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Electromagnetism as a strong interaction*

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In gauge theories of strong, weak, and electromagnetic interactions, the fine-structure constant (α) is a lower bound on the strength of weak and strong couplings. This opens the possibility of physical systems wherein all fundamental couplings are large, and α is a low-energy phenomenon of the system (essentially a collective property of the strong interactions). We construct such a model. At present experimental energies, it agrees with standard (hadron-modified) electrodynamics, while revealing a strongly coupled photon in the extreme asymptotic region.

I. INTRODUCTION

One of the great puzzles of particle physics is the apparent existence of hierarchies of interaction strengths, e.g., $\alpha = e^2/4\pi \simeq 1/137$, while $F = g_{\rho\pi\pi}{}^2/4\pi \simeq 2.5$, etc. It is an old dream that these forces are related in some way. For example, it has recently been proposed[1] that α is the fundamental constant, while F arises from α because the strong interactions are infrared unstable. Here we explore much the opposite hypothesis—that all fundamental couplings are of order F, while the fine-structure constant appears as a low-energy collective phenomenon of the system.

We explicitly construct and solve such a model. Our results can be conveniently and colorfully summarized in terms of an "effective" fine-structure constant $\alpha_{\text{eff}}(t)$. For (energies)^2 $t < e^{1/\alpha}\,\text{GeV}^2$, $\alpha_{\text{eff}}(t) \simeq \alpha$, while at extreme asymptotic energies there is a logarithmic approach to $\alpha_{\text{eff}}(t) \approx F$. In the language of the renormalization group, it appears that the fine-structure constant has an ultraviolet-stable fixed point at a value comparable with typical strong couplings.

Because the dynamics of the models is so unorthodox (at high energies), we carefully checked it against experiment. We were particularly concerned that loops ($g-2$, Lamb shift, etc.) would reflect the growth of α_{eff}. Such turns out not to be the case, the deviations of the model from conventional electrodynamics being of the order expected for hadronic corrections. We conclude that our new viewpoint is in fact viable.

II. CHARGE AND THE FINE-STRUCTURE CONSTANT

Our first important observation is that in gauge theories of strong, weak, and electromagnetic interactions, the fine-structure constant is a lower bound on weak and strong (in fact all other) couplings. The reader is familiar with this in a simple form in the Weinberg-Salam theory of weak and electromagnetic interactions.[2] There

$$e = gg'/(g^2+g'^2)^{1/2} \le g \text{ or } g'. \tag{2.1}$$

This is in fact a general characteristic of gauge theories of vector mesons. The true photon is a linear combination of the neutral vector mesons in the system, including the "bare" photon, and each mixing results in a progressively smaller e relative to the other couplings of the system. In M models[3,4,5] for example, the bare photon mixes not only with B but also with the bare ρ, ω, and ϕ. Relations and conclusions similar to (2.1) are borne out there. Here we want to discuss a general formalism for calculating e in an aggregate of many vector bosons.

The charge operator can be expressed as a linear combination of the group generators F_i:

$$Q = \sum_i a_i F_i, \tag{2.2}$$

where, in general, nonvanishing a_i's are of order one (since eigenvalues of Q are chosen to be integral, or third-integral, etc.). The sum in (2.2) is over neutral F_i's. For example, if $\{F_i\}$ are the generators of SU(3), it is conventional for only a_3 and a_8 to be nonzero.

Once Q is specified we can calculate e as a function of the other couplings.[4] The covariant momentum is

$$\mathcal{P}^\mu = P^\mu + \sum_i g_i F_i V_i^\mu, \tag{2.3}$$

where $\{g_i\}$ is the set of coupling constants for all the vector fields of the system. Our task is to rearrange $\mathcal{P}^\mu$ by an orthogonal transformation on $\{V_i\}$ (and therefore on $\{g_i F_i\}$) so that

$$\mathcal{P}^\mu = P^\mu + e\gamma^\mu Q + \cdots. \tag{2.4}$$

γ^μ, the true photon, is one of the $V_i^{\mu\prime}$ resulting from transforming $\{V_i\}$ so that their mass matrix is diagonal. eQ is the corresponding $(gF)'_i$, so we can write

$$eQ=\sum_i c_i g_i F_i, \quad \sum_i c_i^2=1. \tag{2.5}$$

Comparison of (2.2) and (2.5) gives $c_i = a_i e/g_i$, and the normalization condition then implies

$$\frac{1}{e^2}=\sum_i \frac{a_i^2}{g_i^2}. \tag{2.6}$$

Equation (2.6) is a fundamental result of this section. Note that it agrees with the special case (2.1), where $a_B = a_{A_3} = 1$ ($a_{A_1} = a_{A_2} = 0$). In actual practice, of course, the mixing discussed here parallels the spontaneous breakdown of the system. For simplicity we will continue our discussion with all nonvanishing $a_i = 1$, but generalization will be immediate.

How is the fine-structure constant to be made small in such a model? The *conventional* method is to have the bare photon coupling (say g_0) much smaller than all the others:

$$\frac{1}{e^2}=\frac{1}{g_0^2}+\sum_{n>0}\frac{1}{g_n^2}, \quad e^2 \simeq g_0^2. \tag{2.7}$$

To make sure that the sum over other couplings does not upset this, one either truncates the sum, or, if an infinite number of vectors are present, one needs demand rapid convergence of the sum, e.g., $g_n^2 \sim n^2$.

It is curious that if, e.g., all $g_n = g$, and there were an infinite number, then $\alpha = 0$: Electromagnetism cannot couple to such a system. This is a springboard for our new hypothesis on the origin of α. Suppose that α is small *because* the g_n's do *not* increase so rapidly as $g_n^2 \sim n^2$. Suppose in fact that

$$g_n^2/4\pi \to F n^{1+\alpha/F} \text{ as } n\to\infty. \tag{2.8}$$

This is an unorthodox but consistent solution for the system, having the ordinary value for the fine-structure constant, but with *no* fundamental small couplings. Other similar solutions are possible: In particular, the same effect is obtained if all $g_n = g$ and there are $\sim 1/\alpha$ vector mesons in the mixing. This possibility will be reconsidered at the end of the paper, but we will focus most of our attention on the solution of the definite form (2.8).

