Systematics of crossover effects in inelastic diffraction dissociation*

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The behavior in momentum transfer of the differential cross sections for (exclusive) inelastic diffraction dissociation processes is examined. Cross sections for pairs of reactions related to each other by t-channel charge conjugation (line reversal) may exhibit crossover phenomena near $t = -0.2$ $(\text{GeV}/c)^2$ similar to elastic scattering. Detailed predictions for crossovers are developed based on the pion-exchange Deck model and are contrasted with expectations obtained from an analogy with the Regge phenomenology of elastic scattering. The two sets are not always in agreement. Experimental checks are proposed. Quantitative values of slopes and integrated cross sections are derived from the Reggeized Deck model for $Kp \to K^*\pi p$, and for the charge-exchange reaction $Kp \to K^*\pi n$; these are presented as a function of energy. Selections on decay angles in the rest frame of the low-mass excited system are described as a method for isolating the π-exchange Deck graph.

I. INTRODUCTION

It has been known for some years that the differential cross sections $d\sigma/dt$ for K^-p and K^+p *elastic* scattering at high-energy cross over one another in the neighborhood of momentum transfer $|t| \simeq 0.1$ to 0.3 $(\text{GeV}/c)^2$. Similar effects are observed upon comparing π^-p with π^+p, or $\bar{p}p$ with pp data.[1] In these cases, the incident *antiparticle* $(\pi^-, K^-, \bar{p})$ differential cross sections are greater at $t=0$ than their counterparts, but having larger slopes, cross over and are smaller when $|t| \gtrsim 0.3$ $(\text{GeV}/c)^2$. A sample of data[1] illustrating these points is shown in Fig. 1. The usual interpretation of this phenomenon within a Regge exchange picture is that the t dependence of the (spin-nonflip) exchange contribution, odd under charge conjugation, is such that it vanishes at the crossover point. The odd-C exchange amplitude has a positive (negative) imaginary part at $t=0$ in the *antiparticle*- (particle-) induced processes because antiparticle total cross sections are larger.

Inelastic diffractive (dissociation) processes are known to have some properties in common with elastic scattering, in particular, weak dependence on energy and fairly rapid decrease of their differential cross sections as a function of t. Well-known examples of baryon dissociation are

$$\begin{Bmatrix} \pi^\pm \\ K^\pm \\ p \end{Bmatrix} p \to \begin{Bmatrix} \pi^\pm \\ K^\pm \\ p \end{Bmatrix} (N\pi) \tag{1.1}$$

and

$$\begin{Bmatrix} \pi^\pm \\ K^\pm \\ p \end{Bmatrix} p \to \begin{Bmatrix} \pi^\pm \\ K^\pm \\ p \end{Bmatrix} (N\pi\pi)\,. \tag{1.2}$$

Among the meson-dissociation processes, there

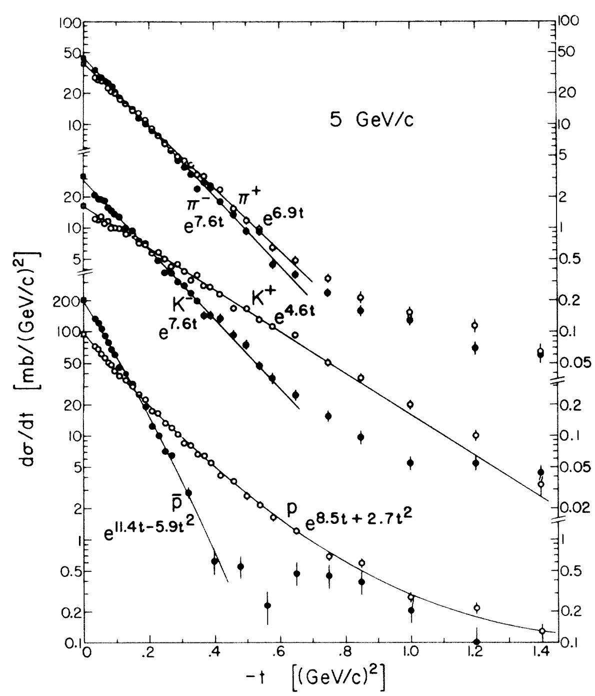

FIG. 1. Elastic differential cross section data at 5 GeV/c for $\pi^\pm p$, $K^\pm p$, pp, and $\bar{p}p$ scattering, Ref. 1.

are

$$\left\{\begin{matrix} K \\ \pi \end{matrix}\right\} p \to \left\{\begin{matrix} (K\pi\pi) \\ (\pi\pi\pi) \end{matrix}\right\} p \,. \qquad (1.3)$$

In Eqs. (1)–(3), the two- and three-particle systems in parentheses have relatively small invariant mass (no more than a few GeV above threshold in traditional diffraction dissociation).

Since inelastic diffraction has been observed as an important reaction mechanism at all energies,[2] understanding in more detail the systematics of this mechanism is of evident interest. From the viewpoint of t-channel exchange phenomenology, one interesting issue is the possible existence and nature of crossovers between differential cross sections of pairs of inelastic reactions. Just as in the case of elastic scattering, crossovers may provide valuable theoretical guidelines. An exploration of this question is the subject matter of the present article. Expectations are developed here along two different lines.

First, in Sec. II, arguments in analogy with elastic scattering are used to construct a set of speculations for crossover systematics. It is presumed that the same set of Regge exchanges is relevant. The dissociation system is treated as a quasiparticle, its internal degrees of freedom being ignored. The extension from elastic to inelastic processes requires several assumptions which should be examined experimentally.

On the other hand, the Deck model,[3] which is a specific approach to inelastic diffraction processes, may be invoked. In its Reggeized form,[4] this model has enjoyed a measure of success in fitting if not interpreting properties of "diffractive" enhancements observed near thresholds of invariant mass spectra in inelastic reactions. Recent efforts have shown that some phase expectations of the Reggeized amplitude are also supported.[5]

The pion-exchange Deck model provides rather clean (asymptotic) predictions for crossover effects, as discussed in Sec. III. Some of these predictions agree with the speculations of the first approach, but there are significant differences in several cases. An experimental verification at momenta of 8 GeV/c and above of which, if either, approach is correct would be instructive. A list of reactions is presented in Table I.

The Deck approach suggests also that there may be important changes of the crossover effect as energy increases (e.g., change of sign at $p_{\text{lab}} \gtrsim 6$ to 8 GeV/c). As a specific model for inelastic processes, it can further serve as a laboratory for investigating the possible influence of reflections in the final state (e.g., what happens when a selection is made to remove a competing non-diffractive Δ^{++} signal). These points are developed quantitatively in Sec. III and IV.

Experimentally, crossover effects have been observed for the following sets of reactions:

$K^0 p \to Q^0 p \to (K^0\pi^+\pi^-)p$ (4 to 12 GeV/c) Ref. 6,

$\overline{K}^0 p \to \overline{Q}^0 p \to (\overline{K}^0\pi^+\pi^-)p$ (4 to 12 GeV/c) Ref. 6,

$K^\pm p \to Q^\pm p \to (K^\pm\pi^+\pi^-)p$ (8 GeV/c) Ref. 7,

$$\left\{\begin{matrix} K^+ p \to Q^+ p \to (K^+\pi^+\pi^-)p \text{ (12 GeV/}c\text{) Ref. 8,} \\ K^- p \to Q^- p \to (K^-\pi^+\pi^-)p \text{ (14.3 GeV/}c\text{) Ref. 9,} \end{matrix}\right\}$$

and

$\pi^\pm p \to A_1 p \to (\pi^\pm\pi^+\pi^-)p$ (16 GeV/c) Ref. 10.

The symbols "Q" and "A_1" are used here as generic labels for enhancements observed in the mass spectra, and carry no implication that resonances are or are not established.

The observed crossovers in the $K^\pm p \to Q^\pm p$ and $\pi^\pm p \to A_1^\pm p$ cases, at the energies listed, agree with both the Deck predictions and elastic-analogy speculations. In the $K^0 p \to Q^0 p$ case, the reported results disagree with the asymptotic pion-exchange Deck model prediction. However, the Deck prediction *in the low momentum 4 to 8* GeV/c *range* may be in accord with the $K^0 p$ data. The issue is one of reflections from the competing Δ production channels: $K^0 p \to K^0\pi^+\Delta^0$ and $\overline{K}^0 p \to \overline{K}^0\pi^-\Delta^{++}$. According to the model, if events with πN invariant mass in the Δ region are not excluded explicitly (before slopes are measured), then the slope of $d\sigma/dt(\overline{K}^0 p)$ is larger than that of $d\sigma/dt(K^0 p)$ in the low-momentum range, as observed experimentally. In other words, the Δ reflection reverses the sign of the crossover at low momentum in essentially all Deck processes.

While not stated as explicitly in Ref. 6 as would seem appropriate, it appears that the Δ events may have been removed from the $K_L^0 p \to K_S^0\pi^+\pi^- p$ data sample before slopes were established. Thus, these data appear to contradict the model. Since the crossover issue is a crucial test of the pion-exchange Deck model, it is essential in view of the model's other successes to reexamine the Δ reflection effect at low momenta in $K^0 p \to Q^0 p$, in $K^\pm p \to Q^\pm p$, and in other reactions as well.

In an attempt to make tests of the Deck predictions an unambiguous as possible, it is advisable to make certain momentum transfer restrictions or selections on decay angles in the rest frame of the diffractively excited system. As discussed in Sec. IV E, a selection on helicity angle ϕ_s

TABLE I. Projectile and target dissociation for proton and neutron targets. Pairs of inelastic reactions are listed in Column 1. One member of each pair is related to the other by line reversal in the t channel. In the remaining columns, I present predictions for the crossover systematics of the production differential cross sections, based on the Deck model. Column 3 lists the *asymptotic* crossover systematics of the differential crosssections $d\sigma/dt'$ for these pairs, according to the *pion-exchange* Deck model. The predictions are strictly valid only in the portion of phase space where pion exchange dominates (e.g., $\cos\phi_s > 0$; see Sec. IV E), but, in certain cases, they may continue to be correct beyond. In Column 2 of parts (a) and (b), the πN charge states appearing in the Deck graph are noted; for the target a dissociation processes in parts (c) and (d), the $a\pi$ charge states are presented. Column 5 lists the asymptotic crossover systematics for the portion of decay phase space ($\cos\phi_s < 0$) controlled by the a^* exchange Deck graph. The net crossover observed in the full data sample ($-1 \le \cos\phi_s \le 1$) depends upon the relative weights of the π-exchange and a^*-exchange graphs.

(a) Projectile dissociation; proton target

	Pion-exchange Deck graph $\cos\phi_s > 0$		The other graph $\cos\phi_s < 0$	
Process [a]	Reflection of the elastic scattering of	Systematics for the slopes of $d\sigma/dt'$ and crossover	Reflection of the elastic scattering of	Systematics for the slopes of $d\sigma/dt'$ and crossover
$K^{\pm}p \to (K^*\pi^{\pm})p$	$\pi^{\pm}p$	$b_{K^-} > b_{K^+}$	K^*p $\bar{K}^*p$	$b_{K^-} > b_{K^+}$
$\pi^{\pm}p \to \rho^0\pi^{\pm}p$	$\pi^{\pm}p$	$b_{\pi^-} > b_{\pi^+}$	ρ^0p	$b_{\pi^+} = b_{\pi^-}$
$pp \to (n\pi^+)p$ $\bar{p}p \to (\bar{n}\pi^-)p$	π^+p π^-p	$b_{\bar{p}} > b_p$	np $\bar{n}p$	$b_{\bar{p}} > b_p$
$pp \to \Delta^{++}\pi^-p$ $\bar{p}p \to \bar{\Delta}^{--}\pi^+p$	π^-p π^+p	$b_p > b_{\bar{p}}$	$\Delta^{++}p$ $\Delta^{--}p$	$b_{\bar{p}} > b_p$
$K^0p \to K^{*+}\pi^-p$ $\bar{K}^0p \to K^{*-}\pi^+p$	π^-p π^+p	$b_{K^0} > b_{\bar{K}^0}$	$K^{*+}p$ $K^{*-}p$	$b_{\bar{K}^0} > b_{K^0}$

[a] Predictions for $\pi^{\pm}p \to f^0\pi^{\pm}p$ and $\pi^{\pm}p \to g^0\pi^{\pm}p$ are the same as for $\pi^{\pm}p \to \rho^0\pi^{\pm}p$.

