Neutral-current effects in the decay $\psi \to \mu^+\mu^-$ *

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We calculate the angular distribution and polarization of muon pairs from the decay of a ψ particle produced in e^+e^- collisions. We show, among other things, that weak neutral-current effects do not produce a forward-backward asymmetry but do give the muons a net helicity which is a sensitive test of both neutral current theories and models of the ψ particles. The possibility of measuring the resulting net helicity in the near future is briefly discussed.

The discovery of two very narrow resonances $\psi(3105)$ and $\psi(3695)$ at SLAC[1] and Brookhaven[2] signals the beginning of a new regime in particle physics. As we increase our understanding of the nature of these new structures we can certainly hope that they will prove to be important, perhaps even crucial, clues to some of the outstanding problems in particle physics. At the very least, however, they would seem to be the harbingers of a whole new stratum of information on the structure of matter. We wish to point out, in the latter vein, that, independently to a large degree of what precisely they are, the existence of the ψ's allows the possibility of seeing neutral-current effects using the present generation of e^+e^- colliding-beam machines.

If, as now seems likely, the ψ's are not directly coupled to leptons, then their decay into muon pairs may proceed either through the intermediation of a photon [Fig. 1(a)] or via the weak neutral current [Fig. 1(b), shown in the local limit]. These two diagrams lead to the two invariant amplitudes

$$T^{\text{em}} = \frac{e^2}{\mu_\psi{}^2(2\pi)^3}\bar{u}(p_-s_-)\gamma_\mu v(p_+s_+)\langle 0|V^\mu_{\text{em}}(0)|\psi(k\lambda)\rangle \tag{1}$$

and

$$T^{\text{nc}} = \frac{G}{2\sqrt{2}(2\pi)^3}\bar{u}(p_-s_-)\gamma_\mu(g^V - g^A\gamma_5)v(p_+s_+) \times\langle 0|V^\mu_{\text{nc}}(0)|\psi(k\lambda)\rangle, \tag{2}$$

where V^μ_{nc} is the vector part of the hadronic weak neutral current coupled to muons and where we have parameterized the relevant sector of the neutral-current interaction in the form

$$H_{\text{nc}} = +\frac{G}{2\sqrt{2}}\bar{\mu}\gamma_\mu(g^V - g^A\gamma_5)\mu V^\mu_{\text{nc}}, \tag{3}$$

with G the Fermi constant. We choose this parameterization, including the explicit factor of $+\frac{1}{2}$, because it corresponds to the weak SU(2) extension of the intermediate vector boson (IVB) model in which the semiweak interaction is

$$H_{\text{IVB}} = g\sum_{i=1}^{4}\bar{D}_i\vec{\tau}\gamma_\mu\left(\frac{1-\gamma_5}{2}\right)D_i\cdot\vec{W}^\mu, \tag{4}$$

where $D_1=(\nu_e, e)$, $D_2=(\nu_\mu, \mu)$, $D_3=(u, d\cos\theta + s\sin\theta)$, $D_4=(c, s\cos\theta - d\sin\theta)$ (Ref. 3), and

$$\frac{g}{2M_W{}^2}=\frac{G}{\sqrt{2}}. \tag{5}$$

We feel that such a model is the most natural successor to the old phenomenological description of the weak interaction, and hence we use it as a frame of reference. If further we parameterize the weak neutral vector current in the quark model according to

$$V^\mu_{\text{nc}} = \alpha\bar{c}\gamma^\mu c + \beta\bar{u}\gamma^\mu u - \gamma\bar{d}\gamma^\mu d - \delta\bar{s}\gamma^\mu s, \tag{6}$$

then in the SU(2) version of the IVB model we have $g^V=1$, $g^A=1$, and $\alpha=\beta=\gamma=\delta=1$. Note that V^μ_{nc} has a structure very similar to that of the electromagnetic current,

$$V^\mu_{\text{em}} = \tfrac{2}{3}\bar{c}\gamma^\mu c + \tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s. \tag{7}$$

The values of g^V, g^A, α, β, γ, and δ are given in Table I for a variety[4] of weak-interaction

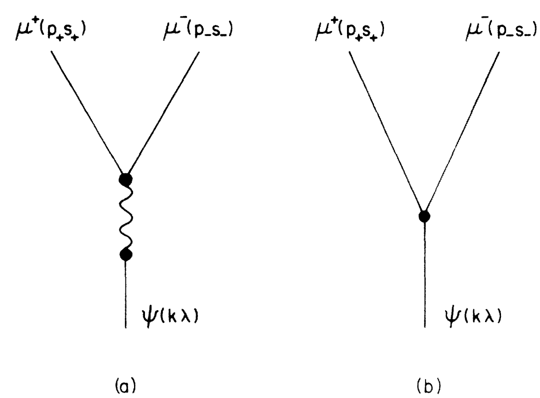

FIG. 1. (a) The electromagnetic amplitude for $\psi \to \mu^+\mu^-$; (b) the weak-neutral-current amplitude for $\psi \to \mu^+\mu^-$.

models.