III. A MODEL

We believe that many models are possible. For simplicity, we will discuss an M model[3,4,5] with an infinite number of vector mesons. The model is schematized in Fig. 1.[6] Referring to that figure, ψ_n are hadronic fermions, ψ_0 leptonic fermions, V_n hadronic vector bosons, and V_0 the bare photon and weak bosons (from now on we use subscripts $l, m, n = 1, 2, \ldots$ and $i, j, k = 0, 1, 2, \ldots$). M_n are the connecting scalars which mediate between leptonic and hadronic worlds. ϕ is an extra Higgs field (like Weinberg's), useful in the non-Abelian case to split $W^\pm$, Z, etc. from γ.

For simplicity we will first take Abelian groups, but internal symmetry can be addended trivially. Explicitly, the Lagrangian is

$$\mathcal{L}=\sum_i\left[-\tfrac{1}{4}F^i_{\mu\nu}F^{\mu\nu}_i+\bar\psi_i(i\not{D}-m_i)\psi_i\right] +\sum_n\left[|D_\mu M_n|^2+\tfrac{1}{2}m_n^2|M_n|^2-\tfrac{1}{2}\lambda_n^2|M_n|^4\right], \tag{3.1}$$

where each V_i is the gauge field of a U(1) subgroup ($F^i_{\mu\nu} \equiv \partial_\mu V^i_\nu - \partial_\nu V^i_\mu$). Since there is only one V_0, ϕ is omitted. The covariant derivatives are given by

$$\begin{aligned} D_\mu\psi_i &= (\partial_\mu + ig_i V^i_\mu)\psi_i, \\ D_\mu M_n &= [\partial_\mu + i(g_n V^n_\mu - g_0 V^0_\mu)]M_n. \end{aligned} \tag{3.2}$$

g_0 is of order 1, and [using (2.6)] we find

$$g_n^2 \approx g_1^2 n^{1+1/[g_1^2(1/e^2-1/g_0^2)]}. \tag{3.3}$$

Here we have been more accurate than in (2.8). The gauge symmetries are broken by $\langle M_n\rangle = \kappa_n/\sqrt{2}$, so $M_n \to (\kappa_n + M_n')/\sqrt{2}$, where κ_n and M_n' are real. Eliminating terms in $\mathcal{L}$ linear in M_n' gives $\lambda_n = m_n/\kappa_n$, and m_n is the mass of M_n'. The mass matrix μ^2_{ij} for the V_i is then given by

$$\begin{aligned} \mu^2_{00} &= g_0^2\sum \kappa_l^2, \\ \mu^2_{0n} &= \mu^2_{n0} = -g_0 g_n \kappa_n^2, \\ \mu^2_{mn} &= \delta_{mn} g_n^2\kappa_n^2. \end{aligned} \tag{3.4}$$

The eigenvalues μ^2_i of the matrix μ^2 are the roots of its characteristic polynomial

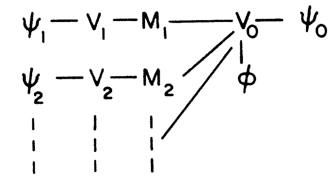

FIG. 1. The model.

$$\det(s-\mu^2)=\prod_m (s-g_m{}^2\kappa_m{}^2)\left[\left(s-g_0{}^2\sum_n \kappa_n{}^2\right)-\sum_n \frac{(-g_0 g_n \kappa_n{}^2)^2}{s-g_n{}^2\kappa_n{}^2}\right]. \tag{3.5}$$

After combining the two summations the determinant is

$$\prod_m (s-g_m{}^2\kappa_m{}^2)s\left(1-g_0{}^2\sum_n \frac{\kappa_n{}^2}{s-g_n{}^2\kappa_n{}^2}\right)=\prod_m (s-g_m{}^2\kappa_m{}^2)sg_0{}^2\left[\frac{1}{g_0{}^2}+\sum_n \frac{1}{g_n{}^2}-\sum_n\left(\frac{\kappa_n{}^2}{s-g_n{}^2\kappa_n{}^2}+\frac{1}{g_n{}^2}\right)\right]$$
$$=\prod_m (s-g_m{}^2\kappa_m{}^2)s\frac{g_0{}^2}{e^2}\left(1-e^2 s\sum_n \frac{1}{g_n{}^2(s-g_n{}^2\kappa_n{}^2)}\right). \tag{3.6}$$

We have used (2.6) in the last identity. This gives $\mu^2{}_0=0$ as the root for the photon; $\{\mu^2{}_n\}$ are the roots of the function

$$A(s)\equiv 1-e^2 s\sum_n \frac{1}{g_n{}^2(s-g_n{}^2\kappa_n{}^2)}. \tag{3.7}$$

We analyze this by breaking the sum into three terms: n less than, equal to, and greater than m, where m is the integer for which $g_m{}^2\kappa_m{}^2$ is closest to s. We then approximate the three terms by $g_n{}^2\kappa_n{}^2$ much less than, approximately equal to, and much greater than s, respectively:

$$A(s)\approx 1-e^2 s\sum_1^{m-1}\frac{1}{g_n{}^2 s}-e^2 s\frac{1}{g_m{}^2(s-g_m{}^2\kappa_m{}^2)}-e^2 s\sum_{m+1}^{\infty}\frac{1}{-g_n{}^4\kappa_n{}^2}$$
$$\approx 1-e^2\left(\frac{1}{e^2}-\frac{1}{g_0{}^2}\right)(1-m^{-e^2/g_1{}^2})-\frac{e^2\kappa_m{}^2}{s-g_m{}^2\kappa_m{}^2}+\frac{e^2}{a}m^{-e^2/g_1{}^2}. \tag{3.8}$$

Equation (3.3) was used in the last approximation. We have also assumed $g_n{}^2\kappa_n{}^2\sim n^a$ $(a>0)$, e.g., $a=1$ is reasonable for V_n as daughters of linear Regge trajectories. Thus for a reasonably greater than zero,

$$A(s)\approx\left(\frac{e^2}{g_0{}^2}+m^{-e^2/g_1{}^2}\right)-\frac{e^2\kappa_m{}^2}{s-g_m{}^2\kappa_m{}^2}. \tag{3.9}$$

Our result is then that

$$\mu^2{}_m\approx g_m{}^2\kappa_m{}^2+e^2\kappa_m{}^2\Big/\left(\frac{e^2}{g_0{}^2}+m^{-e^2/g_1{}^2}\right). \tag{3.10}$$

Therefore, $\mu^2{}_m\approx g_m{}^2\kappa_m{}^2+e^2\kappa_m{}^2$ until m is of the order of e^{137}, and $\mu^2{}_m\approx g_m{}^2\kappa_m{}^2+g_0{}^2\kappa_m{}^2$ for m above the order of 137^{137}.

