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Theoretical evidence for a new weak current*

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We postulate the existence of a new weak or superweak neutral isosinglet current $\mathfrak{X}$, of the chiral form $V+A$ (the usual form is $V-A$). The observed spin-$\frac{1}{2}$ baryons' contributions to $\mathfrak{X}$ are $\mathfrak{X}=\alpha(N^\dagger N+\Xi^\dagger\Xi)+\beta\Lambda^*\Lambda$, with parameters α and β. This form follows deductively from a compensation theory for Feynman diagrams. Observation points to a value $\beta/\alpha=2.007\pm0.005$. By taking $\beta/\alpha=2$ exactly, we derive a mass relation which allows us to calculate in a satisfactory way the mass of the Λ in terms of the masses of the N and Ξ.

I. INTRODUCTION

In a recent series of publications,[1] a peculiar class of divergent Feynman diagrams, not susceptible to renormalization, has been extensively studied. A well-known example of such a diagram is the "bubble" consisting of a baryon-antibaryon pair mediating the decay of a pseudoscalar meson into two leptons. It has been shown in detail that one can arrange for a mutual cancellation of the infinities arising from diagrams of the same order and topology, but with different internal labels, provided certain algebraic constraints are satisfied by the strong couplings and by the baryonic masses. If this compensation process is postulated to be a general phenomenon, one thereby obtains a considerable amount of *a priori* information concerning the elementary particles and their couplings.

The strong Lagrangian is taken to be of the Yukawa type, because of its renormalizability; the hadrons are octets or nonets with spins $\frac{1}{2}$ and 0; quarks are not used. The weaker interactions are described in terms of W bosons coupled to weak, superweak, and electromagnetic currents satisfying an algebra based on chiral SU(3).

In this paper we show that the compensation requirements allow the existence of a new (the eighteenth in this model) weak isoscalar current. The main theoretical evidence for this current is the connection it establishes between hitherto unrelated aspects of compensation theory. Contact with experiment is made through a new mass relation for the spin-$\frac{1}{2}$ baryons.

Previous work in compensation theory made use of seventeen distinct currents and their linear combinations as the carriers of weak, electromagnetic, and superweak interactions. The choice of these currents was motivated by (a) chiral SU(3) and electrodynamics, and (b) a search for larger current algebras. (The latter serves as a heuristic guide and replaces the older notion of a search for higher symmetries.)

The seventeen currents may be listed as follows[2]:

(i) an SU(3), F-type octet of $V-A$ currents $\mathcal{P}^+, \mathcal{P}^0, \mathcal{P}^-, \mathcal{K}^+, \mathcal{K}^0, \overline{\mathcal{K}}^0, \overline{\mathcal{K}}^-, \mathcal{Y}$, respectively transforming like the particle octet $\pi^+, \pi^0, \pi^-, K^+, K^0, \overline{K}^0, \overline{K}^-, \eta$,

(ii) the total baryon-number current, $\mathcal{B}$, of the vector form,

(iii) an isoscalar, $\mathcal{L}$, of the form $V-A$, whose V part expresses the ninth-baryon current,

(iv) an SU(2) triplet $\mathcal{P}'^+, \mathcal{P}'^0, \mathcal{P}'^-$, the $V+A$ counterpart of $\mathcal{P}^+, \mathcal{P}^0, \mathcal{P}^-$,

(v) a triplet $\mathcal{J}^+, \mathcal{J}^0, \mathcal{J}^-$, also $V+A$, which does not conserve CP, and which extends the SU(2) triplet in (iv) to O(4), and

(vi) an isoscalar, $\mathcal{Y}'$, the $V+A$ counterpart of $\mathcal{Y}$.

The explicit form of all these currents is given in I, Appendix D. All together, the seventeen currents form an algebra, several pieces of which commute.

As pointed out in I, Appendix F, the axial-vector form of $\mathcal{B}$ is not allowed by the compensation relations, although it would commute with the algebra we already have. Therefore, the forms $V\pm A$ of $\mathcal{B}$ are both forbidden. Is any other isoscalar current allowed in the theory? This question is at the basis of the present work.

To be incorporated into our chiral algebra, the proposed current, say $\mathfrak{X}$, must be either $V+A$ or $V-A$. However, we see from the above-given list that there is no room for a new $V-A$ current: Commutation with SU(3) would require that such a current be a linear combination of $\mathcal{B}$ and $\mathcal{L}$. Thus $\mathfrak{X}$ must be of the form $V+A$.