We are now in a position to proceed with our calculation. We define

$$\langle 0|V^{\mu}_{\rm em}(0)|\psi(k\lambda)\rangle=\frac{\epsilon^{\mu}(k\lambda)}{(2\pi)^{3/2}}f_{\psi}\,\mu_{\psi}{}^2\,, \quad (8)$$

$$\langle 0|V^{\mu}_{\rm nc}(0)|\psi(k\lambda)\rangle=\frac{\epsilon^{\mu}(k\lambda)}{(2\pi)^{3/2}}g_{\psi}\,\mu_{\psi}{}^2 \quad (9)$$

and anticipate that the dimensionless constants f_ψ and g_ψ are of the same general size. These definitions then lead us to the two amplitudes

$$A^{\rm em}\equiv e^2 f_\psi\,, \quad (10)$$

$$A^{\rm nc}\equiv\frac{G\mu_\psi{}^2 g^A g_\psi}{2\sqrt{2}} \quad (11)$$

which will appear throughout what follows. Next, let the storage-ring magnetic field direction and the momentum of the e^- beam define the $\hat{x}$ and $\hat{z}$ axes of a coordinate system, and call θ the angle that the momentum of the decay μ^- makes with $\hat{z}$ and ϕ the angle its projection into the $\hat{x}$-$\hat{y}$ plane makes with $\hat{x}$. The direction $\hat{x}$ enters into our considerations because the $e^{\mp}$ beam in a storage ring is expected to quickly attain a polarization of $p_{\mp}=\mp\frac{8}{15}\sqrt{3}=\mp 0.924$ with respect to this direction.[5]

In the ultrarelativistic approximation which applies here, it is easy to show that the ψ particles will be produced in one of two coherent polarization states, which we label $\vec{e}_{\pm}$, with probabilities $\frac{1}{2}(1\pm p^2)$:

$$\vec{e}_{\pm}=\frac{A^{\rm em}\vec{e}(k,1)\mp iA^{\rm nc}\vec{e}(k,2)}{[(A^{\rm em})^2+(A^{\rm nc})^2]^{1/2}}\,, \quad (12)$$

where the $\vec{e}(k,\lambda)$ are linear polarization vectors.[6] The amplitudes for this process are of course just the complex conjugates of the amplitudes (1) and (2).

If ψ is produced at a rate R and has a total decay rate Γ, then the differential event rates $dr(\lambda_-,\lambda_+)/d\Omega_-$ for muon pair events leading to a μ^- at angles (θ,ϕ) with helicity λ_- and an oppositely directed μ^+ with helicity λ_+ are

$$\frac{dr(\pm,\pm)}{d\Omega_-}=0\,, \quad (13)$$

$$\frac{dr(\pm,\mp)}{d\Omega_-}=\frac{R}{\Gamma}\frac{p_-}{64\pi^2}[(A^{\rm em})^2\mp 2A^{\rm em}A^{\rm nc}]F(\theta,\phi)\,, \quad (14)$$

where

$$F(\theta,\phi)=1+\cos^2\theta-p^2\sin^2\theta\cos 2\phi\mp\frac{4A^{\rm nc}}{A^{\rm em}}\cos\theta\,. \quad (15)$$

Hence the unpolarized rate is

$$\frac{dr}{d\Omega_-}=\frac{R}{\Gamma}\frac{p_-}{32\pi^2}(A^{\rm em})^2(1+\cos^2\theta-p^2\sin^2\theta\cos 2\phi) \quad (16)$$

and the net helicity is[7]

$$h(\theta,\phi)=-\frac{4A^{\rm nc}}{A^{\rm em}}\left(\frac{1}{2}+\frac{\cos\theta}{1+\cos^2\theta-p^2\sin^2\theta\cos 2\phi}\right). \quad (17)$$

Notice that, in contrast with the off-resonance $e^+e^-\to\mu^+\mu^-$ process, there is no forward-backward asymmetry in the unpolarized rate. This is perhaps just as well in that such an effect need not be due to weak interactions, so that the interpretation of such a measurement has its concomitant difficulties.[8] The predicted helicity $h(\theta,\phi)$ and unpolarized event rate $dr(\theta,\phi)/d\Omega_-$ are shown in Figs. 2 and 3, respectively.

