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Matter distribution in $\pi^{\pm}$ and $K^{\pm}$ and possible universality of meson form factors

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Analysis of the Fermi National Accelerator Laboratory 200-GeV meson-proton elastic scattering data based on the geometrical picture leads to predictions for the matter distribution in $\pi^{\pm}$ and $K^{\pm}$. The result suggests that form factors for these particles are essentially identical. At higher incident energies, form factors calculated from the model will remain fixed, while the effective blackness for the particles increases.

Using the geometrical picture of hadron-hadron collisions[1] advanced some time ago, we calculate the matter distribution inside $\pi^{\pm}$ and $K^{\pm}$. This calculation is similar in spirit to the one performed in Ref. 2, except for the fact that we now use recent experimental data on elastic $\pi^{\pm}p$ and $K^{\pm}p$ differential cross sections. The result of these calculations gives the root-mean-square radius of $K^{\pm}$ and $\pi^{\pm}$ as $(0.62\pm0.02)\times10^{-13}$ cm. Remarkably the radii of $K^{\pm}$ and $\pi^{\pm}$ are the same. In fact the entire form factors for all four particles are quantitatively very close to each other. The observed difference of elastic cross sections, in this picture, is due to the different effective blackness for the particles.

In Sec. I the main ideas of the elastic model and its experimental support will be recapitulated. The computations of meson form factors will then be presented in Sec. II and results discussed in Sec. III.

I. THE GEOMETRICAL MODEL

The geometrical description of hadron-hadron collisions at high energies was developed at first to account for the elastic phenomena, which utilizes three main physical ideas: (a) eikonal approximation, (b) exponentiation of the S matrix, and (c) convolution integral in impact-parameter space for the opaqueness.

In the eikonal approximation, the elastic scattering amplitude a_{AB} for hadrons A and B as a function of the momentum transfer $\vec{k}$ is the two-dimensional Fourier transform [denoted by the symbol $\langle\ \rangle$] of $1-S(\vec{b})$ in the impact-parameter $\vec{b}$ space:

$$a_{AB}(k)=\langle 1-S_{AB}(b)\rangle\,, \tag{1}$$

$$\left(\frac{d\sigma}{dt}\right)_{\rm el}=\pi|a_{AB}|^2\,, \tag{2}$$

where $-t=k^2$ in the usual notation.

It is natural to write the elastic S-matrix element $S_{AB}(b)$ in terms of the opaqueness $\Omega_{AB}(b)$ of the system in exponential form,

$$S_{AB}(b)=\exp[-\Omega_{AB}(b)]\,. \tag{3}$$

Ω_{AB} is assumed to be real and related to the form factors by

$$\Omega_{AB}(b)=(\text{constant})\times\langle F_A(k^2)\rangle * \langle F_B(k^2)\rangle\,, \tag{4}$$

where $*$ designates the folding integral. The constant in (4) represents the effective blackness of the system and is different for different collisions.

With these assumptions, an explicit relation between the elastic amplitude a_{AB} and the form factors can be derived:

$$F_A(k^2)F_B(k^2)=\text{const}\times[a_{AB}+\tfrac{1}{2}a_{AB}*a_{AB}+\cdots]\,. \tag{5}$$

The inverse of (5) can also be obtained.[1]

This model of elastic scattering works extremely well at high energies. Its predictions have been remarkably confirmed by recent CERN-ISR experiments.[3] The following few points related to pp collisions are particularly worth noting:

(i) Given the proton form factor as input, the calculated differential cross section is in good quantitative agreement[4] with observations. Conversely, if the experimental pp differential cross sections are used, (5) yields an excellent fit to the form factor of the proton as measured in ep experiments.

(ii) The change of slope seen at $-t\simeq0.1$ (GeV/c)^2 can be reproduced[4] in this model.

(iii) The dip found recently near $-t\simeq1.4$ (GeV/c)^2 was predicted[5] earlier. The height of the secondary maximum can also be fitted[4] correctly if *experimental* data for the proton form factor are used.

(iv) The slow shrinkage of the diffraction pattern and the increase of total cross section as observed beyond 200 GeV can both be attributed to the increasing opacity of the proton with incoming energy. However, the shape of the opaqueness $\Omega(b)$ as a function of b does not change appreciably.[6]

Thus the model remains useful and can give good quantitative fits to the data over the entire CERN-ISR range, if the constant in (4) is allowed to have a slight energy dependence without changing the form factors.