Dependence on the internal quantum numbers can be restricted as follows. First we require commutation of $\mathfrak{X}$ with O(4), in particular with

$$\vec{\mathcal{J}}=N^\dagger\vec{\tau}N-\Xi^\dagger\vec{\tau}\Xi\pm2i(\vec{\Sigma}^*L-L^*\vec{\Sigma}). \tag{1.1}$$

This means[3] that $\mathfrak{X}$ can contain $\vec{\Sigma}$ and L only in the combination $\vec{\Sigma}^*\cdot\vec{\Sigma}+L^*L$. Next, we note that an arbitrary amount of $\mathcal{Y}'\propto N^\dagger N-\Xi^\dagger\Xi$ may be added.

Therefore, we have, without loss of generality, the form

$$\mathfrak{X} = \alpha(N^\dagger N + \Xi^\dagger \Xi) + \beta\Lambda^*\Lambda + \gamma(\vec{\Sigma}^* \cdot \vec{\Sigma} + L^*L), \tag{1.2}$$

for some parameters α, β, γ. In the following section we demonstrate that $\gamma = 0$ on the basis of the assumption that $g_{\Sigma\Lambda}$, the strong $\Sigma\Lambda\pi$ coupling, is nonzero. We also show that two seemingly unrelated compensation relations point to the same value for β/α. From observational evidence $\beta/\alpha \approx 2$ to an excellent accuracy, and we shall therefore explore the consequences of taking $\beta/\alpha = 2$ exactly.

II. COMPENSATION RELATIONS INVOLVING THE CURRENT $\mathfrak{X}$

We now consider those Feynman diagrams which are of first order in the strong couplings (this is the only case where "pure" compensation must be applied, unencumbered by renormalization) and which have an external W boson coupled through the new current $\mathfrak{X}$, Eq. (1.2). Such diagrams comprise three classes: those which are cutoff-independent by power counting in the integrand; those whose terms, corresponding to different internal particle choices, are individually divergent but compensate each other automatically (e.g., by properties of the isospin coefficients); and those whose total contribution is logarithmically divergent unless some linear homogeneous constraint ("compensation relation") is satisfied by the strong couplings.

To achieve compensation in this third class of diagrams, it is sufficient to require the convergence of the three processes of Fig. 1. The particle makeup of the O(4) current $\mathcal{J}^+_{V+A}$ and the SU(3) current $\mathcal{K}^+_{V-A}$ is recalled[4] to be

$$\mathcal{J}^+ = \sqrt{2}\,(n^*p - \Xi^{-*}\Xi^0) + 2i\,(\Sigma^{-*}L - L^*\Sigma^+), \tag{2.1}$$

$$\mathcal{K}^+ = -\sqrt{3}\,\Lambda^{\omega *}p - \Sigma^{0*}p - \sqrt{2}\,\Sigma^{-*}n + \sqrt{3}\,\Xi^{-*}\Lambda^{\omega} - \sqrt{2}\,\Xi^{0*}\Sigma^+ + \Xi^{-*}\Sigma^0 . \tag{2.2}$$

The mixture Λ^ω refers to

$$\Lambda^\omega = \Lambda\cos\omega + L\sin\omega .$$

We also recall that the mesonic constituents of these currents are not relevant to the class of terms under consideration. The strong-coupling conventions for π, K, and η are shown in I, Appendix A.

Note that some internal labels, while possible *a priori*, are nevertheless omitted in Fig. 1. In Fig. 1(a) these labels (n, p, Ξ^0, Ξ^-) correspond to the real part of the compensation relation, automatically satisfied here because of equal baryon masses at the strong vertex. The imaginary part of the same compensation relation is then used to obtain $\gamma = 0$ in (1.2). Thus, no Σ or L labels have to be considered in Figs. 1(b) and 1(c) at the $\mathfrak{X}$ vertex.

