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Causality, spin, and equal-time commutators

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We study the causality constraints on the structure of the Lorentz-antisymmetric component of the commutator of two conserved isovector currents between fermion states of equal momenta. We discuss the sum rules that follow from causality and scaling, using the recently introduced refined infinite-momentum technique. The complete set of sum rules is found to include the spin-dependent fixed-mass sum rules obtained from light-cone commutators. The causality and scaling restrictions on the structure of the electromagnetic equal-time commutators are discussed, and it is found, in particular, that causality requires the spin-dependent part of the matrix element for the time-space electromagnetic equal-time commutator to vanish identically. It is also shown, in comparison with the electromagnetic case, that the corresponding matrix element for the time-space isovector current equal-time commutator is required, by causality, to have isospin-antisymmetric tensor and scalar operator Schwinger terms.

I. INTRODUCTION

Fixed-mass sum rules have recently been derived[1,2] on the basis of light-cone commutators[3] extracted from the quark model with vector-gluon interaction. In particular, Dicus, Jackiw, and Teplitz[1] obtain six sum rules for the structure functions which characterize the Fourier transform of the matrix element of the commutator of conserved vector currents between fermion states of equal momenta. Three of these sum rules are spin-independent whereas the other three are spin-dependent. The sum rules so obtained were found to correct the sum rules of current algebra derived by use of the conventional infinite-momentum procedure.[4]

In the belief that these corrections to the sum rules of current algebra amounted to a criticism of the conventional infinite-momentum procedure, rather than equal-time algebra, a refinement[5] of this procedure was introduced and was shown[6] to yield, in the forward case, from equal-time current algebra the same spin-independent fixed-mass sum rules as obtained in Ref. 1 from light-cone commutators. In this paper we examine the derivation of the spin-dependent fixed-mass sum rules using the methods of Ref. 6.

To begin with, we discuss, in Sec. II, the causality properties of the spin-dependent structure functions and demonstrate that they are causal. Then using the causal Jost-Lehmann-Dyson (JLD) representation[7] we write, for these structure functions, the sum rules implied by causality and a certain assumption on the asymptotic behavior of the JLD spectral functions. This part of our work is an extension of an earlier analysis by Meyer and Suura[8] on the spinless case.

In Sec. III we make use of a theorem in Ref. 6 on the refined infinite-momentum limit in order to obtain the scaling[9,10,1] form of the variable-mass causality sum rules. On passing to the fixed-mass limit we find that three of the fixed-mass sum rules so obtained coincide with the spin-dependent fixed-mass sum rules derived by Dicus *et al.*[1] from the $(+,\nu)$ light-cone commutators. These sum rules are then consequences of causality and scaling alone.

Toward the end of Sec. III we discuss the electromagnetic time-space and space-space equal-time commutators (ETC's) and observe that their structures are severely restricted by our causality and scaling results. In particular we find that, according to causality, the spin-dependent part of the matrix element for the time-space electromagnetic equal-time commutator should vanish identically. In contrast with the electromagnetic case we also show that causality requires the corresponding matrix element for the isovector current time-space ETC to have *both* tensor and scalar operator Schwinger terms, antisymmetric in the isospin indices. Finally, we devote Sec. IV to some concluding remarks and a summary of the results which we claim to be new.

The necessary conventions, normalizations, and some definitions are collected together for convenience in the Appendix. We also discuss in the Appendix, for the sake of completeness, the causality properties of the spin-independent structure functions. As remarked earlier the original discussion of this topic was previously given by Meyer and Suura.[8]

II. CAUSALITY OF THE STRUCTURE FUNCTIONS

Consider the Fourier transform of the connected diagonal matrix element of the commutator of two conserved isovector currents between fermion states of momenta p ($p^2 = m^2 = 1$),

$$C^{ij}_{\mu\nu}(p, q, s) = \int e^{iqx}\langle p, s|[V^i_\mu(x), V^j_\nu(0)]|p, s\rangle d^4x, \tag{2.1}$$

where s_μ is the fermion spin axial vector

$$s_\mu = \bar{u}(p) i\gamma_\mu\gamma_5 u(p). \tag{2.2}$$

In terms of the fermion momentum and the polarization unit vector $\hat{n}$ specifying the direction of spin quantization, s_μ is given by

$$s_0 = \vec{p}\cdot\hat{n}, \quad \vec{s} = \hat{n} + \frac{\vec{p}\cdot\hat{n}}{p_0 + 1}\vec{p} \qquad (s^2 = -p^2 = -1;\ s\cdot p = 0). \tag{2.3}$$

The tensor $C^{ij}_{\mu\nu}$ may be written in the form[1]

$$\begin{aligned} C^{ij}_{\mu\nu} = {} & (q_\mu q_\nu - q^2 g_{\mu\nu})V^{ij}_1 \\ & + [\nu(p_\mu q_\nu + q_\mu p_\nu) - q^2 p_\mu p_\nu - \nu^2 g_{\mu\nu}]V^{ij}_2 \\ & + \epsilon_{\mu\nu\alpha\beta}s_\alpha q_\beta V^{ij}_3 + q\cdot s\,\epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta V^{ij}_4, \end{aligned} \tag{2.4}$$

where the structure functions V^{ij}_k ($k = 1, \ldots, 4$) depend on $\nu = p\cdot q$ and q^2 only and V^{ij}_1, V^{ij}_2 are related to the usual functions W^{ij}_L, W^{ij}_2 by $V^{ij}_1 = W^{ij}_L/q^2$, $V^{ij}_2 = -W^{ij}_2/q^2$.

Meyer and Suura[8] have shown that the function V^{ij}_1 is causal and satisfies the causality sum rule[11]

$$\int V^{ij}_1 dq_0 = 0. \tag{2.5}$$

Introducing the equal-time commutator

$$E^{ij}_{\mu\nu} = \int e^{iqx}\delta(x_0)\langle p, s|[V^i_\mu(x), V^j_\nu(0)]|p, s\rangle d^4x, \tag{2.6}$$

one finds, on using (2.5), that

$$\begin{aligned} E^{ij}_{00} &= \frac{1}{2\pi}\int C^{ij}_{00}dq_0 \\ &= \frac{1}{2\pi}[p_0^2\vec{q}^2 - (\vec{p}\cdot\vec{q})^2]\int V^{ij}_2 dq_0. \end{aligned} \tag{2.7}$$

Thus provided the causal part of the function V^{ij}_2 satisfies a causality sum rule of the type (2.5) the nonvanishing of the left-hand side of Eq. (2.7), i.e., $E^{ij}_{00} \neq 0$, would imply that V^{ij}_2 must possess a nonvanishing noncausal part.[8] However, this equation does not constrain V^{ij}_3 and V^{ij}_4 since these functions do not contribute to C^{ij}_{00} in any case. In this section we aim to demonstrate that these functions are in fact causal.

