Current-field identities of vector mesons

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A gauge-invariant Lagrangian, leading to a hadronic electromagnetic current which to order e is a linear combination of the vector mesons ρ^0, ϕ, and ω, is constructed when the vector mesons are described by an antisymmetric second-rank tensor field.

In several previous papers the vector mesons were described by means of an antisymmetric second-rank tensor field $T_{\mu\nu}(x)$, which transforms under the representation $(1, 0)\oplus(0, 1)$ of the Lorentz group,[1-4] while the usual vector field $V_\mu(x)$ transforms under the representation $(\frac{1}{2}, \frac{1}{2})$. Various aspects of this formalism were examined and certain advantages of the new description were pointed out. In this paper we want to investigate the possibility of constructing a Lagrangian field theory in which the hadronic electromagnetic current operator is identical with a linear combination of the ρ^0, ϕ, and ω vector-meson fields[5,6] when they are described by tensor fields. Such a construction is a field-theoretic basis of vector-meson dominance.[7,8] The current-field identity summarizes basically the ingredients of vector-meson universality and vector-meson dominance.

We assume that besides the isotriplet vector meson described by the field $\vec{\mathbf{T}}^{\rho}_{\mu\nu}(x)$ there are two isospin-zero vector mesons ϕ and ω described by the fields $T^{\phi}_{\mu\nu}(x)$ and $T^{\omega}_{\mu\nu}(x)$, respectively. Consider the interaction of these mesons with some nonstrange and some strange baryons. Let $\vec{\mathbf{I}}'_\mu$ be the isospin current of these baryons, I^Y_μ be the hypercharge current, and I^B_μ be the baryon current, which can be easily calculated from the free Lagrangians. Also let g_Y, g_B, θ_Y, and θ_B be four real constants, and let us define the currents[9] $\underline{I}^{\phi}_\mu$ and $\underline{I}^{\omega}_\mu$ by the relations[5]

$$\underline{I}^{\phi}_\mu = \frac{1}{\cos(\theta_Y-\theta_B)}\left[(\cos\theta_B)g_Y\underline{I}^Y_\mu + (\sin\theta_Y)g_B\underline{I}^B_\mu\right], \tag{1}$$

$$\underline{I}^{\omega}_\mu = \frac{1}{\cos(\theta_Y-\theta_B)}\left[-(\sin\theta_B)g_Y\underline{I}^Y_\mu + (\cos\theta_Y)g_B\underline{I}^B_\mu\right]. \tag{2}$$

The currents $\underline{I}^{\phi}_\mu$ and $\underline{I}^{\omega}_\mu$ are obviously conserved. We assume that the strong interactions of the mesons ϕ and ω are introduced by the addition of the term

$$\frac{1}{m_\phi}\,\partial_\nu\underline{T}^{\phi}_{\nu\mu}\underline{I}^{\phi}_\mu + \frac{1}{m_\omega}\,\partial_\nu\underline{T}^{\omega}_{\nu\mu}\underline{I}^{\omega}_\mu + \frac{1}{2m_\phi^2}\underline{I}^{\phi}_\mu\underline{I}^{\phi}_\mu + \frac{1}{2m_\omega^2}\underline{I}^{\omega}_\mu\underline{I}^{\omega}_\mu$$

to the free Lagrangians of these mesons, where m_ϕ and m_ω are their masses. The interaction of the ρ mesons is determined by requiring that to certain order in the coupling constant g_I the isospin current is the source of the ρ-meson field. It was shown previously[10] that, when the vector mesons are described by a tensor field, this requirement can be satisfied exactly only by a Lagrangian which is an infinite series in g_I. A Lagrangian was given which satisfies this requirement to order g_I^2, which we shall use in the following. In the absence of electromagnetic interactions our Lagrangian is taken to be the following[11]:

$$\begin{aligned}\underline{\mathfrak{L}}_{\rm st} = {} & \underline{\mathfrak{L}}'_{\rm free} + \frac{1}{2}\left(\partial_\nu\underline{\vec{\mathbf{T}}}^{\rho}_{\nu\mu} + \frac{g_I}{m_\rho}\underline{\vec{\mathbf{I}}}'_\mu\right)\cdot\left(\partial_\lambda\underline{\vec{\mathbf{T}}}^{\rho}_{\lambda\mu} + \frac{g_I}{m_\rho}\underline{\vec{\mathbf{I}}}'_\mu\right) + \frac{m_\rho^2}{4}\underline{\vec{\mathbf{T}}}^{\rho}_{\mu\nu}\cdot\underline{\vec{\mathbf{T}}}^{\rho}_{\mu\nu} + \frac{g_I}{2m_\rho}\left(\partial_\sigma\underline{\vec{\mathbf{T}}}_{\sigma\mu} + \frac{g_I}{m_\rho}\underline{\vec{\mathbf{I}}}'_\mu\right)\cdot\left(\partial_\lambda\underline{\vec{\mathbf{T}}}_{\lambda\nu} + \frac{g_I}{m_\rho}\underline{\vec{\mathbf{I}}}'_\nu\right)\times\underline{\vec{\mathbf{T}}}_{\nu\mu} \\ & + \frac{g_I^2}{2m_\rho^2}(\partial_\tau\underline{\vec{\mathbf{T}}}_{\tau\mu}\times\underline{\vec{\mathbf{T}}}_{\mu\nu})\cdot(\partial_\lambda\underline{\vec{\mathbf{T}}}_{\lambda\sigma}\times\underline{\vec{\mathbf{T}}}_{\sigma\nu}) + \frac{1}{2}\partial_\nu\underline{T}^{\phi}_{\nu\mu}\partial_\lambda\underline{T}^{\phi}_{\lambda\mu} + \frac{m_\phi^2}{4}\underline{T}^{\phi}_{\mu\nu}\underline{T}^{\phi}_{\mu\nu} + \frac{1}{2}\partial_\nu\underline{T}^{\omega}_{\nu\mu}\partial_\lambda\underline{T}^{\omega}_{\lambda\mu} + \frac{m_\omega^2}{4}\underline{T}^{\omega}_{\mu\nu}\underline{T}^{\omega}_{\mu\nu} \\ & + \frac{1}{m_\phi}\partial_\nu\underline{T}^{\phi}_{\nu\mu}\underline{I}^{\phi}_\mu + \frac{1}{m_\omega}\partial_\nu\underline{T}^{\omega}_{\nu\mu}\underline{I}^{\omega}_\mu + \frac{1}{2m_\phi^2}\underline{I}^{\phi}_\mu\underline{I}^{\phi}_\mu + \frac{1}{2m_\omega^2}\underline{I}^{\omega}_\mu\underline{I}^{\omega}_\mu\,,\end{aligned} \tag{3}$$

where $\underline{\mathfrak{L}}'_{\rm free}$ is the free Lagrangian of the spin-$\frac{1}{2}$ particles.

To get a gauge-invariant result we introduce the electromagnetic interactions by the minimal substitution in the above Lagrangian. Also, we add the terms $\underline{A}_\mu\partial_\nu\underline{T}^{i}_{\nu\mu}$, $i=\rho^0, \phi, \omega$, where $\underline{A}_\mu$ is the electromagnetic field. Under the gauge transformation $\underline{A}_\mu\to\underline{A}_\mu+\partial_\mu\Lambda$ such terms change by a total divergence, because of the antisymmetry of $\underline{T}^i_{\nu\mu}$. Therefore, the contribution of such terms to the action is gauge invariant, which allows the addition of these terms to the Lagrangian. The electromagnetic Lagrangian $\underline{\mathfrak{L}}_\gamma$ is taken as follows:

$$\underline{\mathfrak{L}}_\gamma = -\tfrac{1}{4}\underline{F}_{\mu\nu}\underline{F}_{\mu\nu} + e\underline{A}_\mu(\underline{I}^{\rho^0}_\mu + \tfrac{1}{2}\underline{I}^Y_\mu) + e\underline{A}_\mu\left(\frac{m_\rho}{g_I}\partial_\nu\underline{T}^{\rho^0}_{\nu\mu} + \frac{m_\phi\cos\theta_Y}{2g_Y}\partial_\nu\underline{T}^\phi_{\nu\mu} - \frac{m_\omega\sin\theta_Y}{2g_Y}\partial_\nu\underline{T}^\omega_{\nu\mu}\right) + O(e^2), \tag{4}$$

where $\underline{F}_{\mu\nu} = \partial_\mu\underline{A}_\nu - \partial_\nu\underline{A}_\mu$ and the current $\vec{\underline{I}}^\rho_\mu$, which is obtained from the Lagrangian $\underline{\mathfrak{L}}_{st}$, is given by

