PHYSICAL REVIEW D VOLUME 11, NUMBER 11 1 JUNE 1975

# Inclusive ρ effects on two-pion production

J. Randa
*Physics Department, Texas A & M University, College Station, Texas 77843*
(Received 4 November 1974)

The effects of inclusive ρ production on two-pion distributions are investigated. The differential cross section, correlation function, and azimuthal asymmetry are calculated for decay-product pions. Two cases of ρ polarization are considered.

## INTRODUCTION

Inclusive resonance production is receiving increased attention recently,[1] motivated by interest both in production of the resonances themselves and in their effect on the distributions of their decay products. Sasaki and Murai[2] have dealt with single pion spectra resulting from the production of resonances, in particular from $\rho$'s and $\omega$'s. Basetto, Toller, and Sertorio have considered certain two-pion distributions[3] from decaying $\rho$'s, but do not calculate any correlations that are commonly measured (now). In this paper we consider some of the effects of inclusive $\rho$ production on the two-pion spectra, where one expects that its importance might be greater than in single pion production. (In $\pi p\to\pi X$, it is estimated[1] that at 205 GeV/$c$ incident momentum, about 25% of the pions are decay products of $\rho$ mesons.) The program followed in this paper is to use a parameterization of the single $\rho$ production cross section in the central region and compute from this the two-particle spectrum for the pions into which the $\rho$ decays. We compute the differential cross section $d^2\sigma/dy_1dy_2$ and the correlation function $R(y_1, y_2)$ as functions of rapidity difference $(y_1-y_2)$ at fixed $(y_1+y_2)$. We also compute the azimuthal asymmetry as a function of $(y_1-y_2)$.

The process we are considering is $\pi^{\pm}p\to\rho^0X\to\pi^+\pi^-X$. The charges of the pions are chosen for experimental accessibility. What we do will also apply qualitatively to $pp\to\rho^0X\to\pi^+\pi^-X$, the only change (other than normalization) being in the $p_\perp$ distribution of the $\rho$'s, which will, of course, modify the pion spectrum somewhat.

We will use the following notation: $p_\pi, p_\rho, p_1, p_2$ are the lab momenta of the incident pion, the $\rho$, and the two final pions, respectively. Momenta denoted by $q$'s will refer to the Gottfried-Jackson frame. Also, we often use $P=p_1+p_2$ instead of $p_\rho$. With this notation, the invariant cross section for $\pi^-p\to\rho^0X\to\pi^+\pi^-X$ can be written

$$(2\pi)^6 2E_1 2E_2\frac{d^6\sigma}{dp_1{}^3dp_2{}^3} = (2\pi)^3 2E_\rho\frac{d^3\sigma}{dp_\rho{}^3}\frac{|g_{\rho\pi\pi}|^2}{\{[(p_1+p_2)^2-m_\rho{}^2]^2+m_\rho{}^2\Gamma^2\}} \times\sum_{\lambda,\lambda'}(p_1-p_2)\cdot\epsilon(\lambda)(p_1-p_2)\cdot\epsilon^*(\lambda')\rho_{\lambda\lambda'}\,, \tag{1}$$

where $E_\rho d^3\sigma/dp_\rho{}^3$ is the invariant differential cross section for $\pi^-p\to\rho^0X$ (summed over $\rho$ polarizations), $\epsilon$ is the $\rho$ polarization vector, and $\rho_{\lambda\lambda'}$ is the $\rho$ density matrix. In the Gottfried-Jackson frame

$$\sum_{\lambda,\lambda'}(p_1-p_2)\cdot\epsilon(\lambda)(p_1-p_2)\cdot\epsilon^*(\lambda')\rho_{\lambda\lambda'} = 4\vec{q}_1{}^2[\rho_{00}\cos^2\theta+\rho_{11}\sin^2\theta-\sqrt{2}\sin 2\theta\cos\varphi\,\mathrm{Re}\rho_{10}-\sin^2\theta\cos 2\varphi\,\rho_{1-1}]\equiv 4\vec{q}_1{}^2W(\theta,\varphi)\,. \tag{2}$$

To compute the two-pion spectrum then, one need only put in $d^3\sigma/dp_\rho{}^3$ and the $\rho_{\lambda\lambda'}$'s. In this paper we use the parameterization

$$2E_\rho\frac{d\sigma}{dp_\rho{}^3}=Ce^{-6p_{\rho\perp}{}^2}f(y_\rho)\,. \tag{3}$$

The function of rapidity will be ignored, the motivation being that we wish to consider distributions in the central region and take the cross section to be flat in rapidity there. In light of present high-energy data on inclusive $\rho$'s this assumption is as good as and simpler than any other. The $p_\perp$ dependence is chosen to be roughly consistent with current data. For $W(\theta,\phi)$ we consider the two (extreme) cases $W(\theta,\phi)=\cos^2\theta$ and $W(\theta,\phi)=\frac{1}{2}\sin^2\theta$ and plot the results for each.

