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Renormalization of dual models*

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A regularization and renormalization program for dual models in the critical dimension is discussed. The counterterms required for regularization are shown to be multiple insertions of zero-momentum dilatons. A heuristic summation of all such insertions leads to a slope and coupling-constant renormalization, and also to an intercept shift. The faults of the heuristic argument are discussed, and it is proved to lowest order and conjectured in general that the correct treatment leads to no intercept shift but only to renormalizations.

I. INTRODUCTION

In dual models, as in normal quantum field theory, amplitudes in general can be evaluated only in a perturbation expansion. Amplitudes then become sums over Feynman graphs, each one constructed from known vertices and propagators.[1] Just as in field theory, however, some of the amplitudes thus constructed diverge, and it is necessary to regularize the amplitude by subtracting off an infinite piece.[2] In renormalizable field theory these pieces may be considered simply corrections to the bare parameters of the theory.

It has long been believed that the same is true for dual models.[3] In the region where all invariant masses are below the appropriate thresholds, the amplitudes only diverge if there are tadpole loop insertions.[4] These loops may be re-expressed in terms of emission of Pomerons into the vacuum at zero four-momenta. The amplitudes diverge because they are evaluated at a mass squared greater than the lowest Pomeron mass squared. This suggests[5,6] as a regularization procedure first giving an incoming momentum to each tadpole, so that for sufficiently spacelike q^2 the amplitude is well defined. One may then analytically continue around the singularities. If we are in the critical dimension (for which the Pomeron singularities become acceptable poles rather than unitarity-violating cuts) the singularities are poles in q^2. The infinities due to tachyon poles automatically become finite thereby, but there is always a scalar massless Pomeron called the dilaton which gives an infinity as $q^2 \to 0$ simply because we are sitting on the pole of its propagator. Thus, the regularization procedure which I shall use is to analytically continue in q^2 and to subtract all contributions involving a dilaton vanishing into the vacuum. If these subtractions are equivalent to a change in the arbitrary parameters of the theory, that is, the slope and coupling constants, then the theory is renormalizable.

This paper is devoted to a discussion of whether the theory is renormalizable. That is, can we show that the counterterms are equivalent to renormalizations of the slope and coupling constant, the only free parameters of the theory? While having no definite results, I will show how the conjecture of renormalizability is satisfied in the one insertion counterterm and discuss the difficulties in higher orders.

In the next section the divergences are displayed as dilaton insertions, and the regularization procedure is discussed. I give a heuristic summation of the counterterm to all orders in Sec. III. To first order this leads to a slope and coupling-constant renormalization, but in second order (and higher) the heuristic argument indicates the necessity of an infinite intercept shift. In Sec. IV I discuss the two inadequacies of the heuristic argument in higher order, and present a conjecture. The last section corrects one of these inadequacies in second order, and shows that most of the intercept shift is due to improper analytic continuation.

II. THE DIVERGENCES

A general term in the expansion of a dual amplitude corresponds[7] to a two-dimensional manifold with boundaries. Each external particle is represented as a current source at a point on one of the boundaries. The amplitude is then the exponential of the heat generated, integrated over the positions of the current sources and all the conformal invariants of the manifold. If all the products of incoming momenta are sufficiently spacelike, these integrals converge except for the integral over the size of holes which have no incoming momenta. Each hole may be viewed as the emission of Pomerons into the vacuum, as shown in Fig. 1.