The only remaining parameters to be determined are the couplings of the diagonalized vector mesons V'_i. Since $V'_i=\sum_j c_{ij}V_j$, $g_i F_i V_i$ in (2.3) becomes $\sum_j (g_i c_{ji})F_i V'_j$ (c_{ij} is orthogonal), so $g_i c_{ji}$ is the coupling of V'_j to ψ_i (the coupling to M_n is $g_n c_{jn}-g_0 c_{j0}$). Since we also have

$$\sum_{i,j}\mu^2{}_{ij}V_i\cdot V_j\equiv\sum_i \mu^2{}_i V'_i\cdot V'_i,$$

then

$$\sum_j \mu^2{}_{ij}c_{kj}=\mu^2{}_k c_{ki};$$

thus c_{ij} is the jth component of the ith normalized eigenvector of $\mu^2{}_{ij}$. From (3.4) we then have the eigenvector equations

$$\left(g_0{}^2\sum_l \kappa_l{}^2\right)c_{k0}-\sum_n g_0 g_n\kappa_n{}^2 c_{kn}=\mu^2{}_k c_{k0},$$
$$-g_0 g_n\kappa_n{}^2 c_{k0}+g_n{}^2\kappa_n{}^2 c_{kn}=\mu^2{}_k c_{kn}. \tag{3.11}$$

The first equation is redundant since we already have the eigenvalue condition. This is found by solving the second equation for c_{kn} and plugging into the first.[7] The second equation gives

$$c_{kn}=\frac{-g_0 g_n\kappa_n{}^2}{\mu^2{}_k-g_n{}^2\kappa_n{}^2}c_{k0}. \tag{3.12}$$

Notice that this gives $c_{0i}=e/g_i$ as in Sec. II; it also shows that the M_n are neutral: $g_n c_{0n}-g_0 c_{00}=0$. After normalizing the c_{ij} [using (3.7) for $s=\mu^2{}_m$ and (2.6)] we obtain

$$c_{m0}=\left(g_0{}^2\mu^2{}_m\sum_n\frac{\kappa_n{}^2}{(\mu^2{}_m-g_n{}^2\kappa_n{}^2)^2}\right)^{-1/2}$$
$$=\left(-\frac{g_0{}^2}{e^2}+g_0{}^2\mu_m{}^4\sum_n\frac{1}{g_n{}^2(\mu^2{}_m-g_n{}^2\kappa_n{}^2)^2}\right)^{-1/2}. \tag{3.13}$$

Using (3.10) and the method used in deriving (3.10), the $n=m$ term in the second expression above dominates, and we find the following set of coupling constants:

$g_0 c_{00}=e$ (photon to lepton),

$g_n c_{0n}=e$ (photon to nth quark),

$$g_0 c_{n0}\approx\frac{e^2}{g_n(e^2/g_0{}^2+n^{-e^2/g_1{}^2})} \tag{3.14}$$

(nth vector meson to lepton),

$g_n c_{nn}\approx -g_n$ (nth vector meson to nth quark),

$$g_n c_{mn}\approx\frac{e^2}{g_m(e^2/g_0{}^2+m^{-e^2/g_1{}^2})}\,\frac{g_n{}^2\kappa_n{}^2}{g_n{}^2\kappa_n{}^2-g_m{}^2\kappa_m{}^2}$$

($m\neq n$; mth vector meson to nth quark).

Clearly, γ couples universally with e. Notice as in (3.10) that instead of the e^2 expected in conventional vector-dominance models[8] there occurs an

$$e^2/(e^2/g_0^{\,2}+m^{-e^2/g_1^{\,2}})$$

which approximates e^2 for all but extremely large m (i.e., $m \gtrsim e^{1/\alpha}$), where it asymptotically reaches $g_0^{\,2}$.

IV. TREE GRAPHS

The S-matrix elements for fermion-fermion scattering take on a simpler form if they are calculated in terms of the undiagonalized mass matrix[5]: in the Born approximation the invariant amplitude for scattering between the ith and jth fermions is

$$\begin{aligned} M_{ij} &= g_i g_j \left(\frac{1}{t-\mu^2}\right)_{ij} \\ &= g_i g_j \left(c^{-1}\frac{1}{t-\mu^{2\prime}}c\right)_{ij} \\ &= \sum_k (g_i c_{ki})(g_j c_{kj})\frac{1}{t-\mu^2_k}, \end{aligned} \tag{4.1}$$

where $\mu^{2\prime}$ is the diagonalized μ^2. Using (3.4) and (3.7),

$$\begin{aligned} M_{00} &= g_0^{\,2}\prod (t-g_n^{\,2}\kappa_n^{\,2})/\det(t-\mu^2) \\ &= e^2/tA(t), \\ M_{0n} &= g_0 g_n(-g_0 g_n\kappa_n^{\,2})\prod_{m\neq n}(t-g_m^{\,2}\kappa_m^{\,2}) \\ &= eg_n(-eg_n\kappa_n^{\,2})/(t-g_n^{\,2}\kappa_n^{\,2})tA(t), \\ M_{mn} &= g_m g_n(-g_0 g_m\kappa_m^{\,2})(-g_0 g_n\kappa_n^{\,2}) \\ &\quad\times\prod_{l\neq m,n}(t-g_l^{\,2}\kappa_l^{\,2})/\det(t-\mu^2) \\ &= g_m g_n\frac{(-eg_m\kappa_m^{\,2})(-eg_n\kappa_n^{\,2})}{(t-g_m^{\,2}\kappa_m^{\,2})(t-g_n^{\,2}\kappa_n^{\,2})tA(t)} \\ M_{nn} &= g_n^{\,2}\prod_{m\neq n}(t-g_m^{\,2}\kappa_m^{\,2}) \\ &\quad\times\left[\left(t-g_0^{\,2}\sum_l \kappa_l^{\,2}\right)-\sum_{l\neq n} g_0^{\,2}\kappa_l^{\,2}\frac{g_l^{\,2}\kappa_l^{\,2}}{t-g_l^{\,2}\kappa_l^{\,2}}\right]\Big/\det(t-\mu^2) \\ &= g_n^{\,2}/(t-g_n^{\,2}\kappa_n^{\,2}) \\ &\quad+g_n^{\,2}(-eg_n\kappa_n^{\,2})^2/(t-g_n^{\,2}\kappa_n^{\,2})^2 tA(t). \end{aligned} \tag{4.2}$$