$$\vec{\underline{I}}^\rho_\mu = \partial_\lambda\vec{\underline{T}}_{\lambda\nu}\times\vec{\underline{T}}_{\nu\mu} + \left[\frac{g_I}{m_\rho}\left(\partial_\lambda\vec{\underline{T}}_{\lambda\nu} + \frac{g_I}{m_\rho}\vec{\underline{I}}'_\nu\right)\times\vec{\underline{T}}_{\nu\sigma} + \frac{g_I^2}{m_\rho^2}(\partial_\lambda\vec{\underline{T}}_{\lambda\tau}\times\vec{\underline{T}}_{\tau\nu})\times\vec{\underline{T}}_{\nu\sigma} + \frac{g_I}{m_\rho}\vec{\underline{I}}'_\sigma\right]\times\vec{\underline{T}}_{\sigma\mu} + \vec{\underline{I}}'_\mu . \tag{5}$$

From the Lagrangians $\underline{\mathfrak{L}}_{st}$ and $\underline{\mathfrak{L}}_\gamma$ we get the equations of motion

$$\partial_\mu\underline{F}_{\mu\nu} = -e(\underline{I}^{\rho^0}_\nu + \tfrac{1}{2}\underline{I}^Y_\nu) - e\left(\frac{m_\rho}{g_I}\partial_\mu\underline{T}^{\rho^0}_{\mu\nu} + \frac{m_\phi\cos\theta_Y}{2g_Y}\partial_\mu\underline{T}^\phi_{\mu\nu} - \frac{m_\omega\sin\theta_Y}{2g_Y}\partial_\mu\underline{T}^\omega_{\mu\nu}\right) + O(e^2), \tag{6}$$

$$\partial_\mu\partial_\lambda\underline{T}^{\rho^0}_{\lambda\nu} - \partial_\nu\partial_\lambda\underline{T}^{\rho^0}_{\lambda\mu} - m_\rho^2\underline{T}^{\rho^0}_{\mu\nu} = \underline{J}^{\rho^0}_{\mu\nu} - \partial_\mu\underline{J}^{\rho^0}_\nu + \partial_\nu\underline{J}^{\rho^0}_\mu, \tag{7}$$

$$\partial_\mu\partial_\lambda\underline{T}^\phi_{\lambda\nu} - \partial_\nu\partial_\lambda\underline{T}^\phi_{\lambda\mu} - m_\phi^2\underline{T}^\phi_{\mu\nu} = -\partial_\mu\underline{J}^\phi_\nu + \partial_\nu\underline{J}^\phi_\mu, \tag{8}$$

$$\partial_\mu\partial_\lambda\underline{T}^\omega_{\lambda\nu} - \partial_\nu\partial_\lambda\underline{T}^\omega_{\lambda\mu} - m_\omega^2\underline{T}^\omega_{\mu\nu} = -\partial_\mu\underline{J}^\omega_\nu + \partial_\nu\underline{J}^\omega_\mu, \tag{9}$$

where the currents $\underline{J}^{\rho^0}_{\mu\nu}$ and $\underline{J}^i_\mu$, $i=\rho^0, \phi, \omega$, are defined by

$$\underline{J}^{\rho^0}_{\mu\nu} = \frac{\partial\underline{\mathfrak{L}}_{int}}{\partial\underline{T}^{\rho^0}_{\mu\nu}}, \tag{10}$$

$$\underline{J}^{\rho^0}_\mu = \frac{\partial\underline{\mathfrak{L}}_{int}}{\partial(\partial_\nu\underline{T}^{\rho^0}_{\nu\mu})} = \frac{g_I}{m_\rho}\underline{I}^{\rho^0}_\mu + \frac{e m_\rho}{g_I}\underline{A}_\mu + O(eg_I, g_I^3), \tag{11}$$

$$\underline{J}^\phi_\mu = \frac{\partial\underline{\mathfrak{L}}_{int}}{\partial(\partial_\nu\underline{T}^\phi_{\nu\mu})} = \frac{1}{m_\phi}\underline{I}^\phi_\mu + \frac{e m_\phi\cos\theta_Y}{2g_Y}\underline{A}_\mu, \tag{12}$$

$$\underline{J}^\omega_\mu = \frac{\partial\underline{\mathfrak{L}}_{int}}{\partial(\partial_\nu\underline{T}^\omega_{\nu\mu})} = \frac{1}{m_\omega}\underline{I}^\omega_\mu - \frac{e m_\omega\sin\theta_Y}{2g_Y}\underline{A}_\mu . \tag{13}$$

From Eqs. (1) and (2) we get

$$\underline{I}^Y_\mu = \frac{\cos\theta_Y}{g_Y}\underline{I}^\phi_\mu - \frac{\sin\theta_Y}{g_Y}\underline{I}^\omega_\mu . \tag{14}$$

