligned}\tilde{Y}=\int dv\frac{1}{2}\Bigg\{&-\left[\bar{t}_1^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^1+\bar{t}_2^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^1\right]+\left[\bar{t}_1^2\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^2+\bar{t}_2^2\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^2\right]\\&+\left[\bar{t}_1^3\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^3+\bar{t}_2^3\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^3\right]-\left[\bar{t}_3^2\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_3^2+\bar{t}_3^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_3^1\right]\Bigg\}\\&+\left[\bar{t}_1^2\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_1^2+\bar{t}_1^3\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_1^3-\bar{t}_2^1\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_2^1-\bar{t}_3^1\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_3^1\right]\ .\end{aligned} \tag{29}$$

In spite of its complicated appearance, it is easy to see that this expression commutes with (15) and (12): The last four terms are right-handed, and each pair of terms in curved brackets is an isoscalar under the four isospin generators shown in Fig. 3. The multiplets are

$$\begin{pmatrix}t_1^\alpha\\ t_2^\alpha\end{pmatrix}_L\ (\alpha=1,2,3),\quad \begin{pmatrix}t_3^2\\ t_3^1\end{pmatrix}_L\ ,\quad (t_3^3)_L\ , \tag{30}$$

and their right-handed singlet counterparts.

Model 2(a). Here the $\tilde{\tau}_i$ are given by

$$\tilde{\tau}_i=I_i'+V_i''\ , \tag{31}$$

leading to the U(1) generator:

$$\begin{aligned}\tilde{Y}=\int dv\ \tfrac{1}{12}\{&[\bar{t}_1^\alpha\gamma_4(1+\gamma_5)t_1^\alpha+\bar{t}_2^\alpha\gamma_4(1+\gamma_5)t_2^\alpha]-2[\bar{t}_3^\alpha\gamma_4(1+\gamma_5)t_3^\alpha]-[\bar{t}_\alpha^1\gamma_4(1+\gamma_5)t_\alpha^1+\bar{t}_\alpha^3\gamma_4(1+\gamma_5)t_\alpha^3]+2\bar{t}_\alpha^2\gamma_4(1+\gamma_5)t_\alpha^2\}\\&+\tfrac{1}{6}[2\bar{t}_1^\alpha\gamma_4(1-\gamma_5)t_1^\alpha-\bar{t}_2^\alpha\gamma_4(1-\gamma_5)t_2^\alpha-\bar{t}_3^\alpha\gamma_4(1-\gamma_5)t_3^\alpha-2\bar{t}_\alpha^1\gamma_4(1-\gamma_5)t_\alpha^3+\bar{t}_\alpha^3\gamma_4(1-\gamma_5)t_\alpha^3+\bar{t}_\alpha^2\gamma_4(1-\gamma_5)t_\alpha^2]\ .\end{aligned} \tag{32}$$

Again, commutation with (31) is ensured because the last six terms are right-handed, the first three are $SU(3)''$ and $SU(2)_{I'}$ singlets, and the next three are $SU(3)'$ and $SU(2)_{V''}$ singlets. The quark multiplets now become

$$\begin{pmatrix} t_1^3 \\ \frac{1}{\sqrt{2}}\,(t_1^1+t_2^3) \\ t_2^1 \end{pmatrix}_L , \quad \begin{pmatrix} t_3^3 \\ t_3^1 \end{pmatrix}_L , \begin{pmatrix} t_1^2 \\ t_2^2 \end{pmatrix}_L , \quad (t_3^2)_L \,, \quad \frac{(t_2^3-t_1^1)_L}{\sqrt{2}} \,, \tag{33}$$

and corresponding right-handed singlets.