For $i=j$ the same expression for the s channel is added to give the total amplitude. These agree with the S-matrix elements found by treating the off-diagonal part of μ^2 as part of the interaction Lagrangian, since all the V_0-V_n interactions can be expressed in terms of a modified photon propagator $e^2/g_0^{\,2}tA(t)$ (see Fig. 2). They also have the same form as in conventional vector-dominance models, differing only in that

$$\begin{aligned} \frac{e^2}{tA(t)} &= \sum_i (g_0 c_{i0})^2\frac{1}{t-\mu^2_i} \\ &\simeq \frac{e^2}{t}+\sum_n (e^2/g_n)^2\frac{1}{t-\mu^2_n} \end{aligned} \tag{4.3}$$

only at "low energies" [i.e., $(\text{energy})^2=t$ less than some $g_n^{\,2}\kappa_n^{\,2}$ for which $n\ll e^{1/\alpha}$]. At very high energies [t greater than some $g_n^{\,2}\kappa_n^{\,2}$ for $n\gg(1/\alpha)^{1/\alpha}$] there are significant differences, since g_0c_{n0} increases from e^2/g_n to $g_0^{\,2}/g_n$ for very large n.

For example, the form factor [from M_{0n} in (4.2)] is $-g_n^{\,2}\kappa_n^{\,2}/(t-g_n^{\,2}\kappa_n^{\,2})A(t)$, as compared with the form factor $-\mu^2/(t-\mu^2)$ given in conventional vector-dominance models (μ^2 = meson bare-mass $=g_n^{\,2}\kappa_n^{\,2}$) by a photon coupling to a hadron through a meson. These two expressions are approximately equal for low energies, where $A(t)\simeq 1$. At high energies $A(t)\to e^2/g_0^{\,2}$ (see below) so the form factor increases by a factor of $g_0^{\,2}/e^2$; the photon is coupling with strength g_0 instead of e.

The conclusions of the previous paragraph are easily seen in detail. For spacelike $t=-T<0$ [by (3.7) and (3.3)],

$$\begin{aligned} A(-T) &= \frac{e^2}{g_0^{\,2}}+e^2\sum\frac{\kappa_n^{\,2}}{T+g_n^{\,2}\kappa_n^{\,2}} \\ &\approx \frac{e^2}{g_0^{\,2}}+\frac{e^2}{g_1^{\,2}}\int_0^\infty dn\frac{n^{a-1-e^2/g_1^{\,2}}}{n^a+x}, \end{aligned} \tag{4.4}$$

where $x=T/g_1^{\,2}\kappa_1^{\,2}$ and $g_n^{\,2}\kappa_n^{\,2}\sim n^a$. Changing variables, $n^a=xv$,

$$A(-T)=\frac{e^2}{g_0^{\,2}}+\frac{e^2}{g_1^{\,2}a}x^{-e^2/g_1^{\,2}a}\int_0^\infty dv\frac{v^{-e^2/g_1^{\,2}a}}{v+1}. \tag{4.5}$$

Then, using Ref. 9,

$$\begin{aligned} A(-T) &= \frac{e^2}{g_0^{\,2}}+\frac{e^2}{g_1^{\,2}a}x^{-e^2/g_1^{\,2}a}\pi\csc(\pi e^2/g_1^{\,2}a) \\ &\approx \frac{e^2}{g_0^{\,2}}+\left(\frac{T}{g_1^{\,2}\kappa_1^{\,2}}\right)^{-e^2/g_1^{\,2}a}. \end{aligned} \tag{4.6}$$

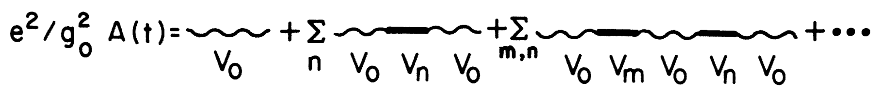

FIG. 2. Modified photon propagator.

For the n for which $g_n{}^2\kappa_n{}^2$ is closest to T, this gives

$$A(-T)\approx \frac{e^2}{g_0{}^2}+n^{-e^2/g_1{}^2},\quad T\approx \mu^2{}_n, \tag{4.7}$$

which is the same factor appearing in (3.10) and (3.14). Thus for T less than order $e^{137}\, g_1{}^2\kappa_1{}^2$ the effective coupling is e [$A(0)=1$ by (3.7)], but it increases to g_0 as T becomes infinite: $-e^2/TA(-T)$ goes from $\approx -e^2/T$ to $\approx -g_0{}^2/T$.

The anomalous asymptotic behavior of the elastic scattering amplitudes, as well as that of the vector masses and coupling constants, can all be summarized as follows [from (3.10), (3.14), and (4.7)]:

$$\mu^2{}_n\approx g_n{}^2\,\kappa_n{}^2+e^2_{\text{eff}}\,(n)\kappa_n{}^2, \tag{4.8}$$

$$g_0 c_{n0}\approx e^2_{\text{eff}}\,(n)/g_n, \tag{4.9}$$

$$-e^2/TA(-T)\approx -e^2_{\text{eff}}\,(n[T])/T, \tag{4.10}$$

where $n[T]$ is the inverse of $T=g_n{}^2\kappa_n{}^2$ (picking the integer n for which T is closest to $g_n{}^2\kappa_n{}^2$), and where

$$e^2_{\text{eff}}\,(n)\approx e^2/(e^2/g_0{}^2+n^{-e^2/g_1{}^2}). \tag{4.11}$$

A relation similar to (4.9) holds for $g_n c_{mn}$ in (3.14), and (4.10) affects the scattering amplitudes (4.2) by replacing $A(t)$ with 1 and e with e_{eff}, for negative t in *all* amplitudes. The behavior of $e_{\text{eff}}(n)$ [by (4.11)] is illustrated in Fig. 3.

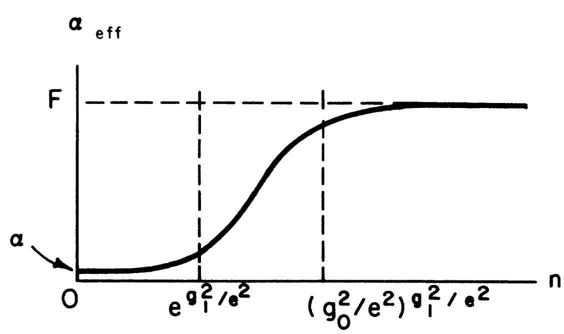

FIG. 3. Effective fine-structure constant.

