Relativistic second-order energy in infinite fermion matter*

M. Bolsterli
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544
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The second-order terms in the energy per unit volume or energy per particle in infinite fermion matter are calculated for scalar, vector, and pseudoscalar meson exchanges.

I. INTRODUCTION

Although it has long been accepted that the interaction between nucleons is mediated by meson fields, except for the early work of Johnson and Teller[1] only relatively recently have attempts been made to develop a meson field theoretic treatment of nuclei. A semirelativistic self-consistent formalism was given in Ref. 2, and a fully relativistic treatment has been attacked in Refs. 3 and 4. Further interest in such theories comes from possible implications of spontaneous symmetry breaking for nuclear matter energies.[5]

This paper is, like Ref. 3, devoted to the problem of a relativistic formalism for infinite nuclear matter. In infinite matter, if meson-nucleon couplings are assumed to be of Yukawa type, the energy per unit volume F is a function only of the density ρ and the squares of various coupling constants,

$$F=F(\rho,g_i^2)\,. \tag{1.1}$$

The assumption of perturbation theory is that it makes sense to expand F about $g_i^2=0$

$$F=F^{(0)}(\rho)+\sum_i g_i^2 F_i^{(2)}(\rho)+\cdots\,. \tag{1.2}$$

In this paper $F^{(0)}$ and the $F_i^{(2)}$ will be calculated for scalar, vector, and pseudoscalar mediating meson fields. The only interest in the procedure comes from the fact that the underlying vacuum field theories have the usual ultraviolet divergences; the manipulations necessary to obtain the correct finite values of $F^{(0)}$ and $F_i^{(2)}$ are not completely trivial.

The usefulness of just the first two terms in the perturbation expansion (1.2) might seem dubious for cases where the coupling constants are as large as they are in nuclei. However, it should be noted that the Hartree-Fock approximation, which is often used to treat nuclear phenomena, is just such a second-order perturbation approximation. The second-order approximation to (1.2) can be expected to be as useful as the Hartree-Fock approximation. Eventually, of course, it will be necessary to compute fourth-order terms in order to get some idea of the convergence of the perturbation expansion.

In Ref. 3 a somewhat different approach was taken, in that some of the perturbation series terms were summed to infinite order by incorporating them in the nucleon effective mass (this is also done in Refs. 4 and 5), but the full second-order terms were not calculated. Here the full second-order terms are given; Sec. VI contains remarks on selective summation of graphs.

This paper gives the formal development only for the scalar meson case. The vector and pseudoscalar cases are similar; only the results are presented for them.

For the field theoretic ideas, the reader is referred to the books by Bogolyubov and Shirkov[6] and Schweber.[7] The metric used here is $(1,-1,-1,-1)$ as in Ref. 7. Of course the units are chosen so that $\hbar=c=1$.

II. DESCRIPTION OF THE PROBLEM

The Hamiltonian that will be considered here describes the interaction of a fermion field $\psi(x)$ with a scalar meson field $\phi(x)$ via a Yukawa coupling:

$$H=\int\{\psi^\dagger(\vec{\alpha}\cdot\vec{p}+\beta m)\psi+\tfrac{1}{2}[\dot{\phi}^2+(\nabla\phi)^2+\mu_S^2\phi^2] -g_S\,\phi(\bar{\psi}\psi-\langle\bar{\psi}\psi\rangle_{00})\}\,d\vec{x}+H_C\,, \tag{2.1}$$

where H_C contains the counterterms that are to be chosen in the usual way so as to renormalize the theory at zero fermion density. The notation $\langle\bar{\psi}\psi\rangle_{\rho 0}$ is used for the expectation value of $\bar{\psi}\psi$ in zeroth order for density ρ; the term $\langle\bar{\psi}\psi\rangle_{00}$ is included because the combination $\bar{\psi}\psi-\langle\bar{\psi}\psi\rangle_{00}$ is normal ordered for $\rho=0$.

