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# Second-quantized non-Abelian field theory for hadrons with quark confinement and scaling deep-inelastic structure functions*

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A four-dimensional second-quantized field theory with quarks bound by "colored" non-Abelian gluons is described which has the following properties: (1) the only physical particles are color singlets composed solely of quarks, (2) the deep-inelastic structure functions have Bjorken scaling, (3) gluon loops and Faddeev-Popov ghost loops are identically zero in any gauge, (4) Regge trajectories are apparently linear on a Chew-Frautschi plot, and (5) constituent motion within hadrons can be nonrelativistic.

## I. INTRODUCTION

After a period of some skepticism the possibility that hadronic interactions might be understood within the framework of quantum field theory is again being seriously considered.[1] This is partly the result of the psychological climate created by the apparently successful unification of weak and electromagnetic interactions in a renormalizable field theory and partly the result of a greater appreciation of the variety of phenomena which can occur in field theories.

In this article we shall describe a field-theoretic model of hadron binding which has two major features: (1) Hadrons only occur as quark-antiquark or three-quark bound states, and (2) quarks behave as quasifree particles within hadrons. We assume that the suggestions of an internal symmetry called color[2] are correct and that the strong interaction consists of the exchange of colored Yang-Mills gluons. The nature of the interaction allows only color singlet states to occur in the gauge-invariant physical particle spectrum and consequently the first feature will be realized by choosing the color group to be SU(3). Since the (Schwinger) mechanism which produces this result is an infrared phenomenon, the second feature is not precluded and the model is essentially free in the ultraviolet region of the quark sector.

Our model is a non-Abelian version of a recently investigated Abelian field theory which had quark confinement and scaling electroproduction structure functions.[3] In that theory the free propagator of the massless gluon field embodying the quark-quark interaction was proportional to

$$\lambda^2/k^4, \tag{1}$$

where $\lambda$ is a constant with the dimensions of mass and $k$ is the gluon four-momentum. As a result the Schwinger mechanism[4] manifestly occurred, and it was shown that any charged particle was totally screened by vacuum polarization effects. In addition, explicit calculations of the deep-inelastic electroproduction structure functions in perturbation theory were in agreement with Bjorken scaling with corrections of $O(q^{-4})$, where $q$ is the virtual photon four-momentum. These features of the Abelian model will also be shown to be true in the non-Abelian version. In addition, we shall argue that the quarks can be nonrelativistic within hadrons and that the spectrum of states has linearly rising Regge trajectories.

In spite of these salutary properties an interaction of the form of Eq. (1) could be questioned because of well-known[5] indefinite-metric difficulties which result in the violation of unitarity. While an optimist may hope that the nonappearance of colored gluons in asymptotic (color singlet) states might eliminate unitarity problems it is almost certain that the approximation techniques which will necessarily be used to find the bound states will lead to the occurrence of negative-metric states. Whether these states are "real" or artifacts of the approximation will not be clear. In view of this we suggested[3] that the gluon propagator be taken in principal value rather than as a Feynman propagator:

$$\mathrm{P}\frac{\lambda^2}{k^4} \equiv \frac{\lambda^2}{2}\left[\frac{1}{(k^2+i\epsilon)^2}+\frac{1}{(k^2-i\epsilon)^2}\right]. \tag{2}$$

As a result unitarity is maintained order by order in perturbation theory. Gluons do not appear in asymptotic states. All components of the vector-gluon propagator are "Coulombized" and the gluon field reduced to the embodiment of a direct quark interaction. There are a number of other decided advantages to principal-value propagators in the present context: (1) no color singlet states composed solely of gluons, (2) the elimination of substantial infrared divergences, (3) the suppression of corrections to Bjorken scaling in the electroproduction structure functions by a factor of $q^2$

vis-à-vis the corresponding Feynman-propagator result which sets the stage for precocious scaling, and (4) the elimination of closed loops of vector gluons and thus the elimination of Faddeev-Popov ghost loops.

In Sec. II we give a brief recapitulation of the Abelian model. In Sec. III we describe the canonical properties of the non-Abelian model. In Sec. IV we describe the qualitative features of the model and describe an approximation technique which appears to be naturally adopted to "solving" the theory. We shall restrict our discussion to the color binding interaction and defer the introduction of other interactions to a later work. The properties of the bound states in the non-Abelian model are currently under study and will be the subject of the next report.

## II. ABELIAN MODEL

The possibility that the physical particle spectrum of a field theory consisted only of neutral states and did not include states of charged fields was first investigated in massless two-dimensional quantum electrodynamics.[6] In that case the absence of the "electron" from the gauge-invariant physical particle spectrum was directly related to the acquisition of a mass by the photon via the Schwinger mechanism. The Schwinger mechanism was manifest in the lowest-order contribution to the vacuum polarization (Fig. 1), and, taking account of the dimensionality of the coupling constant, $e \sim$ mass, could almost be considered a consequence of dimensional analysis. These vacuum polarization effects led to the total screening of the "electronic" charge, and, as a result, the "electron" was removed from the gauge-invariant physical particle spectrum. Our Abelian and non-Abelian models will display a similar pattern of events.

The Lagrangian of the Abelian model contains two gluon fields, $A^1_\mu(x)$ and $A^2_\mu(x)$, and the quark field $\psi(x)$:

$$\mathcal{L} = -\tfrac{1}{2}F^1_{\mu\nu}F^2_{\mu\nu} - \tfrac{1}{2}\lambda^2 A^2_\mu A^2_\mu + \bar{\psi}(i\not\nabla - g\not A^1 - m)\psi\,, \tag{3}$$

where for typographic convenience we denote the inner product of four vectors, $a\cdot b = a_\mu b_\mu = a_0 b_0 - \vec{a}\cdot\vec{b}$ throughout, $\lambda$ is a constant with the dimensions of mass, $g$ is dimensionless, and $F^i_{\mu\nu} = \partial_\nu A^i_\mu - \partial_\mu A^i_\nu$.