We see that in the forward hemisphere one will find an enhancement of the helicity due to the strong transverse polarization of the beams to a value

$$h_{\rm forward}\simeq-\frac{4A^{\rm nc}}{A^{\rm em}}=-\frac{G\mu_\psi{}^2 g^A}{2\sqrt{2}\pi\alpha}\frac{g_\psi}{f_\psi} \quad (18)$$

$$\simeq\begin{cases}-0.17\dfrac{g^A g_{\psi(3105)}}{f_{\psi(3105)}}\times 10^{-2} & \text{for } \psi(3105) \quad (19)\\[2ex] -0.24\dfrac{g^A g_{\psi(3695)}}{f_{\psi(3695)}}\times 10^{-2} & \text{for } \psi(3695). \quad (20)\end{cases}$$

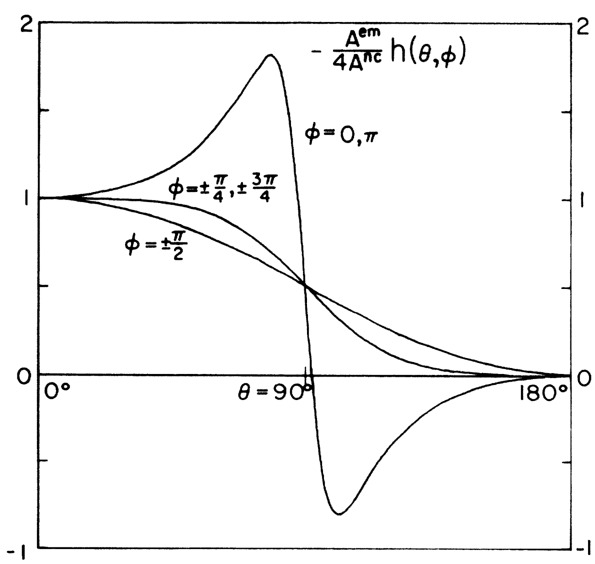

FIG. 2. The predicted helicity of the μ^- in $\psi\to\mu^+\mu^-$.

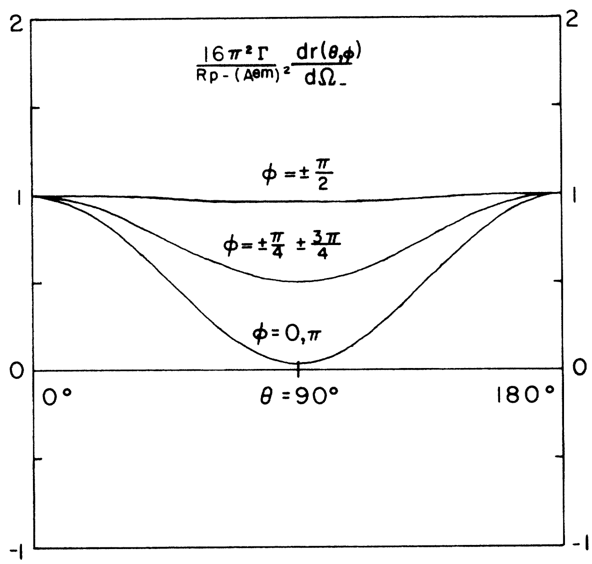

FIG. 3. The angular distribution in $\psi \to \mu^+\mu^-$.

It remains to discuss $g^A g_\psi/f_\psi$, which depends not only on the weak interaction but also, of course, on the nature of the ψ particles. For example, if the ψ's should be relatively pure charm-anticharm vector mesons, then one would expect $g/f = \frac{3}{2}\alpha$ (see Table I). More generally, one can find by expanding V^μ_{nc} and V^μ_{em} in the 16-plet of SU(4) vector currents[9] that if SU(4) were exact, the value of g/f would be determined by the SU(4) classification of ψ according to $(g/f)_3 = \beta + \gamma$, $(g/f)_8 = \beta - \gamma + 2\delta$, $(g/f)_{15} = \frac{1}{2}(-3\alpha + \beta - \gamma - \delta)$, and $(g/f)_0 = 3(\alpha + \beta - \gamma - \delta)$; SU(4) breaking should not change the size of this effect. In fact, as we have already remarked, no matter what the nature of the ψ's, the similarity in the structure of V^μ_{nc} and V^μ_{em} virtually guarantees that $g/f \sim O(1)$.