Consider a one-loop planar diagram for n external particles. The amplitude may be written[8] as

$$\mathfrak{L} = c\langle 0|U^\dagger \Delta_P(0)\, U\Delta_R\, |T(k_1, \ldots, k_n)\rangle\,, \tag{2.1}$$

as shown in Fig. 1(c). Here U is the transition operator which turns a Reggeon into a Pomeron, $T(k_1, \ldots, k_n)$ is the usual Reggeon tree, and Δ_R and Δ_P are Reggeon and Pomeron propagators. As the Pomeron states require a doubled set of oscillators, the operator U takes one from the Reggeon Hilbert space described by a single set of operators $a_{n\mu}$ to a doubled space $A_{n\mu}$, $A'_{n\mu}$. The Pomeron propagator is then

$$\Delta_P(q^2) = \int_0^1 dr\, r^{-3+q^2/4+L_0+L_0'} f^2(r^2)\, \delta_{L_0, L_0'}\,, \tag{2.2}$$

where f is the partition function and L_0' and L_0 are made of primed and unprimed A oscillators, respectively, without zeroth modes. Inserting a complete set of Pomeron states after the Δ_P, we immediately see that the states $|0\rangle$ and $A_1^{\dagger\mu}A_{1\mu}'^{\dagger}|0\rangle$ give contributions which diverge as $r\to 0$, while all other states are finite. By analytic continuation in q^2, the $r^{-3+q^2/4}$ term is also a finite contribution. The infinite part of the loop diagram may therefore be written

$$\mathfrak{L} = \mathfrak{L}_{\text{finite}} - \frac{Z}{8\pi^2}\langle 0|(A_1^\mu A'_{1\mu} - 2)\, U\Delta_R\, |T(k_1, \ldots, k_n)\rangle\,, \tag{2.3}$$

where we have used the expansion $f^2(r^2) = 1 - 2r^2 + \cdots$.

The U operator, as well as doubling the operators, maps the interval (0, 1) onto the unit circle. The A operators then correspond to operators at 0 and infinity, and we may do a Möbius transformation back to the configuration[9] with the Reggeons on the positive real axis. The counterterm corresponds to an insertion of a superposition

$$\mathfrak{D} = D - \frac{d-6}{8\pi}\, T\,, \tag{2.4}$$

where D is the normal-ordered dilaton vertex and T is the tachyon vertex:

$$D = \frac{1}{2\pi}\int \frac{d^2 z}{|z|^2} :\exp\{i(q/2)[Q(z)+Q(z^*)]\} P^\mu(z) P_\mu^\dagger(z^*):, \tag{2.5}$$

$$T = \int d^2 z\, (\operatorname{Im} z)^{(1/2)q^2-2} : \exp\{i(q/2)[Q(z)+Q(z^*)]\}:.$$

In D we have dropped some terms which vanish as $q^2\to 0$. Each of these vertices is integrated over the upper half plane, with a $|z|$ ordering in the positions of the operators relative to the Reggeons. The tachyon vertices were considered by Neveu and Scherk[5] and shown to contribute coupling-constant renormalizations, at least when inserted into tree graphs. We are thus more concerned with the summation over the normal-ordered dilaton.

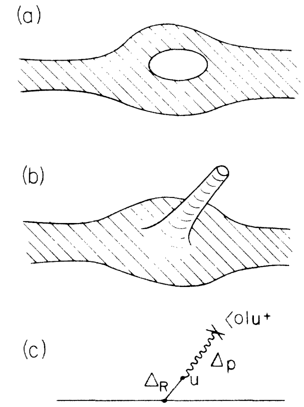

FIG. 1. A planar loop insertion in a diagram (a) may be deformed (b) to look like a tube, or Pomeron, which disappears into the vacuum. In terms of Feynman-like graphs, we may picture this insertion as in (c), with the wavy line a Pomeron propagator. The X represents the coupling for a Pomeron to disappear.

We use a regularization scheme similar to the one suggested by Cremmer and Neveu.[10] The unregulated n-loop graph consists of a counterterm of order g^{2n} corresponding to all the loops connected via a dilaton to the rest of the graph, a sum of graphs involving lower-order counterterms, and a regularized piece. All of the counterterm insertions are proportional, being an infinite constant times a physical dilaton insertion at $q = 0$.