We start by considering the commutator $\tilde{C}^{ij}_{\mu\nu}$ defined by

$$\tilde{C}^{ij}_{\mu\nu} = \langle p, s|[V^i_\mu(x), V^j_\nu(0)]|p, s\rangle. \tag{2.8}$$

Although $\tilde{C}^{ij}_{\mu\nu}$ is causal, i.e., vanishes for $x^2 < 0$, the invariants V^{ij}_k, $k = 1, \ldots, 4$, need not, in general, be causal. However, any noncausal parts in their Fourier transforms must be annihilated by the operators acting on them to give $\tilde{C}^{ij}_{\mu\nu}$. Denoting the Fourier transforms of V^{ij}_k by $\tilde{V}^{ij}_k$ we, therefore, write

$$(-\partial_\mu\partial_\nu + g_{\mu\nu}\Box)\tilde{V}^{ij}_1 - [p\cdot\partial(p_\mu\partial_\nu + p_\nu\partial_\mu) + p_\mu p_\nu\Box + g_{\mu\nu}(p\cdot\partial)^2]\tilde{V}^{ij}_2 + i\epsilon_{\mu\nu\alpha\beta}s_\alpha\partial_\beta\tilde{V}^{ij}_3 - \epsilon_{\mu\nu\alpha\beta}p_\alpha s\cdot\partial\partial_\beta\tilde{V}^{ij}_4 = 0, \quad x^2 < 0. \tag{2.9}$$

From the causality of the spin-averaged matrix element we have

$$(-\partial_\mu\partial_\nu + g_{\mu\nu}\Box)\tilde{V}^{ij}_1 - [p\cdot\partial(p_\mu\partial_\nu + p_\nu\partial_\mu) + p_\mu p_\nu\Box + g_{\mu\nu}(p\cdot\partial)^2]\tilde{V}^{ij}_2 = 0, \quad x^2 < 0. \tag{2.10}$$

Hence (2.9) gives

$$\epsilon_{\mu\nu\alpha\beta}is_\alpha\partial_\beta\tilde{V}^{ij}_3 - \epsilon_{\mu\nu\alpha\beta}p_\alpha s\cdot\partial\partial_\beta\tilde{V}^{ij}_4 = 0, \quad x^2 < 0. \tag{2.11}$$

Alternatively, Eqs. (2.10) and (2.11) can be attributed to causality of the Lorentz-symmetric and -antisymmetric components of $\tilde{C}^{ij}_{\mu\nu}$, respectively.

As Eq. (2.11) is invariant we may analyze it, for convenience, in the fermion rest frame $\vec{p} = 0$. The equation then becomes ($k = 1, 2, 3$)

$$\epsilon_{\mu\nu k\beta}in_k\partial_\beta\tilde{V}^{ij}_3 + \epsilon_{\mu\nu 0\beta}\hat{n}\cdot\vec{\nabla}\partial_\beta\tilde{V}^{ij}_4 = 0, \quad x^2 < 0. \tag{2.12}$$

Since $\hat{n}$ is an arbitrary unit vector specifying the rest-frame spin direction we can choose $\hat{n} \equiv (0, 0, n_3)$, $n_3^2 = 1$. This enables us to write Eq. (2.12) in the form

$$\epsilon_{\mu\nu 3\beta}i\partial_\beta\tilde{V}^{ij}_3 + \epsilon_{\mu\nu 0\beta}\partial_3\partial_\beta\tilde{V}^{ij}_4 = 0, \quad x^2 < 0. \tag{2.13}$$

As the causal parts of $\tilde{V}^{ij}_k$, $k = 3, 4$, vanish for $x^2 < 0$ Eq. (2.13) is equivalent to

$$\epsilon_{\mu\nu 3\beta}i\partial_\beta\tilde{V}^{ij,\mathrm{nc}}_3 + \epsilon_{\mu\nu 0\beta}\partial_3\partial_\beta\tilde{V}^{ij,\mathrm{nc}}_4 = 0, \quad x^2 < 0 \tag{2.14}$$

where $\tilde{V}^{ij,\mathrm{nc}}_3$ and $\tilde{V}^{ij,\mathrm{nc}}_4$ signify possible noncausal parts in the respective functions.

Since Eq. (2.14) holds for all $\mu, \nu = 0, \ldots, 3$ we have for the following different choices of these indices ($x^2<0$):

$$\mu=0,\ \nu=1:\ \partial_2\tilde{V}_3^{ij,\mathrm{nc}}=0, \tag{2.15}$$

$$\mu=0,\ \nu=2:\ \partial_1\tilde{V}_3^{ij,\mathrm{nc}}=0, \tag{2.16}$$

$$\mu=1,\ \nu=2:\ i\partial_0\tilde{V}_3^{ij,\mathrm{nc}}-\partial_3^2\tilde{V}_4^{ij,\mathrm{nc}}=0, \tag{2.17}$$

$$\mu=1,\ \nu=3:\ \partial_2\partial_3\tilde{V}_4^{ij,\mathrm{nc}}=0, \tag{2.18}$$

$$\mu=2,\ \nu=3:\ \partial_1\partial_3\tilde{V}_4^{ij,\mathrm{nc}}=0. \tag{2.19}$$

Equations (2.15) and (2.16) imply that $\tilde{V}_3^{ij,\mathrm{nc}}=f(x_0,x_3)$. But $\tilde{V}_3^{ij,\mathrm{nc}}$ can only depend on the invariants $x\cdot p$ and x^2. Hence it is a function of $x\cdot p=x_0$ only. Consequently on imposing the boundary condition[8,12] $\tilde{V}_3^{ij,\mathrm{nc}}\to 0$ for $|\vec{x}|\to\infty$ we deduce that $\tilde{V}_3^{ij,\mathrm{nc}}$ must vanish identically for $x^2<0$. Therefore, V_3^{ij} is causal.