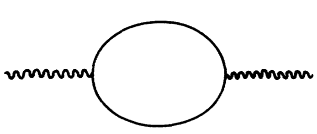

FIG. 1. A vacuum polarization diagram.

Following the canonical procedure we find the equations of motion;

$$\partial_\mu F^1_{\mu\nu} + \lambda^2 A^2_\nu = 0\,, \tag{4}$$

$$\partial_\mu F^2_{\mu\nu} + g J_\nu = 0\,, \tag{5}$$

$$(i\not\nabla - g\not A^1 - m)\psi = 0\,, \tag{6}$$

and nonzero equal-time commutation relations [in the Coulomb gauge $\vec\nabla\cdot\vec{A}^1 = 0$; note $\partial_\mu A^2_\mu = 0$ by Eq. (4)]

$$[F^1_{0i}(x), A^2_j(y)] = i\Delta^{tr}_{ij}(x-y)\,, \tag{7}$$

$$[F^2_{0i}(x), A^1_j(y)] = i\Delta^{tr}_{ij}(x-y)\,, \tag{8}$$

with $i,j = 1,2,3$ and

$$\Delta^{tr}_{ij}(x-y) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}-\vec{y})}\left(\delta_{ij} - \frac{k_i k_j}{|\vec{k}|^2}\right). \tag{9}$$

It is clear from the equations of motion, Eqs. (4) and (5), tnat $A^2_\mu$ may be eliminated to obtain

$$\Box\partial_\mu F^1_{\mu\nu} + g\lambda^2 J_\nu = 0\,. \tag{10}$$

The form of the quark-gluon interaction and Eq. (10) show that only the Green's function of $A^1_\mu$ is relevant to quark-quark scattering. The perturbation theory rules of QED may be used if the photon propagator is replaced with the gluon propagator for $A^1_\mu$:

$$iG^{11}_{\mu\nu}(k) = \frac{i\lambda^2(g_{\mu\nu} - \chi k_\mu k_\nu/k^2)}{k^4}\,, \tag{11}$$

where $\chi$ is constant, determined by the gauge choice.

In Ref. 3 we showed that choosing $G^{11}_{\mu\nu}$ to be a principal-value propagator allowed us to develop a perturbation theory which was unitary order by order:

$$G^{11}_{\mu\nu}(k^2) \equiv \tfrac{1}{2}[G^{11}_{\mu\nu}(k^2 + i\epsilon) + G^{11}_{\mu\nu}(k^2 - i\epsilon)]\,. \tag{12}$$

In addition, the equivalent of the Nambu representation of a Feynman diagram was given and some features of the perturbation theory discussed. Of particular interest was a calculation of the deep-inelastic electroproduction structure functions which scaled in the Bjorken limit. Leading corrections to scaling were of $O(q^{-4})$ as $q^2 \to \infty$ with $q$ being the virtual photon four-momentum, and were given by the diagrams of Fig. 2(b), 2(c), and 2(d). This is to be contrasted with the logarithmic deviations from scaling found in pseudoscalar or vector meson models previously studied.[7]

The Schwinger mechanism manifestly occurred in low orders of perturbation theory. As a result quarks (and all charged objects) are removed from the gauge-invariant spectrum of physical states. The total screening of charge can be seen from the following argument.[3] Consider a spatially bounded system of charge density $\rho$. The total charge is

$$Q = \int d^3x\, \rho(x) \tag{13}$$

$$= \frac{-1}{g\lambda^2}\int d^3x\, \Box \nabla^2 A_0^1 \tag{14}$$

using the equations of motion in the Coulomb gauge. By Gauss's law

$$Q = \frac{-1}{g\lambda^2}\int d\vec{S}\cdot\vec{\nabla}\Box A_0^1 . \tag{15}$$

From the definition of a Green's function, we have

$$A_0^1(x) \equiv \int d^4y\, G_{00}^{11}(x-y)\rho(y) \tag{16}$$

in the Coulomb gauge. If, for simplicity, we choose $\rho$ to describe a static point quark charge and use the free gluon propagator [Eq. (11)], then $Q \neq 0$. However, if we take account of the effect of vacuum polarization processes (the Schwinger mechanism) we find $A_0^1(x)$ is a monotonically decreasing function of $|\vec{x}|$ for large $|\vec{x}|$ and consequently $Q = 0$ in the limit where the integration surface is taken to infinity in Eq. (15). Thus the spectrum of physical states does not include states of nonzero charge. In the next section we shall show that the proof of quark confinement is essentially the same in the non-Abelian model.

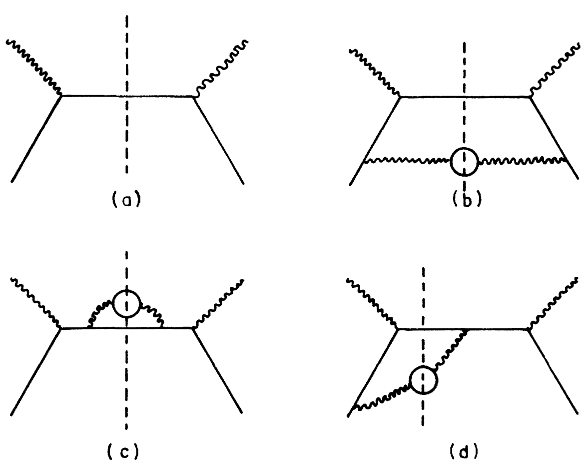


FIG. 2. Lowest-order diagrams contributing to the inelastic electroproduction structure functions. The dashed lines indicate the only contributions to the electroproduction structure functions of the absorptive part of the forward virtual Compton scattering diagram. External "wiggly" lines represent photons while internal "wiggly" lines represent gluons.

