On-mass-shell current algebra and the $\pi^0 \longrightarrow 2\gamma$ decay and partial conservation of axial-vector current anomaly

A. A. Golestaneh*

International Centre for Theoretical Physics, Trieste, Italy, and The Center for Particle Theory, Department of Physics, The University of Texas, Austin, Texas

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Using the algebra of the axial-vector current which is defined as a source for the pion field, we study the $\pi^0 \longrightarrow 2\gamma$ decay rate and the Adler anomaly. Our treatment is identical to the one previously applied to other interactions which are mentioned in the text. It consists of expressing the reduction formula in terms of the above current and evaluating it directly on the mass shell. In the process, we have a set of intermediate states from which we choose the one- and two-meson states. The 3π continuum is taken into account effectively by a pion isobar, π'. Hence the π^0 amplitude is expressed in terms of the parameters of the ρ, ω, ϕ, and π' and a coefficient β which represents the momentum dependence of the coupling constants of these mesons at the $\pi\gamma$ vertex. A feature of our method is that it does not explicitly depend on the anomaly operator. However, by writing our current in terms of the modified PCAC (partial conservation of axial-vector current) operator, the π^0 lifetime is linked with the anomaly via these meson data and two unknown parameters, in addition to the β, which represent the π' contributions in the π^0 lifetime and in the chiral-symmetry–breaking terms underlying the PCAC relation. We give the most appropriate combinations of these parameters for obtaining the observed π^0 lifetime in accord with the anomaly given by either of the existing quark-theoretical models. We find the correction to the Adler anomaly, in terms of the π' parameters and according to the quark model we choose. Assuming that the π' contribution to the decay amplitude is small, our results favor the integral charge quark schemes.

I. INTRODUCTION

The calculation of the decay rate of the real $\pi^0 \to 2\gamma$ has been an interesting problem for the test of various formalisms in elementary particles. Several treatments have been used and their result have been analyzed. A well-known one is the interaction Lagrangian for the first-order Feynman graph consisting of a closed proton-antiproton loop.[1] Another method is based on the vector dominance assumption (VMD) in which $\pi^0 \to \rho + \omega$ followed by $\rho \to \gamma$ and $\omega \to \gamma$.[2] A refined VMD model exists on the basis of the light-cone-dominated mass dispersion relation.[3] Finally we have the current algebra based on the PCAC (partial conservation of axial-vector current) relation which yields zero for the above decay amplitude, unless it is modified by the anomaly operator which Adler has found from the perturbation theory in the presence of the electromagnetism.[4] Yet this modified PCAC gives a π^0 lifetime which is one order of magnitude smaller than the observed one. Adler has, therefore, defined the anomaly operator such that the π^0 amplitude is found proportional to the anomaly parameter S, which is in turn related to the hadron internal charges. Accordingly, the observed π^0 data give $S = 0.44$ close to $\frac{1}{2}$ which is obtained from the recently developed "colored" quark schemes,[5] but in disagreement with $S = \frac{1}{6}$ given by the original quark model based on the fractional charge. This conflict has naturally revived the argument concerning the validity of the PCAC smoothness assumption. In order to obtain the π^0 lifetime in accord with the fractional charge quark, while retaining the PCAC features, Drell has introduced the π' field (a pion isobar representing the 3π continuum) in the modified PCAC operator.[6] Other authors, focusing on the smoothness problem, have introduced the π' pion in the off-shell π^0 amplitude which then is extrapolated to the physical one by the Fubini-Furlan technique.[7] This approach, which requires a relatively small extrapolation, also favors $S = \frac{1}{2}$ unless the field is included in the modified PCAC operator, as in Ref. 6. In view of the important connection which has been developed between the π^0 lifetime and various quark models, and mindful of the general weakness of the PCAC smoothness assumption, we wish to study this decay with a method which avoids extrapolation and does not explicitly depend on the anomaly.