(b) Projectile dissociation; neutron target

$K^{\pm}n \to (K^*\pi^{\pm})n$	$\pi^{\pm}n$ (i.e., $\pi^{\mp}p$)	$b_{K^+} > b_{K^-}$	K^*n $\bar{K}^*n$	$b_{K^-} > b_{K^+}$
$\pi^{\pm}n \to \rho^0\pi^{\pm}n$	$\pi^{\pm}n$ (i.e., $\pi^{\mp}p$)	$b_{\pi^+} > b_{\pi^-}$	ρ^0n	$b_{\pi^+} = b_{\pi^-}$
$pn \to (n\pi^+)n$ $\bar{p}n \to (\bar{n}\pi^-)n$	π^+n (i.e., π^-p) π^-n (i.e., π^+p)	$b_p > b_{\bar{p}}$	nn $\bar{n}n$	$b_{\bar{p}} > b_p$
$pn \to (\Delta^{++}\pi^-)n$ $\bar{p}n \to (\Delta^{--}\pi^+)n$	π^-n (i.e., π^+p) π^+n (i.e., π^-p)	$b_{\bar{p}} > b_p$	$\Delta^{++}n$ $\bar{\Delta}^{--}n$	$b_{\bar{p}} > b_p$
$K^0n \to (K^{*+}\pi^-)n$ $\bar{K}^0n \to (K^{*-}\pi^+)n$	π^-n (i.e., π^+p) π^+n (i.e., π^-p)	$b_{\bar{K}^0} > b_{K^0}$	$K^{*+}n$ $K^{*-}n$	$b_{\bar{K}^0} > b_{K^0}$

(c) Target dissociation; proton target

$K^{\pm}p \to K^{\pm}(n\pi^+)$	$K^{\pm}\pi^+$	$b_{K^-} > b_{K^+}$ likely but depends on the unknown properties of $K\pi$ scattering. See text.	$K^{\pm}n$	$b_{K^-} > b_{K^+}$
$K^{\pm}p \to K^{\pm}(\Delta^{++}\pi^-)$	$K^{\pm}\pi^-$	$b_{K^+} > b_{K^-}$ likely but depends on the unknown properties of $K\pi$ scattering. See text.	$K^{\pm}\Delta^{++}$	$b_{K^-} > b_{K^+}$
$\pi^{\pm}p \to \pi^{\pm}(n\pi^+)$	$\pi^{\pm}\pi^+$	$b_{\pi^-} > b_{\pi^+}$ likely but depends on the unknown properties of $\pi\pi$ scattering. See text.	$\pi^{\pm}n$ ($\pi^{\mp}p$)	$b_{\pi^+} > b_{\pi^-}$
$\pi^{\pm}p \to \pi^{\pm}(\Delta^{++}\pi^-)$	$\pi^{\pm}\pi^-$	$b_{\pi^+} > b_{\pi^-}$ likely but depends on the unknown properties of $\pi\pi$ scattering. See text.	$\pi^{\pm}\Delta^{++}$	$b_{\pi^-} > b_{\pi^+}$
$pp \to p(n\pi^+)$ $\bar{p}p \to \bar{p}(n\pi^+)$	π^+p $\pi^+\bar{p}$ (i.e., π^-p)	$b_{\bar{p}} > b_p$	pn $\bar{p}n$	$b_{\bar{p}} > b_p$
$pp \to p(\Delta^{++}\pi^-)$ $\bar{p}p \to \bar{p}(\Delta^{++}\pi^-)$	π^-p $\pi^-\bar{p}$ (i.e., π^+p)	$b_p > b_{\bar{p}}$	$p\Delta^{++}$ $\bar{p}\Delta^{++}$	$b_{\bar{p}} > b_p$

TABLE I. - *(continued)*

Process [a]	Pion-exchange Deck graph $\cos\phi_s > 0$: Reflection of the elastic scattering of	Pion-exchange Deck graph $\cos\phi_s > 0$: Systematics for the slopes of $d\sigma/dt'$ and cross-over	The other graph $\cos\phi_s < 0$: Reflection of the elastic scattering of	The other graph $\cos\phi_s < 0$: Systematics for the slopes of $d\sigma/dt'$ and cross-over
(d) Target dissociation; neutron target				
$K^{\pm}n \to K^{\pm}(p\pi^-)$	$K^{\pm}\pi^-$	$b_{K^+} > b_{K^-}$ likely; see text.	$K^{\pm}p$	$b_{K^-} > b_{K^+}$
$K^{\pm}n \to K^{\pm}(\Delta^-\pi^+)$	$K^{\pm}\pi^+$	$b_{K^-} > b_{K^+}$ likely; see text.	$K^{\pm}\Delta^-$	$b_{K^-} > b_{K^+}$
$\pi^{\pm}n - \pi^{\pm}(p\pi^-)$	$\pi^{\pm}\pi^-$	$b_{\pi^+} > b_{\pi^-}$ likely; see text.	$\pi^{\pm}p$	$b_{\pi^-} > b_{\pi^+}$
$\pi^{\pm}n \to \pi^{\pm}(\Delta^-\pi^+)$	$\pi^{\pm}\pi^+$	$b_{\pi^-} > b_{\pi^+}$ likely; see text.	$\pi^{\pm}\Delta^-$	?
$pn \to p(p\pi^-)$ $\bar{p}n \to \bar{p}(p\pi^-)$	π^-p $\pi^-\bar{p}$ (i.e., π^+p)	$b_p > b_{\bar{p}}$	pp $\bar{p}p$	$b_{\bar{p}} > b_p$
$pn \to p(\Delta^-\pi^+)$ $\bar{p}n \to \bar{p}(\Delta^-\pi^+)$	π^+p $\pi^+\bar{p}$ (i.e., π^-p)	$b_{\bar{p}} > b_p$	$p\Delta^-$ $\bar{p}\Delta^-$	$b_{\bar{p}} > b_p$

should be effective in separating the region of phase space in which pion exchange is dominant from those regions in which other exchanges (e.g., $\Delta, K^*, \ldots$) would contribute. However, such selections on the decay angles do not alter the expectations of the elastic-analogy approach of Sec. II. Disagreement with the pion-exchange Deck model in the more limited region of phase space would be especially telling.

At asymptotic energies ($p_{\rm lab} \gtrsim 10$ GeV/c), regardless of how the Δ^{++} is handled, the pion-exchange Deck approach predicts that $d\sigma/dt$ for $K^0p \to Q^0p$ is larger at $t=0$ and has larger slope than that for $\bar{K}^0p$, with a crossover in the neighborhood of $t=-0.2$ (GeV/c)^2. This expectation contradicts that of the naive elastic-analogy approach, which would have $d\sigma/dt$ for $\bar{K}^0p$ larger at $t=0$, and it disagrees with the trend of the SLAC K_L^0p data. If data on the crossover systematics of $K^0p \to Q^0p$ in this high-momentum range (and in the restricted-decay angular region) continue to support the SLAC observation, then it will be established that the pion-exchange Deck model is irrelevant for Q production. Likewise, a valuable check on whether the pion-exchange Deck model is correct for $pp \to p\pi^-\Delta^{++}$ would be a study of the crossover systematics of $pp \to p\pi^-\Delta^{++}$ and $\bar{p}p \to \bar{p}\pi^-\Delta^{++}$ for momenta of 10 GeV/c and above.

Inasmuch as excellent relative normalization is required to establish that a crossover exists, it is perhaps useful to remark that a precise determination only of the relative *slopes* of the corresponding $d\sigma/dt$, at the same energy and over the same t range, may be sufficient.

II. ELASTIC ANALOGY APPROACH

In elastic scattering, it is observed that the *antiparticle* scattering processes π^-p, K^-p, and $\bar{p}p$ have differential cross sections with larger slope and larger value at $t=0$ than the corresponding line-reversed or *particle* scattering processes π^+p, K^+p, and pp. The corresponding differential cross sections cross over one another in the vicinity of $t \simeq -0.2$ (GeV/c)^2. This situation (in which *antiparticle*-induced processes have larger values of $d\sigma/dt$ at $t=0$, and larger slopes) will be referred to as the "canonical" situation.

The most naive extension of this observation from elastic to quasielastic or diffraction dissociation processes would be that, again, the antiparticle inelastic diffractive processes are expected to show canonical crossovers between the differential cross sections of pairs of reactions related by line reversal. The $t=0$ value of $d\sigma/dt$ should be larger, and the slope of $d\sigma/dt$ should be larger for the antiparticle- ($\bar{K}^0, K^-, \pi^-, \bar{p}$) induced reactions than for their particle- (K^0, K^+, π^+, p) induced counterparts.

While appealing, perhaps, and to some extent supported by data, the justification of this speculation is by no means obvious once its underlying assumptions are examined. To justify the speculation, one must argue (i) that the t dependences of the Pomeron and Regge (ρ, A_2, f, ω) exchange amplitudes are essentially the same in elastic and quasielastic processes; (ii) that the relative signs of the Pomeron and Regge exchange amplitudes are the same in the two situations; and (iii) that spin-flip amplitudes are still relatively unimportant out to $|t'| \simeq 0.3$ (GeV/c)^2 in quasielastic reactions, even though the diffractively excited system can have high spin values. The assumptions (i)–(iii) may eventually be proven correct experimentally, but at present they are not necessary consequences of well-understood phenomena. To turn the question around, a systematic experimental investigation of differential cross sections and density matrix elements for quasielastic processes would help in a step-by-step evaluation of the validity of the assumptions. To illustrate the role of each of these assumptions, I present a brief review in the Appendix of the crossover situation in elastic scattering.[11,12]

Accepting its speculative nature, one may adopt the analogy with elastic scattering as a working hypothesis to be tested against data. It agrees with data on $\overline{K}^0 p \to \overline{Q}^0 p$ and $K^0 p \to Q^0 p$ over the energy range 4 to 12 GeV/c,[6] with data on $\pi^\pm p \to A_1^\pm p$ at 16 GeV/c,[10] and with data on $K^\pm p \to Q^\pm p$ at 8 GeV/c.[7] The reactions $\pi^\pm p \to \pi^\pm(\pi^+\pi^- p)$ have also been studied at 16 GeV/c.[10] No crossover is found for these proton dissociation processes, contrary to the elastic analogy speculation. The presence of a crossover in $\pi^\pm p \to A_1^\pm p$, but none in $\pi^\pm p \to \pi^\pm N^*$ is surprising in the context of the speculation, since ρ exchange should be the agent in both cases. It implies a surprising significantly suppressed coupling of ρ exchange at the p to $p\pi^+\pi^-$ vertex, when $p\pi^+\pi^-$ is treated as a quasiparticle.