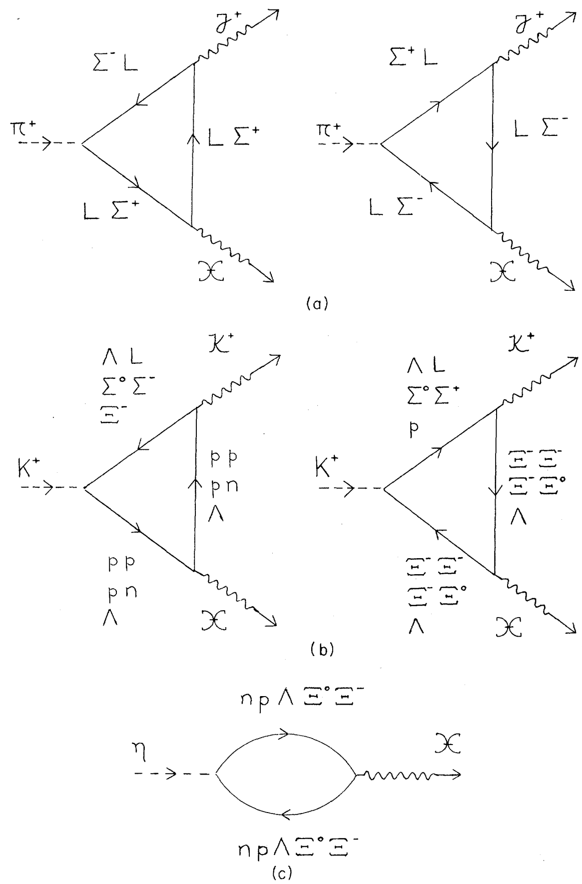

FIG. 1. The complete set of lowest-order one-meson processes which involve the current $\mathfrak{X}$ and which must be required to compensate.

The compensation relations now read as follows. From Fig. 1(a),

$$\gamma(\Sigma + L)g_{\Sigma L} = 0 . \tag{2.3}$$

(We use the particle's symbol for its mass when no confusion is likely; the g's are strong coupling constants.) Note the + sign for the mass L, due to the L's negative parity. Also, we shall use here the essential input

$$g_{\Sigma L} \neq 0, \tag{2.4}$$

which can be obtained from the compensation of quadratic divergences (see III), or from observational hints[5] concerning $g_{\Sigma\Lambda}$, or even as a qualitative assumption justified by its quantitative results.[6] Thus,

$$\gamma = 0 . \tag{2.5}$$

From Fig. 1(b),

$$\alpha(N\bar{g}_{N\Lambda} + \Xi\bar{g}_{\Xi\Lambda} + \sqrt{3}\,Ng_{N\Sigma} - \sqrt{3}\,\Xi\,g_{\Xi\Sigma}) - \beta\Lambda(\bar{g}_{\Xi\Lambda} + \bar{g}_{N\Lambda})\cos^2\omega = 0\,, \quad (2.6)$$

where we have used the no-nonet theorem to set (see I, Sec. 6)

$$g_{N\Lambda} = \bar{g}_{N\Lambda}\cos\omega\,,$$
$$g_{NL} = \bar{g}_{N\Lambda}\sin\omega\,. \quad (2.7)$$

Finally, from Fig. 1(c),

$$\alpha(2Ng_N + 2\Xi g_\Xi) + \beta\Lambda g_\Lambda = 0\,. \quad (2.8)$$

III. INTERNAL CONSISTENCY OF A THEORY WITH $\mathfrak{X}$

In this section we present two arguments leading to the same mass relation. The separate origins of these arguments and the identical nature of their results is what constitutes in our view a compelling motivation for the current $\mathfrak{X}$.

In (2.6), let us substitute the previously found solution [see Eq. (I.11.17)]:

$$\bar{g}_{N\Lambda}:\bar{g}_{\Xi\Lambda}:g_{N\Sigma}:g_{\Xi\Sigma} = \frac{\Xi-\bar{\Lambda}}{\bar{\Lambda}}:\frac{N-\bar{\Lambda}}{\bar{\Lambda}}:\frac{\Sigma-\Xi}{\sqrt{3}\,\Sigma}:\frac{N-\Sigma}{\sqrt{3}\,\Sigma}\,. \quad (3.1)$$

We obtain

$$2\alpha N\Xi\left(\frac{1}{\bar{\Lambda}}-\frac{1}{\Sigma}\right) - \beta\Lambda\frac{\Xi+N-2\bar{\Lambda}}{\bar{\Lambda}}\cos^2\omega = 0\,. \quad (3.2)$$

Previously derived mass relations [(I.4.1), (I.4.2)] allow us to substitute

$$\frac{1}{\bar{\Lambda}} = \frac{4}{N+\Xi} - \frac{1}{\Sigma}\,, \quad (3.3)$$

whence

$$\frac{4\alpha N\Xi}{N+\Xi} = \beta\Lambda\cos^2\omega\,. \quad (3.4)$$

Equation (3.4) is derived from Eq. (2.6) on the basis of information involving K mesons only. We now rederive it on the basis of the η couplings. From the compensation of quadratic divergences in III, we have an independent assignment for these couplings, namely

$$g_N:g_\Xi:g_\Lambda = \frac{1}{N}:\frac{1}{\Xi}:-\left(\frac{1}{N}+\frac{1}{\Xi}\right)\cos^2\omega\,. \quad (3.5)$$

Inserting these in (2.8), we reproduce (3.4).