Summing all counterterm insertions to a given graph gives

$$\lim_{q_i\to 0}\sum_n \frac{Z^n}{n!}\left\langle 0\left| T^*\left(\left\{\prod_{i=1}^n \mathfrak{D}(q_i, z)\right\} G\right)\right|0\right\rangle\,, \tag{2.6}$$

where G is the product of operators which describes the graph without counterterm insertions. The $n!^{-1}$ factor eliminates the overcounting of the insertions in the $|z|$ ordering, ensuring that we count the n-loop insertion with unit weight.

Rewriting the operator in terms of propagators, we see that the effect of the counterterm insertions is to replace each propagator Δ in G by

$$\Delta_R' = \lim_{q_i\to 0}\Delta_R\sum_{n=0}^\infty \prod_{i=1}^n\left\{Z\int_0^\pi d\theta\, \mathfrak{D}(q_i, e^{i\theta})\,\Delta_R\right\}\,. \tag{2.7}$$

III. HEURISTIC SUMMATION OF ALL COUNTERTERMS

I will now present a heuristic argument which shows that the insertion of arbitrary numbers of D's is equivalent to a slope renormalization, coupled with an infinite intercept shift. The argument was discovered independently by Scherk[11] and by Lovelace[12] and an equivalent argument was discovered by Ademollo *et al.*[13] In the $q \to 0$ limit D becomes

$$M_0 = (4\pi)^{-1}\int_0^{2\pi} d\theta : P(e^{i\theta})\, P(e^{-i\theta}) :$$

$$= \tfrac{1}{2} P_0^{\,2} - \sum \tfrac{1}{2} n (a_n^{\dagger 2} + a_n^{\,2}) \ . \tag{3.1}$$

Using[13] the position and momentum of the string

$$x(\tau,\sigma) = \tfrac{1}{2}[Q(z) + Q(z^*)] \ , \qquad (z = e^{\tau + i\sigma}) \ , \tag{3.2}$$
$$\pi(\tau,\sigma) = \tfrac{1}{2}[P(z) + P(z^*)]$$

we find

$$M_0(\tau) = \frac{1}{2\pi}\int_0^{\pi} [\pi^2(\tau,\sigma) - x'(\tau,\sigma)]\, d\sigma \ . \tag{3.3}$$

This is just the Lagrangian of the string,[14] and it seems reasonable that insertions of the free Lagrangian should be equivalent to changing the constant multiplying it, which is just the reciprocal of the Regge slope.

When we sum all such insertions to the propagator (2.7), we find

$$\Delta_R'^{\,-1} = L_0 - 1 - Z M_0 \ . \tag{3.4}$$

The propagator is now quadratic in a's and $a^\dagger$'s We diagonalize with a Bogoliubov transformation

$$B_n^\dagger = a_n^\dagger \cosh\lambda + a_n \sinh\lambda \ ,$$
$$B_n = a_n \cosh\lambda + a_n^\dagger \sinh\lambda \ , \tag{3.5}$$
$$P_0 = e^{-\lambda} P_0 \ ,$$

where

$$\tanh 2\lambda = Z \ . \tag{3.6}$$

In terms of the new L_0

$$\mathcal{L}_0 = L_0(P_0, B, B^\dagger) = \tfrac{1}{2} P_0^{\,2} + \sum n B_n^\dagger B_n \ , \tag{3.7}$$

we find

$$\Delta_R' = (\mathcal{L}_0 - \alpha_0)^{-1} \cosh 2\lambda \ , \tag{3.8}$$

where

$$\alpha_0 = \cosh 2\lambda + (\sinh\lambda)^2\, d \sum_{n=1}^{\infty} n \ . \tag{3.9}$$

Thus, the effect of the M_0 insertions is threefold. It shifts the intercept, introduces a wave-function renormalization (or, equivalently, a coupling-constant renormalization of $\cosh 2\lambda$), and introduces a transformation on the operators. We must therefore transform the vertices as well.