The excited system ($A_1, Q, N^*, \ldots$) is treated in this section as a resonance or "quasiparticle," such that its internal degrees of freedom can be ignored. In this scheme, therefore, the $d\sigma/dt$ are expected to be independent of the decay mode (e.g., $n\pi^+$ or $p\pi^0$ from N^{*+}; $K^{*0}\pi^+$ or $K^{*+}\pi^0$ from Q^+) and of the decay angles θ, ϕ in the rest frame of the excited system. *By contrast, in the Deck-model approach, considered in Secs. III and IV, the excited system is not a quasiparticle and the production differential cross sections $d\sigma/dt$ may depend crucially on both the decay mode and decay angular range selected.*

III. THE REGGEIZED DECK APPROACH

In this section I develop qualitative predictions for crossover systematics based on the Reggeized *pion-exchange* Deck model.[4] The predictions are strongly energy dependent, showing a change of sign of the crossover in the 6 to 8 GeV/c region, if reflections from competing $\Delta(1238)$ production channels in the same final state are not explicitly removed before the slopes of $d\sigma/dt$ are measured. Expectations for beam and target dissociation are also different. Explicit quantitative predictions are presented in Sec. IV. The role of contributions from exchanges other than the pion is examined in Sec. IV E.

A. Beam particle dissociation: Proton target

I consider the process $ap \to A^*p \to a^*\pi p \to a\pi\pi p$. The basic ingredient of the Deck model for this process, as sketched in Fig. 2, is pion exchange, followed by (off-shell) pion-nucleon elastic scattering. The entire πp scattering amplitude is intended in Fig. 2, including resonances, nonasymptotic Regge exchanges and so forth; specifically, not just the "Pomeron." Consequently, many predictions for *inelastic* processes are a direct reflection of known properties of πN elastic scattering.

At high energies the differential cross section at $t=0$ for $\pi^- p$ elastic scattering is greater than that for $\pi^+ p$ elastic scattering, but then crosses over and becomes smaller as t increases (see Fig. 1). Because the πN amplitude is embedded in Fig. 2, similar crossovers should be present in related inelastic processes, when the Deck model is applicable. In $\pi^\pm p$ elastic scattering, above a lab momentum of 3 GeV/c, the position of the crossover is *roughly* energy independent at $t \simeq -0.15$ (GeV/c)^2. Thus, in inelastic processes, the value $t' \equiv t - t_{\max} \simeq -0.15$ (GeV/c)^2 is approximately the expected crossover position in the cross section $d\sigma/dt'$. As will be discussed below, however, this prediction is only unambiguous above lab momentum of 8 GeV/c, or so, in typical inelastic processes such as $Kp \to Qp$ and πp

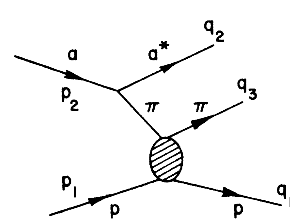

FIG. 2. Pion-exchange Deck graph for the process $ap \to a^*\pi p$. Symbols p_1, p_2, and q_j and four-vector momenta. The shaded oval represents the full pion-nucleon elastic amplitude.

$\to A_1 p$. The overall center-of-mass threshold energy is significantly higher in the inelastic processes because what is relevant is not so much that overall energy be large, but that the πp *subenergy* be large.

In computing $d\sigma/dt'$ for $ap \to A^* p \to a^* \pi p$ from the Deck model one integrates over an interval of the final-state πp mass spectrum. This interval may be the full interval allowed kinematically, by phase space, given the incident lab momentum and mass of A^*. Otherwise, the interval can be chosen to match whatever selections are made in the data with which comparison would be made, or to abide by theoretical prejudices regarding the region of applicability of the model.

The choice of interval in πp mass is crucial for the determination of the slope of $d\sigma/dt'$.

In Fig. 3, a compilation[13] is shown of the slopes of $d\sigma/dt$ for $\pi^\pm p$ elastic scattering. The slopes vary wildly from threshold up to at least $m_{\pi p} \gtrsim 2$ GeV, presumably reflecting resonance structure. Above $m_{\pi p} \gtrsim 2$, the "asymptotic systematics" take over: To wit, $B_{\pi^-} > B_{\pi^+}$. Moreover, only above $m_{p\pi} \simeq 2$ GeV is it always true that $\sigma^{\text{tot}}_{\pi^- p} > \sigma^{\text{tot}}_{\pi^+ p}$; c.f. Ref. 14. Thus, in inelastic processes mediated by the Deck amplitude, predictions for crossover systematics should be simple, direct, and trouble-free so long as either (i) a selection is always made to require $m_{\pi p} \gtrsim 2$ GeV in the final state, or, (ii) the incident energy is large enough so that any contribution to the integral from the region $m_{\pi p} < 2$ GeV is overruled by the contribution from values above 2 GeV.

A simple *estimate* may be made of the incident momentum required if the latter criterion (ii) is accepted. Assume that the integral over the πp masses is taken from threshold to the maximum mass value M allowed kinematically: $M = \sqrt{s} - m_{a^*}$. To overrule the interval $m_{\pi p} \lesssim 2$ GeV, presumably $M - 2 \gtrsim 2$ GeV. Thus, I find

$$\sqrt{s} \gtrsim 4 + m_{a^*} \,.$$

If $a^* = \rho, K^*(890), \Delta^{++}, \ldots,$ then $\sqrt{s} \gtrsim 5$ GeV, or $p_{\text{lab}}^{\text{incident}} \gtrsim 12$ GeV/c. Explicit calculations, reported in Secs. III A and IV, suggest that this limit can be lowered to 8 GeV/c. Consequently, only above 8 GeV/c should one expect a simple (asymptotic) crossover systematics to hold true in inelastic processes, *unless* selections are made on the final πp subenergy. I list a set of such asymptotic ($p_{\text{lab}} \gtrsim 8$ GeV/c) predictions below, and then I discuss in Sec. III A 2 the substantial changes which may occur if values of πp mass below 2 GeV contribute with significant weight in the final state.

1. Simple asymptotic predictions

Our main concern is a comparison of particle and antiparticle induced processes. According to the model, any differences between particle and antiparticle induced reactions comes as a result of the different charges in the πp elastic-scattering part of the diagram. Thus, it is important to follow the charge flow through the diagram of Fig. 2. Diagrams with charge labels for $K^\pm p \to K^{*0} \pi^\pm p$ and for $\pi^\pm p \to \rho^0 \pi^\pm p$ are drawn in Fig. 4. Diagrams for $K_L^0 p \to K^{*\pm} \pi^\mp p$ are presented in Fig. 5.

(a) Neutral bachelor pion. There will be no crossover, and differential cross sections for particle and antiparticle induced reactions will be identical, if reactions are of the type

$$ap \to a^* \pi^0 p$$

with a *neutral* bachelor pion. Examples of these (admittedly difficult) sets of reactions, *with no expected crossover*, are

$$K^\pm p \to (K^{*\pm} \pi^0) p \,,$$

$$\pi^\pm p \to (\rho^\pm \pi^0) p \,,$$

$$\left\{ \begin{array}{l} pp \to (p\pi^0) p \,, \\ \bar{p}p \to (\bar{p}\pi^0) p \,, \end{array} \right\}$$

$$\left\{ \begin{array}{l} pp \to (\Delta^+ \pi^0) p \,, \\ \bar{p}p \to (\bar{\Delta}^- \pi^0) p \,, \end{array} \right\}$$

$$\left\{ \begin{array}{l} pp \to p(\pi^0 p) , \\ \bar{p}p \to \bar{p}(\pi^0 p) . \end{array} \right\}$$

The system in parentheses is always the relatively low mass dissociation system.

(b) Charged bachelor pion. When the bachelor pion in the Deck diagram is charged, its sign is determined by that of the incident hadron. When a change is made from particle to antiparticle in the initial state, the sign of the pion's charge changes. This point is illustrated in Figs. 4 and 5. Predictions for the crossover systematics are therefore automatic. A list is given in Table I. Numerical values of the slopes obtained from an explicit numerical calculation are given in Sec. III A 2. The systematics predicted in Table I are insensitive to model-dependent details, and depend only on the essential assumption that the inelastic processes reflect πp elastic scattering. *If the systematics fail*, especially *after kinematic selections are made to enhance the pion-exchange contribution*, as discussed in Sec. IV E, *then this pion-exchange assumption is wrong*. The terse

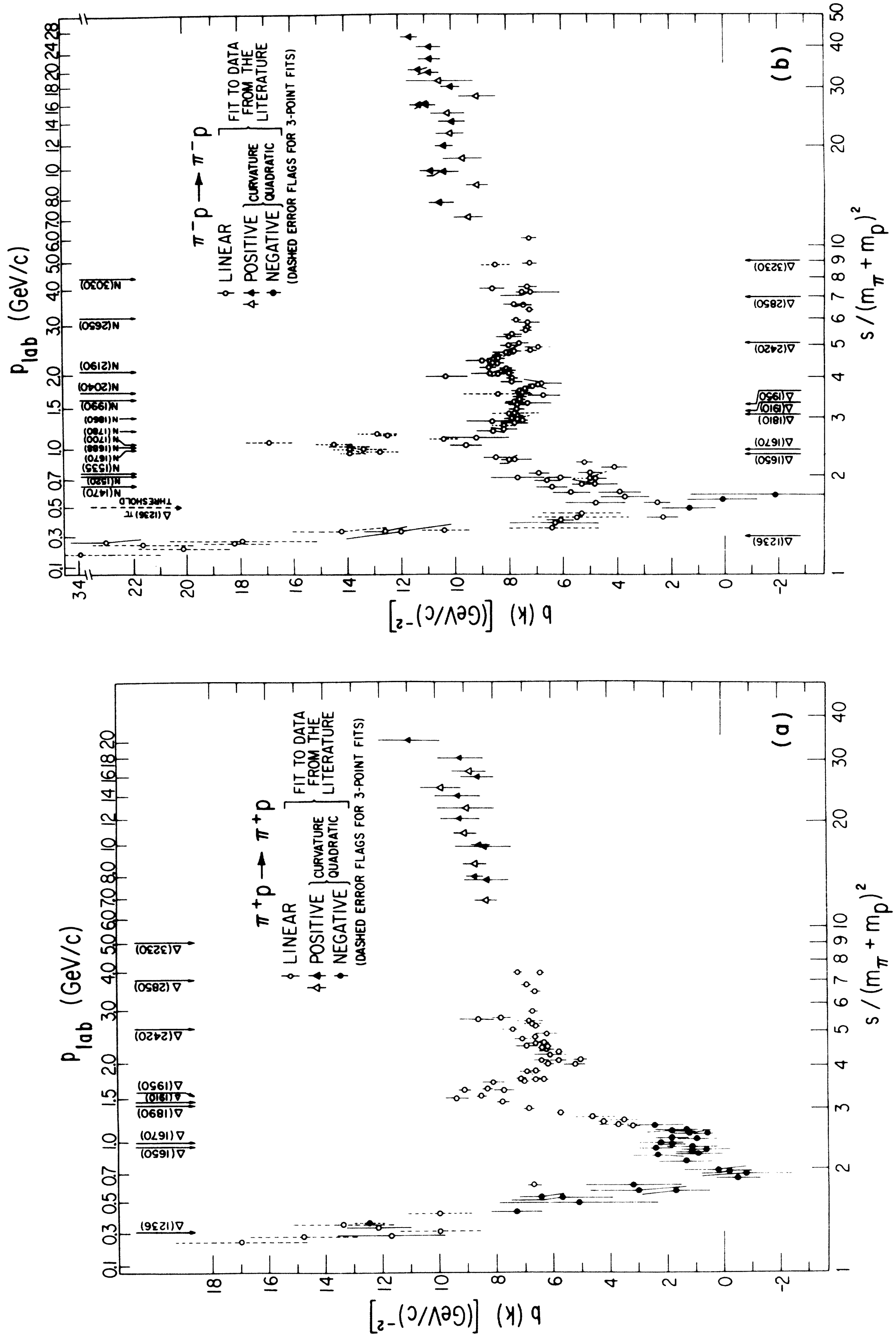

FIG. 3. A compilation of slopes of the experimental differential cross sections $d\sigma/dt$ for elastic $\pi^{\pm}p$ scattering, from Ref. 15.