In order to consider infinite matter at density ρ, it is convenient to set

$$\phi(x)=\phi_\rho(x)+\chi(x)\,, \tag{2.2}$$

where $\phi_\rho(x)$ is a static c-number field (also uniform in infinite uniform matter); then H can be rewritten

$$H=H_{0,\rho}+H_{I,\rho}+H_C\,,$$

$$H_{0,\rho}=\int\psi^\dagger(\vec{\alpha}\cdot\vec{p}+\beta m)\psi + \tfrac{1}{2}\int[\dot{\chi}^2+(\nabla\chi)^2+\mu_S{}^2\chi^2]\,,$$

$$H_{I,\rho}=H_{I0,\rho}+H_{I1,\rho}+H_{I2,\rho}+H_{I3,\rho}\,,$$

$$H_{I0,\rho}=\int\phi_\rho[\tfrac{1}{2}(-\nabla^2+\mu^2)\phi_\rho - g_S(\langle\bar\psi\psi\rangle_{\rho 0}-\langle\bar\psi\psi\rangle_{00})]\,, \tag{2.3}$$

$$H_{I1,\rho}=-g_S\int\chi(\bar\psi\psi-\langle\bar\psi\psi\rangle_{\rho 0})\,,$$

$$H_{I2,\rho}=-g_S\int\phi_\rho(\bar\psi\psi-\langle\bar\psi\psi\rangle_{\rho 0})\,,$$

$$H_{I3,\rho}=\int\chi f\,,$$

$$f=(-\nabla^2+\mu^2)\phi_\rho-g_S(\langle\bar\psi\psi\rangle_{\rho 0}-\langle\bar\psi\psi\rangle_{00})\,.$$

For $\rho=0$, all the terms involving ϕ_0 are just exactly of the same form as some of the counterterms in H_C; it follows that ϕ_0 can be chosen to be zero, and ϕ_ρ is equivalent to $\phi_\rho-\phi_0$.

The eigenvectors of $H_{0,\rho}$ are easily constructed from the plane-wave fermion eigenstates of the operator $\vec{\alpha}\cdot\vec{p}+\beta m$ and from the plane-wave boson states. The state $|\rho,0\rangle$ is the ground state of $H_{0,\rho}$ with fermion density ρ; it corresponds to all negative-energy states filled and positive-energy states filled up to momentum $p_F(\rho)$ with

$$\rho=\langle\psi^\dagger\psi\rangle_{\rho 0}-\langle\psi^\dagger\psi\rangle_{00} = \frac{2\gamma}{(2\pi)^3}\int d_F\vec{p} = \frac{\gamma}{3\pi^2}p_F{}^3\,. \tag{2.4}$$

Here γ is the number of kinds of fermions present: $\gamma=1$ for neutron matter and $\gamma=2$ for nuclear matter. The notation $d_F\vec{p}$ will be used for integrals in which $\vec{p}$ is integrated over the finite Fermi sea,

$$d_F\vec{p}\equiv d\vec{p}\,\theta(p_F-|\vec{p}|)\,; \tag{2.5}$$

if p_0 appears in such an integrand, it is to be understood that p_0 takes the value

$$p_0=\epsilon(\vec{p})=(\vec{p}^2+m^2)^{1/2}\,. \tag{2.6}$$

The expectation value of $H_{0,\rho}$ has divergent terms, but the difference $\langle H_{0,\rho}\rangle_{\rho 0}-\langle H_{0,0}\rangle_{00}$ is finite[8]:

$$E_\rho^{(0)}\equiv\langle H_{0,\rho}\rangle_{\rho,0}-\langle H_{0,0}\rangle_{0,0} = \frac{2\gamma}{(2\pi)^3}\,\Omega\int\epsilon(\vec{p})\,d_F\vec{p}\,, \tag{2.7}$$

where $E_\rho^{(0)}$ is the zeroth-order energy and Ω is the volume of the system.