Our treatment, in Sec. II, is based on the already known algebra of the axial-vector current which is generally defined as the pion source and is given as

$$J_a^{\nu} = A_a^{\nu} + c_a \partial^{\nu} \varphi_a, \quad a = 1 \text{ to } 8 . \tag{1}$$

Here A^{ν} is the usual axial-vector current satisfying the PCAC relation $\partial_{\nu} A^{\nu} = cm\varphi$, so that $\partial_{\nu} J^{\nu} = c(m^2 + \partial^2)\varphi$, and c, m, and φ are the decay constant, mass, and the field component associated to the symmetry index a. The algebra of this current is based on the linear σ-model Lagrangian for the canonical fields.[8] The vector current J^ν is the same one which appears in the commutators of the current A^ν. This J-current formalism has been successfully used in the calculation of the Kp and $\pi\pi$ scattering lengths[9,10] and for the study of the K_{l3} form factors.[11] Here, by reducing the pion and a photon we express the π^0 amplitude in terms of the commutators of the currents J^ν and V^μ, making use of the properties of these currents and the canonical rules. The terms which finally appear in the amplitude are calculated via a set of intermediate states from which the one- and two-meson states are taken into account, while the terms of the order of e^3 and smaller are neglected. The contribution of the 3π continuum is represented by the π' pion. Thus the π^0 amplitude is expressed in terms of the parameters of the ρ, ω, and ϕ mesons and the π' pion. In an appendix we show that the momentum-dependent terms in the coupling constants of these mesons, at the $\pi\gamma$ vertex, are also due to the 3π continuum in the intermediate states. Effectively we take account of these terms by a coefficient β. In Sec. III, we explain why our treatment does not directly depend on the PCAC anomaly. However, by writing the current J in terms of the modified PCAC relation,[4,6] we derive a sum rule for the Adler anomaly, while considering a chiral-symmetry–breaking term proportional to the π' field. Our expressions therefore depend on, in addition to β, two unknown parameters which represent the π' contributions to the π^0 decay amplitude and to the anomaly parameter S. In Sec. IV, we analyze our results and give the most economical combinations of these parameters for which we can obtain the π^0 lifetime and the anomaly S in agreement with either of the existing quark-theoretical models.

II. CALCULATION OF THE $\pi^0 \to 2\gamma$ LIFETIME

The transition matrix element for the $\pi^0 \to 2\gamma$ decay may be written as

$$T_{\pi^0} = T^\mu \epsilon'_\mu , \tag{2a}$$

$$T^\mu = i \int d^4x\, d^4y \exp[i(k\cdot x - p\cdot y)] \times K_y \langle \gamma, q, \epsilon |\, \theta(x_0 - y_0)[V^\mu(x), \varphi_{\pi^0}(y)]|0\rangle , \tag{2b}$$

making use of the reduction formalism. Here $K_y \equiv m^2 + \partial^2$, with m being the pion mass, and V^μ is the electromagnetic current operator in the SU(3) scheme,

$$V^\mu = V_3^\mu + \frac{1}{\sqrt{3}} V_8^\mu . \tag{2c}$$

Also, p is the pion momentum, while (q, ϵ) and (k, ϵ') are the momenta and polarization vectors of the two outgoing photons. We consider the pion source current J^μ as[8]

$$c_a K_y \varphi_a(Y) = \partial_\nu J_a^\nu(y), \quad a = 1 \text{ to } 8 \tag{2d}$$

where c and φ are the pion decay constant and the field, respectively. For the π^0, Eqs. (2d) and (2b) give

$$T^\mu = ic^{-1} \int d^4x\, d^4y \exp[i(k\cdot x - p\cdot y)] \times \langle \gamma, q, \epsilon | \theta(x_0 - y_0)[V^\mu(x), \partial_\nu J_3^\nu(y)]|0\rangle . \tag{3a}$$

To obtain Eq. (3a) we have chosen the σ-model Lagrangian formalism in which V^μ is expressed in terms of the quark, scalar, and pseudoscalar fields, (Q, σ, φ), as[8]

$$V_c^\mu = \tfrac{1}{2}\overline{Q}\lambda_c \gamma^\mu Q + f_{abc}(\partial^\mu \sigma_b \sigma_a + \partial^\mu \varphi_b \varphi_a) . \tag{3b}$$