The expected magnitude of the resulting net helicity is therefore very similar to that which is present in $e^+e^- \to \mu^+\mu^-$ at any nearby energy.[10] However, one of the advantages of using the decay of the ψ particles to see neutral-current effects is that *the large production cross sections for ψ's make the measurement of such a helicity feasible long before it becomes possible at any nearby energy*. Since with a 1.9-MeV resolution[1] the peak cross section to produce muon pairs from $\psi(3105)$ is ~200 nb, one would see more than ten such events per second with a luminosity of 10^{32} cm^{-2} sec^{-1} [luminosities of this magnitude should be available soon at DCI (Orsay) and DORIS (DESY)[11]]. This event rate is more than ample to allow detection of an effect at the 10^{-3} level. Alternatively, if it were possible to define the energies of the electron and positron beams more accurately, one could produce the ψ on resonance with a cross section of ~5 μb, which would give an event rate sufficient to make the expected effect observable at presently available luminosities. Of course, if another such resonance were to be discovered at a significantly higher mass, the enlarged effect could be seen immediately.

Such a measurement would serve many purposes simultaneously: (1) The existence of an effect at any level would indicate the presence of an as yet unseen charged-lepton–hadron weak neutral current, (2) within the context of a given model for the ψ particles, the *value* and *sign* of the helicity would put severe restrictions on models of the neutral currents, and conversely (3) within the context of a given model for the neutral currents the value and sign of the helicity would put severe restrictions on models of the ψ particles (for example, in the charm-anticharm model it measures the charge of the charmed quark).

Considering the exciting issues which could be resolved, we feel that such a measurement should be given serious consideration.

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TABLE I. The values of the parameters g^V, g^A, α, β, γ, and δ in various models.

Model [a]	g^V	g^A	α	β	γ	δ
(A) SU(2)-IVB	1	1	1	1	1	1
(B) SU(2)×U(1)-SB	$1-4\sin^2\theta$	1	$1-\frac{8}{3}\sin^2\theta$	$1-\frac{8}{3}\sin^2\theta$	$1-\frac{4}{3}\sin^2\theta$	$1-\frac{4}{3}\sin^2\theta$
(C) SU(2)×SU(4)-ISB	$-\sqrt{2}$	$-\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	0
(D) U(1)	1	1	a	a	a	a

[a] See Ref. 4 for a description of the models.

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[1]J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974); G. S. Abrams *et al.*, *ibid.* 33, 1453 (1974).

[2]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974).

[3]Here u, d, and s are the three usual fractionally charged quarks and c is the charmed quark with $Q=\frac{2}{3}$, $I=Y=0$.

[4]The models we tabulate are the following. *Model A:* the SU(2) extension of the IVB model discussed in the text. *Model B:* the spontaneously broken SU(2)× U(1)-symmetry model [S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No. 8), edited by N. Svartholm (Wiley, New York, 1969), p. 367] with the GIM mechanism to suppress strangeness-changing neutral currents [S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970)]. *Model C:* an intermediate-scalar-boson model of the weak interactions [R. E. Pugh, Phys. Rev. D 5, 474 (1972); R. L. Penner, *ibid.* 6, 2059 (1972); N. Isgur, Nuovo Cimento 20A, 585 (1974)]. *Model D:* a model in which the neutral hadron current is the baryon current [J. J. Sakurai, Phys. Rev. D 9, 250 (1974)].

[5]A. A. Sokolov and I. M. Ternov, Dokl. Akad. Nauk SSSR 153, 1052 (1963) [Sov. Phys.—Dokl. 8, 1203 (1964)].

[6]Of course, $\vec{e}(k,1)=\hat{x}$, $\vec{e}(k,2)=\hat{y}$, and $\vec{e}(k,3)=\hat{z}$.

[7]As discussed by J. Godine and A. Hankey, Phys. Rev. D 6, 3301 (1972), a net helicity at an angle ϕ if $p \neq 0$ is not a guarantee of parity violation unless $\phi=\pi/2$. However, a net helicity averaged over an angular region symmetric about $\phi=\pi/2$ is a parity-violating effect. In any event such difficulties, which can arise from higher-order electromagnetic effects, are expected to be small.

[8]See, for example, Godine and Hankey (Ref. 8), or D. Dicus, Phys. Rev. D 8, 890 (1973).

[9]We define these currents as follows: V_3^μ and V_8^μ are the ordinary SU(3) currents, while V_0^μ is the SU(4)-singlet baryon-number current; V_{15}^μ is the remaining independent $Q=Y=C=0$ current.

[10]There are, of course, also some significant differences. For example, in the Weinberg-Salam model the net helicity away from the resonance is proportional to $(1-4\sin^2\theta)$, while at a charm-anticharm resonance it would be proportional to $(1-\frac{8}{3}\sin^2\theta)$.

[11]K. Strauch, in *Proceedings of the Sixth International Symposium on Electron and Photon Interactions at High Energy, Bonn, Germany, 1973,* edited by H. Rollnick and W. Pfeil (North-Holland, Amsterdam, 1974), p. 1. Actually, 0.6×10^{32} cm^{-2}sec^{-1} was the design luminosity at $s=3.1$ GeV of the present SPEAR facility, but this has not yet been realized.