The boson Green's function at density ρ is the same as at density zero

$$D_\rho(p)=D_0(p) = D(p) = \frac{i}{(2\pi)^4}\,\frac{1}{p^2-\mu^2+i0}\,, \tag{2.8}$$

while the fermion Green's function, which at zero density is

$$G_0(p)=\frac{i}{(2\pi)^4}\,\frac{1}{\not{p}-m+i0} = \frac{i}{(2\pi)^4}\,\frac{\not{p}+m}{[p_0+\epsilon(\vec{p})-i0][p_0-\epsilon(\vec{p})+i0]}\,, \tag{2.9}$$

becomes at density ρ

$$G_\rho(p)=\frac{i}{(2\pi)^4}\,\frac{\not{p}+m}{[p_0+\epsilon(\vec{p})-i0][p_0-\epsilon(\vec{p})+i\eta(\vec{p})]} = G_0(p)+G_{\rho 0}(p)\,,$$

$$G_{\rho 0}(p)=-\frac{\not{p}+m}{16\pi^3\epsilon(\vec{p})}\,\delta_F(p_0-\epsilon(\vec{p}))\,, \tag{2.10}$$

$$\eta(p)=\begin{cases}0-, & |\vec{p}|\le p_F\\ 0+, & |\vec{p}|>p_F\end{cases}$$

$$\delta_F(p_0-\epsilon(\vec{p}))\equiv\delta(p_0-\epsilon(\vec{p}))\,\theta(p_F-|\vec{p}|)\,.$$

The basic idea now is that the perturbation theory based on $H_{0,\rho}$ and the state $|\rho,0\rangle$ is very like the perturbation theory for the vacuum based on $H_{0,\rho=0}$ and the state $|\rho=0,0\rangle$. The differences between the two involve the following: (1) Some terms in $H_{I,\rho}$ differ from those in $H_{I,0}$; however, these differences are such as to lead to finite results for physical quantities. (2) The fermion Green's function at finite density differs from the vacuum Green's function. This difference is nonzero only inside the Fermi sphere and therefore cannot give ultraviolet divergent integrals other than those that already appear in the vacuum perturbation theory.

III. SECOND-ORDER ENERGY

The calculation of the second-order energy of the system at density ρ illustrates how the divergences cancel. From H, it is clear that this calculation requires that ϕ_ρ be known to first order. In first order, ϕ_ρ is determined by the requirement that the (constant) function f in $H_{I3,\rho}$ be zero so that

$$\phi_\rho^{(1)}=g\rho_S^{(0)}/\mu_S{}^2 \tag{3.1}$$

for uniform matter. The value of $\rho_S^{(0)}$ is given by

$$\rho_S^{(0)} = \langle \bar\psi\psi\rangle_{\rho 0} - \langle \bar\psi\psi\rangle_{00} = \frac{2\gamma}{(2\pi)^3} m \int \frac{d_F\vec{p}}{\epsilon(\vec{p})}. \tag{3.2}$$

Thus, the second-order term in $H_{I0,\rho}$ is

$$-\tfrac{1}{2} g \int \phi_\rho^{(1)} \rho_S^{(0)}. \tag{3.3}$$

To second order, the fermion mass counterterm in H^C, which is

$$-\delta m_C \int (\bar\psi\psi - \langle\bar\psi\psi\rangle_{00}) = -\delta m_C \int (\bar\psi\psi - \langle\bar\psi\psi\rangle_{\rho 0}) - \delta m_C \int (\langle\bar\psi\psi\rangle_{\rho 0} - \langle\bar\psi\psi\rangle_{00}), \tag{3.4}$$

contributes an energy

$$\Delta E_{\delta m}^{(2)} = -\delta m_C^{(2)} \int \rho_S^{(0)} d\vec{x} = -\delta m_C^{(2)} \rho_S^{(0)} \Omega, \tag{3.5}$$

where $\delta m_C^{(2)}$, the second-order mass counterterm, is obtained by computing the vacuum self-energy part of Fig. 1 whose S-matrix contribution is

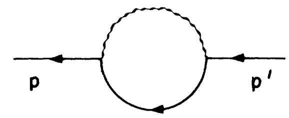

FIG. 1. The second-order self-energy graph.

$$-(2\pi)^4 i \Sigma_0^{(2)}(\not p)\delta^{(4)}(p-p') = [-(2\pi)^4 ig]^2 \delta^{(4)}(p-p') \times \int d^4q\, D(p-q) G_0(q). \tag{3.6}$$

When $\Sigma^{(2)}(p)$ is expanded about $\not p = m$,

$$\Sigma_0^{(2)}(\not p) = \Sigma_0^{(2)}(m) + (\not p - m)\Sigma_{0,1}^{(2)}(\not p), \tag{3.7}$$

the first term must be canceled by the graph arising from the first term on the right-hand side of (3.4); this gives

$$\delta m_C^{(2)} = \Sigma_0^{(2)}(m). \tag{3.8}$$

The term $\Sigma^{(2)}(m)$ diverges in the limit as regulator masses become infinite, but it will be seen that $\delta m_C^{(2)}$ does not contribute to the final expression for $\Delta E^{(2)}$.