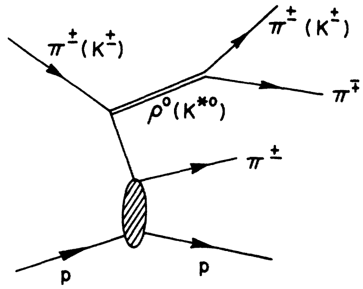

FIG. 4. Pion-exchange Deck graphs for the processes $\pi^\pm p \to \rho^0\pi^\pm p$ and $K^\pm p \to K^{*0}\pi^\pm p$.

notation in the table is simple. The remark $b_x > b_{\bar{x}}$ means that $d\sigma/dt'(x)$ is greater at $t=0$ than $d\sigma/dt'(\bar{x})$, that the slope b of $d\sigma/dt'(x)$ is greater than that of $d\sigma/dt'(\bar{x})$, and that the value of $d\sigma/dt'(x)$ is less than the value of $d\sigma/dt'(\bar{x})$ for $|t'| \geq 0.3$ (GeV/c)^2. The crossovers should all occur near $t' \simeq -0.2$ (GeV/c).

(c) Comments. Some observations may be made about the predictions listed.

(1). In the reaction $K^\pm p \to Q^\pm p$, the behavior of $d\sigma/dt'$ should not depend on the decay properties of $Q^\pm$, if the Q system is essentially a resonance. The Deck model suggests otherwise, giving a crossover if the decay mode is all charged, $Q^\pm \to K^{*0}\pi^\pm$, but no crossover for the neutral mode $Q^\pm \to K^{*\pm}\pi^0$.

(2). For the "$N^*(1400)$," the situation is more dramatic. In comparing $\bar{p}p \to \bar{N}^*p$ with $pp \to N^*p$, the Deck model suggests a canonical crossover $b_{\bar{p}} > b_p$ if the decay mode is $N^* \to n\pi^+$, no crossover if $N^* \to p\pi^0$, and a noncanonical $b_{\bar{p}} < b_p$ if the decay mode is $N^* \to \Delta^{++}\pi^-$. By contrast, the production distribution $d\sigma/dt'$ should not depend on decay properties if the N^* is a "resonance."

(3). The first three crossover predictions listed in Table I(a) are canonical, in the sense that antiparticle-induced reactions have the larger cross section at $t'=0$, and the larger slope. These predictions agree with the elastic-scattering analogy speculations listed in Sec. II, and with meager data.[7-10] However, the processes $pp \to N^*p \to \Delta^{++}\pi^-p$ and $K^0p \to Q^0p \to K^*\pi^-p$ (see Fig. 5) are predicted to be noncanonical, with particle-induced cross sections larger at $t'=0$ and having larger slope. No relevant data above 12 GeV/c are available. An experimental comparison of $\bar{p}p \to \bar{\Delta}^{--}\pi^+p$ with $pp \to \Delta^{++}\pi^-p$ (or of $K^0p \to Q^0p$ with $\bar{K}^0p \to \bar{Q}^0p$) in the 10 to 20 GeV/$c$ range would resolve the conflict and, if contrary to the simple Deck prediction, would be a crucial piece of evidence against the pion-exchange model for these reactions.

(4). In Table I(b), predictions are made for pro-

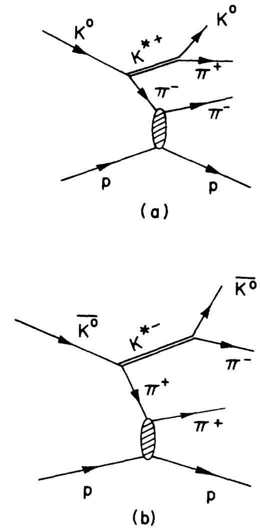

FIG. 5. Pion-exchange Deck graphs for (a) $K^0p \to K^{*+}\pi^-p$ and (b) $\bar{K}^0p \to K^{*-}\pi^+p$.

jectile dissociation reactions on a neutron target. These are obtained trivially once it is recalled that for elastic-scattering processes $d\sigma/dt(\pi^\pm p) = d\sigma/dt(\pi^\mp n)$, according to isospin invariance. Note that the signs of all crossovers are predicted to be opposite to those of Table I(a). The fact that the crossover for the π-exchange graph changes sign when one passes from $\pi^\pm p \to A_1^\pm p$ to $\pi^\pm n \to A_1^\pm n$ agrees with the elastic-scattering analogy predictions given in Sec. II. A change from a proton to a neutron target is a rotation in t'-channel isospin, changing the sign of the t'-channel $I=1$ amplitudes. For $\pi^\pm N \to A_1^\pm N$, this I-spin rotation is equivalent to charge conjugation of the ρ exchange amplitude, which determines the crossover in both the Deck-model and elastic-analogy approaches. However, for $K^\pm N \to Q^\pm N$, the π-exchange-Deck-model and elastic-scattering-analogy predictions disagree. In the elastic analogy, $b_{K^-} > b_{K^+}$ whether the target is a proton or a neutron. In the t' channel there are both $I=0(\omega)$ and $I=1(\rho)$ exchanges, odd under C. The ω contribution is larger than the ρ term and does not change sign under $p \to n$. However, in the Deck model, there is no ω contribution. Thus, the conversion from p to n forces a rotation of C and a sign change of the crossover. A comparison of the crossover properties of $K^\pm p \to Q^\pm p$ with those for $K^\pm n \to Q^\pm n$ at, say, 15 GeV/c, provides, therefore, a good check on the *isospin* implications of the π-exchange Deck model.

2. Low energy, Δ not excluded

Suppose the overall center of mass energy $\sqrt{s}$ is below 6 GeV, and that no selection is made to remove events with $m_{p\pi} < 2$ GeV. The Deck-model predictions are no longer unambiguous. An examination of Fig. 3 reveals that the major source of difficulty is the prominent $\Delta(1238)$ resonance. In the interval $m_{\pi p} < 1.4$ GeV, the slopes of $\pi^{\pm}p$ elastic scattering are very large (and similar, since isospin $\frac{3}{2}$ dominates). However, the elastic cross section favors $\pi^+ p$ by a substantial factor (≈ 9). Any integral over the $\pi^+ p$ final-state mass spectrum which includes the Δ^{++} mass region receives a heavy contribution from this resonance, with its large slope b. Therefore, *if the* Δ^{++} *contribution dominates, all of the predictions of Table I(a) are reversed.* In particular, $d\sigma/dt'$ for $\pi^+ p \to A_1^+ p$ will be greater at $t' = 0$ than that for $\pi^- p \to A_1^- p$, and will have a larger slope. If a crossover occurs at all, it will tend to be moved out in $|t|$, to relatively large values. The reaction $\pi^{\pm} p \to A_1^{\pm} p$ will tend to become noncanonical at low momenta if the Δ region is included.

The Deck model suggests, therefore, that there are potentially important correlations between the values of slopes b and the interval of πp final-state subenergy. This is a new correlation, different from the well-known mass-slope correlation in $ap \to A^* p \to a^* \pi p$, according to which b decreases systematically as the mass of A^* increases.

As a second example, if the Δ is not excluded $d\sigma/dt'$ for $\bar{K}^0 p \to \bar{Q}^0 p$ at $t = 0$ will be greater than that for $K^0 p \to Q^0 p$ and will have a larger slope. The $K^0 p \to Q^0 p$ process "becomes canonical" at low momenta, as a result of the Δ inclusion. The presence of a Δ^{++} signal in the final state of the $\bar{K}^0$ process increases the $t = 0$ value and slope of $d\sigma/dt$ for this reaction above that for the $K^0 p$ case.

These remarks may be illustrated by an explicit Deck-model calculation, described in Sec. IV. Differential cross sections $d\sigma/dt'$ were calculated for $K^0 p \to K^{*+}\pi^- p$ and $\bar{K}^0 p \to K^{*-}\pi^+ p$ at several momenta, and fitted with the form $d\sigma/dt' = c\exp(-bt')$, over the range $0.02 < |t'| < 0.5$ $(\mathrm{GeV}/c)^2$. This is the same range used in treating the SLAC data.[6] The $K^*\pi$ mass was restricted to be less than 1.5 GeV, as in the data. Slopes b obtained with Δ included and excluded are presented in Fig. 6. With Δ excluded, the integrated cross sections shown in Fig. 7 for $K^0 p \to Q^0 p$ and $\bar{K}^0 p \to \bar{Q}^0 p$ are nearly equal, both in the data and model.

3. *Low energy; Δ excluded*

Once events with πp mass in the Δ region (e.g., $m_{\pi p} < 1.4$ GeV) are explicitly removed, the major uncertainty in the Deck prediction would seem to be eliminated. The ambiguity would be entirely removed if at masses higher than the $\Delta(1238)$ it were true that $b_{\pi^- p} > b_{\pi^+ p}$ and $\sigma_- > \sigma_+$. Above 1.4 GeV, these inequalities appear to fail only in the neighborhood of the $\Delta(1960)$ resonance. Thus, provided that the selection $m_{p\pi} > 1.4$ GeV is made and the interval of πp masses is always large enough on both sides of the $\Delta(1960)$ to wash out the noncanonical behavior of the $\Delta(1960)$ region, the "asymptotic" predictions of Table I should be reliable down to quite low momenta ($p_{\mathrm{lab}} \gtrsim 3$ GeV/c).

It is interesting to speculate that Δ reflections may be responsible for the cross over effect seen in the SLAC $K_L^0 p$ data.[6] However, a careful reading of Ref. 6 suggests that events with πp mass in the Δ region were excluded from the $K_L^0 p \to (K_L^0 \pi^+\pi^-)p$ data sample in the momentum range 4 to 12 GeV/c before slopes were determined.

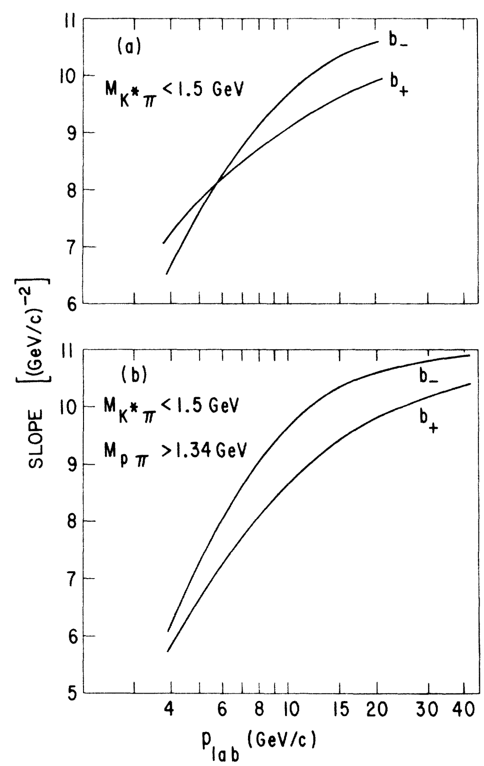

FIG. 6. Slopes b for the differential cross section $d\sigma/dt' = c\exp(-bt')$ obtained from the Reggeized Deck-model calculations described in the text, as a function of incident kaon lab momentum. Slope b_- is for $K^- p \to K^{*0}\pi^- p$ or for $K^0 p \to K^{*+}\pi^- p$. Slope b_+ is for $K^+ p \to K^{*0}\pi^+ p$ or $\bar{K}^0 p \to K^{*-}\pi^+ p$. In (a) and (b), the selection is made which restricts the $K^*\pi$ invariant mass to be below 1.5 GeV, in the "Q region." In (b), an additional selection restricts the final πp invariant mass to be greater than 1.34 GeV. All slopes are fitted over the range $0.02 < |t'| < 0.5$ $(\mathrm{GeV}/c)^2$.