## III. NON-ABELIAN MODEL

The non-Abelian model for the color sector of hadronic interactions is a direct generalization of the model of the last section.[8] There are two colored Yang-Mills fields, $A^1_{\mu a}(x)$ and $A^2_{\mu a}(x)$, which when regarded as vectors in the adjoint representation of the color group are denoted $\underline{A}^1_\mu$ and $\underline{A}^2_\mu$. The Lagrangian is

$$\begin{aligned}\mathcal{L} = {}& \tfrac{1}{2}\underline{F}^1_{\mu\nu}\cdot\underline{F}^2_{\mu\nu} - \tfrac{1}{2}\underline{F}^2_{\mu\nu}\cdot(\partial_\mu\underline{A}^1_\nu - \partial_\nu\underline{A}^1_\mu + g\underline{A}^1_\mu\times\underline{A}^1_\nu)\\ & - \tfrac{1}{2}\underline{F}^1_{\mu\nu}\cdot(\partial_\mu\underline{A}^2_\nu - \partial_\nu\underline{A}^2_\mu + g\underline{A}^1_\mu\times\underline{A}^2_\nu - g\underline{A}^1_\nu\times\underline{A}^2_\mu)\\ & - \tfrac{1}{2}\lambda^2\underline{A}^2_\mu\cdot\underline{A}^2_\mu + \bar\psi(i\not\nabla + g\not{A}^1 - m)\psi \end{aligned}\tag{17}$$

$$= \mathcal{L}_0 + \bar\psi(i\not\nabla + g\not{A}^1 - m)\psi , \tag{18}$$

with $\psi$ being the quark field.

It is invariant under the local gauge transformation

$$\psi' = S^{-1}\psi , \tag{19}$$

$$A^{1\prime}_\mu = S^{-1}A^1_\mu S + \frac{i}{g}S^{-1}\partial_\mu S , \tag{20}$$

$$A^{2\prime}_\mu = S^{-1}A^2_\mu S , \tag{21}$$

$$F^{1\prime}_{\mu\nu} = S^{-1}F^1_{\mu\nu}S , \tag{22}$$

$$F^{2\prime}_{\mu\nu} = S^{-1}F^2_{\mu\nu}S , \tag{23}$$

where $S$ is an element in the gauge group $G$ [which is color SU(3) in our case], and $A^1_\mu$ is a matrix in the defining representation of $G$ formed from

$$A^1_\mu = \underline{A}^1_\mu\cdot\underline{T} . \tag{24}$$

$T_a$ is a matrix in the defining representation of $G$ satisfying

$$[T_a, T_b] = i\, t_{abc}T_c , \tag{25}$$

and $\underline{T}$ is a vector formed from such matrices. We note that the homogeneity of the gauge transformation of $A^2_\mu$ allows a mass term to occur in $\mathcal{L}$ without breaking the gauge symmetry. We shall see that the natural gauge-fixing term to add to the Lagrangian has the form

$$-\frac{1}{\beta}\partial_\mu\underline{A}^1_\mu\cdot\partial_\nu\underline{A}^2_\nu . \tag{26}$$

The Euler-Lagrange equations of motion are obtained in the canonical manner:

$$(\partial_\mu + g\underline{A}^1_\mu\times)\underline{F}^1_{\mu\nu} - \lambda^2\underline{A}^2_\nu = 0 , \tag{27}$$

$$(\partial_\mu + g\underline{A}^1_\mu\times)\underline{F}^2_{\mu\nu} + g\underline{A}^2_\mu\times\underline{F}^1_{\mu\nu} + g\underline{J}_\nu = 0 , \tag{28}$$

$$\underline{F}^1_{\mu\nu} = \partial_\mu\underline{A}^1_\nu - \partial_\nu\underline{A}^1_\mu + g\underline{A}^1_\mu\times\underline{A}^1_\nu , \tag{29}$$

$$\underline{F}^2_{\mu\nu} = \partial_\mu\underline{A}^2_\nu - \partial_\nu\underline{A}^2_\mu + g\underline{A}^1_\mu\times\underline{A}^2_\nu - g\underline{A}^1_\nu\times\underline{A}^2_\mu , \tag{30}$$

$$(i\not\nabla + g\not{A}^1 - m)\psi = 0 . \tag{31}$$

The antisymmetry of $\underline{F}^1_{\mu\nu}$ and $\underline{F}^2_{\mu\nu}$ leads to two conservation laws,

$$\partial_\nu(g\underline{A}^1_\mu \times \underline{F}^1_{\mu\nu} - \lambda^2 \underline{A}^2_\nu) = 0\,, \tag{32}$$

$$\partial_\nu(\underline{A}^1_\mu \times \underline{F}^2_{\mu\nu} + \underline{A}^2_\mu \times \underline{F}^1_{\mu\nu} + \underline{J}_\nu) = 0\,, \tag{33}$$

which can be reexpressed as

$$(\partial_\nu + g\,\underline{A}^1_\nu \times)\underline{A}^2_\nu = 0 \tag{34}$$

and

$$(\partial_\nu + g\underline{A}^1_\nu \times)\underline{J}_\nu = 0 \tag{35}$$

using the equations of motion. The first of these relations acts in effect as a gauge-fixing term for $A^2_\mu$ if a gauge is chosen for $A^1_\mu$. The second relation has the familiar form of current-conservation equations in conventional Yang-Mills theories.