Model 2(b). Using expression (23) for $\tilde{T}_3$, we find

$$\begin{aligned} \tilde{Y} = \int dv \, \frac{1}{2}\Bigg(\Bigg\{ & -\left[\bar{t}_1^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^1+\bar{t}_2^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^1\right]+\left[\bar{t}_1^2\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^2+\bar{t}_2^2\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^2\right] \\ & +\frac{1}{2}\left[\bar{t}_1^3\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_1^3+\bar{t}_2^3\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_2^3\right]-\frac{1}{2}\left[\bar{t}_3^1\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_3^1+\bar{t}_3^3\gamma_4\left(\frac{1+\gamma_5}{2}\right)t_3^3\right]\Bigg\} \\ & +\left[\bar{t}_1^3\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_1^3+\bar{t}_1^2\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_1^2-\bar{t}_2^1\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_2^1-\bar{t}_3^1\gamma_4\left(\frac{1-\gamma_5}{2}\right)t_2^1\right]\Bigg) \,. \end{aligned} \tag{34}$$

The first four pairs of terms are singlets under the four separate SU(2) transformations generated by the charges of this model, and the last four terms are right-handed. Thus (27) is satisfied in this scheme also. The multiplet structure is

$$\begin{pmatrix} t_1^\alpha \\ t_2^\alpha \end{pmatrix}_L \ (\alpha=1,2,3), \ \begin{pmatrix} t_3^3 \\ t_3^1 \end{pmatrix}_L , \quad (t_3^2)_L \,. \tag{35}$$

One important criterion for a gauge theory is that it should be free of anomalies, which is expressed by the condition[17]

$$\sum_{i\,=\,\text{fermions}} (2\tilde{T}_3^i)^2 Q_i = 0 \,. \tag{36}$$

It can be easily checked that this sum vanishes for the Han-Nambu quarks in all of the models discussed in this section. This necessitates the lepton contributions separately adding up to zero. If, in addition we demand that the neutral heavy intermediate boson couple to leptons as suggested by recent neutrino experiments,[4,5] the simplest leptonic multiplet structure is

$$\psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad \psi_R = \begin{pmatrix} E^+ \\ E^0 \\ E^- \end{pmatrix} , \quad \psi_L' = \begin{pmatrix} E^+ \\ E^0 \end{pmatrix}_L . \tag{37}$$

Thus we need two new heavy leptons of both electronic and muonic types. This is the "2-3 model" in Appendix A of Ref. 18.

Another immediate test is provided by the process $\pi^0 \to 2\gamma$.[19] As pions are SU(3)″ singlets, we must only take the I_3' part of our neutral currents in the calculation. Then all the above models give

$$\sum_{\text{quarks}} (I_3' Q^2) = \tfrac{1}{2} \,, \tag{38}$$

as required by experiment.

V. NEUTRAL CURRENTS IN INCLUSIVE NEUTRINO-NUCLEON SCATTERING

Within the framework of a gauge theory, the strength and the form of the coupling between the neutral quark currents and neutrino currents are uniquely determined. Though the results for these neutral currents will involve one free parameter, i.e., the Weinberg angle[15] θ_W, it is still of interest to see if the rates predicted by the above models agree with the recent data for an acceptable value of $\sin\theta_W$. As there is no evidence of production of SU(3)″ nonsinglet final states at Fermilab neutrino energies, we will use the SU(3)″ singlet parts of our currents. We will also make the usual approximations of neglecting the quark-antiquark pairs in the nucleon and of taking $\sin\theta_C = 0$. The latter approximation corresponds in case 2(a) to ignoring the mixing in Eq. (17). This is not possible for the models with the mixing angle of 41°. In these cases we will take it as 45° to simplify our calculations. Thus we have to use the quark currents (16), (20), and (24), which become identical in this approximation. The effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}\gamma_\mu(1+\gamma_5)\nu] \times 2(j_{3\mu} - \sin^2\theta_W j_\mu^{\text{em}}) \,, \tag{39}$$

where G_F is the usual Fermi constant and only the SU(3)″ singlet part of the electromagnetic current enters j_μ^{em}. The experimental quantities of interest are

$$R_\nu \equiv \frac{\sigma(\nu_\mu + N \to \nu_\mu + X)}{\sigma(\nu_\mu + N \to \mu^- + X')} \tag{40}$$

and $R_{\bar{\nu}}$. N denotes an isoscalar nucleon target. Defining the quantity $\chi \equiv 1 - 2\sin^2\theta_W$ and using the standard methods of the quark-parton model,[20] we find

$$R_\nu = \tfrac{1}{54}(10\chi^2 + 7\chi + 10)\,, \tag{41}$$

$$R_{\bar\nu} = \tfrac{1}{18}(10\chi^2 - 11\chi + 10)\,. \tag{42}$$

These are consistent with CERN-Gargamelle[4] results if $\sin^2\theta_W \cong \frac{1}{3}$. In this case,