Thus, the observation of a canonical crossover in these data would appear to contradict the pion-exchange Deck model, the Δ-reflection argument notwithstanding. Since the crossover is such a clean consequence of the assumption of π-exchange dominance, it is essential to confirm this observation in other reactions as well, to establish whether Δ reflections are at all relevant. Furthermore, experimental momentum transfer or angular selections discussed in Sec. IV E to enhance the pion-exchange graph would be very instructive.

At low energy, a selection on $Kp \to K\pi\pi p$ data to eliminate all events with $m_{p\pi} < 1.4$ GeV also eliminates "true Q events," and reduces the "true Q cross section." Experimental techniques are used therefore to repopulate the aborted regions of phase space. The Deck-model exercise suggests that the proper technique is one in which the *slopes* of $d\sigma/dt'$ are determined from the reduced sample of data, in which no events with $m_{p\pi} < 1.4$ GeV contribute. Only in this way can the observed crossover systematics be expected to show their asymptotic character.

B. Target dissociation

For the cases treated in Sec. III A, the target is always a nucleon and the Deck graph incorporates πN elastic scattering.

In the case of target dissociation, the elastic-scattering part of the Deck diagram involves a pion scattering from the projectile. If the projectile is a nucleon or an antinucleon, the relevant crossover predictions are again straightforward reflections of the known πN systematics. However, if the projectile is a π (or K), it is $\pi\pi$ (or $K\pi$) scattering which determines $d\sigma/dt'$. The relevant graph is drawn in Fig. 8.

1. Neutral bachelor pion

Differential cross sections for particle and antiparticle induced reactions will be identical if reactions are of the type

$$ap \to a\pi^0 N^*$$

with a neutral bachelor pion. Examples are

$$\pi^\pm p \to \pi^\pm(\pi^0 p),$$

$$K^\pm p \to K^\pm(\pi^0 p),$$

$$\pi^\pm p \to \pi^\pm(\pi^0 \Delta^+),$$

$$\begin{Bmatrix} pp \to p(\pi^0 p), \\ \bar p p \to \bar p(\pi^0 p). \end{Bmatrix}$$

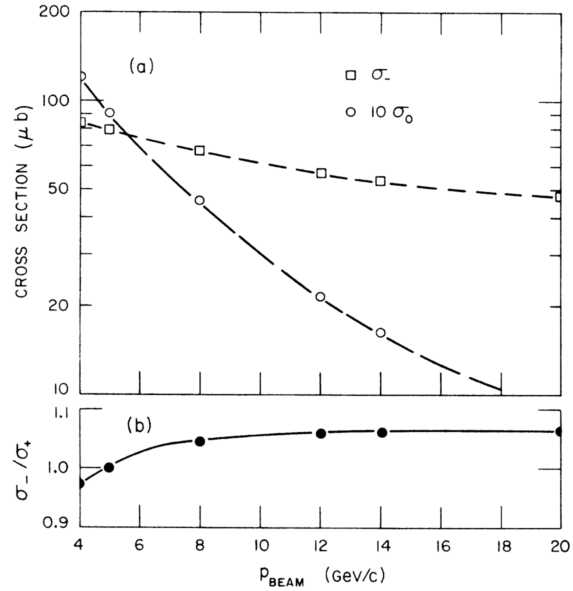

FIG. 7. Cross sections $\sigma_\pm$ and σ_0 obtained from the Reggeized Deck model are presented as a function of incident kaon lab momentum. σ_- is the cross section for $K^-p \to \bar K^{*0}\pi^-p$, with $\bar K^* \to \bar K^0\pi^+$, only, or for $K^0p \to K^{*+}\pi^-p$ $(K^{*+} \to K^0\pi^+)$; σ_+ is the cross section for $\to K^+p \to K^{*0}\pi^+p$ $(K^{*0} \to K^+\pi^-)$ or for $K^0p \to K^{*-}\pi^+p\,(K^{*-} \to \bar K^0\pi^+)$; and σ_0 is the cross section for the charge-exchange process $K^-p \to K^{*-}\pi^+n \to \bar K^0\pi^-\pi^+n$. For all results, the two selections imposed are mass $(K^*\pi) < 1.5$ GeV and mass $(N\pi) > 1.34$ GeV, where N denotes the final nucleon.

2. Charged bachelor pion

The systematics of $\pi\pi$ and $K\pi$ elastic scattering at high c.m. energies (above the resonance region) are essentially unknown. It is unclear whether a crossover exists when $d\sigma/dt$ for $\pi^+\pi^-$ elastic scattering is compared with $d\sigma/dt$ for $\pi^-\pi^-$. Speculations may be made, of course. Using duality and t-channel "factorization," we may argue that $d\sigma/dt$ for $\pi^+\pi^-$ is greater at $t=0$, and has greater slope,

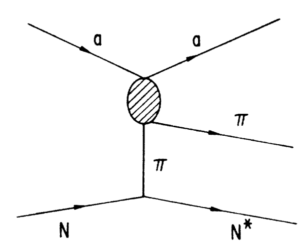

FIG. 8. Pion-exchange Deck graphs appropriate to the target fragmentation process $aN \to a\pi N^*$, where N^* denotes the neutron or a baryonic resonance. The shaded oval represents the full $a\pi$ scattering amplitude.

than $d\sigma/dt$ for $\pi^-\pi^-$. If this is true in the elastic case, then, by reflection, predictions are easily made for relevant inelastic processes.

However, "factorization" as used above might well be inaccurate. If the zero near $t \simeq -0.15$ $(\mathrm{GeV}/c)^2$ of the spin-nonflip odd-C amplitude in meson-nucleon and nucleon-nucleon scattering is an absorption or other s-channel effect,[11,12] then it could well be absent from the odd-C amplitudes in $\pi\pi$ and $K\pi$ scattering, or moved out to large $|t|$. This uncertainty regarding the crossover situation in elastic meson-meson scattering means, by extension, that no firm guesses are possible for inelastic processes for which the Deck model requires the elastic input. Nevertheless, relying on finite-energy sum rule[15] arguments and the fact that the prominent $\pi\pi$ and $K\pi$ resonances provide a zero of the scattering amplitude near $t=-0.2$ $(\mathrm{GeV}/c)^2$, I adopt as "likely" that the high energy $\pi\pi$ and $K\pi$ elastic $d\sigma/dt$ data will show a canonical crossover near -0.2 $(\mathrm{GeV}/c)^2$. The consequences for inelastic processes are listed in parts (c) and (d) of Table I.

Although predictions are less firm, an experimental determination of the existence or nonexistence of crossover effects, and their systematics, in meson-induced target dissociation processes remains interesting. Just as for the beam dissociation processes indicated above, a (new) correlation is expected between the slopes of $d\sigma/dt'$ for $aN \to a(\pi N^*)$ and the mass of $a\pi$. As before, care must be taken in the analysis to avoid nonasymptotic effects due to reflections of prominent resonances in the πa subsystem.

In an experimental study of $\pi^\pm p \to \pi^\pm(p\pi^\pm\pi^-)$ at 16 GeV/c, no crossover is found.[10] Via the pure pion-exchange Deck model, this would suggest the absence of a crossover in elastic $\pi^\pm\pi^-$ scattering. More interesting, however, is the possibility that the absence of a crossover is a result of compensation between the pion-exchange and the baryon-exchange graphs sketched in Fig. 9. Well-known difficulties with baryon-exchange calculations make it impossible to predict the relative cross sections of Figs. 9(a) and 9(b). However, it is qualitatively clear that the baryon-exchange process would yield a canonical crossover, with $b_{\pi^-} > b_{\pi^+}$, as a reflection of the expected properties of $\pi^\pm\Delta^{++}$ elastic scattering. By contrast, the π-exchange process should provide $b_{\pi^+} > b_{\pi^-}$, because it is the π^+-induced reaction which has the nonexotic $\pi^+\pi^-$ combination in the final state.

Events corresponding to the pion- and (proposed) baryon-exchange graphs for $\pi^\pm p \to \pi^\pm(\pi^+\pi^- p)$ might be separable, if judicious momentum transfer or angular selections are made. This issue is treated further in Sec. IV E. After the event selection is made to enhance pion exchange, it would be valuable to ascertain whether $b_{\pi^+} > b_{\pi^-}$ as expected, and whether $b_{\pi^-} > b_{\pi^+}$ obtains in the baryon-exchange segment.

IV. NUMERICAL CALCULATIONS WITH THE REGGEIZED DECK MODEL

As an explicit illustration of the crossover systematics expected, I present results of a numerical calculation of the process $ap \to A^*p \to a^*\pi N$ according to the Reggeized pion-exchange Deck model. Both the charge-exchange ($N=n$) and the non-charge-exchange ($N=p$) cases are treated.

Normalization is such that the integrated cross section in millibarns is

$$\sigma = \frac{0.3893}{F}\phi_3\left(\sum|\mathfrak{M}|^2\right). \qquad (4.1)$$

The flux factor F is

$$F = 4m_p p^{\mathrm{lab}}_{\mathrm{inc.}}, \qquad (4.2)$$

and $\sum|\mathfrak{M}|^2$ is the usual square of the absolute value of the invariant matrix element, summed over final spins and averaged over initial spins. The three-particle phase-space element is

$$\phi_3(1) = \frac{1}{(2\pi)^5}\prod_{i=1}^{3}\frac{d^3\vec{q}_i}{2q_{i0}}\delta^4(q_1+q_2+q_3-p_1-p_2). \qquad (4.3)$$

The final- and initial-particle four-vector momenta are

$$q_i = (\vec{q}_i, q_{i0}) \text{ and } p_j = (\vec{p}, p_{j0}), \qquad (4.4)$$

as illustrated in Fig. 2.

A. Matrix element

The squared matrix element is

$$\sum|\mathfrak{M}|^2 = g^2\cdot\frac{[\frac{1}{2}(s_2-u_2)]^{2\alpha_\pi}}{(m_\pi^2-t_2)^2}\sum|\mathfrak{M}_{\pi p}|^2. \qquad (4.5)$$

Kinematic variables are $t_2=(q_2-p_2)^2$, $s_2=(q_2+q_3)^2$, $u_2=(q_3-p_2)^2$, $t_1=(p_1-q_1)^2$, and $s_1=(q_3+q_1)^2$. The

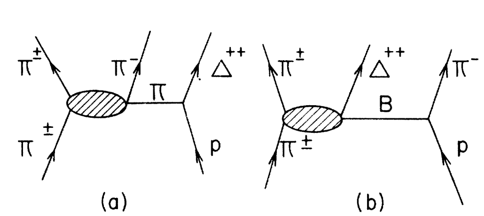

FIG. 9. (a) Pion-exchange Deck diagram for the reactions $\pi^\pm p \to \pi^\pm(\pi^-\Delta^{++})$. The shaded oval represents $\pi^\pm\pi^-$ elastic scattering. (b) Baryon-exchange Deck diagram for $\pi^\pm p \to \pi^\pm(\pi^-\Delta^{++})$, with the shaded oval representing $\pi^\pm\Delta^{++}$ scattering.

pion trajectory is

$$\alpha_\pi = \alpha'_\pi (t_2 - m_\pi^2), \qquad (4.6)$$

where α'_π is the trajectory slope, which I fix at 0.9, in this article. The coupling constant g at the $\pi a a^*$ vertex is discussed below. This is the only factor in Eq. (4.5) which changes for the different processes listed in parts (a) and (b) of Table I (other than input mass values). The quantity $\sum|\mathfrak{M}_{\pi p}|^2$ appearing in Eq. (4.5) is the (off-shell) $\pi p \to \pi N$ scattering amplitude, squared and summed, as usual. If the π were on shell, the differential πp elastic cross section in millibarns would be

$$\frac{d\sigma}{d\Omega} = \frac{0.3893}{64\pi^2 s} \sum |\mathfrak{M}_{\pi p}|^2 . \qquad (4.7)$$

Inasmuch as $\mathfrak{M}_{\pi p}$ is employed at fairly low πN subenergy, I do not use an asymptotic Regge approximation. Rather, below the mass $M_{p\pi} = 2.45$ GeV, I construct $\mathfrak{M}_{\pi p}$ from phase-shift analysis results.[16] I use a multipole Regge fit for subenergies above this value.[17] The amplitude $\mathfrak{M}_{\pi p}$ is evaluated at a value of mass $M_{p\pi}$ and of πN rest system scattering angle θ_{pN} determined by the inelastic kinematics. No other corrections are made for the fact that one pion is off shell.