Setting $\tilde{V}_3^{ij,\mathrm{nc}}=0$ in Eq. (2.17) the resulting equation, together with (2.18) and (2.19), will imply

$$\partial_3\tilde{V}_4^{ij,\mathrm{nc}}=f(x_0). \tag{2.20}$$

Integrating this equation with respect to x_3, one obtains

$$\tilde{V}_4^{ij,\mathrm{nc}}=x_3f(x_0)+g(x_0,x_1,x_2). \tag{2.21}$$

Since $f(x_0)$ and $g(x_0,x_1,x_2)$ are arbitrary constants of integration the two terms on the right-hand side of (2.21) should be independently invariant. But $\tilde{V}_4^{ij,\mathrm{nc}}$ can only depend on $x\cdot p$ and x^2. Therefore, we must require

$$f(x_0)=0,\qquad g(x_0,x_1,x_2)\equiv g(x\cdot p=x_0), \tag{2.22}$$

i.e., $\tilde{V}_4^{ij,\mathrm{nc}}$ depends on $x\cdot p=x_0$ only. Thus if it is to vanish as $|\vec{x}|\to\infty$ (see Refs. 8 and 12) it must vanish identically for $x^2<0$. Hence V_4^{ij} is causal.[13]

The above arguments can also be applied to Eq. (2.10) in order to show that V_1^{ij} is causal and that V_2^{ij} can have a noncausal part. This is done in the Appendix. Although these results on V_1^{ij} and V_2^{ij} were previously obtained by Meyer and Suura,[8] we have elected, for the sake of completeness, to prove them using the above procedure.

III. CONSEQUENCES OF CAUSALITY AND SCALING

A. Causality sum rules

Having shown in the last section that V_3^{ij} and V_4^{ij} are causal we can, therefore, write for them the JLD representations[7] ($k=3,4$)

$$V_k^{ij}=\int_0^\infty ds\int d^4u\,\epsilon(q_0-u_0)\delta((q-u)^2-s)\psi_k^{ij}(u,s). \tag{3.1}$$

Assuming the possibility of interchanging the orders of integration we then find that

$$\int V_k^{ij}dq_0=0,\quad k=3,4. \tag{3.2}$$

It has been shown by Meyer and Suura[8] that such a causality sum rule holds provided that

$$\lim_{s\to\infty}\psi_k^{ij}(u,s)=0.$$

We remark that this condition is not equivalent to the interchange of the order of integration between (3.1) and (3.2) and is, moreover, a sufficient as well as a necessary condition for Eq. (3.2) to hold when this equation is interpreted as[8]

$$\lim_{\Lambda\to\infty}\int_{-\infty}^{\infty}e^{-q_0^2/\Lambda^2}V_k^{ij}dq_0=0.$$

A detailed discussion of this point is given in the appendix of Ref. 8. We also note that a model in which Eq. (3.2) holds is the original quark model of Gell-Mann, as is verified by considering the explicit equal-time commutators of this model. It then follows that in such a model the above asymptotic condition on $\psi_k^{ij}(u,s)$ is satisfied.

Next we observe that in addition to Eq. (3.2) one also has ($k=3,4$)

$$\int q_0V_k^{ij}dq_0=\int\psi_k^{ij}(u,s)d^4u\,ds, \tag{3.3}$$

$$\int q_0^2V_k^{ij}dq_0=\int u_0\psi_k^{ij}(u,s)d^4u\,ds, \tag{3.4}$$

provided that $\lim_{s\to\infty}s\psi_k^{ij}(u,s)=0$. Since the spectral functions $\psi_k^{ij}(u,s)$ are Lorentz-invariant the right-hand sides of (3.3) and (3.4) must transform like a scalar and a time component of a Lorentz vector, respectively.[14] These equations may therefore be written in the form

$$\int q_0V_k^{ij}dq_0=b_k^{ij}, \tag{3.5}$$

$$\int q_0^2V_k^{ij}dq_0=c_k^{ij}p_0, \tag{3.6}$$

where b_k^{ij} and c_k^{ij} are constants. Note that $b_k^{ij}=0$ if $\psi_k^{ij}(-u,s)=-\psi_k^{ij}(u,s)$ and $c_k^{ij}=0$ if $\psi_k^{ij}(-u,s)=\psi_k^{ij}(u,s)$. In fact (the brackets (ij) and $[ij]$ denote ij-symmetric and -antisymmetric parts, respectively)

$$\psi_3^{(ij)}(-u,s)=\psi_3^{(ij)}(u,s),\qquad \psi_3^{[ij]}(-u,s)=-\psi_3^{[ij]}(u,s), \tag{3.7}$$

implying that $b_3^{[ij]}=0$, $c_3^{(ij)}=0$;

$$\psi_4^{(ij)}(-u,s)=-\psi_4^{(ij)}(u,s),\qquad \psi_4^{[ij]}(-u,s)=\psi_4^{[ij]}(u,s), \tag{3.8}$$

implying that $b_4^{(ij)}=0$, $c_4^{[ij]}=0$. These results can be arrived at upon writing the JLD representations (3.1) in the form

$$V_k^{ij}(\nu, q^2) = \tfrac{1}{2}\int \frac{1}{|q_0 - u_0|}\{\delta(q_0 - u_0 - [s + (\vec{q} - \vec{u})^2]^{1/2}) - \delta(q_0 - u_0 + [s + (\vec{q} - \vec{u})^2]^{1/2})\}\psi_k^{ij}(u, s)d^4u\,ds\,,$$

$$= \tfrac{1}{2}\int \frac{1}{|q_0 + u_0|}\{\delta(q_0 + u_0 - [s + (\vec{q} + \vec{u})^2]^{1/2}) - \delta(q_0 + u_0 + [s + (\vec{q} + \vec{u})^2]^{1/2})\}\psi_k^{ij}(-u, s)d^4u\,ds\,.$$

Hence we can write

$$V_k^{ij}(\nu, q^2) = \tfrac{1}{4}\int \Big[\frac{1}{|q_0 - u_0|}\{\delta(q_0 - u_0 - [s + (\vec{q} - \vec{u})^2]^{1/2}) - \delta(q_0 - u_0 + [s + (\vec{q} - \vec{u})^2]^{1/2})\}\psi_k^{ij}(u, s)$$
$$+ \frac{1}{|q_0 + u_0|}\{\delta(q_0 + u_0 - [s + (\vec{q} + \vec{u})^2]^{1/2}) - \delta(q_0 + u_0 + [s + (\vec{q} + \vec{u})^2]^{1/2})\}\psi_k^{ij}(-u, s)\Big]d^4u\,ds\,. \qquad (3.9)$$

Using the crossing relations

$$V_k^{(ij)}(\nu, q^2) = \eta_{(k)} V_k^{(ij)}(-\nu, q^2)\,,$$
$$V_k^{[ij]}(\nu, q^2) = -\eta_{(k)} V_k^{[ij]}(-\nu, q^2)\,,$$
$$(\eta_{(3)} = -\eta_{(4)} = -1) \qquad (3.10)$$

in Eq. (3.9) we immediately get the results (3.7) and (3.8).