$$R_\nu = 0.250, \quad R_\nu^{\exp} = 0.23 \pm 0.04\,,$$

$$R_{\bar\nu} = 0.413, \quad R_{\bar\nu}^{\exp} = 0.43 \pm 0.12\,.$$

The quantity measured at Fermilab[5] is

$$0.63 R_\nu + 0.37 R_{\bar\nu} = 0.20 \pm 0.05\,,$$

for which we predict with the above numbers

$$0.63 R_\nu + 0.37 R_{\bar\nu} = 0.31\,.$$

The discrepancy indicated by the Fermilab result is because of the same discrepancy between CERN and Fermilab measurements and the fact that we have chosen $\sin\theta_W$ to fit the CERN data.

VI. SU(3)″ NONSINGLET THRESHOLD EFFECTS IN INCLUSIVE NEUTRINO-NUCLEON REACTIONS

If we assume SU(3)″ octet states can be produced at finite energy, we will get different results than the usual ones for nucleon structure functions in deep-inelastic electron-nucleon and neutrino-nucleon scattering above this threshold. For example, in the former case, the prethreshold form of F_2^{eN} is

$$F_2^{eN} \equiv \tfrac{1}{2}(F_2^{ep} + F_2^{en}) = \frac{5x}{18}\left[u_p(x) + d_p(x)\right]\,, \tag{43}$$

where u_p and d_p are up- and down-quark distributions in the proton. With the natural assumption that a quark distribution for each triplet is $\frac{1}{3}$ the corresponding one-triplet model distribution, we can calculate F_2^{eN} above the 8″ threshold to find

$$F_2^{eN} = \frac{x}{2}\,(u_p + d_p)\,, \tag{44}$$

almost twice its prethreshold value.

Now we will discuss color excitation effects in inclusive neutrino-nucleon scattering

$$\nu_\mu(\bar\nu_\mu) + N \to \mu^-(\mu^+) + X \tag{45}$$

with the currents of models 1(b), 2(a), and 2(b). Factors corresponding to $\sin\theta_C$ and antiquarks in the nucleon will be neglected as is usual in such quark-parton model calculations.

Model 1(b). Here we will take the mixing angle as 45°. Then the charge-raising current (12) becomes

$$j_{+\mu}(\text{mix}) = \frac{1}{\sqrt2}\,\bar t^{\,2}_{3} t^{1}_{3} - \frac{1}{\sqrt2}\,\bar t^{\,2}_{3} t^{1}_{2} + \frac{1}{\sqrt2}\,\bar t^{\,1}_{1} t^{1}_{2} + \frac{1}{\sqrt2}\,\bar t^{\,1}_{1} t^{1}_{3} + \bar t^{\,2}_{1} t^{2}_{2} + \bar t^{\,3}_{1} t^{3}_{2}\,, \tag{46}$$

where we have left out the $\gamma_\mu[(1+\gamma_5)/2]$ between triplet fields. The terms with an initial t_3^α quark will not contribute as we are ignoring the strange quarks in the nucleon. Thus for $\nu_\mu + t_2^\alpha \to \mu^- + t_\beta^\delta$ via (46), we get $(\frac{1}{2} + \frac{1}{2} + 1 + 1) = 3$, equal to the prethreshold contribution of the I'_+ current. The same result holds for $\bar\nu_\mu + t_1^\alpha \to \mu^+ + t_\gamma^\lambda$, hence $F_2^{\nu N}, F_2^{\bar\nu N}$ will remain unchanged above the 8″ threshold.

Model 2(a). Above the color threshold, the currents V''_+ would contribute through the processes $(t_2^2 \leftarrow t_2^1)$ and $(t_1^3 \leftarrow t_1^1)$. The current $(t_3^3 \leftarrow t_3^1)$ will not be involved as there are no strange quarks in the nucleon. These have to be added onto the contribution of the three components of the isospin-changing current from each triplet; thus $F_2^{\nu N}$ and $F_2^{\bar\nu N}$ will be increased by a factor $(\frac{3}{3} + \frac{2}{3}) = \frac{5}{3}$. Hence the ratio $\sigma_{\bar\nu}/\sigma_\nu \cong 1/3$ will remain constant although both cross sections will exhibit a discontinuous change as a function of neutrino energy. Also, $F_2^{eN}/F_2^{\nu N}$ will change from $\frac{5}{18}$ to $\frac{3}{10}$.