Finally there are the second-order energy represented by the graph of Fig. 2 and the corresponding constant counterterm in H_C, which has just the value of the graph of Fig. 2 calculated with the vacuum Green's function. Thus, combining these gives

$$-2\pi i \delta^{(1)}(0)[\Delta E_{G,\rho}^{(2)} - \Delta E_{G,\rho=0}^{(2)}] = -\tfrac{1}{2}[-(2\pi)^4 ig]^2 \delta^{(4)}(0)\, \mathrm{Tr} \int [G_\rho(p)G_\rho(q) - G_0(p)G_0(q)] D(p-q)\, d^4p\, d^4q, \tag{3.9}$$

$$\Delta E_{G,\rho}^{(2)} - \Delta E_{G,\rho=0}^{(2)} = (2\pi)^4 ig^2 \Omega\, \mathrm{Tr} \int [G_{\rho 0}(p)G_0(q) + \tfrac{1}{2} G_{\rho 0}(p) G_{\rho 0}(q)] D(p-q)\, d^4p\, d^4q. \tag{3.10}$$

From (3.6) and (3.7) it follows that

$$-(2\pi)^4 ig^2 \int G_0(q) D(p-q) = \Sigma_0^{(2)}(p) = \delta m_C^{(2)} + (\not p - m)\Sigma_{0,1}^{(2)}(p), \tag{3.11}$$

so that the first term in (3.10) is

$$\Omega\, \mathrm{Tr} \int \frac{\not p + m}{(2\pi)^3 2\epsilon(\vec{p})} \delta_F(p_0 - \epsilon(\vec{p})) \times [\delta m_C^{(2)} + (\not p - m)\Sigma_1^{(2)}(p)] = \Omega \delta m_C^{(2)} \frac{2\gamma m}{(2\pi)^3} \int \frac{d_F\vec{p}}{\epsilon(\vec{p})} = \rho_S^{(0)} \delta m_C^{(2)} \Omega, \tag{3.12}$$

and this cancels the $\Delta E_{\delta m}^{(2)}$ term of (3.5).

The remaining term is

$$-\frac{g^2\Omega}{8(2\pi)^6} \int \frac{d_F\vec{p}\, d_F\vec{q}}{\epsilon(\vec{p})\epsilon(\vec{q})} \frac{\mathrm{Tr}[(\not p + m)(\not q + m)]}{(p-q)^2 - \mu_S^2} = \frac{\gamma}{2} \frac{g^2}{(2\pi)^6} \Omega \int \frac{d_F\vec{p}\, d_F\vec{p}}{\epsilon(\vec{p})\epsilon(\vec{q})} \frac{(pq) + m^2}{\mu_S^2 - (p-q)^2}, \tag{3.13}$$

where, as noted after Eq. (2.5), it is to be understood that the subscript F on $d_F\vec{p}$ implies that p_0 takes the value $\epsilon(\vec{p})$, and similarly for q_0.

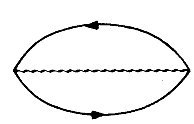

FIG. 2. The second-order energy graph.

Combining the second-order terms gives

$$\frac{E_\rho^{(2)}}{\Omega}=F_{\rho,D}^{(2)}+F_{\rho,\mathrm{Ex}}^{(2)}, \tag{3.14}$$

with

$$F_{\rho,D,S}^{(2)}=-\tfrac{1}{2}g_S\,\phi_\rho^{(1)}\rho_S^{(0)},$$

$$F_{\rho,\mathrm{Ex},S}^{(2)}=\frac{\gamma}{2}\frac{g_S{}^2}{(2\pi)^6}\int\frac{d_F\vec{p}\,d_F\vec{p}}{\epsilon(\vec{p})\,\epsilon(\vec{q})}\frac{(pq)+m^2}{\mu_S{}^2-(p-q)^2}, \tag{3.15}$$