We turn now to the derivation of the perturbation-theory rules in the gluon sector. We consider the vacuum-vacuum transition amplitude in the presence of external sources[9]:

$$W(\underline{J}^1_\mu, \underline{J}^2_\mu) = \int \prod_x dA^1_\mu\, dA^2_\nu \exp\left[i \int d^4x \left(\mathcal{L}_0 - \frac{1}{\beta}\partial_\mu \underline{A}^1_\mu \cdot \partial_\nu \underline{A}^2_\nu + \underline{A}^1_\mu \cdot \underline{J}^1_\mu + \underline{A}^2_\mu \cdot \underline{J}^2_\mu\right)\right]. \tag{36}$$

After some functional translations we find

$$W(\underline{J}^1_\mu, \underline{J}^2_\nu) \equiv \exp\left\{-i \int d^4x\, d^4y \left[\underline{J}^1_\mu(x) \cdot G^{12}_{\mu\nu}(x-y) \cdot \underline{J}^2_\nu(y) + \tfrac{1}{2}\underline{J}^1_\mu(x) \cdot G^{11}_{\mu\nu}(x-y) \cdot \underline{J}^1_\nu(y)\right]\right\}, \tag{37}$$

where we have dropped an irrelevant factor independent of $\underline{J}^1_\mu$ and $\underline{J}^2_\mu$ on the right-hand side, and

$$G^{12}_{\mu\nu\, ab}(x) = -\delta_{ab} \int \frac{d^4k\, e^{-ik\cdot x}}{(2\pi)^4 k^2}\left[g_{\mu\nu} + (\beta - 1)\frac{k_\mu k_\nu}{k^2}\right] \tag{38}$$

and

$$G^{11}_{\mu\nu\, ab}(x) = \frac{\lambda^2 \delta_{ab}}{(2\pi)^4} \int \frac{d^4k\, e^{-ik\cdot x}}{k^4}\left[g_{\mu\nu} + (\beta^2 - 1)\frac{k_\mu k_\nu}{k^2}\right], \tag{39}$$

with $a$ and $b$ labeling color indices. The free propagators corresponding to the time-ordered products are

$$\langle TA^1_{\mu a}(x) A^1_{\nu b}(y)\rangle = i\, G^{11}_{\mu\nu ab}(x-y) \tag{40}$$

and

$$\langle TA^1_{\mu a}(x) A^2_{\nu b}(y)\rangle = i\, G^{12}_{\mu\nu ab}(x-y)\,. \tag{41}$$

The somewhat unusual Green's functions of Eqs. (40) and (41) have their origin in the canonical equal-time commutation relations which we shall now find.

From Eqs. (27)–(30) we obtain the equations of motion

$$\partial_0 \underline{A}^1_k = \underline{F}^1_{0k} + \partial_k \underline{A}^1_0 + g\underline{A}^1_k \times \underline{A}^1_0\,, \tag{42}$$

$$\partial_0 \underline{A}^2_k = \underline{F}^2_{0k} + \partial_k \underline{A}^2_0 + g\underline{A}^2_k \times \underline{A}^1_0 - g\underline{A}^2_0 \times \underline{A}^1_k\,, \tag{43}$$

$$\partial_0 \underline{F}^1_{0k} = (\partial_i + g\underline{A}_i \times)\underline{F}^1_{ik} - g\underline{A}^1_0 \times \underline{F}^1_{0k} + \lambda^2 \underline{A}^2_k\,, \tag{44}$$

$$\partial_0 \underline{F}^2_{0k} = (\partial_i + g\underline{A}^1_i \times)\underline{F}^2_{ik} - g\underline{A}^1_0 \times \underline{F}^2_{0k} - g\underline{A}^2_\mu \times \underline{F}^1_{\mu k} - g\underline{J}_k\,, \tag{45}$$

and equations of constraint

$$\underline{F}^1_{ik} = \partial_i \underline{A}^1_k - \partial_k \underline{A}^1_i + g\underline{A}^1_i \times \underline{A}^1_k\,, \tag{46}$$

$$\underline{F}^2_{ik} = \partial_i \underline{A}^2_k - \partial_k \underline{A}^2_i + g\underline{A}^1_i \times \underline{A}^2_k - g\underline{A}^1_k \times \underline{A}^2_i\,, \tag{47}$$

$$(\partial_i + g\underline{A}^1_i \times)\underline{F}^1_{i0} + \lambda^2 \underline{A}^2_0 = 0\,, \tag{48}$$

$$(\partial_i + g\underline{A}^1_i \times)\underline{F}^2_{i0} + g\underline{A}^2_i \times \underline{F}^1_{i0} - g\underline{J}_0 = 0\,. \tag{49}$$

The Lagrangian indicates that the canonical momenta are

$$\underline{\Pi}^1_j = \underline{F}^2_{0j} \tag{50}$$

and

$$\underline{\Pi}^2_j = \underline{F}^1_{0j}\,, \tag{51}$$

for $j = 1, 2, 3$ with $\underline{\Pi}^i_j$ conjugate to $\underline{A}^i_j$, and $\underline{A}^i_0$ having no conjugate momentum for $i = 1, 2$. However, the equations of constraint indicate that not all components are independent. We now find the independent components. Let us define

$$\underline{F}^a_{0i} = \underline{F}^{aT}_{0i} + \underline{F}^{aL}_{0i} \tag{52}$$

and

$$\underline{F}^{aL}_{0i} = \partial_i \underline{\phi}^a\,, \tag{53}$$

where

$$\partial_i \underline{F}^{aT}_{0i} = 0\,. \tag{54}$$

Then Eq. (48) gives

$$(\partial_i + g\underline{A}^1_i \times)\partial_i \underline{\phi}^1 - \lambda^2 \underline{A}^2_0 = -g\underline{A}^1_i \times \underline{F}^{1T}_{i0} \tag{55}$$

and Eq. (49) gives

$$(\partial_i + g\underline{A}^1_i \times)\partial_i \underline{\phi}^2 + g\underline{A}^2_i \times \partial_i \underline{\phi}^1 = g\underline{A}^1_i \times \underline{F}^{2T}_{i0} + g\underline{A}^2_i \times \underline{F}^{1T}_{0i} - g\underline{J}_0\,. \tag{56}$$