## CALCULATION

To obtain $d^2\sigma/dy_1dy_2$ for the decay pions one integrates over transverse momenta in Eq. (1),

$$\frac{d^2\sigma}{dy_1dy_2}(\pi^-p\to\rho^0X\to\pi^+\pi^-X)=\frac{C}{(2\pi)^3}|g_{\rho\pi\pi}|^2\int dp_{1\perp}{}^2dp_{2\perp}{}^2e^{-6(p_{1\perp}+p_{2\perp})^2}\frac{\vec{q}_1{}^2}{(P^2-m_\rho{}^2)^2+m_\rho{}^2\Gamma^2}\begin{pmatrix}\cos^2\theta\\ \frac{1}{2}\sin^2\theta\end{pmatrix}, \tag{4}$$

where

$$\cos\theta=(P^2-4\mu^2)^{-1/2}\left(\frac{(p_\rho\cdot p_\pi)^2}{P^2}-\mu^2\right)^{-1/2}\times p_\pi\cdot(p_2-p_1), \tag{5}$$

$$\vec{q}_1{}^2=\tfrac{1}{4}P^2-\mu^2,$$

and $\mu$ is the pion mass. In the calculation we use $g_{\rho\pi\pi}{}^2/4\pi=2.13$.

Equation (4) gives the contribution of the process $\pi p\to\rho X\to\pi\pi X$ to the two-pion spectrum. If we want the effect of $\rho$ production on the inclusive pion cross section, we must also consider the possibility of producing two $\rho$ mesons and detecting one pion from each. Thus the full effect of $\rho$ production on the two-pion cross section is

$$\left(\frac{d^2\sigma}{dy_1dy_2}\right)_\rho=\frac{d^2\sigma}{dy_1dy_2}(\pi p\to\rho X\to\pi_1\pi_2X)+\frac{d^2\sigma}{dy_1dy_2}(\pi p\to\rho\rho X\to\pi_1\pi_2X). \tag{6}$$

In reality there will also be "interference" terms where one $\pi$ meson is produced directly and one is a $\rho$ decay product. These are neglected for now because they are not attributable solely to $\rho$ production and because their treatment requires relatively detailed input for the direct production of $\pi$ mesons. Their inclusion awaits a detailed model.

As a first approximation we will ignore $\rho$-$\rho$ correlations and write

$$\frac{d^2\sigma}{dy_1dy_2}(\pi^-p\to\rho\rho X\to\pi_1\pi_2X)=\frac{4}{\sigma_{\text{incl}}(\pi^-p)}\frac{d\sigma}{dy_1}(\pi p\to\rho X\to\pi_1X)\times\frac{d\sigma}{dy_2}(\pi p\to\rho X\to\pi_2X), \tag{7}$$

where

$$\frac{d\sigma}{dy_i}(\pi p\to\rho X\to\pi_iX)=\int dy_k\frac{d^2\sigma}{dy_idy_k}(\pi p\to\rho X\to\pi_i\pi_kX). \tag{8}$$

The factor of 4 is due to the fact that each of the final pions can come from either of two charge states of the $\rho$. (An implicit assumption is that central $\rho$ production cross sections are independent of the $\rho$'s charge.) As a result of our choice that the cross section for $\pi p\to\rho X$ be independent of rapidity in the central region, the cross section $d\sigma/dy_1(\pi p\to\rho X\to\pi_1X)$ also is flat in rapidity, and consequently the cross section of Eq. (6) differs from $d^2\sigma/dy_1dy_2(\pi p\to\rho X\to\pi_1\pi_2X)$ by an additive constant (in the central region).

In order to have some indication of the importance of $\rho$'s on the two-pion spectrum one needs to estimate the over-all normalization, or $C$ in Eq. (3). In fact, to evaluate $(d^2\sigma/dy_1dy_2)_\rho$ at all $C$ must be determined since it enters our cross section for $\pi p\to\rho X\to\pi\pi X$ linearly and that for $\pi p\to\rho\rho X\to\pi\pi X$ quadratically. An approximate normalization can be obtained from the requirement

$$\langle n_{\rho^0}\rangle\sigma_{\text{inel}}(\pi^-p)=\int dp_{\rho\perp}{}^2dy_\rho\frac{d^2\sigma}{dy_\rho dp_{\rho\perp}{}^2} \tag{9}$$

plus the preliminary 205 GeV/$c$ data presented in Ref. 1, $\sigma_{\text{inel}}(\pi^-p)=20.9$ mb and $\langle n_{\rho^0}\rangle=0.54$. This in itself does not fix $C$ very well since Eq. (3) is only intended for the central region, measurement of $\langle n_{\rho^0}\rangle$ does not include the full range of $y_\rho$, and our estimate of the $p_\perp$ slope is rough at best. (The fact that there is a single slope is itself a simplifying approximation.) Hence our value of $C\simeq 7$ mb should be regarded with appropriate skepticism. We shall point out which results de-