Thus, in summary, one has the causality sum rules ($k = 3, 4$)

$$\int V_k^{ij} dq_0 = 0\,, \qquad (3.11)$$

$$\int q_0 V_k^{ij} dq_0 = b_k^{ij}\,, \qquad (3.12)$$

$$\int q_0^2 V_k^{ij} dq_0 = c_k^{ij}\, p_0\,, \qquad (3.13)$$

with

$$b_3^{[ij]} = b_4^{(ij)} = c_3^{(ij)} = c_4^{[ij]} = 0\,. \qquad (3.14)$$

The sum rules (3.11)–(3.13) are general causality sum rules which do not depend on any specific assumption about current commutators. In particular, they are not in any sense, model-dependent. In the next subsection we apply the methods of Ref. 6 to obtain the scaling form of these sum rules.

B. Scaling

On the grounds that $\tilde{V}_3^{ij}$ and $\tilde{V}_4^{ij}$ possess canonical leading light-cone singularities of the type

$$\tilde{V}_3^{ij}(x) \sim \epsilon(x_0)\delta(x^2)h_3^{ij}(x\cdot p, 0) + \text{less singular terms},$$
$$\tilde{V}_4^{ij}(x) \sim \epsilon(x_0)\theta(x^2)h_4^{ij}(x\cdot p, 0) + \text{less singular terms}, \qquad (3.15)$$

where h_k^{ij} ($k = 3, 4$) are matrix elements of nonsingular bilocal operators, one can[9,10,1] deduce the scaling behavior ($\nu \to \infty$ with $-q^2/2\nu = \omega$ fixed);

$$\nu V_3^{ij} \sim F_3^{ij}(\omega)\,, \qquad (3.16a)$$
$$\nu^2 V_4^{ij} \sim F_4^{ij}(\omega)\,. \qquad (3.16b)$$

We mention at this point that a theorem due to Leutwyler, Otterson, and Stern[15] states that the scaling laws for causal functions scaling as in (3.16) are valid for $q^2 \to \infty$ as well as $q^2 \to -\infty$, provided that these laws reflect canonical leading light-cone singularities of the type (3.15).

In order to incorporate scaling in Eqs. (3.11)–(3.13) we first start by introducing the variables

$$\alpha = p_0^{-1}, \quad \xi = -p_0^{-2}\vec{p}\cdot\vec{q}, \quad \eta = \vec{q}^{\,2} - p_0^{-2}(\vec{p}\cdot\vec{q})^2\,. \qquad (3.17)$$

These parameters vary such that

$$0 \le \alpha \le 1, \quad -\infty < \xi < \infty, \quad \eta \ge \xi^2/(1-\alpha^2)\,, \qquad (3.18)$$

and when $\alpha = 1$, $\xi = 0$, and $\eta \ge 0$.

Changing the integration variable in Eqs. (3.11)–(3.13) from q_0 to ν we may write these equations as ($k = 3, 4$):

$$\int V_k^{ij}(\nu, \alpha^2\nu^2 - 2\xi\nu - \eta)d\nu = 0\,, \qquad (3.19)$$

$$\alpha^2 \int \nu V_k^{ij}(\nu, \dots)d\nu = b_k^{ij}\,, \qquad (3.20)$$

$$\alpha^4 \int \nu^2 V_k^{ij}(\nu, \dots)d\nu = 2\xi b_k^{ij} + c_k^{ij}\,. \qquad (3.21)$$

In the method of the refined infinite-momentum limit[6] one considers an integral of the form

$$I = \int_{-\infty}^{\infty} V(\nu, \alpha^2\nu^2 - 2\xi\nu - \eta)d\nu\,. \qquad (3.22)$$

Dividing the integration region into the intervals $(-\infty, -R)$, $[-R, R]$, and (R, ∞), one assumes that it is possible to interchange the limit $\alpha \to +0$ and the integration in the range $[-R, R]$. In the other intervals the variable ν is changed to ξ', where

$$\nu = -2\alpha^{-2}(\xi' - \xi)\,. \qquad (3.23)$$

One then obtains

$$\lim_{\alpha^2\to 0} I = \int_{-R}^{R} V(\nu, -2\xi\nu - \eta)d\nu + \lim_{\alpha^2\to 0}\left[\int_{-\infty}^{\xi-(1/2)\alpha^2 R} + \int_{\xi+(1/2)\alpha^2 R}^{\infty}\right][2\alpha^{-2}V(-2\alpha^{-2}(\xi'-\xi), 4\alpha^{-2}\xi'(\xi'-\xi)-\eta)]d\xi' . \tag{3.24}$$

In the second term the integral is evaluated in the scaling region. We may therefore use scaling behavior in this integral. Since the lowest value of $|\nu|$ in this term is $|\nu| = R$ we must choose $R \geq R_0$, where R_0 is the value of ν at which scaling behavior sets in. Thus letting $\alpha^2 \to 0$ and then proceeding to $R \to \infty$, we obtain

$$\lim_{\alpha^2\to 0} I = \int_{-\infty}^{\infty} V(\nu, -2\xi\nu - \eta)d\nu - \mathrm{P}\int \frac{F(\xi')}{\xi' - \xi}\, d\xi' , \tag{3.25}$$

where $\nu V \sim F$ in the scaling limit.

Next we apply Eq. (3.25) to the sum rules (3.19)–(3.21), taking the scaling behavior (3.16) into account. From Eqs. (3.19) we obtain

$$\int_{-\infty}^{\infty} V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = \mathrm{P}\int \frac{F_3^{ij}(\xi')}{(\xi' - \xi)}\, d\xi' , \tag{3.26}$$

$$\int_{-\infty}^{\infty} V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 . \tag{3.27}$$

Turning now to Eq. (3.20) for V_3^{ij} we observe that as $\alpha^2 \to 0$

$$\int \nu V_3^{ij}(\nu, \ldots)d\nu \sim \int_{-\infty}^{\infty} \nu V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu + 2\alpha^{-2}\int F_3^{ij}(\xi')d\xi' \sim \alpha^{-2} b_3^{ij} . \tag{3.28}$$

Thus the assumption that the integral

$$\int_{-\infty}^{\infty} \nu V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu \tag{3.29}$$

exists gives

$$b_3^{ij} = 2\int F_3^{ij}(\xi')d\xi' , \tag{3.30}$$

$$\int_{-\infty}^{\infty} \nu V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 . \tag{3.31}$$