Such a current and the canonical commutation rules for the above fields, lead to

$$\delta(x_0)[\varphi_3, V_c^\mu(x)] = \delta(x_0)[\partial^0\varphi_3, V_c^\mu(x)] = 0, \quad \text{for } c = 3 \text{ and } 8 , \tag{3c}$$

and thus

$$K_y \theta(x_0 - y_0)[V^\mu(x), \varphi_3(y)] = \theta(x_0 - y_0)[V^\mu(x), \partial_\nu J_3^\nu(y)] . \tag{3d}$$

By introducing a set of intermediate states in Eq. (3a) and dropping $(2\pi)^4\delta^4(p - q - k)$, we find

$$T^\mu = T_1^\mu + T_2^\mu , \tag{4a}$$

$$T_1^\mu = ic^{-1} \sum_n \frac{\langle \gamma, q, \epsilon | V^\mu |n\rangle\langle n| p_{n\nu} J_3^\nu |0\rangle \delta^3(\vec{p}_n - \vec{p})}{p_{n0} - p_0} , \tag{4b}$$

$$T_2^\mu = ic^{-1} \sum_n \frac{\langle \gamma, q, \epsilon | (q - p_n)_\nu J^\nu |n\rangle\langle n| V^\mu |0\rangle \delta^3(\vec{p}_n + \vec{k})}{p_{n0} + k_0} , \tag{4c}$$

where p_n is the four-momentum of the intermediate states n. To evaluate Eq. (4) we assume the pole dominance of the one- and two-meson states among the intermediate states in the present decay. Considering parity, gauge invariance, and $\partial_\mu V^\mu = 0$, we find the states n. For the V_3^μ part of the V^μ operator, in Eq. (4b) n represents π' mesons (i.e., 3π continuum, or pion isobars) and $|\rho, \gamma'\rangle$, which consists of a ρ meson and a photon γ'; while in Eq. (4c) n denotes ρ and $|\pi', \gamma'\rangle$. For the V_8^μ part of the V^μ, on the other hand, we have ω and ϕ mesons in Eq. (4c), and (ω, γ') and (ϕ, γ')

in Eq. (4b). The two-particle intermediate states (the Z graphs) are evaluated by an approximation, such as

$$\langle\gamma|V^{\mu}|\rho,\gamma'\rangle\langle\gamma',\rho|J_3^{\nu}|0\rangle \simeq \langle\gamma|\gamma'\rangle\langle 0|V^{\mu}|\rho\rangle \times\langle\rho|J_3^{\nu}|\bar{\gamma}\rangle . \tag{4d}$$

The matrix elements which we require here are[12]

$$\begin{aligned} N_n\langle n|V^{\mu}|0\rangle &= ic_n\epsilon_n^{\mu} , \\ N_{\pi'}\langle\pi'|V^{\mu}|0\rangle &= ic'p_{\pi'}^{\mu} , \\ N_{n\gamma}\langle\gamma|J_3^{\nu}|n\rangle &= ig_{n\pi\gamma}(t_n)\epsilon^{\alpha\beta\sigma\nu}\epsilon_{\alpha}q_{\beta}\epsilon_{n\sigma} , \\ N_{\pi'\gamma}\langle\gamma|J_3^{\nu}|\pi'\rangle &= ig_{\pi'\gamma\pi}(t_{\pi'})\epsilon^{\alpha\beta\sigma\nu}\epsilon_{\alpha}q_{\beta}p_{\pi'\sigma} , \end{aligned} \tag{5a}$$

where here and henceforth n denotes ρ, ω, and ϕ mesons, the N's are the normalization factors, while c_n and ϵ_n are the decay constant and polarization for the particle n. Also c' and $p_{\pi'}$ are the same as c and p, but for the π' pion. The coupling constants $g_{i\pi\gamma}$ depend on the momentum transfer squared,

$$t_i = (p_i - q)^2, \quad i = n \text{ and } \pi' . \tag{5b}$$

By combining Eqs. (2c), (5), (4), and (2) and summing over spins of the states n, we find, in the rest frame of the decaying pion,

$$T = N\frac{(2\pi)^4}{c}\delta^4(p-q-k)(\mathfrak{M}+\mathfrak{M}') , \tag{6a}$$