Model 2(b). The mixed charge-raising current is now

$$j_{+\mu}(\text{mix}) = \frac{1}{\sqrt2}\,\bar t^{\,1}_{1} t^{1}_{2} - \frac{1}{\sqrt2}\,\bar t^{\,3}_{3} t^{1}_{2} + \bar t^{\,2}_{1} t^{2}_{2} + \bar t^{\,3}_{1} t^{3}_{2} + \frac{1}{\sqrt2}\,\bar t^{\,1}_{1} t^{1}_{3} + \frac{1}{\sqrt2}\,\bar t^{\,3}_{3} t^{1}_{3}\,. \tag{47}$$

Just as in case 1(b), this does not allow SU(3)″ nonsinglet threshold effects to be seen in $F_2^{\nu N}$ and $F_2^{\bar\nu N}$.

VII. THE NONLEPTONIC HAMILTONIAN AND THE $\Delta T = \frac{1}{2}$ RULE

It has been shown[21] that only the octet part of the nonleptonic Hamiltonian is effective in weak transitions between SU(3)″ singlet baryons. However, this argument rests upon the antisymmetry of the SU(3)″ part of the baryon wave function. It is therefore interesting to investigate whether the rule could be derived as a property of the Hamiltonian itself for a new set of currents. Unfortunately, the answer appears to be negative for the models presented here, as will be shown below.

In their mixed forms, the currents of models 1(b), 2(a), and 2(b) have the structure (8′, 1″) + (1′, 8″) + (8′, 8″). Thus the SU(3)″ singlet nonleptonic Hamiltonian involved in hadronic weak decays of observed strongly interacting particles will in general have a piece (8′⊗8′, 1″) originating from the term (8′, 8″)⊗(8′, 8″). Therefore it must be seen if the combination of the former with the term $(8'\otimes 8', 1''\otimes 1'') = (8'\otimes 8', 1'')$ results in an effective Hamiltonian[22] transforming as $(8', 1'')$. Briefly, the reason this fails is that the $1''$ terms obtained from the decomposition of $8''\otimes 8''$ are approximately one order of magnitude smaller than the usual part of the Hamiltonian, and thus cannot significantly change its transformation properties. We will present the argument using case 2(a), but the same conclusion holds for 1(b), 2(b), and the model of paper II.

The singlet part of a product of octets $T^{\alpha}{}_{\beta}T^{\mu}{}_{\nu}$ is equal to

$$(\tfrac{1}{8}\delta^{\alpha}{}_{\nu}\delta^{\mu}{}_{\beta} - \tfrac{1}{24}\delta^{\alpha}{}_{\beta}\delta^{\mu}{}_{\nu})T^{\lambda}{}_{\rho}T^{\rho}{}_{\lambda} . \tag{48}$$

Of course, in terms of quark fields,

$$\begin{aligned} T^{\lambda}{}_{\rho}T_{\lambda}{}^{\rho} &= [q^*_{\lambda}q_{\rho} - \tfrac{1}{3}\delta^{\lambda}{}_{\rho}(q^*_{\gamma}q_{\gamma})][q^*_{\rho}q_{\lambda} - \tfrac{1}{3}\delta_{\lambda}{}^{\rho}(q^*_{\delta}q_{\delta})] \\ &= [q^*_{\lambda}q_{\rho}q^*_{\rho}q_{\lambda} - \tfrac{1}{3}(q^*_{\gamma}q_{\gamma})(q^*_{\delta}q_{\delta})] . \end{aligned} \tag{49}$$