In a detailed fit to data, one might want to include, for example, the freedom of an adjustable form factor in t_2 {of the type $\exp[2\lambda(t_2 - m_\pi^2)]$} in Eq. (4.5). In the present article, I am concerned primarily with the systematics of $d\sigma/dt_1$. Any change in the t_2 input structure feeds through kinematically and affects dependence on t_1, but in a fashion which is independent of the *charge* of the bachelor pion. Thus, I forego any use of a form factor. The t_1 systematics are also little affected by the presence of the Regge factor $(s_2 - u_2)^{2\alpha_\pi}$ in Eq. (4.5). Note that $\frac{1}{2}(s_2 - u_2) = s_2 + \frac{1}{2}(t_2 - t_1 - m_{a^*}^2 - m_a^2 - m_\pi^2)$.

I computed distributions for the Reggeized Deck model using a standard Monte Carlo event generator. Results are described below.

B. The reaction $Kp \to K^*\pi N$

For definiteness, I specialize to $Kp \to K^*\pi N$. This choice affects the overall normalization, primarily. Shapes of distributions and crossover systematics are more general.

For the $K\pi K^*$ vertex in Fig. 5, the coupling constant g [Eq. (4.5)] is evaluated easily in terms of the full width Γ of the (spin-1) $K^*(890)$ as

$$g^2 = \frac{48\pi\Gamma m_{K^*}^3}{[m_{K^*}^2 - (m_K + m_\pi)^2]^{1/2}[m_{K^*}^2 - (m_K - m_\pi)^2]^{1/2}} . \qquad (4.8)$$

However, this value must be reduced by isospin factors appropriate to the charge states considered. For the reaction $K^+p \to K^{*0}\pi^+p$, if only the charged decay mode $K^{*0} \to K^+\pi^-$ is accepted, the isospin factor is $\frac{4}{9}$. For $K^0p \to K^{*+}\pi^-p$, with only $K^{*+} \to K^0\pi^+$ accepted, the factor is again $\frac{4}{9}$. For $K^-p \to K^{*-}\pi^+n$, with only the mode $K^{*-} \to \bar{K}^0\pi^-$ accepted, the factor is $\frac{2}{9}$.

At 8 GeV/c, the distribution $d\sigma/dM_{K^*\pi}$ for $K^-p \to K^{*0}\pi^-p$ (or, equivalently, for $K^0p \to K^{*+}\pi^-p$) computed from Eq. (4.5) is presented in Fig. 10. A selection is made in the calculation to require $M_{p\pi} > 1.34$ GeV. The usual "Q" enhancement near the $K^*\pi$ threshold is visible. While this distribution may not provide a good fit to published data[7-9] (it is too broad, rises from threshold too rapidly, and falls from its maximum too slowly), it provides a reasonable qualitative representation.

I do not dwell here on distributions in the variables $s_2 = M_{K^*\pi}^2$, t_2, $s_1 = M_{p\pi}^2$, and their correlations,[4] nor various angular distributions. I rely on previous work in which general qualitative agreement between Deck-model distributions and data has been established,[4] and restrict further attention here to overall cross sections and to the distribution in momentum transfer variable t_1.

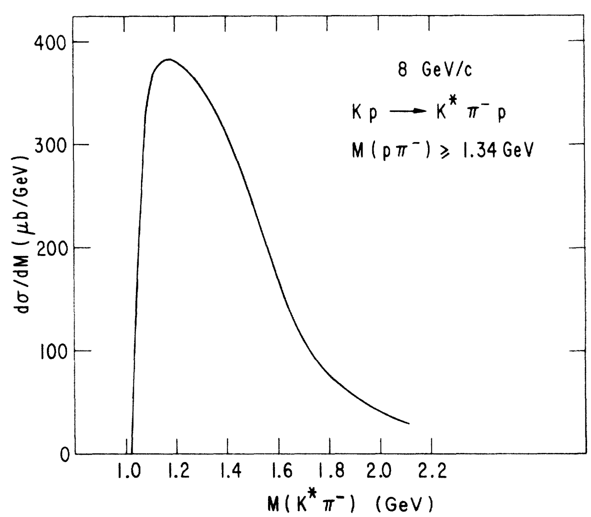

FIG. 10. The differential cross section $d\sigma/dM_{K^*\pi}$ for $K^-p \to \bar{K}^{*0}\pi^-p$ or for $K^0p \to K^{*+}\pi^-p$ calculated from the Reggeized Deck model at 8 GeV/c is shown as a function of the $K^*\pi^-$ invariant mass. The selection is imposed that the invariant mass of $p\pi^-$ be greater than 1.34 GeV. The curve should be scaled by the isospin factor $\frac{4}{9}$ for the decays $K^{*+}_{\ 0} \to K^{0}_{+}\pi^{+}$, or by $\frac{2}{3}$ if all K^* decay modes are included.

C. Total cross sections

To simplify the discussion, I adopt the notation

$$\sigma_+ = \sigma(K^+p \to K^{*0}\pi^+p \to K^+\pi^-\pi^+p)$$
$$= \sigma(\bar{K}^0p \to K^{*-}\pi^+p \to \bar{K}^0\pi^-\pi^+p); \quad (4.9)$$
$$\sigma_- = \sigma(K^-p \to \bar{K}^{*0}\pi^-p \to K^-\pi^+\pi^-p)$$
$$= \sigma(K^0p \to K^{*+}\pi^-p \to K^0\pi^+\pi^-p); \quad (4.10)$$

and

$$\sigma_0 = \sigma(K^-p \to K^{*-}\pi^+n \to K^0\pi^-\pi^+n). \quad (4.11)$$

Note the equalities $\sigma(K^+p) = \sigma(\bar{K}^0p)$ and $\sigma(K^-p) = \sigma(K^0p)$. These are obvious once one writes down the relevant Deck graphs. They are a particular consequence of the Deck model, and need not be true in a context more general than the model.

For the *restricted region* $M_{K^*\pi} < 1.5$ GeV and $M_{N\pi} > 1.34$ GeV, the integrated cross section predicted by the model is shown as a function of incident kaon momentum in Fig. 7. The selection on $M_{N\pi}$ is made to eliminate the Δ region; the selection $M_{K^*\pi} < 1.5$ GeV defines the "*Q*" region.

As will be noted, there is a slight decrease of the predicted cross section σ_- with increasing energy. The energy dependence of the production amplitude is controlled by that of the πp elastic amplitude in the final πp subenergy variable. As incident kaon momentum increases, so does the typical final πp subenergy. Asymptotically, σ_- and the π^-p elastic-scattering cross sections should have the same energy dependence (but with an appropriate shift of threshold). The charge-exchange cross section σ_0 falls more rapidly with increasing energy, as expected, because it reflects elastic charge exchange in the πN subenergy.[18] It is an order of magnitude smaller than σ_- at $p_{lab} \simeq 6$ GeV/c.

Note that $\sigma_\pm$ and σ_0 are cross sections for specific charge configurations in the final state. They are not summed over all K^* decay modes.

The calculated ratio σ_-/σ_+ is shown in Fig. 7(b) as a function of incident momentum. With the Δ region excluded, this ratio hovers just above unity.

Precise comparisons with data are difficult because experimental conditions and kinematic selections vary widely. Nevertheless, σ_+ and σ_- are roughly 60 to 70% of the values reported in Fig. 14 of Ref. 6, in the 6 to 8 GeV/c range, and σ_- at 14 GeV/c is again $\gtrsim 60\%$ of that measured in Ref. 9. Therefore, whereas the pion-exchange Deck graph appears not to provide the entire observed cross sections, the present results show that it is consistent with a good fraction. In this connection, it should be borne in mind that the Deck graph contains no *resonant* $2^+K^*(1420)$ component, and it provides only a natural parity exchange amplitude (at the nucleon vertex). Resonant 2^+ and unnatural parity exchange are present in the data[19] and presumably should be removed before cross section comparisons are made. Moreover, the produced $K\pi$ system (=K^*) is imagined here to be purely a spin-1 system, whereas in the data there is also an s-wave (κ?) contribution.[19] In a more sophisticated Deck-model calculation, the K^* could be replaced by the entire $K\pi$ scattering amplitude, Fig. 11(a). Finally, the cross-section discrepancy, and possible energy-dependent discrepancies of the slope predictions, discussed below, might indicate the need to include a graph with K exchange, as shown in Fig. 11(b). *A priori*, however, this graph should not provide much cross section, because the K is so far off shell. Moreover, before entertaining this extension, it would be well to search for other more direct manifestations of the K-exchange graph in the data themselves; e.g., by examining the final state Kp and $\pi\pi$ mass distributions. Angular selections which should help to isolate effects associated with the pion-exchange graph are discussed in Sec. IV E.

D. Slopes

Keeping the same notation of Eqs. (4.9)–(4.11), I define slopes b_+ and b_- for the differential cross sections $d\sigma_+/dt'$ and $d\sigma_-/dt'$, respectively, by fitting calculated spectra to the form

$$d\sigma/dt' = c\exp(-bt'). \quad (4.12)$$

The momentum transfer variable t' is the usual difference $(t_1 - t_{1\,max})$, where t_1 and $t_{1\,max}$ are the momentum transfer, and maximum value thereof, between the incident and final nucleon at a given val-

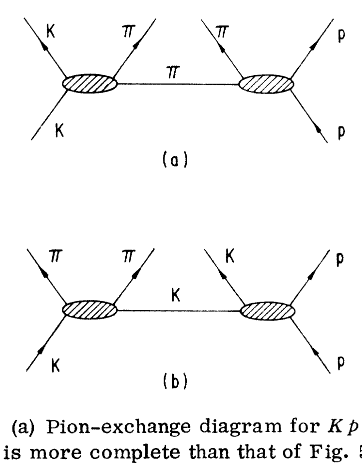

FIG. 11. (a) Pion-exchange diagram for $Kp \to (K\pi)(\pi p)$. The graph is more complete than that of Fig. 5 in that the full $K\pi$ amplitude is retained, not just the $K^*(890)$. (b) Kaon-exchange diagram for $Kp \to (\pi\pi)(Kp)$.

ue of the mass of recoiling $K\pi\pi$ system (reevaluated for each event). In most cases the simple exponential form in Eq. (4.12) provides a good fit to the calculated spectra, although χ^2 would improve in some cases were a more complicated parametrization to be used.