with

$$\mathfrak{M} = \epsilon^{\alpha\beta\mu\nu}\epsilon_{\alpha}q_{\beta}\epsilon'_{\mu}\sum_n\frac{c_n p_{n\nu}}{2p_{n0}}\left[\frac{g_n(t_n^-)}{p_{n0}-q_0}\delta^3(\vec{p}_n-\vec{q}) + \frac{g_n(t_n^+)}{p_{n0}+q_0}\delta^3(\vec{p}_n+\vec{q})\right], \tag{6b}$$

where N is the over-all normalization factor, g_n denotes $g_{n\pi\gamma}$, and

$$t_n^{\pm} = m_n^2 \pm mm_n . \tag{6c}$$

Also, $\mathfrak{M}'$ represents the contribution of the pion isobars and is obtained by replacing the parameters of the particle n by those of the π' pion in Eq. (6b); i.e.,

$$\mathfrak{M}' = \mathfrak{M}(c_n \to c',\ p_n \to p_{\pi'},\ m \to m';\ t_n \to t_{\pi'}) , \tag{6d}$$

where m' is the π' mass.

The transition probability for the π^0 decaying into two identical photons is found to be

$$w = \frac{m^3}{64\pi c}\left[F' + \sum_n\frac{a_n c_n}{m_n}\left(1-\beta\frac{m_n^2}{m^2}\right)g_n(0)\right]^2 , \tag{7a}$$

where

$$F' = \frac{c'}{2m'}\left[g_{\pi'}(t_{\pi'}^+) + g_{\pi'}(t_{\pi'}^-)\right] \tag{7b}$$

is the direct contribution of the π' pion, and we have used Eq. (A6) of the Appendix in which $\beta_n(t_n^{\pm})$ are, on the average, taken as β for all mesons n, in the range of $t_n = t_n^-$ to t_n^+ given by Eq. (6c). This parameter β represents an indirect contribution of the pion isobar, which comes from the coupling constants g_n.[13,14] In Eqs. (7) we have also

$$a_\rho = 1, \quad a_\omega = a_\phi = \frac{1}{\sqrt{3}} \tag{7c}$$

and $t_{\pi'}^{\pm}$ is given by Eq. (6c) in which $m_n \to m'$. From the vector-meson data and work in Ref. 15 we have

$$c_n g_n(0) = b_n m_n^2 , \tag{7d}$$

with $b_\rho = 2.56 \pm 0.11$, $b_\omega = 2.95 \pm 0.41$, and $b_\phi = -0.55 \pm 0.38$ in units of $10^{-2}\, m^{-1}$, and $m_\rho = 5.45m$, $m_\omega = 5.6m$, $m_\phi = 8.5m$, and $c = 0.65m$. Using these data, there are two unknown parameters F' and β left in Eq. (7a). We will find a relation between these parameters and estimate their values below, by finding the connection between our formalism and the PCAC anomaly constant which may be determined by either one of the quark models. What we know now is that these parameters are not simultaneously negligible, since by setting $F' = \beta = 0$ we get

$$\begin{aligned} \tau^{-1} &= w \\ &= 0.11\times 10^{-16}\ \text{sec}^{-1} \\ &\cong 60\ \text{eV} , \end{aligned} \tag{8a}$$

unacceptably different from

$$\tau^{-1} = (7.8 \pm 0.9)\ \text{eV} , \tag{8b}$$

which is the latest observed π^0 lifetime.[16]

III. CONNECTION WITH THE PCAC ANOMALY

A feature of the on-mass-shell treatment is that it gives the decay rate of the physical π^0 without the necessity of dealing explicitly with the PCAC anomaly. We first explain this point and then derive a sum rule for the anomaly parameter as follows.

We note that at the soft-pion limit $p^2 = 0$ the reduction formula expressed in terms of the J current is equivalent with that written in terms of the usual axial-vector current A. To keep this property in the presence of the electromagnetism, the two currents J and A must be modified by the same anomaly operator. Now the modified PCAC operator, for the neutral pion, may be written as

$$\partial_\nu A_3'^{\nu} = cm^2\varphi_3 + \partial_\nu R_3^{\nu} \tag{9a}$$

where the anomaly current operator is defined as

$$\partial_\nu R_3^\nu = \frac{\alpha}{4\pi} S\partial_\nu v_\beta \partial_\sigma v_\xi \epsilon^{\alpha\beta\sigma\xi} + c' m'^2 \varphi_3' . \tag{9b}$$