If we define $\underline{\rho}^0_\mu$, $\underline{\phi}_\mu$, and $\underline{\omega}_\mu$ by the relations

$$\underline{\rho}^0_\mu = \frac{1}{m_\rho}(\partial_\nu\underline{T}^{\rho^0}_{\nu\mu} + \underline{J}^{\rho^0}_\mu), \tag{15}$$

$$\underline{\phi}_\mu = \frac{1}{m_\phi}(\partial_\nu\underline{T}^\phi_{\nu\mu} + \underline{J}^\phi_\mu), \tag{16}$$

$$\underline{\omega}_\mu = \frac{1}{m_\omega}(\partial_\nu\underline{T}^\omega_{\nu\mu} + \underline{J}^\omega_\mu), \tag{17}$$

and use Eqs. (11) and (14), Eq. (6) becomes

$$\partial_\mu\underline{F}_{\mu\nu} = -\frac{e m_\rho^2}{g_I}\underline{\rho}^0_\nu - \frac{e m_\phi^2\cos\theta_Y}{2g_Y}\underline{\phi}_\mu + \frac{e m_\omega^2\sin\theta_Y}{2g_Y}\underline{\omega}_\mu + O(e^2), \tag{18}$$

while Eqs. (7)–(9) give after differentiation

$$\partial_\mu\left(\partial_\mu\underline{\rho}^0_\nu - \partial_\nu\underline{\rho}^0_\mu - \frac{1}{m_\rho}\underline{J}^{\rho^0}_{\mu\nu}\right) - m_\rho^2\underline{\rho}^0_\nu = -g_I\underline{I}^{\rho^0}_\nu + O(e, g_I^3), \tag{19}$$

$$\partial_\mu(\partial_\mu\underline{\phi}_\nu - \partial_\nu\underline{\phi}_\mu) - m_\phi^2\underline{\phi}_\nu = -\underline{I}^\phi_\nu + O(e), \tag{20}$$

$$\partial_\mu(\partial_\mu\underline{\omega}_\nu - \partial_\nu\underline{\omega}_\mu) - m_\omega^2\underline{\omega}_\nu = -\underline{I}^\omega_\nu + O(e). \tag{21}$$

Equation (18) expresses the current-field identity to order e, while Eqs. (19)–(21) indicate that to lowest order the sources of $\underline{\rho}^0_\nu$, $\underline{\phi}_\nu$, and $\underline{\omega}_\nu$ are the currents $g_I\underline{I}^{\rho^0}_\nu$, $\underline{I}^\phi_\nu$, and $\underline{I}^\omega_\nu$, respectively.

The alternative proposal[8,12] that the isovector part of the hadronic electromagnetic current is the same as the current which generates the ρ-meson field can be approximately incorporated in the tensor description of vector mesons, and more specifically in the above model, by dropping the term $(e m_\rho/g_I)\underline{A}_\mu\partial_\nu\underline{T}^{\rho^0}_{\nu\mu}$ from the Lagrangian $\underline{\mathfrak{L}}_\gamma$. As we argued before, the gauge invariance is not affected. Similarly, by dropping the terms

$$e\underline{A}_\mu\left(\frac{m_\phi \cos\theta_Y}{2g_Y}\,\partial_\nu \underline{T}^\phi_{\nu\mu} - \frac{m_\omega \sin\theta_Y}{2g_Y}\,\partial_\nu \underline{T}^\omega_{\nu\mu}\right)$$

from $\underline{\mathcal{L}}_Y$ the isoscalar part of the electromagnetic current becomes to order e the same as a linear combination of the sources of the vector-meson fields ϕ and ω.

In our tensor formalism of spin-one mesons the interaction Hamiltonian in the interaction representation $\mathcal{H}_{\rm int}$ is given by[1,2,13]

$$\mathcal{H}_{\rm int} = -\mathcal{L}_{\rm int} + \frac{1}{2}(\vec{J}^\rho_\mu \cdot \vec{J}^\rho_\mu + J^\phi_\mu J^\phi_\mu + J^\omega_\mu J^\omega_\mu) + \frac{1}{4m_\rho^2}\vec{J}^\rho_{\mu\nu}\cdot\vec{J}^\rho_{\mu\nu}, \tag{22}$$