Slopes were determined by fitting in the range $0.02 \leq t' \leq 0.5$ $(\text{GeV}/c)^2$. This is the same range used in fitting the data in Ref. 6. Values of $b_\pm$ are presented in Fig. 6, as a function of incident kaon momentum. The selection $M_{K^*\pi} \leq 1.5$ GeV was made, defining the Q region. In addition, for values in Fig. 6(b), the selection $M_{p\pi} \geq 1.34$ GeV was imposed. No such anti-Δ selection was made in obtaining the slopes presented in Fig. 6(a).

A comparison of the values of b_+ in Fig. 6(a) and (b) shows how significant the Δ^{++} reflection is, especially for momenta below 10 GeV/c. The Δ^0 reflection is much weaker, of course, as can be seen upon comparing the values of b_-.

The slopes $b_\pm$ increase gradually and approach a common value as incident kaon momentum increases. Again, this results in the model as a reflection of the behavior of the slopes of $\pi^\pm p$ elastic scattering.

In Fig. 6(b), for Δ excluded, it is observed that $b_- > b_+$ at all momenta. Inasmuch as $\sigma_-/\sigma_+ \simeq 1$, there must be a crossover of the differential cross sections. In Fig. 12, the calculated difference $(d\sigma_-/dt' - d\sigma_+/dt')$ is shown explicitly. The crossover occurs near $|t'| = 0.2$ $(\text{GeV}/c)^2$.

Examination of Fig. 6(a), with no anti-Δ selection, reveals that the inequality $b_- > b_+$ is true above 6 GeV/c or so, but that it reverses sign below this momentum. Because of the presence of a strong Δ^{++} signal in σ_+, the slope of $d\sigma_+/dt'$ is greater than that of $d\sigma_-/dt'$ at sufficiently low energies. This effect was discussed qualitatively in Sec. III. In order for the asymptotic Deck expectation to hold true at low energies, it is necessary to exclude the Δ regions of phase space.

The dependence of $b_\pm$ on the mass of the $K^*\pi$ system at 8 GeV/c is shown in Fig. 13. The usual decrease of b with mass is obtained. In the model, this is a kinematic effect,[4] largely, resulting from a strong kinematic coupling between t_1 and t_2 at small $M_{K^*\pi}$.

The values of $b_\pm$ shown in Fig. 6 are in good agreement with data[7] on $K^\pm p \to Q^\pm p$ at 8.25 GeV/c. The $K^*\pi$ mass dependences of $b_\pm$ presented in Fig. 13 also agree with these data.[7] Although Q-region definitions differ for the various experiments, published values[8,9,20-22] of slope parameters for $K^+p \to Q^+p$ and for $K^-p \to Q^-p$ at other energies are also in acceptable accord with Fig. 6.

Clear disagreement is the case, however, for the reactions $K^0p \to K^{*+}\pi^-p$ and $\bar{K}^0p \to K^{*-}\pi^+p$. The

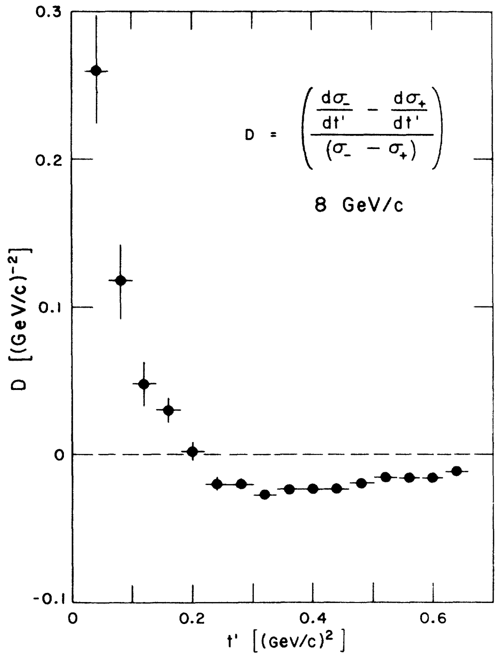

FIG. 12. The difference of the differential cross sections $(d\sigma_-/dt' - d\sigma_+/dt')$ for $K^\pm p \to K^{*0}\pi^\pm p$ (divided by the difference of their integrated values) computed from the Reggeized Deck model is shown as a function of t' at 8 GeV/c. The two selections imposed are mass $K^*\pi < 1.5$ GeV and mass $p\pi > 1.34$ GeV.

Fig. 9 of Ref. 6 shows that $b(\bar{K}^0) = b_+$ is greater than $b(K^0) = b_-$ throughout the range 4 to 12 GeV/c. The average difference $(b_+ - b_-)$ is 4 ± 1, whereas a negative value (≈ -1) is predicted in Fig. 6 of this article. The positive experimental difference

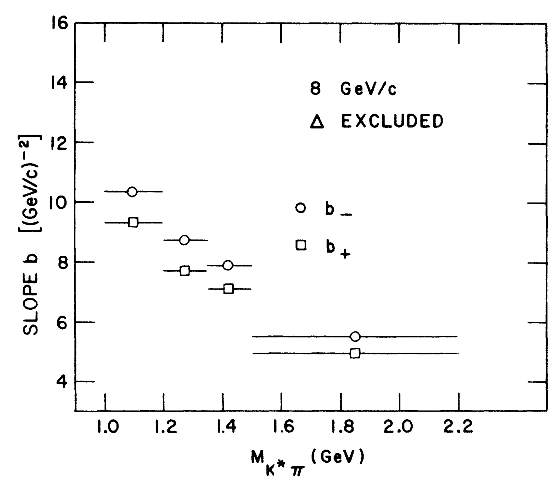

FIG. 13. Slopes $b_\pm$ of $d\sigma/dt'$ predicted by the Reggeized Deck model for $K^\pm p \to (K^{*0}\pi^\pm)p$ [or, equivalently, for $K_L^0 p \to (K^{*\mp}\pi^\pm)p$] are shown as a function of $K^*\pi$ invariant mass at 8 GeV/c. The selection mass $p\pi > 1.34$ GeV is imposed.

is characteristic of all $K\pi\pi$ mass values in the range 1 to 1.8 GeV. As discussed above, it would appear that Δ reflections have been excluded and, therefore, that this possible explanation of the discrepancy with theory must be rejected. Assuming that these $K_L^0 p$ data are otherwise correct, one must conclude that the pion-exchange Deck model for Q production is inadequate. If it fails for $K_L^0 p \to Q^0 p$, then its apparent success for $K^{\pm} p \to Q^{\pm} p$ should also be fortuitous. *The possibility that other Deck graphs contribute, and a more stringent test of the pion-exchange graph are discussed below.*

E. Other exchanges and selections to enhance pion exchange

Before concluding that the failure of the Deck crossover prediction for $K^0 p \to Q^0 p$ means that the model is irrelevant, one may entertain the possibility that other Deck graphs contribute significantly to Q production. Graphs with exchange of K mesons [c.f. Fig. 11(b)] or of K^*'s instead of π are obvious candidates. These have the advantage that they would provide a crossover which agrees qualitatively with what is observed. In $K_L^0 p \to K_S^0 \pi^+ \pi^- p$, the crossover would become a reflection of $K^{*\pm} p$ or of $K_S^0 p$ elastic scattering, which presumably have canonical crossovers. If both π-exchange and K^*- (or K-) exchange graphs contribute, the $K^0 p \to Q^0 p$ data require that the K^* (or K) graph overcompensate for the wrong sign from π exchange alone. In $K^{\pm} p \to Q^{\pm} p$, the K^* (or K) graphs again give canonical crossovers, reinforcing the correct prediction of the π-exchange graph.

The difficulty with K^*- or K- exchange graphs, by contrast with the pion-exchange graphs, is that it is not possible to obtain reliable quantitative values for integrated cross sections and slopes. The exchanges are too far off their mass shells. In any case, before attempting such calculations it would be useful to seek more clear evidence in the data for the π and other possible exchanges.

The pion-exchange graph, Fig. 2, should dominate in that region of phase space where momentum transfer t_{aa^*} is "small," whereas the a^*-exchange graph is appropriate in the region where $t_{a\pi}$ is small. Thus far in this article, selections on t_{aa^*} or on $t_{a\pi}$ have not been discussed. Unfortunately, at values of $M_{\pi a^*}$ not far above threshold, there is considerable overlap between small t_{aa^*} and small $t_{a\pi}$. Indeed, for $M_{K^*\pi} < 1.5$ GeV, the distribution $d\sigma/dt_{K\pi}$ for $Kp \to K^*\pi p$ at 8 GeV/c obtained from the Deck model discussed in this section has a pronounced maximum near $t_{K\pi} = 0$, although no K^* exchange (and only π exchange) is present. Restrictive selections on t_{aa^*}, e.g., $|t| < 0.2$ (GeV/c)^2, should nevertheless be attempted, to the extent that statistics allow, to check whether the pion-exchange model's crossover predictions are more correct for the restricted data sample. Selections on πa^* decay angles discussed below may be more convenient, however.

It is plausible intuitively that a selection on $\cos\theta_{GJ}$ would be effective in separating the regions of phase space in which π and a^* exchanges are dominant. Here $\cos\theta_{GJ}$ is the cosine of the t_2-channel (Gottfried-Jackson) scattering angle between a and a^* in the final-state (πa^*) rest frame. However, in $Kp \to K^*\pi p$, for $M_{K^*\pi} < 1.5$ GeV, the distribution $d\sigma/d\cos\theta_{GJ}$ for $K^{\pm} p \to K^{*0}\pi^{\pm} p$ is predicted to be essentially *flat* in the range $-1 \leq \cos\theta_{GJ} \leq 1$. No spectacular π-exchange peak is evident near $\cos\theta_{GJ} = 1$. This flat behavior corresponds to a well-known kinematic argument,[4,23,24] and explains why the spin-parity content of A_1 and Q threshold enhancements is dominantly $J^P = 1^+$. The tendency of the pion propagator to generate a peak at $\cos\theta_{GJ} = +1$ is countered by the $s_{\pi p}$ character of the πp elastic amplitude, which favors $\cos\theta_{GJ} \to -1$. The effects balance because

$$s_{\pi p}(m_\pi^2 - t_2)^{-1} \simeq \text{constant} .$$

(In the charge-exchange mode, $K^- p \to K^*\pi n$, a peak near $\cos\theta \simeq +1$ is clearly expected from the π-exchange graph since in this case the πN amplitude behaves as $s_{\pi p}^{1/2}$, not $s_{\pi p}$, and cancellation does not occur.) Similar kinematic arguments for a^* exchange in $ap \to a^*\pi p$ lead us to expect rather flat distributions for $d\sigma/d\cos\theta_{GJ}$ from this graph also, rather than a pronounced peak near $\cos\theta_{GJ} \simeq -1$, only. The variable $\cos\theta_{GJ}$ is therefore not the best discriminator, especially since a selection to keep $\cos\theta_{GJ}$ near $+1$ would severely reduce cross section at large values of the final πp invariant mass, from which the bulk of the Deck effect comes.