We find similar high-energy behavior (for the same reason) in lepton-antilepton total cross section: Using the optical theorem, with widths given by the prescription $s\to se^{i\theta}$ (except for the photon) the cross section is

$$\sigma=\frac{s+2m^2}{[s(s-4m^2)]^{1/2}}\left[-\operatorname{Im}\left(g_0{}^2\left(\frac{1}{se^{i\theta}-\mu^2}\right)_{00}-e^2\,\frac{1}{se^{i\theta}}\right)\right]$$

$$\simeq g_0{}^2 s\,\sin\theta\sum c_{n0}{}^2\,\frac{1}{s^2-2\mu^2{}_n s\cos\theta+\mu_n{}^4}$$

$$\to\frac{g_0{}^2}{s}\sin\theta\sum c_{n0}{}^2=\frac{g_0{}^2}{s}\sin\theta(1-c_{00}{}^2)$$

$$=\frac{g_0{}^2-e^2}{s}\sin\theta \tag{4.12}$$

for asymptotic s ($\gg\alpha^{-1/\alpha}\,\text{GeV}^2$). For less than these asymptotic energies the cross section is of order e^4, as observed. Again this huge asymptotic limit is unexpected in conventional models.

It is an amusing check on the asymptotic form (4.12) that, with the help of (2.6), it can be rewritten

$$\sigma_T\to\frac{g_0{}^2-e^2}{s}\sin\theta=\frac{e^4\sin\theta}{s}\;\frac{1}{1-e^2\sum\limits_1^\infty \frac{1}{g_n{}^2}}\left(\sum_1^\infty\frac{1}{g_n{}^2}\right) \tag{4.13}$$

Thus, in a conventional theory (with, say, $g_0\sim e$ and $g_1\neq 0$, all others zero) σ_T would be $O(e^4)$ for all energies.

V. HIGHER-ORDER CORRECTIONS

Because of the anomalous high-energy behavior of our model it seems particularly important to study loop corrections, which are sensitive to all energies. As examples of higher-order processes we will calculate the lepton anomalous magnetic moment, Lamb shift, and electromagnetic mass differences.

To calculate the *lepton anomalous magnetic moment,* we need to evaluate the part of the integral (see Fig. 4; $p'^2=p^2=m^2$)

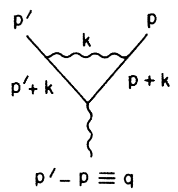

FIG. 4. Relevant graph for leptonic $g-2$ and Lamb shift.

$$g^2 \int i \frac{d^4k}{(2\pi)^4} \gamma^\nu \frac{(\not p' + \not k + m)\gamma^\mu(\not p + \not k + m)}{[(p'+k)^2 - m^2][(p+k)^2 - m^2]} \gamma^\sigma \frac{-g_{\nu\sigma}}{k^2-\mu^2}$$
$$\equiv [F_1(q^2)-1]\gamma^\mu + F_2(q^2) i\sigma^{\mu\nu} q_\nu/2m, \tag{5.1}$$

which gives $F_2(0)$ (Feynman gauge). The result is (in units $m=1$)

$$F_2(0) = \frac{1}{2\pi}\left(\frac{g^2}{4\pi}\right)\left[1 - 2\mu^2 + \mu^2(\mu^2-2)\ln\mu^2 - \mu^2(\mu^4 - 4\mu^2 + 2)\frac{1}{(\mu^4-4\mu^2)^{1/2}} \times \ln\left(\frac{\mu^2 + (\mu^4-4\mu^2)^{1/2}}{\mu^2 - (\mu^4-4\mu^2)^{1/2}}\right)\right]$$
$$= \begin{cases} \dfrac{1}{2\pi}\left(\dfrac{g^2}{4\pi}\right) \text{ for } \mu^2 = 0 \\ \dfrac{1}{2\pi}\left(\dfrac{g^2}{4\pi}\right)\dfrac{2}{3\mu^2}\left[1 + O\left(\dfrac{\ln\mu^2}{\mu^2}\right)\right] \text{ for } \mu^2 \gg 1. \end{cases} \tag{5.2}$$

Using (3.10) and (3.14) to sum the hadronic contribution, we get

$$F_2^{\text{had}}(0) \approx \sum \frac{1}{2\pi}\left(\frac{(e^2/g_n)^2}{4\pi}\right)\frac{2}{3g_n^2\kappa_n^2}$$
$$= \frac{e^4}{12\pi^2}\sum \frac{1}{g_n^4\kappa_n^2}. \tag{5.3}$$

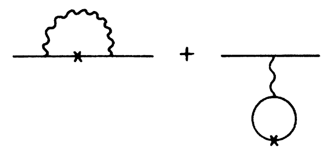

FIG. 5. Other relevant graphs for Lamb shift.

Since $\sum 1/g_n^4\kappa_n^2$ converges rapidly [the terms for which the

$$e^2/g_0^2 + n^{-e^2/g_1^2}$$

factor in (3.14) is important are negligible], the whole sum is of the same order as the first term, which is the hadronic contribution found in standard treatments (due to ρ, ω, ϕ mesons etc.).

The graphs contributing to lowest-order *Lamb shift*[10] hadronic corrections are shown in Fig. 5. These are the graphs for lowest-order Lamb shift with photons replaced by vector mesons. The hadronic vacuum-polarization contribution is found by replacing the photon propagator and coupling by those of the vector mesons:

$$e^2/\vec{q}^2 \to (e^2/g_n)^2/(\vec{q}^2 + \mu^2{}_n).$$

This adds to the photon's correction to the external potential $(\alpha/15\pi)(\vec{q}^2/m^2)(e^2/\vec{q}^2)$ the term

$$\frac{\alpha}{15\pi}\frac{\vec{q}^2}{m^2}\frac{\vec{q}^2}{\vec{q}^2+\mu^2{}_n}\frac{e^2}{g_n^2}\frac{e^2}{\vec{q}^2}. \tag{5.4}$$

Since $\vec{q}^2 \ll \mu^2{}_n$ in an atom, the sum over all vector mesons is $\sim\sum 1/g_n^2 g_n^2\kappa_n^2$. This again converges rapidly, and so is of the size expected from standard hadronic contributions.[11] The vertex correction includes an anomalous magnetic moment contribution, treated above, and a contribution from $F_1'(0)$

$$F_1(q^2) \approx Z_1^{-1} + q^2 F_1'(0).$$

That part of the integral for Fig. 4 gives

$$F_1'(0) = -\frac{1}{48\pi}\left(\frac{g^2}{4\pi}\right)\left[2\mu^2 + 23 + \frac{12}{\mu^2-4} - (\mu^4 + 10\mu^2 + 4)\ln\mu^2 + \left(\mu^6 + 8\mu^4 - 18\mu^2 - 24 - \frac{24}{\mu^2-4}\right)\frac{1}{(\mu^4-4\mu^2)^{1/2}}\ln\left(\frac{\mu^2+(\mu^4-4\mu^2)^{1/2}}{\mu^2-(\mu^4-4\mu^2)^{1/2}}\right)\right]$$
$$= -\frac{1}{\pi}\left(\frac{g^2}{4\pi}\right)\left[\frac{1}{6}\frac{\ln\mu^2}{\mu^2} - \frac{79}{72}\frac{1}{\mu^2} + O\left(\frac{\ln\mu^2}{\mu^4}\right)\right] \quad \text{for } \mu^2 \gg 1. \tag{5.5}$$