Next we consider the V_4^{ij} sum rule in Eq. (3.20). We get

$$\lim_{\alpha^2\to 0}\int \nu V_4^{ij}(\nu, \ldots)\, d\nu = \int_{-\infty}^{\infty} \nu V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu - \mathrm{P}\int \frac{F_4^{ij}(\xi')d\xi'}{\xi' - \xi} = \lim_{\alpha^2\to 0} \alpha^{-2} b_4^{ij} . \tag{3.32}$$

The assumption that this limit exists implies that

$$b_4^{ij} = 0 . \tag{3.33}$$

Then (3.20), for $k = 4$, will give

$$\int_{-\infty}^{\infty} \nu V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = \mathrm{P}\int \frac{F_4^{ij}(\xi')}{\xi' - \xi}\, d\xi' . \tag{3.34}$$

Considering the V_3^{ij} sum rule in (3.21) we observe that as $\alpha^2 \to 0$

$$\int \nu^2 V_3^{ij}(\nu, \ldots)d\nu \sim \int_{-\infty}^{\infty} \nu^2 V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu - 4\alpha^{-4}\int (\xi' - \xi)F_3^{ij}(\xi')d\xi' \sim \alpha^{-4}(2\xi b_3^{ij} + c_3^{ij}) . \tag{3.35}$$

Assuming that the integral

$$\int \nu^2 V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu \tag{3.36}$$

exists, we have

$$2\xi b_3^{ij} + c_3^{ij} = 4\int (\xi - \xi')F_3^{ij}(\xi')d\xi' , \tag{3.37}$$

which gives again Eq. (3.30) for b_3^{ij} as well as

$$c_3^{ij} = -4\int \xi' F_3^{ij}(\xi')d\xi' . \tag{3.38}$$

From (3.37) and (3.35) we then have

$$\int \nu^2 V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 . \tag{3.39}$$

Finally Eq. (3.21) for V_4^{ij} gives, on assuming that the integral

$$\int \nu^2 V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu \tag{3.40}$$

exists, the result (3.33) as well as

$$c_4^{ij} = 0 , \tag{3.41}$$

$$\int F_4^{ij}(\xi')d\xi' = 0 , \tag{3.42}$$

$$\int \nu^2 V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 . \tag{3.43}$$

C. The fixed-mass limit

The main sum rules derived in the previous subsection are

$$\int V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = \mathrm{P}\int \frac{F_3^{ij}(\xi')}{\xi' - \xi}\, d\xi' , \qquad (3.26)$$

$$\int V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 , \qquad (3.27)$$

$$\int \nu V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = \mathrm{P}\int \frac{F_4^{ij}(\xi')d\xi'}{\xi' - \xi} , \qquad (3.34)$$

$$\int \nu V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 , \qquad (3.31)$$

$$\int \nu^2 V_3^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 , \qquad (3.39)$$

$$\int \nu^2 V_4^{ij}(\nu, -2\xi\nu - \eta)d\nu = 0 . \qquad (3.43)$$

In addition we also had

$$b_3^{ij} = 2\int F_3^{ij}(\omega)d\omega , \qquad (3.30)$$

$$b_4^{ij} = 0 , \qquad (3.33)$$

$$c_3^{ij} = -4\int \omega F_3^{ij}(\omega)d\omega , \qquad (3.38)$$

$$c_4^{ij} = 0 , \qquad (3.41)$$

$$\int F_4^{ij}(\omega)d\omega = 0 . \qquad (3.42)$$

To obtain the fixed-mass sum rules one proceeds to the limit $\xi \to 0$. If one simply sets[6] $\xi = 0$ in Eqs. (3.26), (3.27), and (3.34) one gets the nontrivial sum rules

$$\int_0^\infty V_3^{[ij]}(\nu, q^2)d\nu = \mathrm{P}\int_0^1 \frac{F_3^{[ij]}(\omega)}{\omega}\, d\omega , \qquad (3.44)$$

$$\int_0^\infty V_4^{(ij)}(\nu, q^2)d\nu = 0 , \qquad (3.45)$$

$$\int_0^\infty \nu V_4^{[ij]}(\nu, q^2)d\nu = \mathrm{P}\int_0^1 \frac{F_4^{[ij]}(\omega)}{\omega}\, d\omega , \qquad (3.46)$$

where $q^2 \leq 0$. These relations are the spin-dependent fixed-mass sum rules derived in Ref. 1 from light-cone commutators.[16] Setting $\xi = 0$ in Eqs. (3.31), (3.39), and (3.43) gives an additional set of three fixed-mass sum rules. Whereas the sum rules arising from (3.31) and (3.39) are not satisfied in the free-quark-model Born approximation of the amplitudes, where[17]

$$V_3^{ij} = \frac{i\pi}{2} f^{ijk}\lambda_k[\delta(q^2 + 2\nu) + \delta(q^2 - 2\nu)] + \frac{\pi}{2} d^{ijk}\lambda_k[\delta(q^2 + 2\nu) - \delta(q^2 - 2\nu)] ,$$

$$V_4^{ij} = 0 ,$$

the one from (3.43) is trivially satisfied. Compared to (3.44)–(3.46) the sum rules obtained from (3.31) and (3.43) are less likely to converge on the basis of the Regge model.[1] The expectation for the Regge convergence of the sum rule that results on setting $\xi = 0$ in Eq. (3.39) is even more remote than that for the sum rules from (3.31) and (3.43). This could be an indication that the assumption made in connection with the existence of the integral (3.36) is unjustifiable. Since the relation (3.38) follows also from this assumption, its validity is equally doubtful. But in any case these latter results which depend on the existence of the integral (3.36) will not be needed for later use in this paper.

Finally, considering Eqs. (3.42) and (3.30) we first remark that (3.42) is the scaling version of (3.45).[1,18] The significance of Eq. (3.30), on the other hand, will be explored in the next subsection.

D. Electromagnetic equal-time commutators

Considerable attention is being currently devoted to the study of the structure of the electromagnetic equal-time commutators. Here we aim to investigate the restrictions imposed by the results obtained so far, on the structure of these commutators.