Rewriting Eqs. (42) and (43) after taking the divergence with respect to spatial components gives

$$(\partial_0 + g\underline{A}^1_0 \times)\partial_k \underline{A}^1_k = (\partial_k + g\underline{A}^1_k \times)\partial_k \underline{A}^1_0 + \partial_k \partial_k \underline{\phi}^1 \tag{57}$$

and

$$(\partial_0 + g\underline{A}^1_0 \times)\partial_k \underline{A}^2_k + g\underline{A}^2_0 \times \partial_k \underline{A}^1_k = \partial_k \partial_k \underline{A}^2_0 + g\underline{A}^2_k \times \partial_k \underline{A}^1_0 + gA^1_k \times \partial_k \underline{A}^2_0 + \partial_k \partial_k \underline{\phi}^2 \tag{58}$$

If we choose the Coulomb gauge, $\vec{\nabla}\cdot\underline{A}^1=0$, then

$$\partial_k\partial_k\underline{A}_0^1+g\underline{A}_k^1\times\partial_k\underline{A}_0^1+\partial_k\partial_k\,\underline{\phi}^1=0 \tag{59}$$

and

$$(\partial_0+g\underline{A}_0^1\times)\partial_k\underline{A}_k^2-\partial_k\partial_k\underline{A}_0^2-g\underline{A}_k^2\times\partial_k\underline{A}_0^1-g\underline{A}_k^1\times\partial_k\underline{A}_0^2 -\partial_k\partial_k\,\underline{\phi}^2=0\,, \tag{60}$$

thus determining $\underline{A}_0^1$ and $\underline{A}_0^2$. Suppose we now define

$$\vec{\underline{A}}^2=\vec{\underline{A}}^{2T}+\vec{\underline{A}}^{2L}\,, \tag{61}$$

$$\vec{\underline{A}}^{2L}=\vec{\nabla}\underline{\phi}^3\,, \tag{62}$$

with

$$\vec{\nabla}\cdot\vec{\underline{A}}^{2T}=0 \tag{63}$$

Taking the divergence of Eq. (44) leads to our final equation for dependent variables

$$\lambda^2\partial_k\partial_k\,\underline{\phi}^3=\partial_0\partial_k\partial_k\,\underline{\phi}^1+g\partial_k(\underline{A}_\mu^1\times\underline{F}_{\mu k}^1)\,. \tag{64}$$

The independent dynamical variables are thus seen to be $F_{0i}^{1T}$, $F_{0i}^{2T}$, $A_i^{1T}$, and $A_i^{2T}$. Their equal-time commutation relations are

$$[F_{0ia}^{1T}(x),A_{jb}^2(y)]=i\delta_{ab}\Delta_{ij}^{\mathrm{tr}}(x-y)\,, \tag{65}$$

$$[F_{0ia}^{2T}(x),A_{jb}^1(y)]=i\delta_{ab}\Delta_{ij}^{\mathrm{tr}}(x-y)\,, \tag{66}$$

with $i,j=1,2,3$, $\Delta_{ij}^{\mathrm{tr}}$ given by Eq. (9), and $a$ and $b$ are color indices. All other commutators of the forms $[A^1,A^1]$, $[A^2,A^2]$, $[F^1,F^1]$, $[F^2,F^2]$, $[F^1,F^2]$ are zero.

We return to our development of perturbation-theory rules. The cubic and quartic gluon vertices of our model are given by (see Fig. 3)

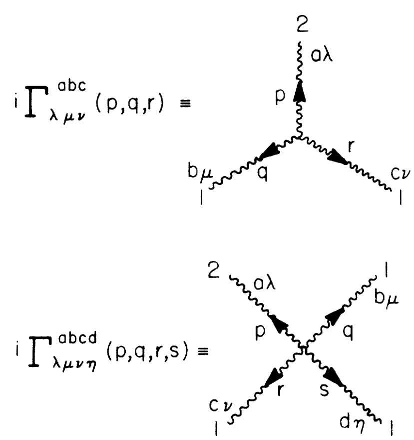


FIG. 3. Cubic and quartic vertices which are given in Eqs. (67) and (68). They introduce $1/r$ potentials in the model and may have an important effect in the baryon spectrum. The numbers 1 and 2 indicate fields $\underline{A}_\mu^1$ and $\underline{A}_\mu^2$, respectively, while $p$, $q$, $r$, and $s$ are momenta, and $a$, $b$, $c$, and $d$ are color indices.

$$i\Gamma_{\lambda\mu\nu}^{abc}(p,q,r)=t^{abc}[\,g_{\nu\lambda}(r_\mu-p_\mu)+g_{\mu\lambda}(p_\nu-q_\nu) +g_{\mu\nu}(q_\lambda-r_\lambda)]\,, \tag{67}$$

with $p+q+r=0$, and

$$i\Gamma_{\lambda\mu\nu\eta}^{abcd}(p,q,r,s)=-i\,t^{abf}t^{cdf}(g_{\lambda\mu}g_{\nu\eta}-g_{\eta\mu}g_{\lambda\nu}) -i\,t^{acf}t^{bdf}(g_{\lambda\eta}g_{\nu\mu}-g_{\nu\lambda}g_{\mu\eta}) -i\,t^{adf}t^{bcf}(g_{\lambda\eta}g_{\nu\mu}-g_{\lambda\mu}g_{\nu\eta})\,, \tag{68}$$

with $p+q+r+s=0$.

The Faddeev-Popov ghost loops will not be relevant to our line of development so we omit their discussion. The necessity for their introduction[10] is closely related to the requirement of unitarity in Yang-Mills theories. In the present model unitarity will be necessarily violated irrespective of the ghost loops if the Green's functions [Eqs. (38) and (39)] pole ambiguities are resolved by using Feynman's $i\epsilon$ procedure. To avoid unitarity violation we have suggested an alternative procedure where the Green's function singularities are taken in principal value,

$$G_{\mu\nu ab}^{kL}(k^2)=\tfrac{1}{2}[\,G_{\mu\nu ab}^{kL}(k^2+i\epsilon)+G_{\mu\nu ab}^{kL}(k^2-i\epsilon)]\,, \tag{69}$$

in momentum space (cf. the Appendix). This choice has the advantage stated in the Introduction. The effects are the same as in the Abelian model[3] and may be summarized as: (1) Only states composed solely of quarks contribute to unitarity sums, (2) gluons do not appear in asymptotic states, (3) unitarity is achieved but at the price of possible advanced effects whose range is limited to hadronic dimensions and thus apparently unobservable, and (4) nonscaling corrections to Bjorken scaling in the deep-inelastic electroproduction structure functions are suppressed by a factor of $q^2$ vis-à-vis the corresponding result using Feynman propagators with $q$ being the virtual photon four-momentum.