We must now apply the mixing (17) on the $\Delta S=1$ nonleptonic Hamiltonian

$$\mathcal{H}_{WK}^{\Delta S=1} = \frac{G}{\sqrt{2}}[\bar{t}_1^{\alpha}\gamma_{\mu}(1+\gamma_5)t_1^{\alpha}][\bar{t}_{\beta}^{1}\gamma_{\mu}(1+\gamma_5)t_{\beta}^{3}] \tag{50}$$

and extract the SU(3)$''$ singlet terms from the result. These arise from $1''\otimes 1''$ and $8''\otimes 8''$ as mentioned. In the latter case we have to use Eqs. (48) and (49) in the SU(3)$''$ space. After repeated applications of the Fierz transformation to simplify the answer and dropping one term approximately 0.3% of the leading one, we obtain

$$\begin{aligned} \mathcal{H}_{WK}^{\Delta S=1}(\text{eff}) = \frac{G}{\sqrt{2}}\bigg\{&\left(\frac{2\sin\beta(\cos\beta+2)}{9} - \frac{\sin\beta\cos\beta}{8}\right)\bar{t}_1^{\alpha}\gamma_{\mu}(1+\gamma_5)t_2^{\alpha}\bar{t}_3^{\beta}\gamma_{\mu}(1+\gamma_5)t_1^{\beta} \\ &- \frac{\sin\beta}{24}[2\cos\beta\,\bar{t}_3^{\alpha}\gamma_{\mu}(1+\gamma_5)t_2^{\alpha}\bar{t}_3^{\beta}\gamma_{\mu}(1+\gamma_5)t_3^{\beta} - 2\bar{t}_3^{\alpha}\gamma_{\mu}(1+\gamma_5)t_2^{\alpha}\bar{t}_2^{\beta}\gamma_{\mu}(1+\gamma_5)t_2^{\beta} \\ &+ \cos\beta\,\bar{t}_3^{\alpha}\gamma_{\mu}(1+\gamma_5)t_2^{\alpha}\bar{t}_1^{\beta}\gamma_{\mu}(1+\gamma_5)t_1^{\beta}]\bigg\} . \end{aligned} \tag{51}$$

The first term corresponds to the usual

$$\frac{G}{\sqrt{2}}\sin\theta_C\cos\theta_C(\bar{p}n)(\bar{\lambda}p)$$

of the one-triplet model with Cabibbo currents, and the rest are peculiar to our model. These are roughly 18%, 13%, 13%, and 7% of the first, thus not affecting its transformation properties very appreciably. The weights of the $\Delta T=\frac{1}{2}$ and $\Delta T=\frac{3}{2}$ pieces now become approximately

$$\mathcal{H}^{W}(\tfrac{1}{2}, -\tfrac{1}{2}) + \frac{0.8}{\sqrt{2}}\mathcal{H}^{W}(\tfrac{3}{2}, -\tfrac{1}{2})$$

instead of the usual

$$\mathcal{H}^{W}(\tfrac{1}{2}, -\tfrac{1}{2}) + \frac{1}{\sqrt{2}}\mathcal{H}^{W}(\tfrac{3}{2}, -\tfrac{1}{2}) .$$

Hence the modification does not explain the $\Delta T=\frac{1}{2}$ rule. The same conclusion has been found to apply in cases 1(b), 2(b), and in paper II.

VIII. SUMMARY AND DISCUSSION

Using the mechanism given in paper II as a theoretical framework, we have presented four models of weak currents based on Han-Nambu quarks. It is possible to suppress first-order $\Delta S=1$ neutral currents in all of these models, but as we have shown in Sec. II, only models 1(b) and 2(b) avoid conflict with experiment when induced second-order effects are considered. The mechanism and the currents have the theoretically desirable property of being convertible to gauge theories of weak and electromagnetic interactions and are compatible with existing data on $\sin\theta_C$ and on neutral neutrino currents. A crucial experimental test of these schemes would be provided if the precise magnitude of the radiative corrections to G_v/G_{μ} were known in these gauge theories. This is also of interest in view of the closeness of $1/3\sqrt{2}$ to experimental values of $\sin\theta_C$, but this closeness may very well be fortuitous. We have seen that the neutral-current data fix the Weinberg angle of the theory so that $\sin^2\theta_W \cong 0.33$, approximately the same value as in other popular gauge theories. Thus the mass of the neutral intermediate vector boson M_z must not be much larger than 100 GeV, and this makes it unlikely that radiative corrections attain the required magnitude through typical logarithmic dependence on M_z.[23] Therefore, the smallness of the suppression factor corresponding to $\cos\theta_C$ appears to be the most problematic aspect of our models. Still, these calculations are model-dependent and in fact involve undetermined constant terms not expressible as logarithms of the cutoff M_z. Hence it should be a fair statement that models 1(b) and 2(b) cannot be conclusively ruled out by the measurements of G_v/G_{μ}. Finally, in Sec. VII we proved that the $\Delta T=\frac{1}{2}$ rule cannot be understood in the context of this theory alone.