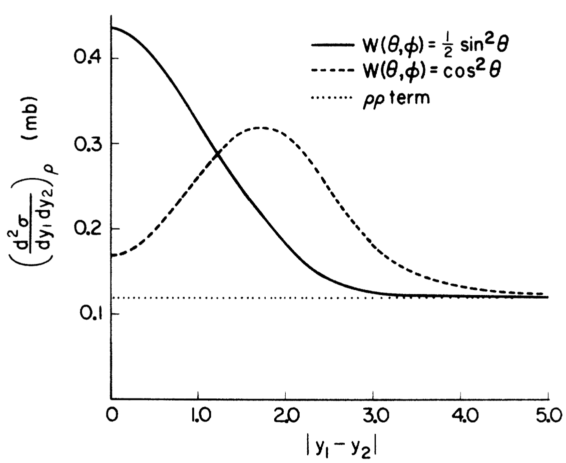


FIG. 1. Differential cross section as a function of $|y_1-y_2|$ at fixed $(y_1+y_2)$.

pend on $C$.

The integrations in Eqs. (4) and (8) are then performed. One of the angular integrations is trivial since the only angular dependence in the integrand is of the form $\vec{p}_{1\perp}\cdot\vec{p}_{2\perp}$. The integrand contains no singularities in the region of integration, and the integrations were done numerically by using the trapezoidal rule and neglecting contributions from $p_{1\perp}$ or $p_{2\perp}$ greater than 4 GeV. The results were checked by shifting the points at which the integrand was evaluated and comparing to the previous results. The largest variation for any integral was less than 1%. Steeper $p_\perp$ distributions were also used ($e^{-12p^2}$), and no difficulties were encountered. Having performed the integrations, we use Eqs. (6) and (7) to obtain the results for $d^2\sigma/dy_1dy_2$ shown in Fig. 1. The two curves correspond to the cases of $W(\theta,\phi)=\cos^2\theta, \frac{1}{2}\sin^2\theta$. The dotted line is the contribution of $d^2\sigma/dy_1dy_2(\pi p\to\rho\rho X\to\pi\pi X)$. Its size relative to $d^2\sigma/dy_1dy_2(\pi p\to\rho X\to\pi\pi X)$ is proportional to $C$ and therefore is not to be taken too seriously. The same holds true for the over-all normalization, which is also proportional to $C$. The correlation function

$$R=\sigma_{\text{inel}}\left(\frac{d^2\sigma}{dy_1dy_2}\Big/\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2}\right)-1$$

$$=\sigma_{\text{inel}}\left(\frac{d^2\sigma}{dy_1dy_2}(\pi p\to\rho X\to\pi\pi X)\Big/\frac{d\sigma}{dy_1}\frac{d\sigma}{dy_2}\right) \qquad (10)$$

is then obtained also and is plotted in Fig. 2. The only dependence on $C$ is that the normalization is inversely proportional to $C$.

The final quantity of interest is the azimuthal correlation

$$A=\frac{N(\phi>90^\circ)-N(\phi<90^\circ)}{N(\phi>90^\circ)+N(\phi<90^\circ)}, \qquad (11)$$

where $\phi$ is the angle between $\vec{p}_{1\perp}$ and $\vec{p}_{2\perp}$. The effect of the process $\pi p\to\rho\rho X\to\pi\pi X$ depends on the dynamics of two-$\rho$ production (which we have ignored) and to kinematic effects involving the $\rho$'s. Therefore, we only consider the effect of single $\rho$ production. And since in any experiment this will be competing with other processes, such as direct production of pions, we weight the asymmetry with the differential cross section in order to get a more realistic idea of its effect on actual asymmetries. Figure 3 is a plot of $A_\phi d^2\sigma/dy_1dy_2(\pi p\to\rho X\to\pi\pi X)$ as a function of $|y_1-y_2|$. The normalization is proportional to $C$ since $d^2\sigma/dy_1dy_2(\pi p\to\rho X\to\pi\pi X)$ is.