This equation is not applicable for $\mu^2 = 0$, where infrared divergences necessitate a partly nonrelativistic treatment, as for the photon in lowest-order Lamb shift. The leading hadronic contribution is $\sim\sum(\ln g_n^2\kappa_n^2)/g_n^4\kappa_n^2$, again rapidly convergent: All hadronic Lamb-shift corrections in this model agree with the conventional theory.

To study *calculable electromagnetic mass dif-*

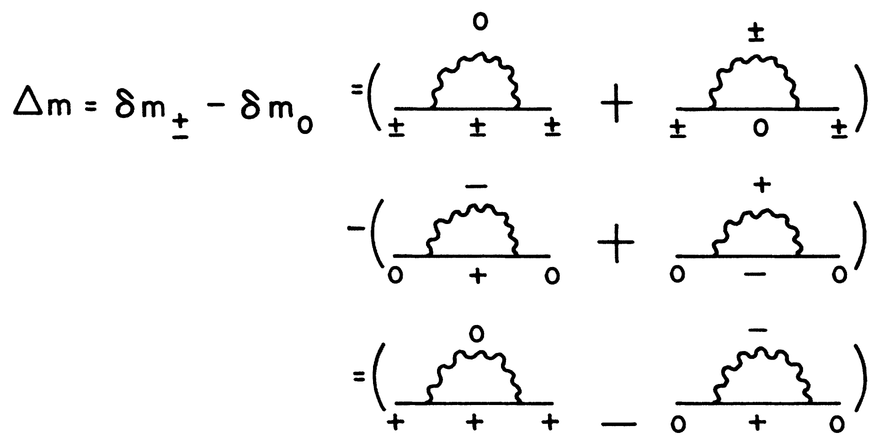

FIG. 6. Graphs for electromagnetic mass differences.

ferences, a higher symmetry than U(1) is needed: The simplest is SU(2). We modify the Lagrangian (3.1) by making each V_i an isovector and do the same for each fermion. The M_n become four-component (real) representations of SU(2)⊗ SU(2) [~SO(4)], most simply written as 2×2 matrices $M_n = M_n^0 + i\vec{\tau}\cdot\vec{M}_n$.[12] A real isovector $\vec{\phi}$ gives the weak vector bosons masses.[13] All isospin breaking is due to $\vec{\phi}$; since it couples directly only to V_0, hadronic isospin is only broken to order α. This group structure is not particularly physical, but provides a simple illustration of magnitudes expected in general.

The diagrams contributing to the mass difference between (any) charged and neutral fermions is given by Fig. 6. The integrals in general are of the form

$$g^2 \int i\,\frac{d^4q}{(2\pi)^4}\,\gamma^\mu\,\frac{(\not p - \not q)+m}{(p-q)^2-m^2}\,\gamma^\nu\,\frac{-g_{\mu\nu}}{q^2-\mu^2} \tag{5.6}$$

(with $p^2 = m^2$, m = fermion mass). This is found to equal ($m = 1$)

$$\frac{1}{2\pi}\left(\frac{g^2}{4\pi}\right)\left[\tfrac{3}{2}\ln\Lambda^2 + 1 + \tfrac{1}{2}\mu^2 - \tfrac{1}{4}\mu^4\ln\mu^2 + \mu^2(\tfrac{1}{4}\mu^4 - \tfrac{1}{2}\mu^2 - 2)\,\frac{1}{(\mu^4-4\mu^2)^{1/2}}\ln\left(\frac{\mu^2+(\mu^4-4\mu^2)^{1/2}}{\mu^2-(\mu^4-4\mu^2)^{1/2}}\right)\right]$$
$$= \begin{cases} \dfrac{1}{2\pi}\left(\dfrac{g^2}{4\pi}\right)(\tfrac{3}{2}\ln\Lambda^2+1) & \text{for } \mu^2 = 0 \\ \dfrac{1}{2\pi}\left(\dfrac{g^2}{4\pi}\right)[\tfrac{3}{2}\ln\Lambda^2 - \tfrac{3}{2}\ln\mu^2 + \tfrac{1}{4} + O(\ln\mu^2/\mu^2)] & \text{for } \mu^2 \gg 1. \end{cases} \tag{5.7}$$

We now specialize to the case of *calculable lepton mass differences*. Instead of using (3.14) we will examine the mass differences more carefully by using (4.2) and the identity

$$\ln\Lambda^2 - \ln\mu^2 = \int_0^{\Lambda^2} dx\,\frac{1}{\mu^2+x}\,. \tag{5.8}$$

Then we find, from (5.7),

$$\Delta m \approx \frac{1}{2\pi}\left(\frac{g_0^{\,2}}{4\pi}\right)\left\{\left[\tfrac{3}{2}\int_0^{\Lambda^2} dx\left(\left(\frac{1}{\mu^2+x}\right)_{00} - \frac{e^2/g_0^{\,2}}{x}\right) + \frac{1}{4} + \frac{e^2}{g_0^{\,2}}(\tfrac{3}{2}\ln\Lambda^2 + \tfrac{3}{4})\right] - \tfrac{3}{2}\int_0^{\Lambda^2} dx\left(\frac{1}{\tilde{\mu}^2+x}\right)_{00}\right\}. \tag{5.9}$$

This form now includes the sums over all vector mesons. Here $\tilde{\mu}^2$ is the charged vector-boson mass matrix. $\tilde{\mu}^2$ differs from μ^2 as given by (3.4) only in that $\tilde{\mu}^2{}_{00} = \mu^2{}_{00} + g_0^{\,2}\lambda^2$ owing to $\langle\vec{\phi}\rangle = \lambda\vec{e}_3$. The extra terms

$$\frac{e^2}{g_0^{\,2}}\left(-\tfrac{3}{2}\int_0^{\Lambda^2}\frac{dx}{x} + \tfrac{3}{2}\ln\Lambda^2 + \tfrac{3}{4}\right) \tag{5.10}$$

inside the square brackets are due to the photon contribution, to which (5.8) does not apply because $\mu^2{}_0 = 0$.