The effect of the Bogoliubov transformation on the vertices is particularly simple if we use external physical "photons," with vertices

$$V(k, \epsilon, 1) = e^{ikQ(1)}\, \epsilon \cdot P(1) \ . \tag{3.10}$$

Because $k^2 = 0$ and $\epsilon \cdot k = 0$, there is no need for normal ordering, and we find that, using

$$Q(1, a, a^\dagger) = e^{-\lambda} Q(1, B, B^\dagger) \ , \tag{3.11}$$

$$P(1, a, a^\dagger) = e^{\lambda} P(1, B, B^\dagger) \ , \tag{3.12}$$

the photon vertex transforms as a simple rescaling of the momenta

$$V(k, \epsilon, 1, a, a^\dagger) = V(e^{-\lambda} k, e^{\lambda}\epsilon, 1, B, B^\dagger) \ . \tag{3.13}$$

Notice that the external momenta transform by the same scale as the momentum operator. Thus, these changes amount simply to a rescaling of all momenta, or, equivalently, a redefining of the unit for measuring momenta, i.e., a rescaling of the Reggeon slope $\alpha' \to e^{2\lambda}\alpha'$.

We have still not discussed the action of the Bogoliubov transformation on the vacuum state. To lowest order in λ there is no effect but thereafter it is ill defined. We may avoid such problems by factoring a larger amplitude at poles on each end. This gives us an amplitude which is the same as the original one except that the slope and coupling constant have been redefined and the intercept has been shifted, because the new propagator is $(\mathcal{L}_0 - \alpha_0)^{-1}$.

IV. DEFECTS OF THE HEURISTIC ARGUMENT

The infinity in α_0 cannot be handled as a renormalization because the intercept is not a free parameter in the unrenormalized theory, but must be 1 in order to have the Ward identities which permit the Pomerons to be physical particles.[15] Even if α_0 were finite, we would still have an unacceptable new amplitude because it is constructed with the normal vertices but with a shifted propagator, which ruins dual behavior. We shall see that this intercept shift is a result of the inadequacies of the heuristic argument and not a real defect of the model. There are two problems with our approach which arise when more than one dilaton is inserted.

The first is that we have used the Lorentz-invariant dilaton insertion, which is not a physical-state vertex. At the single-insertion level this does not matter because the extra, spurious piece will give zero contribution if all the external particles are physical. At the two-insertion level this cancellation is no longer automatic and one should really use

$$D=\frac{1}{2\pi}\int\frac{d^2z}{|z|^2}:\exp\{i(q/2)[Q(z)+Q(z^*)]\}\times P_\mu(z)\,\epsilon^{\mu\nu}P_\nu(z^*): , \tag{4.1}$$

with ϵ symmetric and $q_\mu\epsilon^{\mu\nu}=0$. The extra, spurious contributions are not constrained to be gauge invariant or even Möbius invariant, so might easily cause the kind of intercept shift we have here.

The second problem, which is actually the source of the *infinite* intercept shift, is that by first taking each $q_i\to 0$, the integral expression for the two-dilaton amplitude must diverge, because we are above the lowest pole in $q^2=(q_1+q_2)^2$. Having factorized in the crossed channel this divergence is manifest in the divergent sum on n. Our regularization procedure tells us that we should evaluate the amplitude for arbitrary spacelike q^2, subtract out the dilaton insertion (which has already been lumped into the higher-order corrections to Z), and analytically continue around the tachyon pole.

As these difficulties arise only to second order in Z, we may trust the first-order results. Thus, to lowest order the counterterms effect a change of slope

$$\alpha_R'=(1+Z+\cdots)\,\alpha_0' \tag{4.2}$$

(from the normal-ordered dilatons) and a change of coupling constant

$$g'=[1-\tfrac{1}{8}(d-6)Z+\cdots]g_0 \tag{4.3}$$

from the tachyon piece.[15] There is no change of the intercept to lowest order.

This leads us to speculate that the correctly done summation of counterterms to all orders will result only in a renormalization of g and α'. Of course, the necessity of evaluating the n-insertion amplitude with arbitrary momenta before analytically continuing ruins the summability we saw in the heuristic argument, so it is a difficult conjecture to prove.