Here, the first term is the anomaly operator defined in Ref. 4, and the last term is due to a chiral-symmetry–breaking term introduced in Ref. 6. Also S is the Adler anomaly constant, and v_β and φ' are, respectively, the photon and the π' meson fields. According to definitions (9) and (2d) we can write

$$J_a'^\nu = J_a^\nu + R_a^\nu \delta_{a3} , \tag{10}$$

where $J_3'^\nu$ is the modified third component of the octet current J_a^ν which is given in Ref. 8. We have verified that if v and φ' fields are canonical,[17] using the definition (10) and thus changing J_3^ν to $J_3'^\nu$ in the previous part does not affect our result in Eq. (7). Nevertheless, by this change we can find a connection between the π^0 lifetime given by Eq. (7) and the Adler anomaly, since by reducing only the pion in Eq. (2a) and using Eq. (10), we can also have

$$T = \frac{(2\pi)^4}{cm^2}\delta^4(p-q-k)\langle 2\gamma|\partial_\nu R_3^\nu|0\rangle . \tag{11}$$

The Adler anomaly for the physical pion can be found by evaluating the transition rate with this amplitude and definition (9), and comparing it with Eq. (7). This gives

$$S = \frac{2\pi^2}{m^2}\left[\sum_n \frac{a_n}{m_n}(m^2 - \beta m_n{}^2) c_n g_n(0) + F' - F''\right] \tag{12a}$$

in units of $(4\pi\alpha)^{-1}$. Here

$$F'' = c' m' (m'^2 - m^2)^{-1} g_{\pi'\gamma\gamma} \tag{12b}$$

is due to the φ' field in the anomaly current operator defined by Eq. (9a). An interesting case is to assume that the entire contribution of the 3π continuum in the amplitude (11) is generated by the R_3^μ operator. In this case, by evaluating the transition rate from the amplitude (11) and comparing it with that given by Eq. (7), we find $F' = F''$ and

$$S = \frac{2\pi^2}{m^2}\sum_n \frac{a_n}{m_n}(m^2 - \beta m_n{}^2) c_n g_n(0) . \tag{12c}$$

The numerical analysis of Eqs. (7) and (12) will be seen below.

IV. DISCUSSION AND CONCLUSION

The essential feature of this formalism is that all particles are kept on the mass shell, and thus the soft-pion extrapolation which troubles the PCAC approach is avoided. Moreover, as stated earlier, the modification of the third component of the PCAC operator by the canonical operators, as in Eqs. (9), does not upset the algebra of the current J. It changes the J_3 component to J_3' given in Eq. (10), with no effect on the result in Eq. (7). Nevertheless, by having J_3', we have linked the π^0 amplitude with the Adler anomaly S in terms of the vector-meson and π' parameters. Note that the β parameter represents the 3π continuum which is the only contributing source of the momentum dependent terms in the coupling constants $g_{n\pi\gamma}$, while F' gives the contribution of the same intermediate state in the π^0 amplitude. We recall that the π' pion is, at present, a device to account for the 3π continuum in the intermediate states of the matrix elements of our current products. Nevertheless, we have considered the possible existence of such a field in the chiral breaking terms as done in Ref. 6; thereby F'' has appeared in our anomaly expression. We discuss our results in terms of β, F', and F'' parameters in Eqs. (A6), (7b), and (12b), rather than in terms of the more numerous parameters of the π' pion.

We first consider the numerical results with the assumption that has led to $F' = F''$ and Eq. (12c). In this case, there is a set of $F' = F''$ and β, for which Eqs. (7) and (12c) give

$$\tau^{-1} = (7.8\text{–}9.1)\ \text{eV}, \quad S = \tfrac{1}{2} \tag{13a}$$

and another which yields

$$\tau^{-1} = (7.8\text{–}9.1)\ \text{eV}, \quad S = \tfrac{1}{6} \tag{13b}$$

in agreement with the "colored" quark statistics and the usual quark model, respectively, as seen in Table I.

For the general case $F' \neq F''$, Eqs. (7) and (12a), with the vector-meson data in Sec. II and w in eV

TABLE I. Selected solutions of Eqs. (7) and (12) with the dimensionless parameters β, F', and F'' in units of 10^{-2}.