A novel feature of the use of principal-value propagators in non-Abelian models is the elimination of closed loops composed solely of gluons. If we consider a subdiagram consisting of a gluon loop with $p$ lines, then Eq. (51) of Ref. 3 gives the Feynman parameter representation

$$I=\int_{-\infty}^{\infty}\prod_{j=1}^{p}\alpha_j\,d\alpha_j\,\frac{\epsilon(\alpha_1\alpha_2\cdots\alpha_p\,C)}{C^2}\,Ne^{iD/C}\,, \tag{70}$$

where $C$ is a polynomial consisting of Feynman parameters only, while $D$ contains scalar products of external momenta, $N$ symbolizes appropriate numerator factors, and $\epsilon(\alpha)=\pm1$ if $\alpha\gtrless0$. Since $N$ can be written as a sum of terms each of which is homogeneous in the Feynman parameters, we can take $N$ to be homogeneous without loss of generality. Then scaling all parameters with $u$, assuming

$$N(u\alpha_1, u\alpha_2, \ldots, u\alpha_p) = u^r N(\alpha_1, \alpha_2, \ldots, \alpha_p), \quad (71)$$

with $r$ an integer, and using

$$\int_0^\infty \frac{du}{u}\,\delta\left(1 - \frac{|\alpha_1 + \alpha_2 + \cdots + \alpha_p|}{u}\right) = 1 \quad (72)$$

we find

$$I = \Gamma(r + 2p - 2L) \times \int_{-\infty}^{\infty} \frac{\prod_{j=1}^{p} \alpha_j\, d\alpha_j \epsilon(\alpha_1\alpha_2\cdots\alpha_p C) N \delta\left(1 - \left|\sum_k \alpha_k\right|\right)}{C^2(-iD/C)^{r+2p-2L}}, \quad (73)$$

with $L$ = number of loops = 1. Suppose we let $\alpha_j \to -\alpha_j$ for all $j$ in $I$. Then we find $I = -I$ or

$$I = 0. \quad (74)$$

Thus any closed loop containing only principal-value propagators is zero. Since Faddeev-Popov ghosts appear only in closed loops and consistency[11] requires we use principal-value propagators for them if we use such propagators for gluons, we see that ghosts do not appear in our model. Physically we can understand this result if we remember that ghost loops were introduced to cure problems arising from contributions to unitarity sums of "opened" gluon loops.[10] In our model "opened" loops do not contribute to unitarity sums in any case so the raison d'être for ghosts is lacking.

We now derive the Ward-Takahashi-Slavnov identities using functional methods. Since we take our gluon propagators in principal value it might appear that our use of functional techniques is unjustified. We shall take the view that the functional representation of the vacuum-vacuum transition amplitude embodies the combinatorics of perturbation theory and acts as a generating function for identities, such as the Ward-Takahashi-Slavnov identities. Thus, questions of convergence of functional integrals are irrelevant—the important question is whether identities are valid in perturbation theory.

We define $W(J)$, the vacuum-vacuum transition amplitude, by

$$W(J) = \int \prod_x dA_\mu^1\, dA_\mu^2\, d\psi\, d\bar\psi \exp\left(i\int \tilde{\mathfrak{L}}\, dx\right), \quad (75)$$

with

$$\tilde{\mathfrak{L}} = \mathfrak{L} - \frac{1}{\beta}\partial_\mu \underline{A}_\mu^1 \cdot \partial_\nu \underline{A}_\nu^2 + \underline{A}_\mu^1 \cdot \underline{J}_\mu^1 + \underline{A}_\mu^2 \cdot \underline{J}_\mu^2 + \bar\psi\eta + \bar\eta\psi,$$

with $\mathfrak{L}$ given by Eq. (17). Under the infinitesimal gauge variation

$$\underline{A}_\mu^1 \to \underline{A}_\mu^1 - (\partial_\mu + g\underline{A}_\mu^1\times)\underline{\theta}, \quad (76)$$

$$\underline{A}_\mu^2 \to \underline{A}_\mu^2 - g\underline{A}_\mu^2 \times \underline{\theta}, \quad (77)$$

$$\psi \to \psi - ig\theta\psi, \quad (78)$$

$$\bar\psi \to \bar\psi + ig\bar\psi\theta, \quad (79)$$

with $\theta = \underline{T}\cdot\underline{\theta}$, $\mathfrak{L}$ is invariant but the remaining terms in $\tilde{\mathfrak{L}}$ lead to

$$\delta\tilde{\mathfrak{L}} \equiv \frac{1}{\beta}[(\partial_\mu + g\underline{A}_\mu^1\times)\partial_\nu\partial_\mu \underline{A}_\nu^2 + g\underline{A}_\nu^2 \times \partial_\mu\partial_\nu \underline{A}_\mu^1]\cdot\underline{\theta} - (\partial_\mu + g\underline{A}_\mu^1\times)\underline{J}_\mu^1\cdot\underline{\theta} - g\underline{J}_\mu^2 \times \underline{A}_\mu^2 \cdot \underline{\theta} + ig\bar\psi\theta\eta - ig\bar\eta\theta\psi. \quad (80)$$