The reasons for this highly nontrivial agreement (and for similar "coincidences" encountered in previous work) remain somewhat mysterious. The situation may someday be clarified if a symmetric Lagrangian is found whose spontaneously broken form happens to be the one used here to obtain physical results. In any case, the present agreement helps confirm the close relation between the compensation requirements for the K and η interactions, as well as between the logarithmically and quadratically divergent processes.

IV. NUMERICAL CONSEQUENCES

The experimental values of the appropriate masses, together with our previous results, now enable us to calculate β/α in Eq. (3.4). We get

$$\frac{\beta}{\alpha} = \frac{4N\Xi}{(N+\Xi)\Lambda\cos^2\omega}\,. \quad (4.1)$$

From (I.4.11),

$$1/\cos^2\omega = 1.021 \pm 0.001\,, \quad (4.2)$$

where the stated uncertainty arises from the electromagnetic mass spread of the baryons. Similarly,

$$\frac{N\Xi}{(N+\Xi)\Lambda} = 0.4915 \pm 0.0007\,. \quad (4.3)$$

Thus, from (4.1),

$$\beta/\alpha = 2.007 \pm 0.005\,. \quad (4.4)$$

[The electromagnetic deviation of the Λ mass cannot be determined without a model, but it would affect the right-hand side of (4.4). Also, the uncertainties in (4.2) and (4.3) are not really independent, since the same experimental parameters are used to evaluate both numbers.]

This result for β/α is very suggestive, especially if we expect the coefficients in the weak currents to be simple integers. We now pursue the consequences of assuming $\beta = 2\alpha$ exactly, i.e., of the new form of Eq. (1.2):

$$\mathfrak{X} = N^\dagger N + \Xi^\dagger\Xi + 2\Lambda^*\Lambda\,. \quad (4.5)$$

From (3.4)

$$\Lambda\cos^2\omega = 2N\Xi/(N+\Xi)\,. \quad (4.6)$$

After some algebra, and using the three relations [(I.4.4), (I.4.2), (I.4.1)]

$$\bar{\Lambda} = \Lambda\cos^2\omega - L\sin^2\omega\,, \quad (4.7)$$

$$\frac{3}{\bar{\Lambda}} + \frac{1}{\Sigma} = \frac{2}{N} + \frac{2}{\Xi}\,, \quad (4.8)$$

$$\frac{6}{N+\Xi} = \frac{1}{N} + \frac{1}{\Xi} + \frac{1}{\Sigma}\,, \quad (4.9)$$

we obtain

$$L\sin^2\omega = \bar{\Lambda}\left(\frac{\Xi-N}{\Xi+N}\right)^2\,. \quad (4.10)$$

If in addition we use [see (I.4.3)]

$$L\bar{\Lambda} = \Sigma^2\,, \quad (4.11)$$

the result (4.10) becomes

$$|\sin\omega| = \frac{\bar{\Lambda}}{\Sigma}\left(\frac{\Xi-N}{\Xi+N}\right)\,, \quad (4.12)$$

allowing an accurate *a priori* determination of ω and Λ in terms of, say, N and Ξ only. Taking (see the numerical fit in I) the neutral masses, $N = 940\ \mathrm{MeV}/c^2$, $\Xi = 1315\ \mathrm{MeV}/c^2$, we get

$$|\sin\omega| = 0.1483\,. \tag{4.13}$$

Thus, from Eq. (4.6),

$$\Lambda = 1121.0\ \mathrm{MeV}/c^2\,, \tag{4.14}$$

which, if taken seriously, implies an electromagnetic deviation of $-5.4\ \mathrm{MeV}/c^2$, not an unreasonably large amount.

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[1]M. Wellner, Ann. Phys. (N.Y.) 73, 180 (1972); 76, 549 (1973); 86, 331 (1974). These will be cited as I, II, and III, respectively. The notation (e.g.) (I.11.17) will mean Eq. (11.17) of paper I.

[2]See I, Sec. 7.

[3]We recall (see I) that the L is a postulated high-mass baryon with negative parity, but otherwise with all the quantum numbers of the Λ. The L's "strong" couplings $g^2/4\pi$ are damped by a typical factor $\sin^2\omega \approx 0.02$, so that this particle should be at the edge of observability.

[4]See I, Appendix D.

[5]J. Engels, Nucl. Phys. B51, 269 (1973).

[6]In I it seemed necessary to have $g_{\Sigma\Lambda} = g_{\Sigma L} = 0$. The new features of the theory which force us to take $g_{\Sigma\Lambda} \neq 0$ and $g_{\Sigma L} \neq 0$ are developed in III; see especially Sec. (III.5d).