with $\rho_S^{(0)}$ and $\phi_\rho^{(1)}$ given by (3.1) and (3.2); the subscript S is used to denote that the interaction is via scalar meson field. Similar calculations for vector and pseudoscalar fields give (3.14), with

$$F_{\rho,D,V}^{(2)}=\tfrac{1}{2}g_V\,V_\rho^{(1)}\rho_V^{(0)},$$

$$V_\rho^{(1)}=g_V\,\rho_V^{(0)}/\mu_V{}^2,$$

$$\rho_V^{(0)}=\rho,$$

$$F_{\rho,D,P}^{(2)}=0,$$

$$F_{\rho,\mathrm{Ex},V}^{(2)}=-\frac{\gamma}{2}\frac{g_V{}^2}{(2\pi)^6}\int\frac{d_F\vec{p}\,d_F\vec{q}}{\epsilon(\vec{p})\epsilon(\vec{q})}\frac{4m^2-2(pq)}{\mu_V{}^2-(p-q)^2},$$

$$F_{\rho,\mathrm{Ex},P}^{(2)}=\frac{\gamma}{2}\frac{g_P{}^2}{(2\pi)^6}\int\frac{d_F\vec{p}\,d_F\vec{q}}{\epsilon(\vec{p})\epsilon(\vec{q})}\frac{(pq)-m^2}{\mu_P{}^2-(p-q)^2}.$$

In these expressions F_D is the direct interaction energy while F_{Ex} is the exchange energy. In the nonrelativistic or low-density limit $p_F \ll \mu$, these terms are equal to those calculated previously in the nonrelativistic theory of interactions mediated by fields[2]; in this limit the ratio of pseudoscalar interaction to scalar or vector interaction goes to zero. Note also that the ratio of exchange energy to direct energy has the value $-(2\gamma)^{-1}$ in this limit.

In order to explore the relative size of the direct and exchange energies, numerical computations were done with $\mu=800$ MeV, $m=938.9$ MeV for various values of p_F. For scalar or vector alone, the ratio F_{Ex}/F_D starts at $-(2\gamma)^{-1}$ for $p_F\to 0$ and decreases to $-(3\gamma)^{-1}$ or $-(4\gamma)^{-1}$, respectively, at $p_F=3\ \mathrm{fm}^{-1}$ (corresponding to about ten times nuclear density). For combined scalar and vector fields with equal mass $\mu=800$ MeV and coupling-constant ratio $g_S{}^2/g_V{}^2=1.5$, the ratio F_{Ex}/F_D starts at $-(2\gamma)^{-1}$ and *increases* to about $-\gamma^{-1}$ at $p_F=3\ \mathrm{fm}^{-1}$. Clearly, F_{Ex} can be as important as F_D.

IV. SINGLE-PARTICLE SPECTRUM TO SECOND ORDER

In second order, the proper self-energy part $\Sigma_\rho^{(2)}(p)$ has a contribution from $H_{I2,\rho}$ of $-g\phi_\rho$ and a contribution from the graph in Fig. 1 of

$$-(2\pi)^4 ig^2\int D(p-q)\,G_\rho(q)\,d^4q=\Sigma_0^{(2)}(\not p)-(2\pi)^4 ig^2\int D(p-q)\,G_{\rho 0}(q)\,d^4q\,; \tag{4.1}$$

the counterterms in H_C that are quadratic in ψ and $\bar\psi$ renormalize $\Sigma_0^{(2)}(\not p)$ to $\Sigma_{0,\mathrm{ren}}^{(2)}(\not p)$, where

$$\Sigma_0^{(2)}(\not p)=\Sigma_0^{(2)}(m)+(\not p-m)\,\Sigma_0^{(2)\prime}(m)+\Sigma_{0,\mathrm{ren}}^{(2)}(\not p), \tag{4.2}$$

so that the total second-order self-energy part is

$$\Sigma_\rho^{(2)}(p)=\Sigma_{\rho,1}^{(2)}(p)+\Sigma_{0,\mathrm{ren}}^{(2)}(\not p), \tag{4.3a}$$

$$\Sigma_{\rho,1}^{(2)}(p)=-g\phi_\rho-(2\pi)^4 ig^2\int D(p-q)\,G_{\rho 0}(q)\,d^4q. \tag{4.3b}$$