II. MESON-PROTON SCATTERING

Akerlof *et al.* have recently measured[7] the elastic differential cross sections for $\pi^{\pm}p$, $K^{\pm}p$, and pp collisions at 100 and 200 GeV energies. We now analyze these data with Eq. (5), assuming the purely imaginary character of the scattering amplitude. To facilitate computations, experimental data at 200 GeV are fitted by a sum of three exponentials:

$$a_{AB}=\sum_{i=1}^{3}\alpha_i e^{\beta_i t}\ (\text{GeV}/c)^{-2}. \tag{6}$$

The parameters α_i, β_i for various processes are listed in Table I. As the data are preliminary and accurately measured values are not available to us, we have not attempted a least-squares fit. However, we find that our fit gives a successful representation of data through the range $0<-t<1.2$ $(\text{GeV}/c)^2$. Substitution of (6) into (5) gives the normalized form-factor products. From these products one readily obtains the various meson and proton form factors. The results are plotted in Fig. 1 and also tabulated in Table II.

III. RESULTS AND DISCUSSIONS

(i) The K^+ and K^- have essentially the same form factor. So do approximately the π^+ and π^- [see (vi) below]. Since it may usually be assumed that charge-conjugate particles have similar matter distributions, this result tends to confirm the validity of the whole calculation.

(ii) It is seen that the pions' form factors are very close to the kaons'. This fact is suggestive of the possibility that the matter distributions for all four mesons have nearly the same shape.

(iii) The calculated rms radii for $K^{\pm}$ and $\pi^{\pm}$ are approximately 0.62 ± 0.02 F. The near equality of the rms radii can also be seen from the similar slopes of the meson form factor curves at $k=0$. We also note that the present value of pion rms radius and in fact the entire form factor are not too different from a previous result[2] obtained by extrapolating the low-energy πp differential cross sections.

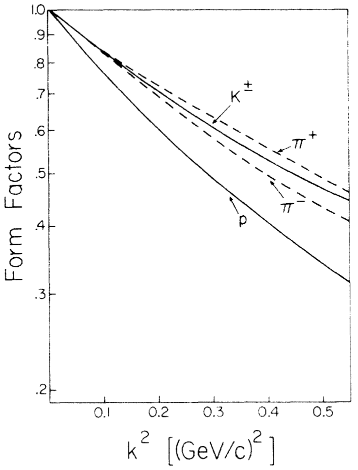

FIG. 1. Calculated proton and meson form factors. The $\pi^{\pm}$ form factors are shown as the dashed lines, the proton and kaon form factors as the solid lines.

(iv) To compare our results of kaon form factors with experiments is not possible at present owing to lack of data. Information about the pion form factor and its rms radius is at best scanty. The form factor in the spacelike region can be inferred in a model-dependent way from the pion electroproduction process.[8] Clearly such a result is sensitive to theoretical interpretations; nevertheless our calculated form factor seems to be consistent with these measurements. The first direct measurement of the pion charge radius was made by the Dubna-UCLA collaboration by scattering high-energy pions off the atomic electrons. Their published report[9] gave the charge radius of π^- as 0.78 ± 0.10 F, which appears to be quite large. Later,

TABLE I. Parameters obtained from a fit to the differential cross sections for various processes at 200 GeV. (For K^-p, only the 100 GeV data are available.)

	α_1	α_2	α_3	β_1	β_2	β_3
π^-p	0.271	3.35	1.18	8.0	5.9	2.5
π^+p	0.723	2.26	1.72	13.5	4.8	3.0
K^-p	1.13	1.36	1.36	7.0	6.0	2.5
K^+p	0.271	2.08	1.36	10.0	6.0	2.6
pp	0.723	1.99	5.15	15.0	9.0	4.7

TABLE II. List of the meson and the proton form factors.

k^2 [(GeV/c)^2]	F_p	F_{π^-}	F_{π^+}	F_{K^-}	F_{K^+}
0	1.000	1.000	1.000	1.000	1.000
0.1	0.764	0.830	0.839	0.838	0.839
0.2	0.605	0.690	0.725	0.708	0.711
0.3	0.491	0.580	0.634	0.608	0.610
0.4	0.407	0.497	0.557	0.532	0.532
0.5	0.342	0.433	0.491	0.472	0.469
0.6	0.291	0.382	0.433	0.424	0.418

a reanalysis[10] of the same data with a different method yielded a lower value of 0.71 ± 0.05 F, closer to our calculated result. We notice, incidentally, that the ρ-dominance model gives a pion form factor which is quantitatively similar to our prediction. However, the spirits of the two calculations are very much different.