Using (3.7) and (4.2) and the analogous expressions for $\tilde{\mu}^2$ (found by the same method) we find

$$\Delta m \approx \frac{3}{16\pi^2} e^2 \left[\int_0^{\Lambda^2} dx \frac{e^2 \sum 1/g_n^2(x + g_n^2\kappa_n^2)}{1 - e^2 x \sum 1/g_n^2(x + g_n^2\kappa_n^2)} + (\ln \Lambda^2 + \tfrac{1}{2}) - \int_0^{\Lambda^2} dx \frac{1}{x[1 - e^2 x \sum 1/g_n^2(x + g_n^2\kappa_n^2)] + e^2\lambda^2} \right]. \tag{5.11}$$

We evaluate the integrals by expanding in e^2, treating $e^2\lambda^2$ ($\approx \tilde{\mu}^2{}_0$) as $O(1)$. The $\ln \Lambda^2$ terms can be seen to cancel in Δm before using (5.8) [then they are explicitly $(1/2\pi)(g_0{}^2/4\pi)\frac{3}{2}\ln \Lambda^2 - (1/2\pi) \times (g_0{}^2/4\pi)\frac{3}{2}\ln \Lambda^2$]. The zeroth-order part of the second integral gives $-\ln(e^2\lambda^2)$; combining this with the photon contribution gives the nonstrong part of Δm to be $\approx (3/16\pi^2)e^2(\ln e^2\lambda^2 + \frac{1}{2})$.

This is just the standard contribution to fermionic mass differences from the weak-boson system.[14] That is, we would obtain just this in a model with the same group structure and no hadronic V_n's, and g_0 replaced by e. [Note that, by the same method as for (3.10), $M_W{}^2 \simeq e^2\lambda^2$.] The order e^2 terms of the two integrals give terms of similar n dependence and opposite sign. The slowest-decreasing terms cancel, leaving terms which decrease too fast in n to affect the order in e^2[as, e.g., $\sum 1/g_n{}^2 = O(1/e^2)$ would]. Higher-order terms in e^2 in the two integrals also converge quickly, so (showing m dependence explicitly)

$$\frac{\Delta m}{m} = \frac{3\alpha}{4\pi}\{[\ln(e^2\lambda^2/m^2) + \tfrac{1}{2}] + O(\alpha)\}. \tag{5.12}$$

The mass difference is the sum of a nonstrong part,[14] which agrees with a conventional gauge theory of nonstrong interactions, plus a strong part, which is of the same order as in a standard vector-dominance model, $O(\alpha^2)$.

Using the same method for *hadronic* (quark) *electromagnetic mass differences,* we find that the $g_n{}^2/(t - g_n{}^2\kappa_n{}^2)$ term in $M_{nn} = g_n{}^2(1/(t-\mu^2))_{nn}$ cancels [using (4.2) for the equation analogous to (5.9)]. This is due to the fact that the matrix elements for charged boson exchange differ from those for neutrals in (4.2) only in that $tA(t)$ is replaced by $e^2\lambda^2 + tA(t)$ (this again follows by using the same steps as in Secs. III and IV for $\tilde{\mu}^2$ instead of μ^2). The other term in M_{nn} can be rewritten as

$$g_n{}^2(-eg_n\kappa_n{}^2)^2/(t - g_n{}^2\kappa_n{}^2)^2 tA(t)$$
$$= e^2/tA(t) - \frac{e^2(t - 2g_n{}^2\kappa_n{}^2)}{(t - g_n{}^2\kappa_n{}^2)^2 A(t)}. \tag{5.13}$$

The first term of (5.13) gives a contribution to the quark mass difference of the same form as the total lepton mass difference. Computations for the second term in (5.13) are similar to those for the leptons. We then have the nth quark mass difference

$$\frac{\Delta m_n}{m_n} = \frac{3\alpha}{4\pi}\left[\ln\left(\frac{e^2\lambda^2}{m_n{}^2}\right) + \frac{1}{2}\right] + \frac{3\alpha}{4\pi}\left(\frac{1}{r-1} - \frac{\ln r}{(r-1)^2}\right) + O(\alpha^2), \tag{5.14}$$

where $r \equiv g_n{}^2\kappa_n{}^2/e^2\lambda^2$.[15] Therefore, we again have an order-α mass difference, so the anomalous high-energy behavior of the model does not appear in *any* of the higher-order corrections considered, to within present experimental accuracy. Note that in M models, order-α mass-splittings can come both from strong (vector mass splits) and nonstrong interactions.[14]

VI. CONCLUSIONS

We have explicitly constructed and solved a simple model in which electromagnetism and the weak interactions can be considered as strong forces, arising in their known form only as "low-energy" phenomena. Although highly unconventional, the model agrees with electrodynamic phenomenology in present experimental ranges, and predicts deviations from conventional QED only at extreme asymptotic energies ($s > e^{1/\alpha}$ GeV^2).

Other models are possible which have nonstandard behavior at energies more accessible to experiment, e.g., a model with a finite number (of the order of $1/\alpha$) of vector bosons with the same size coupling. Such a model might be associated with a single large non-Abelian gauge group with only one coupling constant. In such a model, one expects deviation from conventional QED at energies comparable to α^{-1} GeV^2.

Many other vector-meson-aggregate models are possible. For example, consider Fig. 7, where extra vector mesons have been added "horizontally." This model, in distinction to the basic model of the paper, will have dipole form factors[5] for the hadronic fermions. It is con-

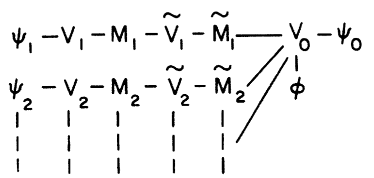

FIG. 7. Modified model with rapidly falling form factors.

venient to separate out our "vertical" model (which is supporting the unconventional interpretation of α) from many "horizontal" vectors in a chain. For such a chain of vectors V_n ($n=0, 1, 2, \ldots, N$), the form factors are easily calculated in hadron-lepton collisions:

$$g_0 g_N\left(\frac{1}{t-\mu^2}\right)_{0N} = e^2 \frac{\prod_{m=1}^{N} \mu^2{}_m}{\prod_{k=0}^{N}(t-\mu^2{}_k)} = \frac{e^2}{t} F(t), \tag{6.1}$$

$$F(t) = \prod_{m=1}^{N} \frac{\mu^2{}_m}{\mu^2{}_m - t}.$$

Thus, an N-link chain has N-pole form factors.[16]