Define

$$E_{\mu\nu} = \int e^{iqx}\delta(x_0)\langle p, s|[V_\mu(x), V_\nu(0)]|p, s\rangle\, d^4x , \qquad (3.47)$$

where $V_\mu(x)$ is the electromagnetic current and $|p, s\rangle$ denotes, for definiteness, a nucleon state. In terms of $\mu\nu$-symmetric and -antisymmetric components,

$$E_{\mu\nu} = E_{(\mu\nu)} + E_{[\mu\nu]} , \qquad (3.48)$$

where

$$E_{\mu\nu} = \frac{1}{2\pi}\int C_{\mu\nu}dq_0 , \qquad (3.49)$$

with

$$C_{(\mu\nu)} = (q_\mu q_\nu - q^2 g_{\mu\nu})V_1 + [\nu(p_\mu q_\nu + q_\mu p_\nu) - q^2 p_\mu p_\nu - \nu^2 g_{\mu\nu}]V_2 , \qquad (3.50)$$

and

$$C_{[\mu\nu]} = \epsilon_{\mu\nu\alpha\beta}s_\alpha q_\beta V_3 + q\cdot s\,\epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta V_4 . \qquad (3.51)$$

With these preliminaries out of the way we now discuss the time-space and space-space ETC's.

1. Time-space ETC

The restrictions placed by the causality and scaling properties of V_1 and V_2 on the structure of $E_{(0k)}$ have recently been studied by Taha,[19] who found that (a) causality requires $E_{(0k)}$ to be of the form

$$E_{(0k)} = -\vec{p}\cdot\vec{q}s_2 p_k + (s_1 + p_0^2 s_2)q_k ,$$

where s_1 and s_2 are constants; (b) if W_L scales

$$s_1 = \frac{1}{4\pi}\int \frac{F_L(\omega)}{\omega^2}\, d\omega ,$$

and if νW_2 scales $s_2 = 0$. In what follows we discuss the restrictions imposed by our results on the structure of $E_{[0k]}$.

From Eqs. (3.49) and (3.51)

$$E_{[0k]} = \frac{1}{2\pi}\, \epsilon_{klm}\Big(s_l\, q_m \int V_3 dq_0 - \vec{q}\cdot\vec{s} p_l\, q_m \int V_4 dq_0 + s_0 p_l\, q_m \int q_0 V_4 dq_0\Big) . \qquad (3.52)$$

Using the causality sum rules (3.11) and (3.12) in this equation, we have [note $b_4^{(ij)} = 0$ from (3.14)]

$$E_{[0k]} = 0 , \qquad (3.53)$$

which means that causality implies for E_{0k} the structure it requires for $E_{(0k)}$, i.e., the form given in (a) and discussed fully in Ref. 19.

Let us next consider, for the sake of comparison with the electromagnetic case, the equal-time commutator $E^{ij}_{[0k]}$, $i \neq j$. The equation corresponding to (3.52) will then give, on using the causality sum rules (3.11) and (3.12) with $b_4^{(ij)} = 0$,

$$E^{ij}_{[0k]} = \frac{1}{2\pi}\, \epsilon_{klm} s_0 p_l\, q_m b_4^{[ij]} ; \qquad (3.54)$$

i.e., unlike $E_{[0k]}$ the ETC $E^{ij}_{[0k]}$, $i \neq j$, does not vanish identically on the basis of causality alone. Further elucidation of the structure of this ETC can be gained upon using the identity (see the Appendix)

$$i\epsilon_{klm}\gamma_0\gamma_5 = \gamma_l\gamma_m\gamma_k + \gamma_m g_{kl} - \gamma_l g_{mk} - \gamma_k g_{lm} . \qquad (3.55)$$

This enables us to write Eq. (3.54) as[20]

$$E^{ij}_{[0k]} = \frac{1}{2\pi}\, \bar{u}(p)[(\vec{\gamma}\cdot\vec{p})(\vec{\gamma}\cdot\vec{q})\gamma_k - \vec{p}^2 q_k]\, b_4^{[ij]} u(p) , \qquad (3.56)$$

in which one observes that the first and second terms correspond to tensor and scalar operator Schwinger term contributions, respectively.

Lastly, an identically vanishing right-hand side in Eq. (3.56) is obtained if $b_4^{[ij]} = 0$. As is clear from Eqs. (3.32) and (3.33) this would be the case if one makes the assumption that the $\alpha^2 \to 0$ limit of the integral,

$$\int \nu V_4^{ij}(\nu, \ldots)\, d\nu ,$$

exists.

2. *Space-space ETC*

Using Eq. (A10) of the Appendix and the causality and scaling sum rules (3.11)–(3.14), (3.41), and (3.30) we obtain

$$E_{[ij]} = \frac{1}{\pi}\, \bar{u}(p)\gamma_0\Big[(\gamma_i\gamma_j - g_{ij})\int F_3(\omega) d\omega\Big] u(p) . \qquad (3.57)$$

On use of the identity

$$\gamma_0\gamma_i\gamma_j - \gamma_0 g_{ij} = i\epsilon_{ijk}\gamma_k\gamma_5 , \qquad (3.58)$$

Eq. (3.57) can be rewritten as

$$E_{[ij]} = \frac{1}{\pi}\, \epsilon_{ijk} s_k \int F_3(\omega) d\omega . \qquad (3.59)$$

Note that this result is a consequence of causality and scaling.

Now in the algebra of fields $E_{[ij]} = 0$, implying that

$$\int F_3(\omega) d\omega = 0 , \qquad (3.60)$$

which is the sum rule of Hey and Mandula.[10]

On the other hand, according to the free-quark model, or the quark model with local four-fermion interaction,[21]

$$\delta(x_0)[V_i(x), V_j(0)] - \delta(x_0)[V_j(x), V_i(0)] = 2i\epsilon_{ijk}\Big(d_{33c} + \frac{2}{\sqrt{3}}\, d_{38c} + \frac{1}{3}\, d_{88c}\Big) A_k^c(0)\delta^4(x) , \qquad (3.61)$$

with

$$c = 0, \ldots, 8 ;$$

$$d_{ab0} = (\tfrac{2}{3})^{1/2}\, \delta_{ab} .$$

Hence in this model

$$E_{[ij]} = i\epsilon_{ijk}\Big(d_{33c} + \frac{2}{\sqrt{3}}\, d_{38c} + \frac{1}{3}\, d_{88c}\Big) \times \langle p, s|A_k^c(0)|p, s\rangle = \epsilon_{ijk}\Big(d_{33c} + \frac{2}{\sqrt{3}}\, d_{38c} + \frac{1}{3}\, d_{88c}\Big)\Gamma^c s_k . \qquad (3.62)$$

Combining this equation with (3.59) we have

$$\left(d_{33c} + \frac{2}{\sqrt{3}}\, d_{38c} + \frac{1}{3}\, d_{88c}\right)\Gamma^c = \frac{2}{\pi}\int_0^1 F_3(\omega)d\omega\,, \tag{3.63}$$

which relates the axial-vector coupling constant to the spin-odd deep-inelastic electroproduction cross section. This result, which was originally derived by Bjorken,[22] corresponds to Eq. (5.13) in the work of Dicus *et al.*[1]

IV. CONCLUDING REMARKS AND SUMMARY OF RESULTS

In this paper we have found that a study of the causality properties of the spin-dependent electroproduction structure functions not only sheds light on the origin of results that have previously been obtained in the light-cone approach but also furnishes information which allows one to constrain the forms of the equal-time commutators. We summarize in the following what we regard to be the new results of our investigations:

(1) First, we have demonstrated[23] that the spin-dependent electroproduction structure functions V_3^{ij} and V_4^{ij} are causal. To our knowledge, this has not been done before.