(v) The proton form factor falls off more rapidly than that for the mesons, as is clear from Table II. The calculated form factor using the 200-GeV pp data from Fermilab as input coincides with the G_E form factor of the proton. Hence, it is in good agreement with the result of a similar analysis[6,11] based on the CERN-ISR data in the energy range of 200 to 1500 GeV.

(vi) It is mentioned in (i) that to the zeroth approximation K^+ and K^- have the same form factor. But why are the form factors for π^+ and π^- slightly different? Although we cannot give a definite answer to this question, we tend to believe that the difference is probably caused by the inadequacy at low energies of certain assumptions used in our model, such as the convolution approximation of opaqueness. But at high enough energies, it is likely that curves for π^+ and π^- will approach each other and converge possibly toward the kaon form factor which lies in between the two.

(vii) In connection with the question discussed in (vi) above, it is perhaps useful to comment on the energy region in which our model is expected to be valid. In the original proposal the model was formulated in the infinite-energy limit for the sake of making certain concepts precise. It was later experimentally observed[12] that (a) the total cross sections, instead of staying flat, rise logarithmically with energy, and (b) the forward elastic peaks shrink slowly with rising energy. Thus it is not fruitful to discuss infinite-energy limits. However, as discussed in Sec. I, effects of (a) and (b) cancel when one computes the *form factor* of the proton. Thus it is possible that in the region of 100 to 3000 GeV, and maybe higher, it is a good approximation to take fixed form factors for the hadrons, but fit the experiments with increasing blackness. If this view is correct, the kaon form factor will approach a limiting form earlier than $\pi^{\pm}$, because the trend of increasing total cross section is already rather obvious for the Kp interaction at 200 GeV. Then the slight difference in form factors for $\pi^{\pm}$ appearing in the present calculation will also be understandable.

(viii) We suggest in (ii) the possibility that the pions and kaons might possess identical form factors. Since $\pi^{\pm}$ and $K^{\pm}$ belong to the same SU(3) multiplet, we are tempted to speculate further that perhaps other supermultiplets of hadrons also share the same property. If these speculations are found true, then hadrons must be pictured[13] as a family of droplets with the same shape but different opaqueness for each member. How to give an example of such a droplet model for hadrons would be a challenging problem. But at the present stage, direct measurement of meson form factors would be most important and useful, for it would provide a crucial test for our proposal.

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[1] T. T. Chou and C. N. Yang, Phys. Rev. 170, 1591 (1968).

[2] T. T. Chou and C. N. Yang, Phys. Rev. 188, 2469 (1969).

[3] A. Böhm *et al.*, Phys. Lett. 49B, 491 (1974).

[4] M. Kac, Nucl. Phys. B62, 402 (1973).

[5] T. T. Chou and C. N. Yang, Phys. Rev. Lett. 20, 1213 (1968).

[6] A. W. Chao and C. N. Yang, Phys. Rev. D 8, 2063 (1973).

[7] C. W. Akerlof *et al.*, paper (492) submitted to XVII International Conference on High Energy Physics, Imperial College, London, 1974 (unpublished).

[8] A. Sofair *et al.*, Nucl. Phys. B42, 369 (1972); C. N. Brown *et al.*, Phys. Rev. D 8, 92 (1973).

[9] G. T. Adylov *et al.*, Phys. Lett. 51B, 402 (1974).

[10] S. Dubnicka and O. V. Dumbrajs, Phys. Lett. 53B, 285 (1974).

[11] T. T. Chou (unpublished).

[12] U. Amaldi *et al.*, Phys. Lett. 44B, 112 (1973); S. R. Amendolia *et al.*, *ibid.* 44B, 119 (1973).

[13] C. N. Yang, in *Fundamental Problems of the Elementary Particle Theory* (Institute for Theoretical Physics, Academy of Sciences, Kiev, 1970).