In this way we are led to interpret "horizontal" or chain structure ($V_1, \tilde{V}_1, \ldots$) as *ordinary hadrons* (ρ, ρ', etc.), just as in Ref. 5. What then is the physical interpretation of the vertical structure ($V_2, V_3, \ldots$)? We are tempted to identify these with the recently discovered ψ particles.[17] The couplings of $V_2, V_3, \ldots$ to V_1 (hadrons) is quite *arbitrary* and can be controlled by a term of the form $\sum a_{mn}|M_m|^2|M_n|^2$ in the Lagrangian (3.1) (and implied loops). Hadronic widths can thus be adjusted by hand to the surprising narrowness. To get the leptonic widths correctly, the following kind of group classification is appropriate: Take each V_n as a nonet in a repeated SU(3) scheme. V_1 contains (ρ, ω, ϕ) as in standard M models, while V_2 contains ψ(3105), ψ(3695), and ψ(4100) as the three neutrals of Suzuki's scheme.[18] New higher-mass ψ's would recur as V_3, V_4, etc.

We are clear on the fact that our choice to interpret the V_m's ($m \geq 2$) as ψ's is arbitrary and not required: A choice of large a_{mn}'s would make the V_m's into more ordinary hadrons, like ρ'.

The $e \to g_0$ behavior at high energies is analogous to the renormalized coupling $\to$ bare coupling behavior of ordinary quantum electrodynamics at high energy.[19] (As mentioned above, such unconventional behavior is also like having an ultraviolet-fixed point in ordinary QED.) The mass-matrix diagonalization is a *zeroth*-order renormalization, using trees instead of loops. The effective coupling e_{eff} has a logarithmic behavior at low energies, analogous to the behavior of asymptotically free theories, but the situation is reversed, since e_{eff} *increases* with energy. It would be interesting to see if such tree renormalization has applications in other contexts. Higher-order charge renormalization in this model might also have interesting consequences: We query whether the model provides (or allows) a self-consistency condition (perhaps in the spirit of the finite QED of Johnson, Baker, and Willey[20]) for the actual calculation of the fine-structure constant. It is also an intriguing question whether such behavior can be found in more dynamical models (such as quark or dual models) where, as here, the photon is expected to mix with an infinite number of vector mesons.

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[6]The symmetry group is $G_0 \otimes G_1 \otimes G_2 \otimes \cdots$, where ψ_i and V_i transform under the subgroup G_i, M_n under $G_n \otimes G_0$, and ϕ under G_0.

[7]The ambiguity of $\sum \kappa_i{}^2 = \infty$ is resolved by treating the case where μ^2 is an $N \times N$ matrix, and since the resulting expressions are N-independent the limit $N \to \infty$ can trivially be taken.

[8]See, e.g., R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972), p. 82.

[9]*Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi *et al.* (McGraw-Hill, New York, 1953), Vol. 1, Eqs. 1.5(2) and 1.2(6).

[10]See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), p. 177.

[11]Similarly, the vacuum-polarization correction due to a photon propagator with a hadronic fermion loop is $\sim \sum 1/m_n{}^2$ owing to the $1/m^2$ factor for an electron: The convergence properties depend on the choice of

hadron masses m_n. For $m_n \sim n^a$ this requires $a > 1$. In any case, this choice is independent of the model since it depends only on the existence of charged fermions and so is also required in a more conventional theory. One could also choose the number of fermions to be finite and not affect any of the results of this paper.

[12]These satisfy $\tau_2 M^* \tau_2 = M$, as do 2×2 unitary matrices, so M stays of this form under $SU(2)\otimes SU(2)'$ transformations $M \to UMU'^{-1}$.

[13]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 28, 1494 (1972). Owing to a discrete symmetry on ϕ ($\phi \longleftrightarrow -\phi$), no counterterms $\bar\psi\psi\phi$ can appear in the Lagrangian because of renormalization.

[14]S. Weinberg, Phys. Rev. D 7, 2887 (1973).

[15]The expression in (5.14) is well defined for $r=1$ by taking the limit $r \to 1$. Approximation involving $\tilde\mu^2$ must be handled with care for $g_n{}^2\kappa_n{}^2 = e^2\lambda^2$, e.g., we then find that $\tilde\mu^2{}_n = g_n{}^2\kappa_n{}^2 + O(e)$ instead of the usual $\tilde\mu^2{}_n = g_n{}^2\kappa_n{}^2 + O(e^2)$. In standard vector-dominance models, this corresponds to doing degenerate perturbation theory instead of ordinary perturbation theory: The diagonal masses are $e^2\lambda^2$ and $g_n{}^2\kappa_n{}^2$, the off-diagonals are $-eg_n\kappa_n{}^2$. In such a case, the model does not have a distinct weak vector boson; instead, it has two bosons which act partially like weak bosons and partially like hadrons.

[16]We want to point out an interesting feature of this explicit form factor. Suppose there are an infinite number of vector mesons whose masses go like $\mu^2{}_n \sim n^\beta$. Then the form factors are finite only for $\beta > 1$. The case of $\beta = 1$ (daughters of linear trajectories) is just divergent, yielding for the leading piece

$$F(t) \simeq \exp\left(+t \sum_1^\infty \frac{1}{n}\right).$$

This divergent-Gaussian structure is just the kind of form factor one obtains in dual models. The solution to this problem is easy to see here: Allow $\beta > 1$ (curved trajectories). In fact, eliminating exponential growth at large $|t|$ requires $\beta \geq 2$.

[17]J. J. Aubert *et al.*, Phys. Rev. Lett. 33, 1404 (1974); J.-E. Augustin *et al.*, Phys. Rev. Lett. 33, 1406 (1974).

[18]The fermions ψ_1 are then "fourth triplet quarks": see M. Suzuki, Phys. Lett. 56B, 165 (1975).

[19]S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory* (Harper and Row, New York, 1961), p. 555.

[20]K. Johnson, M. Baker, and R. S. Willey, Phys. Rev. Lett. 11, 518 (1963); Phys. Rev. 136, B1111 (1964); K. Johnson, R. S. Willey, and M. Baker, *ibid.* 163, 1699 (1967); M. Baker and K. Johnson, *ibid.* 183, 1292 (1969); Phys. Rev. D 3, 2516 (1971); 3, 2541 (1971).