(2) Next, we have shown that a number of the polarized electroproduction results which have recently been obtained from the analysis of the $(+,\nu)$ light-cone commutators, e.g., the spin-dependent fixed-mass sum rules involving V_3^{ij} and V_4^{ij}, do in fact follow from causality and scaling alone *provided* the conventional infinite-momentum procedure is abandoned in favor of the recently introduced refined infinite-momentum technique of Taha.[6] Thus with this proviso one would also deduce these light-cone results from any causal formalism such as that of equal-time commutators. This not only confirms the conclusions reached by Taha[6] in connection with the derivation of the spin-independent electroproduction fixed-mass sum rules but is also in agreement with the recent work of Keppel-Jones[24] and Ward.[25] These authors have proposed a treatment of the equal-time commutator, in which the difficulties of the conventional infinite-momentum limit are circumvented. Their considerations show that the neutrino-nucleon scattering fixed-mass sum rules of light-cone commutators are already contained in equal-time algebra. Further work by Ward[26] investigates the assumptions under which the refined infinite-momentum limit is valid. It is shown that while the refined limit satisfactorily handles the Z graphs that are neglected in the conventional limit, it, nonetheless, misses the class II states, as does the light-cone approach.[27] It is, however, noted[26] that in principle the refined procedure allows the inclusion of all classes of intermediate states and the formalism is extended to explicitly demonstrate this.

(3) Finally, we have used the causality properties of V_3^{ij} and V_4^{ij} to study the structure of the spin-dependent part of the matrix element of the electromagnetic ETC. Denoting the Fourier transform of this matrix element by $E_{[\mu\nu]}$ we found that

$$E_{[0k]} = 0\,. \tag{4.1}$$

Consequently, according to causality, the structure of the time-space electromagnetic ETC between spin-$\frac{1}{2}$ states of equal momenta is identical to that between spinless states and is therefore given by the form of Ref. 19, i.e.,

$$E_{0k} = -\vec{p}\cdot\vec{q}p_k s_2 + (s_1 + p_0{}^2 s_2)q_k\,, \tag{4.2}$$

where s_1 and s_2 are constants. In comparison with Eq. (4.1) we have also shown that causality requires $E_{[0k]}^{ij}$, $i \neq j$, to possess both tensor and scalar operator Schwinger terms, antisymmetric in $\{ij\}$. Specifically we obtained

$$E_{[0k]}^{ij} = \frac{1}{2\pi}\,\bar{u}(p)[(\vec{\gamma}\cdot\vec{p})(\vec{\gamma}\cdot\vec{q})\gamma_k - \vec{p}^{\,2}q_k]\,b_4^{[ij]}u(p)\,, \tag{4.3}$$

in which the first and second terms correspond to tensor and scalar operator Schwinger term contributions, respectively. Furthermore, we have noted that the right-hand side of (4.3) would vanish if the $\alpha^2 \to 0$ limit of the integral

$$\int \nu V_4^{ij}(\nu, \alpha^2\nu^2 - 2\xi\nu - \eta)d\nu \tag{4.4}$$

exists.

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APPENDIX

1. Conventions and normalizations

In this article the γ matrices obey

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}\,, \tag{A1}$$

$$\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0\,, \tag{A2}$$

$$\begin{aligned}\gamma_5 &= -i\gamma_0\gamma_1\gamma_2\gamma_3 \\ &= -\frac{i}{4!}\,\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\,.\end{aligned} \tag{A3}$$

The positive-energy spinor $u(p)$ satisfies the Dirac equation

$$(\not p - 1)u(p) = 0, \tag{A4}$$

and is normed so that

$$\bar{u}(p)u(p) = 2. \tag{A5}$$

From Eqs. (A4) and (A1) one readily shows

$$\bar{u}(p)\gamma_\mu u(p) = p_\mu \bar{u}(p)u(p). \tag{A6}$$

Using Eqs. (A1)–(A3) one can deduce the identities

$$6i\epsilon_{\mu\nu\alpha\beta}\gamma_5 = \sigma_{\mu\nu}\sigma_{\alpha\beta} + \sigma_{\beta\mu}\sigma_{\alpha\nu} + \sigma_{\nu\beta}\sigma_{\alpha\mu} + \sigma_{\alpha\mu}\sigma_{\nu\beta} + \sigma_{\beta\alpha}\sigma_{\nu\mu} + \sigma_{\nu\alpha}\sigma_{\mu\beta}, \tag{A7}$$

$$3i\epsilon_{\mu\nu\alpha\beta}\gamma_\alpha\gamma_5 = \sigma_{\mu\nu}\gamma_\beta + \sigma_{\nu\beta}\gamma_\mu + \sigma_{\beta\mu}\gamma_\nu, \tag{A8}$$

where

$$\sigma_{\mu\nu} = \tfrac{1}{2}[\gamma_\mu, \gamma_\nu]. \tag{A9}$$

Choosing $\mu=0$, $\nu=k$, $\alpha=l$, $\beta=m$ in Eq. (A7) and multiplying by γ_0 one gets Eq. (3.55) of the text. Making use of Eqs. (A7)–(A9), (A1), (A4), and (A6) in Eq. (3.51) of the text one obtains

$$C_{[\mu\nu]} = \bar{u}(p)\{(\not q\gamma_\mu\gamma_\nu - \nu g_{\mu\nu} + p_\mu q_\nu - q_\mu p_\nu)V_3 + [(\nu\not q - q^2)\gamma_\mu\gamma_\nu + (q^2-\nu^2)g_{\mu\nu} + \nu(p_\mu q_\nu - q_\mu p_\nu) + \not q(q_\mu\gamma_\nu - \gamma_\mu q_\nu)]V_4\}u(p). \tag{A10}$$