Since a transformation of the integration variables does not change the value of the functional integral, the variation of $W$ with respect to $\theta$ can be taken to be zero and our equivalent of the Ward-Takahashi-Slavnov identity is

$$\left\{\frac{1}{\beta}\left[D_\nu\left(\frac{\delta}{i\delta \underline{J}_\alpha^1}\right)\partial_\nu\partial_\mu \frac{\delta}{i\delta \underline{J}_\mu^2} + g\frac{\delta}{i\delta \underline{J}_\nu^2}\times\partial_\nu\partial_\mu\frac{\delta}{i\delta \underline{J}_\mu^1}\right] + D_\mu\left(\frac{\delta}{i\delta \underline{J}_\alpha^1}\right)\underline{J}_\mu^1 - g\underline{J}_\mu^2 \times \frac{\delta}{i\delta \underline{J}_\mu^2} + g\underline{T}\eta\frac{\delta}{\delta\eta} - g\bar\eta\underline{T}\frac{\delta}{\delta\bar\eta}\right\} W = 0, \quad (81)$$

with

$$D_\mu\left(\frac{\delta}{i\delta \underline{J}_\alpha^1}\right) = \partial_\mu + g\frac{\delta}{i\delta \underline{J}_\mu^1}\times. \quad (82)$$

In order to investigate the structure of the gluon propagators we shall obtain the proper vertex identity equivalent to Eq. (81). We focus on the novelties of the gluon sector and neglect the quark field terms in $\tilde{\mathfrak{L}}$ and Eq. (81). Let us define

$$W(J) = e^{iZ(J)}, \quad (83)$$

$$\underline{B}_\mu^i = -\frac{\delta Z(J)}{\delta \underline{J}_\mu^i}, \quad i = 1, 2 \quad (84)$$

$$\Gamma(B) = Z(J) + \int d^4x (\underline{J}_\mu^1 \cdot \underline{B}_\mu^1 + \underline{J}_\mu^2 \cdot \underline{B}_\mu^2), \quad (85)$$

where $\Gamma(B)$ is the generating functional of proper vertices. An immediate consequence is

$$\underline{J}_\mu^i = \frac{\delta\Gamma}{\delta \underline{B}_\mu^i}\,, \quad i = 1, 2 \tag{86}$$

and as a result Eq. (81) can be rewritten in the form

$$\frac{1}{\beta}\left[\Box\partial_\mu \underline{B}_\mu^2 - g\underline{B}_\nu^1 \times \partial_\nu\partial_\mu \underline{B}_\mu^2 - g\underline{B}_\nu^2 \times \partial_\mu\partial_\nu \underline{B}_\mu^1 + g\frac{\delta}{i\delta \underline{J}_\nu^1}\times\partial_\nu\partial_\mu \underline{B}_\mu^2 + g\frac{\delta}{i\delta \underline{J}_\nu^2}\times\partial_\nu\partial_\mu \underline{B}_\mu^1\right] - \partial_\mu\frac{\delta\Gamma}{\delta \underline{B}_\mu^1} + \underline{B}_\mu^1 \times \frac{\delta\Gamma}{\delta \underline{B}_\mu^1} + \underline{B}_\mu^2 \times \frac{\delta\Gamma}{\delta \underline{B}_\mu^2} = 0\,. \tag{87}$$

If we apply $\delta/\delta \underline{B}_\alpha^1$ to Eq. (87) and set $\underline{B}_\mu^i = 0$ afterwards, we find

$$-\partial_\mu \frac{\delta^2\Gamma}{\delta \underline{B}_\alpha^1 \delta \underline{B}_\mu^1}\bigg|_{\underline{B}^1 = \underline{B}^2 = 0} = 0\,. \tag{88}$$

The second-order functional derivative of $\Gamma$ is the inverse of the full propagator $G_{\mu\nu ab}^{11\prime}$ and Eq. (88) implies that the proper part of $(G_{\mu\nu ab}^{11\prime})^{-1}$ is purely transverse. We note that the "free" propagator (Eq. 39) contribution to $(G_{\mu\nu ab}^{11\prime})^{-1}$ is not one-particle irreducible and thus not constrained by Eq. (88). Therefore we find the general form

$$G_{\mu\nu ab}^{11\prime}(k) = \delta_{ab}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)G^{11}(k^2) + \delta_{ab}\beta^2\lambda^2\frac{k_\mu k_\nu}{k^6}\,, \tag{89}$$

so that the longitudinal part of the full propagator is not renormalized.

The longitudinal part of the full propagator $G_{\mu\nu ab}^{12\prime}(k)$ is also not renormalized. This may be seen by applying $\delta/\delta \underline{B}_\mu^2$ to Eq. (87) and setting $\underline{B}_\mu^i = 0$ afterwards:

$$\frac{1}{\beta}\Box\partial_\mu \delta^4(x-y) - \partial_\nu \frac{\delta^2\Gamma}{\delta \underline{B}_\mu^2 \delta \underline{B}_\nu^1}\bigg|_{B^1 = B^2 = 0} = 0\,. \tag{90}$$

This implies

$$(G_{\mu\nu ab}^{12\prime})^{-1} = \frac{\delta_{ab}(g_{\mu\nu} - k_\mu k_\nu/k^2)}{G^{12}} - \frac{k_\mu k_\nu \delta_{ab}}{\beta} \tag{91}$$

or

$$G_{\mu\nu ab}^{12\prime}(k) = \delta_{ab}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)G^{12}(k) - \beta\delta_{ab}\frac{k_\mu k_\nu}{k^4}\,. \tag{92}$$

Having now developed the general form of the propagators we now will define the gluon vacuum polarization tensors,

$$\Pi_{\mu\nu ab}^{11}(k) = [G_{\mu\nu ab}^{11\prime}(k)]^{-1} - [G_{\mu\nu ab}^{11}(k)]^{-1}\,, \tag{93}$$

$$\Pi_{\mu\nu ab}^{12}(k) = [G_{\mu\nu ab}^{12\prime}(k)]^{-1} - [G_{\mu\nu ab}^{12}(k)]^{-1}\,, \tag{94}$$

which are transverse by our previous discussion:

$$k_\mu \Pi_{\mu\nu ab}^{11} = k_\mu \Pi_{\mu\nu ab}^{12} = 0\,. \tag{95}$$

Rather than write the Schwinger-Dyson equations for our polarization tensors we have given a diagrammatic representation in Fig. 4.