τ^{-1}		$S=\frac{1}{2}$		$S=\frac{1}{6}$	
(eV)	β	F'	F''	F'	F''
7.8	0	−19.0	−0.18	−19.0	1.5
	3.16	−0.18	−0.18		
	3.19	0	−0.18	0	1.5
	3.45			1.5	1.5
9.1	0	−18.8	0	−18.8	1.7
	3.16	0	0	0	1.7
	3.45			1.7	1.7

units, give

$$\frac{S}{2\pi^2}+F''=(8.50\times10^{-3})w^{1/2}$$
$$=F'+0.214-5.96\beta. \qquad (14)$$

Thus one of the parameters F' and β is free in the formalism. However, we expect these parameters to be small since the coupling constants g_n are known to be slowly varying functions of t_n in the range given by Eq. (6c), and the 3π continuum is a second-order correction in the π^0 amplitude. We know from Eqs. (8) that $\beta=F'=0$ is unacceptable. On the other hand, we see in Table I that $\beta=0$, which implies a complete smoothness of the vector-meson coupling constants requires an undesirably large F', while $F'=0$ and β in the same range as in the case $F'=F''$ yield either of solutions (13).

In dealing with the formalism based on the PCAC operator modified by the anomaly expression (9b), an important point to be considered is the fraction contribution of F'' in relations (14). This fraction, which may be regarded as the extrapolation correction, is very small, 0–4%, for $S=\frac{1}{2}$, but appreciably large, 64–67%, for $S=\frac{1}{6}$ making use of Eq. (8b), the observed τ^{-1} in relations (14). It follows that if the 3π continuum contribution in this decay is small, as is the case for the Goldberger-Treiman relation reported in Ref. 6, our best solution corresponds to the expression of S in Eq. (12c), which is based on $F'=F''$, and on the "colored" quark schemes that gives $S=\frac{1}{2}$.

We conclude that the on-mass-shell current algebra offers a successful analysis of the π^0 lifetime and the Adler anomaly constant, in terms of the parameters of the dominant vector mesons and the 3π continuum in this decay. It leads to the anomaly operator R_3^μ for the modification of the PCAC relation which accounts for the pion electromagnetic interactions, and produces the PCAC extrapolation correction corresponding to whichever quark-theoretical model we choose.

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APPENDIX

Here we evaluate the coupling constants $g_{n\pi\gamma}$ for n, which represents ρ, ω, and ϕ mesons, by writing the unsubtracted dispersion relation

$$g_{n\pi\gamma}(t_n)=\pi^{-1}\int\frac{\operatorname{Im}g_{n\pi\gamma}(t')}{t'-t_n}\,dt'\ , \qquad (\mathrm{A1})$$

where $t'=(p_n'-q)^2$, with $p_n'^2=p_n^2=m_n^2$. To obtain the imaginary part $\operatorname{Im}g_n(t')$ we consider the quantity R defined as

$$R\equiv\langle\gamma,q,\epsilon\,|\partial_\nu J_\pi^\nu(x)|n,p_n,\epsilon_n\rangle$$
$$=i\int d^4y\,e^{ip_n\cdot y}K_y\langle\gamma,q,\epsilon\,|T\{\partial_\nu J_\pi^\nu(x),v_n(y)\}\,|0\rangle\,, \qquad (\mathrm{A2})$$

where v_n is the field for the vector meson n. From Eq. (A2) we find

$$\operatorname{Im}R=\frac{(2\pi)^4}{2}\sum_I\delta^4(p_I-q-p_n)$$
$$\times\langle\gamma|\epsilon_n^\mu V_{n\mu}|I\rangle\langle I|p_{n\nu}J_\pi^\nu|0\rangle\ , \qquad (\mathrm{A3})$$

where p_I is the four-momentum of the intermediate states I, and we have defined $K_y\,v_n(y)=\epsilon_n^\mu V_{n\mu}(y)$ in which V_n is the vector current source. Considering the properties of the operators V_n and J_π^ν, gauge invariance, etc., we find that the only contributing states I in Eq. (A3) are 3π, 5π, ... states, which are called the "3π continuum." Hence for such states we write