2. Causal properties of V_1^{ij} and V_2^{ij}

Consider Eq. (2.10) of the text. Since the causal parts of $\tilde{V}_1^{ij}$ and $\tilde{V}_2^{ij}$ vanish for $x^2<0$ we can write

$$(-\partial_\mu\partial_\nu + g_{\mu\nu}\Box)\tilde{V}_1^{ij,\,\mathrm{nc}} - [p\cdot\partial(p_\mu\partial_\nu + p_\nu\partial_\mu) + p_\mu p_\nu\Box + g_{\mu\nu}(p\cdot\partial)^2]\tilde{V}_2^{ij,\,\mathrm{nc}} = 0, \quad x^2<0. \tag{A11}$$

In the rest frame $\vec{p}=0$ this equation reduces to

$$(-\partial_\mu\partial_\nu + g_{\mu\nu}\Box)\tilde{V}_1^{ij,\,\mathrm{nc}} - [\partial_0(g_{\mu 0}\partial_\nu + g_{\nu 0}\partial_\mu) + g_{\mu 0}g_{\nu 0}\Box + g_{\mu\nu}\partial_0{}^2]\tilde{V}_2^{ij,\,\mathrm{nc}} = 0, \quad x^2<0. \tag{A12}$$

For the following different values of μ and ν we then obtain

$$\mu=1,\ \nu=2:\ \partial_2\partial_1\tilde{V}_1^{ij,\,\mathrm{nc}} = 0, \tag{A13}$$

$$\mu=1,\ \nu=3:\ \partial_3\partial_1\tilde{V}_1^{ij,\,\mathrm{nc}} = 0, \tag{A14}$$

which together imply

$$\partial_1\tilde{V}_1^{ij,\,\mathrm{nc}} = f(x_0, x_1). \tag{A15}$$

Integrate with respect to x_1 to get

$$\tilde{V}_1^{ij,\,\mathrm{nc}} = g(x_0, x_1) + h(x_0, x_2, x_3). \tag{A16}$$

Consequently Lorentz invariance requires

$$g(x_0, x_1) \equiv g(x_0 = x\cdot p), \quad h(x_0, x_2, x_3) \equiv h(x_0 = x\cdot p). \tag{A17}$$

Therefore, the boundary condition $\tilde{V}_1^{ij,\,\mathrm{nc}} \to 0$ as $|\vec{x}| \to \infty$ implies that $\tilde{V}_1^{ij,\,\mathrm{nc}}=0$ for $x^2<0$, i.e., V_1^{ij} is causal. Note that one reaches the same conclusion if $\partial_1\tilde{V}_1^{ij,\,\mathrm{nc}}=0$.

Taking the result $\tilde{V}_1^{ij,\,\mathrm{nc}}=0$ into account we next have

$$\mu=1,\ \nu=1:\ \partial_0{}^2\tilde{V}_2^{ij,\,\mathrm{nc}} = 0, \tag{A18}$$

$$\mu=1,\ \nu=0:\ \partial_1\partial_0\tilde{V}_2^{ij,\,\mathrm{nc}} = 0, \tag{A19}$$

$$\mu=2,\ \nu=0:\ \partial_2\partial_0\tilde{V}_2^{ij,\,\mathrm{nc}} = 0, \tag{A20}$$

$$\mu=3,\ \nu=0:\ \partial_3\partial_0\tilde{V}_2^{ij,\,\mathrm{nc}} = 0. \tag{A21}$$

Suppose that $\partial_0\tilde{V}_2^{ij,\,\mathrm{nc}} \neq 0$. Then these equations will imply that it is a constant $=K$, say. Hence

$$\tilde{V}_2^{ij,\,\mathrm{nc}} = x_0 K + C(x_1, x_2, x_3). \tag{A22}$$

Now the two terms on the right-hand side of this equation can be separately Lorentz-invariant, viz.,

$$x_0 K \equiv x\cdot pK, \tag{A23}$$

$$C(x_1, x_2, x_3) \equiv C((x\cdot p)^2 - x^2). \tag{A24}$$

Consequently the boundary condition $\tilde{V}_2^{ij,\,\mathrm{nc}} \to 0$ as $|\vec{x}| \to \infty$ does *not* require $\tilde{V}_2^{ij,\,\mathrm{nc}}$ to vanish identically for $x^2<0$. Hence V_2^{ij} may possess a noncausal part.

If $\partial_0\tilde{V}_2^{ij,\,\mathrm{nc}}=0$, then $\tilde{V}_2^{ij,\,\mathrm{nc}}$ depends on x_1, x_2, and x_3 only, i.e., it is a function of $(x\cdot p)^2 - x^2$. Hence again the boundary condition does *not* imply that $\tilde{V}_2^{ij,\,\mathrm{nc}}$ vanishes identically for $x^2<0$ and consequently V_2^{ij} may have a noncausal component.

Finally, one can verify that these results are consistent with the equations arising from the remaining choices for μ and ν. In particular we note that $\mu=\nu=0$ gives, on using (A18) and $\tilde{V}_1^{ij,\,\mathrm{nc}}=0$, the equation

$$\vec{\nabla}^2\tilde{V}_2^{ij,\,\mathrm{nc}} = 0. \tag{A25}$$

On the other hand, Meyer and Suura[8] identify the noncausal part in $\tilde{V}_2^{ij}$ as ($\vec{p}=0$; $x^2<0$):

$$\tilde{V}_2^{ij,\,\mathrm{nc}} = 2i\epsilon^{ijk}\Gamma_k|\vec{x}|^{-1}, \tag{A26}$$

where Γ_k is a constant. Clearly this is a solution of (A25).

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$$\psi_3^{ij}(u,s)=\frac{i\pi}{2} f^{ijk}\lambda_k[\delta(u+p)-\delta(u-p)]\delta(s-1) + \frac{\pi}{2} d^{ijk}\lambda_k[\delta(u+p)+\delta(u-p)]\delta(s-1),$$

whereas for $V_4^{ij}=0$ we have $\psi_4^{ij}=0$. Thus in this model the asymptotic conditions $\lim_{s\to\infty}\psi_k^{ij}(u,s)=0$ and $\lim_{s\to\infty} s\psi_k^{ij}(u,s)=0$ are satisfied and consequently the corresponding causality sum rules (3.2)–(3.4) are valid. Since in this free-quark model field theory direct and Z diagrams — but not class II states — are included the validity of the asymptotic conditions indicates that class II states are perhaps not essential for the validity, in general, of the asymptotic conditions or the corresponding causality sum rules. A firm conclusion must, however, hinge on calculations in a more realistic field theory.

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