The conjecture is that each unintegrated propagator x^{L_0-1} in the expression for G is converted by the counterterm insertions into

$$U(x,\lambda)=x^{(\mathfrak{L}_0-1)/\cosh 2\lambda}=x^{L_0-ZM_0-c(\lambda)}, \tag{4.4}$$

where $\mathfrak{L}_0=L_0(B,B^\dagger)$ and

$$c(\lambda)=\frac{1-d\sinh^2\lambda\,\sum n}{1+2\sinh^2\lambda} . \tag{4.5}$$

Our heuristic argument gave $c(\lambda)=1$. If the conjecture is correct we can scale $\tau=\ln x\to\tau\cosh 2\lambda$, and do the Bogoliubov transformation on each vertex by scaling the momenta $k_i\to e^{-\lambda}k_i$. We are then left with an amplitude exactly like G except for an overall factor of $\cosh 2\lambda$ for each propagator, corresponding to a wave-function renormalization or, equivalently, a coupling-constant renormalization.

In the next section the effect of dilaton insertions on the propagator is evaluated to second order. The failure to project out spurious states will leave some finite piece in the intercept shift, but it will be shown that the infinite $\sum n$ is a manifestation of improper analytic continuation.

V. RENORMALIZATION TO SECOND ORDER

The insertion of two dilatons onto an unintegrated propagator x^{L_0-1} gives an operator

$$\int_R\frac{d^2z_1\,d^2z_2}{|z_1|^2\,|z_2|^2}\,\theta(|z_1|-|z_2|)\,D(q_1,z_1)\,D(q_2,z_2) , \tag{5.1}$$

where R is the region $x<|z|<1$, $0<\arg z<\pi$. If our conjecture is correct, this must be equal to

$$\frac{1}{2}\frac{\partial^2}{\partial Z^2}U(x,\lambda)\bigg|_{Z=0}x^{1-L_0}=\int_x^1\frac{d\rho_2}{\rho_2}\int_{\rho_2}^1\frac{d\rho_1}{\rho_1}M_0(\rho_1)\,M_0(\rho_2)+\tfrac{1}{4}(2+d\sum_n n)\ln x . \tag{5.2}$$

The normal-ordered product of the M_0's or of the D's have no singularities and we may take the $q_i\to 0$ limits immediately. We will not consider the spurious-state problem in this paper so we may take D's with $\epsilon_{\mu\nu}\to g_{\mu\nu}$. The $q\to 0$ limit of these is just M_0, so the normal-ordered terms coincide. The remaining terms in (5.2) give

$$\int_x^1\frac{d\rho_2}{\rho_2}\int_{\rho_2}^1\frac{d\rho_1}{\rho_1}\left\{\sum_{n=1}^\infty\frac{n^2}{4}[a_n{}^2\rho_1{}^{-2n},a_n^{\dagger 2}\rho_2{}^{2n}]\right\}+\left(\frac{1}{2}+\frac{d}{4}\sum n\right)\ln x =-\frac{1}{4}\sum_{n=1}^\infty[2n\ln x+(1-x^{2n})]\,a_n^\dagger a_n-\frac{d}{8}\sum(1-x^{2n})+\frac{1}{2}\ln x . \tag{5.3}$$

Before we take the $q_i\to 0$ limit in (5.1) we must extract the singular pieces as $z_1\to z_2$. These terms arise from the expansion of the products of the operators

$$:\exp\{i(q_1/2)[Q(z_1)+Q(z_1^*)]\}\,P(z_1)\cdot\epsilon_1\cdot P(z_1^*)::\exp\{i(q_2/2)[Q(z_2)+Q(z_2^*)]\}\,P(z_2)\cdot\epsilon_2\cdot P(z_2^*)$$

$$=\left|\frac{(z_1-z_2)(z_1-z_2^*)}{z_1z_2}\right|^{q_1q_2/2}:\exp\{i(q_1/2)[Q(z_1)+Q(z_1^*)]+i(q_2/2)[Q(z_2)+Q(z_2^*)]\}\,O:\ , \quad (5.4)$$

where O consists of 119 terms arising from the normal ordering of the P's of one vertex with the P's and Q's of the other.