$$\langle\gamma,q,\epsilon\,|V_n^\mu|I,p_I\rangle=g_{In\gamma}(t_I)\epsilon^{\alpha\beta\sigma\mu}\epsilon_\alpha q_\beta p_{I\sigma}\ ,$$
$$\langle I,p_I|\partial_\nu J_\pi^\nu|0\rangle=ic_I m_I^2\ , \qquad (\mathrm{A4})$$

where p_I, c_I, and $g_{In\gamma}$ are the momentum, decay, and coupling constants of the state $|I\rangle$, $p_I^2=m_I^2$, and $t_I=(p_I-q)^2$. By obtaining $\operatorname{Im}R$ from the matrix element of the current J in Eq. (5a) of the text and comparing it with Eq. (A3) in which Eqs. (A4) are used, we find

$$\operatorname{Im}g_n(t')=\frac{(2\pi)^4}{2}\sum_I G_I(t_I=m_n^2)$$
$$\times\delta(m_I^2-t')\delta^3(\vec{p}_I-\vec{q}+\vec{p}_n)\ , \qquad (\mathrm{A5})$$

where G_I is a function associated with the state I. Using Eqs. (A5) and (A1), we obtain

$$g_{n\pi\gamma}(t_n)=\left[1-\beta_n(t_n)\frac{t_n}{m^2}\right]g_{n\pi\gamma}(0)\ , \qquad (\mathrm{A6})$$

where the coefficient

$$\beta_n(t_n)=\sum_I(t_n-m_I^2)^{-1}m^2 \qquad (\mathrm{A7})$$

is a measure of the effect of the 3π continuum to the coupling constant $g_{n\pi\gamma}$.

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[12]In general, using gauge invariance, we have

$$\langle\gamma|J_\pi^\nu|n\rangle = [g_1 + q\cdot(p_n - q)g_2 + p_n\cdot(p_n-q)g_3] \times \epsilon^{\alpha\beta\sigma\nu}\epsilon_\alpha q_\beta \epsilon_{n\,\sigma},$$

where g's are function of $t_n=(p_n-q)^2$. However, since $q^2=0$ one can define $g_{n\pi\gamma}(t_n)=g_1+\frac{1}{2}(g_3-g_2)m_n{}^2+\frac{1}{2}(g_3+g_2)t_n$ and get Eq. (5a).

[13]Note that the β coefficient, which represents the effective off-shell correction of our coupling constants, has been eliminated in Ref. 7 at the price of introducing the anomaly in the evaluation of the physical π^0 amplitude. The Σ parameter in Ref. 7 must play effectively the combined roles of our parameters β and F'.

[14]The feature of having the physical amplitude in terms of the off-shell vertex functions $g_n(0)$ is a peculiarity of our method and in general that of any treatment based on finite momentum saturation of the retarded product.

[15]M. Gourdin, in *Hadronic Interaction of Electrons and Photons*, edited by J. Cumming and H. Osborn (Academic, New York, 1971), p. 395. Note (a) that in this reference f_n and $f_{n\pi\gamma}$ are related to our c_n and $g_{n\pi\gamma}$ by $c_n = em^2 f_n{}^{-1}$ and $g_{n\pi\gamma}=ef_{n\pi\gamma}m^{-1}$, and (b) that we choose $c_\phi g_\phi = 0.3\, m_\phi{}^2 \times 10^{-2}$ in agreement with this reference and as it is used in Ref. 7.

[16]Particle Data Group, Rev. Mod. Phys. 43, S1 (1971).

[17]From the σ model in Ref. 8 we have

$$J_a^\nu = \tfrac{1}{2}\overline{Q}\lambda_a\gamma^\nu\gamma^5 Q + d_{abc}\sigma_b\overleftrightarrow{\partial}^\nu\varphi_c\,.$$

Using this and the properties of the structure tensors d_{3bc} and f_{3bc}, for the canonical fields v and φ' in Eqs. (9), we find that

$$[J_a^0, J_3'(x)]\delta(x_0) = [J_a^0, J_3^\nu(x)]\delta(x_0) = if_{3ab}V_b^\mu(x) + \text{S.T.}$$

is unchanged.