Substitution gives

$$\Sigma_{\rho,1,S}^{(2)}(p)=-g_S\,\phi_\rho+\frac{g_S{}^2}{2(2\pi)^3}\int\frac{\not q+m}{\mu_S{}^2-(p-q)^2}\frac{d_F\vec{q}}{\epsilon(\vec{q})} \tag{4.4}$$

for the scalar case. For the case of a conserved vector field

$$\Sigma_{\rho,1,V}^{(2)}(p)=g_V\,V_\rho\beta-\frac{g_V{}^3}{2(2\pi)^3}\int\frac{(4m-2q)}{\mu_V{}^2-(p-q)^2}\frac{d_F\vec{q}}{\epsilon(\vec{q})}. \tag{4.5}$$

Finally, for a pseudoscalar field

$$\Sigma_{\rho,1,P}^{(2)}=\frac{g_P{}^2}{2(2\pi)^3}\int\frac{\not q-m}{\mu_P{}^2-(p-q)^2}\frac{d_F\vec{q}}{\epsilon(\vec{q})}. \tag{4.6}$$

The single-particle spectrum is given by the values $p_0(\vec{p})$ for which the operator

$$\not p-m-\Sigma_\rho^{(2)}(p) \tag{4.7}$$

has zero for an eigenvalue. Since $\Sigma_{0,\mathrm{ren}}^{(2)}(\not p)$ is proportional to $(\not p-m)^2$ and (4.7) shows that $p-m$ is of order g^2, it follows that the contribution of $\Sigma_{0,\mathrm{ren}}^{(2)}$ is of order g^4; it is neglected in the following. Moreover, the value of p_0 that makes the operator of (4.7) vanish differs from $\epsilon(\vec{p})$ by an amount of order g^2. Therefore, in $\Sigma_\rho^{(2)}(p)$, p_0 can be replaced by $\epsilon(\vec{p})$. Now $\Sigma_\rho^{(2)}(p_0=\epsilon(\vec{p}),\vec{p})$ can be written in the form $A(\vec{p}^2)+B(\vec{p}^2)\gamma_0$

$+C(\vec{p}^2)\vec{\gamma}\cdot\vec{p}$ and the operator (4.7) becomes

$$\gamma_0[p_0-B(\vec{p}^2)]-[m+A(\vec{p}^2)]-[1+C(\vec{p}^2)]\vec{\gamma}\cdot\vec{p}\,, \tag{4.8}$$

with solutions for p_0 given by

$$p_0(\vec{p}^2)=B(\vec{p}^2)\pm\{[m+A(\vec{p}^2)]^2+[1+C(\vec{p}^2)]^2p^2\}^{1/2}\,, \tag{4.9}$$

where, if all meson masses are equal,

$$A(\vec{p}^2)=-\frac{g_S^2\rho_S^{(0)}}{\mu_S^2}+\frac{g_S^2-4g_V^2-g_P^2}{16\pi^3}\int\frac{m}{\epsilon(\vec{q})}\,\frac{d_F\vec{q}}{\mu^2-(p-q)^2}\,,$$

$$B(\vec{p}^2)=\frac{g_V^2V_\rho^{(0)}}{\mu_V^2}+\frac{g_S^2+2g_V^2+g_P^2}{16\pi^3}\int\frac{d_F\vec{q}}{\mu^2-(p-q)^2}\,, \tag{4.10}$$

$$C(\vec{p}^2)=-\frac{g_S^2+2g_V^2+g_P^2}{16\pi^3\vec{p}^2}\int\frac{\vec{p}\cdot\vec{q}}{\epsilon(\vec{q})}\,\frac{d_F\vec{q}}{\mu^2-(p-q)^2}\,;$$

here p_0 is to be replaced by $\epsilon(\vec{p})$ in the integrands. If the masses are unequal the appropriate generalization of (4.10) is obvious.