Although less intuitive, a decay angular variable with much better technical advantages is ϕ_s, the s-channel helicity angle of the $(a^*\pi)$ decay.[25] For $ap \to a^*\pi N$, using the notation of Fig. 2, I define, in the rest frame of $a^*\pi$,

$$\cos\phi_s = \frac{(\vec{q}_N \times \vec{q}_{a^*}) \cdot (\vec{q}_N \times \vec{p}_a)}{|\vec{q}_N \times \vec{q}_{a^*}||\vec{q}_N \times \vec{p}_a|} . \quad (4.13)$$

Variable ϕ_s has the same value if all vectors are defined in the overall center-of-mass frame. The angle ϕ_s is a conjugate variable to t_{aa^*}, in the sense that

$$t_{aa^*} = A(s_{\pi N}, s_{\pi a^*}, t_{pN}) + B(s_{\pi N}, s_{\pi a^*}, t_{pN}) \cos\phi_s , \quad (4.14)$$

where functions A and B are independent of t_{aa*} and ϕ_s. In the absence of explicit dependence on t_{aa*} in the invariant amplitude for $ap \to a^*\pi N$, the distribution $d\sigma/d\phi_s$ is flat (regardless of functional dependence on $s_{\pi N}$, $s_{\pi a^*}$, and t_{pN}). A pion-propagator falloff in t_{aa*} causes a clear peak near $\phi_s = 0$ in $d\sigma/d\phi_s$, in Monte Carlo simulations. Correspondingly, a^* exchange should provide a peak near $\phi_s = 180°$.

A clean way to select events corresponding to the pion-exchange Deck graph would appear to be the restriction $\cos\phi_s > 0$. This selection should eliminate no more than one half of the data sample in the low-mass region. Correspondingly, $\cos\phi_s < 0$ favors a^*-exchange events.

For $K^\pm p \to K^{*0}\pi^\pm p$, the slopes should obey the rule $b_- > b_+$ in both the sectors $\cos\phi_s \gtrless 0$. However, for $K_L^0 p \to K^{*\pm}\pi^\mp p$, the pion-exchange prediction $b(K^0) > b(\overline{K}^0)$ should be true in the pion-exchange sector $\cos\phi_s > 0$, whereas the opposite $b(\overline{K}^0) > b(K^0)$ result should obtain for $\cos\phi_s < 0$. It would be very instructive to see if selections on $\cos\phi_s$ result in such important changes in the SLAC $K_L^0 p$ results. If the Q is a quasiparticle or resonance, as described in Sec. II, the slope systematics would not depend on ϕ_s.

For $\pi^\pm p \to \pi^\pm(\pi^-\Delta^{++})$ discussed in Sec. III, $\cos\phi_s$ is the angle conjugate to $t_{p\Delta}$, in the $\pi^-\Delta^{++}$ rest frame. The result $b_+ > b_-$ should hold in the sector $\cos\phi_s > 0$, whereas $b_- > b_+$ when $\cos\phi_s < 0$.

All predictions of Table I are appropriate only in the pion-exchange sector $\cos\phi_s > 0$.

V. CONCLUSIONS

The systematics of differential cross sections for pairs of inelastic diffractive processes related to each other by line reversal have been discussed in some detail. The lessons to be gained warrant experimental investigations of inelastic processes as precise as those of Ref. 1 for the elastic reactions.

Quantitative expectations of the Deck model have been presented. The crossover systematics test more clearly than has been possible thus far the essential assumption that it is pion exchange, followed by πN elastic scattering, which mediates the inelastic processes. The test is essentially an examination of the spin-nonflip amplitude, since the crossover is a property of this amplitude. A good test was proposed earlier of spin dependence, involving both flip and nonflip πN amplitudes. This is the Berger-Fox polarization prediction.[26]

Insofar as t distributions are concerned, an inclusive study of $ap \to Xp$ and $\bar{a}p \to \overline{X}p$ might be sufficient. However, as described in Secs. III and IV, the Deck model predicts that the slopes of production distributions and their crossover properties differ for different decay modes of X (e.g., $X \to K^{*0}\pi^+$ or $K^{*+}\pi^-$), and that they vary with decay angle selections in the X rest frame. Experimentally, therefore, it would be most valuable to study the related pairs of reactions with an apparatus which allows examination of the composition of system X.

In several approaches to multiparticle production ($n > 4$), dating back to the ABFST model,[27] as well as the more recent cluster production ansatz,[28] pion exchange is imagined to be repeated along a multiperipheral chain.[29] Detailed investigations of the role of pion exchange in well-defined exclusive inelastic processes, by means of the decay angle selections described in Sec. IV E, might therefore have important consequences beyond the Deck model.

ACKNOWLEDGMENTS

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My interest in the crossover systematics of inelastic processes, particularly with regard to the comparison of proton and antiproton induced reactions, was renewed during discussions with A. Fridman, H. Braun, and their colleagues in Strasbourg. I have also benefited from conversations with Y. Goldschmidt-Clermont, D. R. O. Morrison, J. A. J. Matthews, G. Brandenburg, and R. Diebold.

APPENDIX

In this Appendix, I review the phenomenological understanding of crossovers in elastic scattering, with a view towards applying the same methodology to inelastic diffractive processes.

It is assumed that elastic scattering is mediated at high energy by a set of t-channel exchanges whose amplitudes in a given state of s-channel helicity are A_P (Pomeron), A_ρ, A_ω, A_f, and A_{A_2}. Lower lying Regge singularities are ignored. The amplitudes are different for different reactions.

Specializing to $K^\pm p$ scattering, we write for each s-channel helicity state

$$A_- = A_P + A_\omega + A_f + A_\rho + A_{A_2}, \quad (A1)$$

and

$$A_+ = A_P - A_\omega + A_f - A_\rho + A_{A_2}. \quad (A2)$$

The ω and ρ amplitudes change sign because they represent exchanges in the t channel which are odd under charge conjugation. Ignoring all but the s-channel spin-nonflip term, we can express the difference of differential cross-sections as

$$\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p) = 4\mathrm{Im}(A_P + A_f + A_{A_2})[\mathrm{Im}(A_\rho + A_\omega)] + 4\mathrm{Re}(A_P + A_f + A_{A_2})[\mathrm{Re}(A_\rho + A_\omega)] + \text{similar terms for spin-flip amplitudes}\,. \tag{A3}$$

By definition, the dominant amplitude in Eqs. (A1) and (A2) at sufficiently high energy is A_P. Because the ratio $\mathrm{Re}A_\pm/\mathrm{Im}A_\pm$ is observed to be small, one may safely assert that A_P is purely imaginary at small t. Thus, to first order, at small enough t,

$$\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(K^+p) \cong 4\mathrm{Im}A_P[\mathrm{Im}(A_\rho + A_\omega)]\,. \tag{A4}$$

Although derived with elastic scattering in mind, Eq. (A4) would be equally appropriate at small enough t for the difference of inelastic reactions $[d\sigma/dt(K^-p \to Q^-p) - d\sigma/dt(K^+p \to Q^+p)]$, as long as it is assumed that the same set of Regge exchanges are present, and that, at high energy, A_P is dominant and primarily imaginary in the inelastic reactions as well. The functions A_P, A_ρ, and A_ω would not necessarily be the same.

The arguments may diverge, however, when the positivity and t dependence of the right-hand side of Eq. (A4) are considered.

In elastic scattering, the optical theorem and the positive difference of the *total* cross sections $(\sigma_-^{\text{tot}} - \sigma_+^{\text{tot}})$ assures that both $\mathrm{Im}A_P$ and $\mathrm{Im}(A_\rho + A_\omega)$ are positive. The positive nature of $\mathrm{Im}(A_\rho + A_\omega)$ is interpreted phenomenologically in terms of duality. In K^+p elastic scattering, there are no resonances, and therefore the scattering amplitude is *real*, save for nonresonant diffractive effects represented by A_P; $\mathrm{Im}(-A_\omega + A_f - A_\rho + A_{A_2}) = 0$. In K^-p, resonances provide an amplitude with nonzero imaginary part. This imaginary part is *positive* in an elastic situation, where the coupling constant enters quadratically. Thus, $\mathrm{Im}(A_\omega + A_f + A_\rho + A_{A_2}) > 0$. Combining the two expressions gives

$$\mathrm{Im}(A_\omega + A_\rho) > 0\,.$$

For the inelastic processes $Kp \to Qp$, there is no optical theorem. Is $\mathrm{Im}A_P$ again positive? The role of duality is also obscure. There obviously are resonances in the final states of *both* $K^\pm p \to K^\pm\pi^+\pi^-p$; e.g., $\Delta(1238)$, $K^*(890)$, ρ, ... Even if these resonances cooperate so as to yield a net real Regge amplitude for $K^+p \to Q^+p$ (save for A_P) and a large imaginary part for $K^-p \to Q^-p$, simply duality cannot be used to provide the *sign* of the imaginary part, because the channel is no longer elastic. In the inelastic case, the sign of the right-hand side of Eq. (A4) would not appear to be predictable *a priori*.

The t dependence of Eq. (A4) is examined next. Because the Pomeron term is largely imaginary at small t, and the Regge term is real in K^+p elastic scattering,

$$\frac{d\sigma}{dt}(K^+p) = |A_P|^2 + |A_\rho + A_\omega - A_f - A_{A_2}|^2 \to (\mathrm{Im}A_P)^2 \tag{A5}$$

at high enough energy, and small t. To first approximation,

$$\mathrm{Im}A_P \sim \exp(ct)\,, \tag{A6}$$

since $d\sigma/dt$ is roughly exponential in the small t range. From Eq. (A4), the t dependence of $\mathrm{Im}(A_\rho + A_\omega)$ is then to first order

$$\mathrm{Im}(A_\rho + A_\omega) \propto \frac{(d\sigma/dt)(K^-p) - (d\sigma/dt)(K^+p)}{4[(d\sigma/dt)(K^+p)]^{1/2}}\,. \tag{A7}$$

Elastic data[1] show that this is a function which falls rapidly, passes through zero near $t = -0.2$ $(\mathrm{GeV}/c)^2$, and has a minimum near $t = -0.6$ $(\mathrm{GeV}/c)^2$, before rising again toward zero. The behavior is similar to that of a Bessel function $J_0(r\sqrt{-t})$, with $r \simeq 1$ fm.[11] Most of the strength of $\mathrm{Im}(A_\rho + A_\omega)$ is concentrated about the peripheral impact parameters of order 1 fm. By contrast, $\mathrm{Im}A_P$ is dominated by low partial waves.

Whether A_P and $A_\rho + A_\omega$ should have similar dependences on t in inelastic reactions depends strongly on how their t dependences in the elastic case are interpreted. If A_P, A_ρ, and A_ω are essentially Regge pole amplitudes, then the t dependences are presumably the same in inelastic and elastic processes. However, if as seems more likely, the zero near $t = -0.2$ $(\mathrm{GeV}/c)^2$ is an s-channel absorption effect, $A_\rho + A_\omega$ is a complicated combination of pole plus branch-point singularities. The elastic and inelastic t dependences could be very different, since absorption effects in the elastic and inelastic channels need not be the same. The nature of the t dependence of A_P is also difficult to predict. It need no longer be primarily nonperipheral in the inelastic reactions. This ignorance of the expected t dependences of A_P and $A_\rho + A_\omega$ in the inelastic processes makes it impossible to *predict* a crossover.

Finally, the issue of spin complication arises since the Q and A_1 systems, for example, have spin components of 1^+ and greater.[5,19] This allows more flip terms on the right-hand side of Eq. (A3) and renders less reliable predictions which ignore them.

In a study[6] of $K^0p \to Q^0p$ and $\overline{K}^0p \to \overline{Q}^0p$, the equivalents of Eq. (A4) to (A7) have been used to obtain $\mathrm{Im}A_P$ and $\mathrm{Im}(A_\rho + A_\omega)$. One result of this analysis is the finding that the ratio $\mathrm{Im}(A_\rho + A_\omega)/\mathrm{Im}A_P$ is roughly twice as large at small t in the inelastic reaction as in the corresponding elastic reaction at the same over-all c.m. energy. This result is understood qualitatively in terms of the Deck model discussed in Secs. III and IV. In this model, the ratio is determined by the observed result in πp elastic scattering but at an effective final state πp *subenergy* which is much smaller than the over-all c.m. energy.

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