## COMMENTS

A point worth reiterating is that the magnitude of $C$ is not reliable. Use of our value of $C$ leads to $d^2\sigma/dp_\perp^2(\pi p\to\rho X)\simeq 21$ mb/GeV$^2$ at $p_\perp=0$ and $p_{\text{inc}}=205$ GeV/$c$. This may be indicating that the value for $C$ should be lower. A smaller $C$ would decrease the $\rho\rho$ term in Fig. 1, decrease the normalization in Figs. 1 and 3, and increase it in Fig. 2. The inaccuracy of $C$ notwithstanding, there is indication of an appreciable structureless (i.e., any structure arises from dynamics of $\rho\rho$ production) background resulting from $\pi p\to\rho\rho X\to\pi\pi X$ which lies under the structure of $\pi p\to\rho X\to\pi\pi X$ in the cross section. This background is not present in the correlation function $R$. Also, it is absent from $(d^2\sigma/dy_1dy_2)A_\phi$ because we have eliminated it by hand. Our results suggest that $\rho$ production could be the mechanism responsible for differences between $R_{+-}$ and $R_{--}$ or $R_{++}$ and between $A_\phi(+-)$ and $A_\phi(++)$ or $A_\phi(--)$,[4] since it would contribute to $\pi^+\pi^-$ correlation and asymmetry but not to the $\pi^+\pi^+$ or $\pi^-\pi^-$ cases. The size of $R$ is artificially large since for $[(d\sigma/dy_1)(d\sigma/dy_2)]$ we use only the $\rho$ contribution to pion production, whereas for $\sigma_{\text{incl}}$ we use an experimentally measured value. To obtain an approximation of the

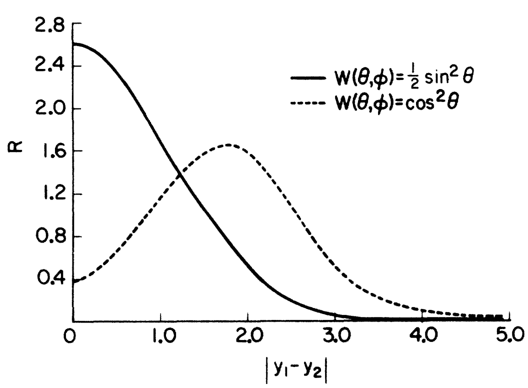


FIG. 2. Correlation function $R$ as a function of $|y_1-y_2|$ at fixed $(y_1+y_2)$.

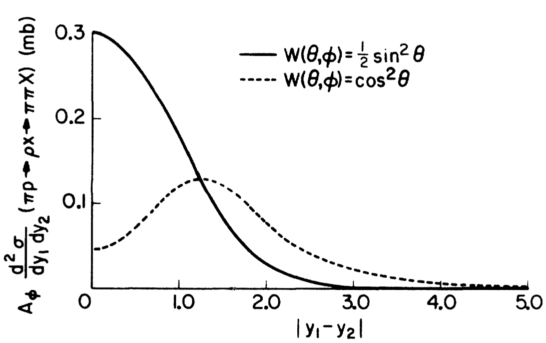


FIG. 3. Azimuthal asymmetry multiplied by the differential cross section plotted as a function of $|y_1-y_2|$ for fixed $(y_1+y_2)$.

absolute contribution of $\rho$ production to $R$ at 205 GeV/$c$, multiply by 2.52 mb$^2/[(d\sigma/dy_1)(d\sigma/dy_2)]$ using experimental values for the one-pion spectra. To obtain an estimate of the effect of $\rho$ production on $A_\phi$ at 205 GeV/$c$, the results of Fig. 3 should be divided by the differential cross section $d^2\sigma/dy_1dy_2(\pi^-p \to \pi^+\pi^-X)$.

The cross section and correlations resulting from the two polarizations considered are quite distinguishable. It is also interesting to note that a combination of the two polarizations could produce correlations over considerable distances in rapidity ($|\Delta y| = 2.5$).

## ACKNOWLEDGMENTS

The author thanks D. Sivers and D. Snider for helpful conversations. He is also grateful for the hospitality of Argonne National Laboratory, where this investigation was begun, and to Professor S. Drell for the hospitality of SLAC where most of the calculations were performed.

---

[1]F.C. Winkelmann, in *Experimental Meson Spectroscopy—1974*, proceedings of the Boston conference, edited by David A. Garelick (American Institute of Physics, New York, 1974).

[2]Tateaki Sasaki and Nobuyuki Murai, Phys. Rev. D 9, 3070 (1974).

[3]A. Bassetto, M. Toller, and L. Sertorio, Nucl. Phys. B34, 1 (1971).

[4]See, for example, H. Bøggild and T. Ferbel, University of Rochester Report No. UR-479, 1974 (unpublished); Winston T. Ko, in *Proceedings of the XVII International Conference on High Energy Physics, London, 1974*, edited by J. R. Smith (Rutherford Lab., Chilton, Didcot, Berkshire, England, 1974), p. I–76; J. Whitmore, Phys. Rep. 10C, 273 (1974).