In A and B, the first term is the direct interaction term; the remaining term in each of A, B, and C is the exchange term. Again the exchange terms are of the same order of magnitude as the direct terms. For example, in the nonrelativistic limit $p_F\ll\mu$, $|\vec{p}|\ll\mu$, A takes the value

$$A_{\rm NR}=\rho\left[-\frac{g_S^2}{\mu_S^2}+\frac{1}{4\gamma}\left(\frac{g_S^2}{\mu_S^2}-4\frac{g_V^2}{\mu_V^2}-\frac{g_P^2}{\mu_P^2}\right)\right]\,. \tag{4.11}$$

In this limit C is smaller than A/m by a factor p_F^2/μ^2; if C is neglected, the effective mass from (4.9) is

$$m^*=m+A$$

and it is clear that the exchange terms in (4.11) are not small.

V. FURTHER REMARKS

In general, the function f in (2.3) must be chosen so that the proper graphs of the type of Fig. 3(a) cancel to the order of perturbation theory desired. In this way ϕ_ρ is determined as a sum of terms in odd powers of g. In Fig. 3(a), C stands for the coefficient of the counterterm in H_C that is linear in χ. It is determined by an equation like Fig. 3(a) with $f=0$ and all graphs evaluated using vacuum parameters. It follows that f is determined by the equation shown graphically in Fig. 3(b), where only differences (graph-vacuum graph) appear.

VI. GRAPH SUMMATION

Once the theory is developed in terms of Feynman graphs with associated factors, it is always possible to choose various (possibly infinite) subsets of the set of all graphs for selective summation. For example, the results of Ref. 3 are obtained by summing all graphs that contain insertions of interactions due to $H_{I2,\rho}$ and no others. However, as noted at the end of Sec. III, the exchange energy is at least $(2\gamma)^{-1}$ of the direct interaction energy; it certainly is not reasonable to neglect it in such a summation.

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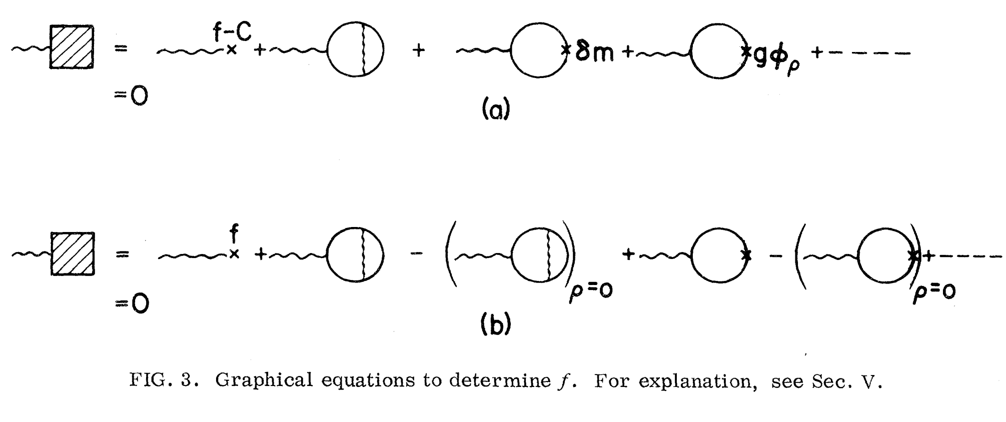

FIG. 3. Graphical equations to determine f. For explanation, see Sec. V.

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[8]The Hamiltonian of Ref. 3 does not have this property. If the fermion mass in H_0 is replaced by an effective mass $m^*(\rho)$ that depends on ρ, then the contribution to $\langle H_{0,\rho}\rangle_{00} - \langle H_{0,0}\rangle_{00}$ from the filled negative-energy states is

$$-\frac{2\gamma\Omega}{(2\pi)^3}\int[(\vec{p}^2+m^{*2})^{1/2}-(\vec{p}^2+m^2)^{1/2}]d\vec{p},$$

where the integral runs over all values of $\vec{p}$. It can be shown that the linearly divergent term in the above expression is canceled by a divergent second-order correction, but I was not able to find terms to cancel the logarithmic divergence. In any case, it seems that a theory in which the zeroth-order energy per particle is infinite is more difficult to deal with than the theory presented here.