ACKNOWLEDGMENTS

I am grateful to Dr. Y. Nambu for directing me to this work and for suggesting most of the central ideas in it. More generally, I would also like to express here my deep appreciation for his generous support. I thank Dr. J. Arafune for his patient and pedagogical discussions on the material of Secs. IV and VII. Finally, I have a great debt of gratitude to Dr. Suha Gürsey and Dr. Feza Gürsey for their hospitality during a summer visit, and for their unfailing encouragement and interest without which the completion of my studies might not have been possible.

---

*Submitted to the Department of Physics in partial fulfillment of the requirements for the Ph.D. Degree. Work supported in part by the National Science Foundation under Contract No. NSF GP 32904 X.

[1]M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965); Y. Nambu, in *Preludes in Theoretical Physics*, edited by A. de-Shalit *et al*. (North-Holland, Amsterdam, 1966), p. 133; Y. Nambu and M. Y. Han, Phys. Rev. D 10, 674 (1974).

[2]B. Stech, Phys. Lett. 49B, 471 (1974).

[3]J. S. Bell and R. Jackiw, Nuovo Cimento 60, 47 (1967); S. L. Adler, Phys. Rev. 177, 2426 (1969); C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Lett. 38B, 519 (1972); D. J. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972).

[4]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973); Nucl. Phys. B73, 1 (1974).

[5]A. Benvenuti *et al.*, Phys. Rev. Lett. 32, 800 (1974); 32, 1457 (1974).

[6]J. D. Bjorken, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energies*, edited by H. Rollnik and W. Pfeil (North-Holland, Amsterdam, 1974), p. 25.

[7]J. M. Rawls and L. P. Yu, Phys. Rev. D 7, 2463 (1973); Y. Achiman, Nucl. Phys. B56, 635 (1973); Y. Achiman *et al.*, Nuovo Cimento 21A, 45 (1974).

[8]N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. B6, 255 (1968); A. Sirlin, *ibid*. B71, 29 (1974).

[9]See Sec. III of the third paper of Ref. 1.

[10]W. Pauli, Nuovo Cimento 6, 204 (1957); F. Gürsey, *ibid*. 7, 411 (1958).

[11]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[12]M. Roos, Nucl. Phys. B77, 420 (1974); L. M. Chounet, J. M. Gaillard, and M. K. Gaillard, Phys. Rep. 4C, No. 5 (1972).

[13]B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D 7, 510 (1973).

[14]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[15]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8)*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[16]P. W. Higgs, Phys. Lett. 12, 132 (1964); Phys. Rev. Lett. 13, 508 (1964); Phys. Rev. 145, 1156 (1966).

[17]B. Zumino, in Lectures at the Cargèse Summer Institute, 1972, CERN Report No. CERN TH 1550 (unpublished).

[18]J. D. Bjorken and C. H. Llewellyn Smith, Phys. Rev. D 7, 887 (1973).

[19]M. Gell-Mann, in *Elementary Particle Physics*, proceedings of the XI Schladming conference, edited by P. Urban (Springer, Berlin, 1972) [Acta Phys. Austriaca Suppl. 9 (1972)], p. 733; W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, in *Scale and Conformal Symmetry in Hadron Physics*, proceedings of the Topical Meeting on the Outlook for Broken Conformal Symmetry in Elementary Particle Physics, Frascati, 1972, edited by R. Gatto (Wiley, N. Y., 1973), p. 129; M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972*, edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 333.

[20]J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975 (1969); J. Kuti and V. F. Weisskopf, Phys. Rev. D 4, 3418 (1971).

[21]J. C. Pati and C. H. Woo, Phys. Rev. D 3, 2020 (1971).

[22]R. L. Kingsley, Phys. Lett. 40B, 387 (1972).

[23]See the last paper of Ref. 8.