Of these, the ones which are analogous to the ones which occur in $M_0 M_0$ are those involving no contractions of P's with the exponential factors. This leaves the terms with no contractions, with one contraction between two P's, and one with all P's contracted. The first is nonsingular, and the second has a singularity as $z_1 \to z_2$ which vanishes on angular integration, so that both of these terms give exactly the result of the corresponding terms in $M_0 M_0$. For the last term, however, we have a contribution given by

$$\frac{\mathrm{Tr}(\epsilon_1\epsilon_2)}{8\pi^2}\int_R d^2z_1\,d^2z_2\left|\frac{(z_1-z_2)(z_1-z_2^*)}{z_1z_2}\right|^{q_1q_2/2}(|z_1-z_2|^{-4}+|z_1-z_2^*|^{-4})$$

$$\times\exp\{i(q_1/2)[Q(z_1)+Q(z_1^*)]\}\exp\{i(q_2/2)[Q(z_2)+Q(z_2^*)]\}\ : \quad (5.5)$$

The two terms in parentheses are equivalent as the rest of the integrand is symmetric under $z_1 \leftrightarrow z_1^*$, or $z_2 \leftrightarrow z_2^*$. For the first we must expand the exponential around $z_1 = z_2$, discarding terms with enough powers of $q_1 - q_2$ and enough powers of $z_1 - z_2$ to vanish in the limit $q_1, q_2 \to 0$. What is left of the exponential is

$$1+\frac{|z_1-z_2|^2}{4|z_2|^2}:q_1\cdot P(z_2)\,q_1\cdot P(z_2^*):\ . \quad (5.6)$$

The first piece in the expansion, 1, gives a contribution

$$-\tfrac{1}{8}\mathrm{Tr}(\epsilon_1\epsilon_2)\sum_{n=1}^{\infty}(1-x^{2n})\ . \quad (5.7)$$

The second piece gives a form quadratic in q_i but divided by q^2 because it is a contribution to the dilaton pole. There are many other such terms coming from terms in O which involve contractions of one or two of the P's with Q's. If $q_i\cdot\epsilon_i = 0$, then explicit calculation shows that all these terms add up to give the dilaton pole piece. As internal dilaton insertions have already been subtracted out by the regularization procedure, this piece should be thrown away.

Reverting to the Lorentz-invariant dilaton $\epsilon \to g$, we see that the correctly continued (5.1) agrees with our conjecture (5.3) except for the term $\frac{1}{2}\ln x$. Such a term would amount to a finite intercept shift (for finite Z) if we believed it. I believe, however, that it is instead a result of not having used physical dilatons. The physical dilaton insertions (with finite Z) are on-shell Pomerons and must give a dual amplitude. Now that we see that we have a well-defined amplitude (with no infinite parameters) the answer should certainly be dual, which means there cannot be a finite intercept shift. The residual $\frac{1}{2}\ln x$ term is a manifestation of the spurious states. A proof that this is true will have to wait for a detailed investigation of the gauge properties of the counterterms.

VI. COMMENTS AND ACKNOWLEDGMENTS

I have presented a proof that the dual model in the critical dimension is renormalizable to lowest order in g^2, and have argued that this is also true to higher orders. This implies the renormalization is not responsible for the shift of trajectories which we need to get rid of all tachyons in dual theories. I believe this shift will have to await an understanding of the breakdown of the vacuum against tachyon emission.

During his investigation of the renormalizability the author has benefited from conversations with many people, in particular with Claud Lovelace, who first interpreted his lowest-order results in terms of the Bogoliubov transformation. He also wishes to thank John Schwarz for informing him of the work of Ademollo *et al.*, and Stafano Sciuto of that group for correspondence concerning their approach. The author fixed upon the analytic continuation as the essential fault of the heuristic approach while enjoying the very stimulating atmosphere of the dual models workshop of the Aspen Center for Physics.

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