where $J^{\rho i}_{\mu\nu}$ and $J^{\rho i}_\mu$, $i=1,2$, are defined in a way similar to $J^{\rho^0}_{\mu\nu}$ and $J^{\rho^0}_\mu$ of Eqs. (10) and (11). From Eqs. (3)–(5), (14), and (22) we get[9]

$$\begin{aligned}\mathcal{H}_{\rm int} =\ & \tfrac{1}{4}(\partial_\nu\vec{\rho}_\mu - \partial_\mu\vec{\rho}_\nu - g_I\vec{\rho}_\mu\times\vec{\rho}_\nu)^2 - \tfrac{1}{4}(\partial_\nu\vec{\rho}_\mu - \partial_\mu\vec{\rho}_\nu)^2 - g_I\vec{\rho}_\mu\cdot\vec{I}'_\mu - \phi_\mu I^\phi_\mu - \omega_\mu I^\omega_\mu \\ & + \frac{e^2}{2}\left[\left(\frac{m_\rho}{g_I}\right)^2 + \left(\frac{m_\phi\cos\theta_Y}{2g_Y}\right)^2 + \left(\frac{m_\omega\sin\theta_Y}{2g_Y}\right)^2\right]A_\mu A_\mu - \frac{e m_\rho^2}{g_I}A_\mu\rho^0_\mu - eA_\mu\left(\frac{m_\phi^2\cos\theta_Y}{2g_Y}\,\phi_\mu - \frac{m_\omega^2\sin\theta_Y}{2g_Y}\,\omega_\mu\right) \\ & + \frac{eg_I}{m^2}A_\mu\{(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\times[\vec{\rho}_\sigma\times(\partial_\sigma\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\sigma)]\}_0 + eg_I A_\mu[\vec{\rho}_\nu\times(\vec{\rho}_\mu\times\vec{\rho}_\nu)]_0,\end{aligned} \tag{23}$$

where the terms of order e^2, eg_I^2, g_I^3, and higher have been dropped except the "photon mass terms," and the fields $\vec{\rho}_\mu$, ϕ_μ, and ω_μ are interaction-representation fields, obeying the usual commutation relations. The perturbation-theory calculations proceed in the well-known way with the $\mathcal{H}_{\rm int}$ of Eq. (23). We observe that the part of the $\mathcal{H}_{\rm int}$ which describes the strong interactions of the ρ mesons is the interaction Hamiltonian of a Yang-Mills theory, in which the vector field is coupled to the isospin current $\vec{I}'_\mu$.[6] Also, the direct photon–vector-meson couplings are accompanied by corresponding "photon mass terms" as expected.[5]

[1] E. Kyriakopoulos, Phys. Rev. 183, 1318 (1969).
[2] E. Kyriakopoulos, Phys. Rev. D 1, 1697 (1970).
[3] E. Kyriakopoulos, Phys. Rev. D 6, 2202 (1972).
[4] E. Kyriakopoulos, Phys. Rev. D 6, 2207 (1972).
[5] N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).
[6] T. D. Lee and B. Zumino, Phys. Rev. 163, 1667 (1967).
[7] Y. Nambu, Phys. Rev. 106, 1366 (1957).
[8] J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).
[9] We use underlined letters for quantities in the Heisenberg representation and nonunderlined letters for quantities in the interaction representation.
[10] E. Kyriakopoulos, Phys. Rev. D 8, 4345 (1973).
[11] The coupling constant $2g$ of Ref. 10 is replaced by g_I/m_ρ in Eq. (3).
[12] M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).
[13] In Refs. 1 and 2 an expression like that of Eq. (22), which is the covariant part of the interaction Hamiltonian, is called the *effective* interaction Hamiltonian. In general, in Ref. 2 we called the effective interaction Hamiltonian its covariant part. The importance of this covariant part is based on the fact that besides the interaction Hamiltonian the propagators of the spin-one particles contain noncovariant terms, but the S matrix is covariant and can be calculated by dropping the noncovariant terms from both the interaction Hamiltonian and the propagators. Therefore, the expression (22) is an effective interaction Hamiltonian with the understanding that as propagators of the spin-one particles we shall use only the covariant parts of these propagators. We want to point out that when people talk about an effective (or phenomenological) interaction Hamiltonian and Lagrangian they usually mean the interaction Hamiltonian or the interaction Lagrangian which to lowest order gives the results of current algebra. It is obvious that the meaning they give to the word "effective" and the meaning we give to this word in Refs. 1, 2, etc., are completely different, and, for example, the expression we call effective interaction Hamiltonian in Ref. 10 will not generate the current-algebra results.