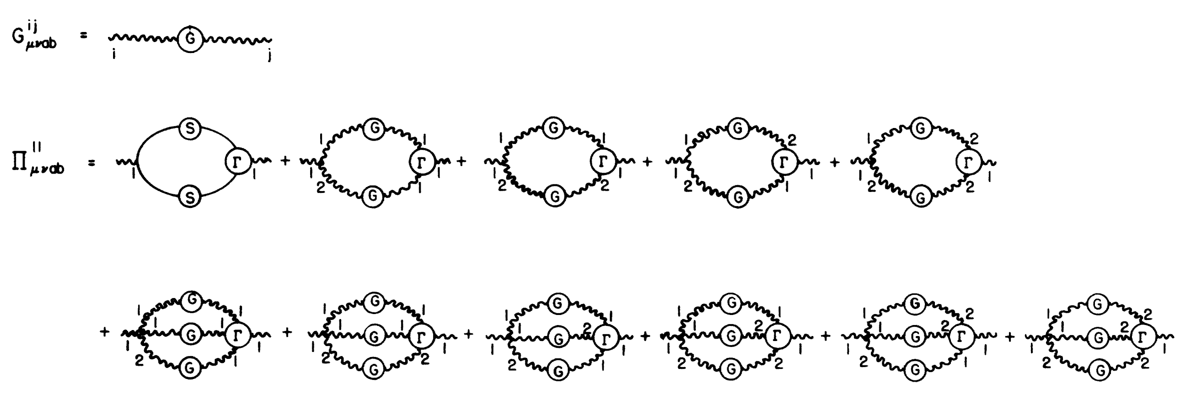


FIG. 4. Diagrammatic representation of the Schwinger-Dyson equation for the proper gluon self-energy, $\Pi_{\mu\nu ab}^{11}$. The numbers at the end of a gluon line specify whether $\underline{A}_\mu^1$ or $\underline{A}_\mu^2$ correspond to that end. The quark propagator is denoted $S$ while $\Gamma$ denotes the appropriate proper (one-particle irreducible) vertex function. A similar diagrammatic expression can be written for $\Pi_{\mu\nu ab}^{12}$.

## IV. OBSERVATIONS

The Schwinger mechanism forces quark confinement to bound color singlet states in a manner which is identical to the Abelian case as described in Sec. II. In order to demonstrate that only color singlets exist in the gauge-invariant physical particle spectrum it is sufficient to show

$$\underline{Q}\psi_{\text{phys}} = 0\,, \tag{96}$$

where

$$\underline{Q} = \int d^3x\, \underline{J}_0(x) \tag{97}$$

for any physical state $\psi_{\text{phys}}$ corresponding to a spatially localized distribution of quarks. We consider a single static quark located at the origin and choose to work in the Coulomb gauge ($\vec{\nabla}\cdot\vec{A}^1 = 0$). Then the time components of the equations of motion [Eqs. (27) and (28)] lead to (at large distance)

$$\Box\nabla^2\underline{A}_0^1 = g\lambda^2\underline{J}_0 \tag{98}$$

if we take into account the elimination of gluons' degrees of freedom through the choice of principal-value propagators and their consequent inability to act as sources. We may now repeat the arguments of Eqs. (13)–(16) for the Abelian case after noticing the occurrence of the Schwinger mechanism in the non-Abelian case which can be verified in low orders of perturbation theory for $\Pi^{11}_{\mu\nu ab}$. Thus the expectation value of the charge in the one-quark state is zero. Since the one-quark state is a charge eigenstate, we find Eq. (96) to be true in this case and more generally through the additivity of the charge operator. Thus only color singlet bound states of quarks are physical.[12]

While the infrared behavior of the theory leads to quark confinement, the ultraviolet behavior allows the quarks to appear quasifree. This is particularly noticeable when we take $\lambda^2 = 0$ in our Lagrangian and examine the corresponding perturbation theory. Taking $\lambda^2 = 0$ is equivalent to examining the short-distance behavior of the theory. The only diagrams which exist in this limit are given in Fig. 5. The quark sector of the theory is free. The only nontree structures are one-quark-loop diagrams for the scattering of gluons associated with $A_\mu^2$ (which of course can only be generated by a hypothetical external source). (As a point of comparison we have shown in Fig. 6 the additional diagrams which would occur in the even that Feynman propagators were used—these diagrams necessarily involve gluon loops which principal-value propagators force to be zero.) The vital role of the $\lambda^2 A_\mu^2 A_\mu^2$ term in the Lagrangian in generating the interacting theory and the fact that $\lambda^2$ has the dimensions of (mass)$^2$ allow a natural approximation procedure in this model. This is perhaps best seen within the context of deep-inelastic electroproduction. Just as in the Abelian case we find that the structure functions scale with leading corrections of $O(q^{-4})$, where $q$ equals the virtual photon four-momentum. We can establish a parton picture of scattering wherein the photon is absorbed on one of the quasifree nucleon constituents [as in Fig. 2(a)] if $|q^2| \gg g^2\lambda^2$. Then leading corrections to such a picture [e.g., the diagrams of Fig. 2(b)–2(d)] will be suppressed by $(g^2\lambda^2/q^2)^2$. Thus the dimensional nature of the effective coupling constant allows a particularly simple picture to exist of the region of large spacelike virtual photon mass and the parton picture emerges as a natural approximation.

The $k^{-4}$ form of the quark interaction also appears to have decidedly good features as far as the bound-state structure is concerned. Ignoring the numerator tensor (which does not affect our conclusions), we find the Fourier transform of the gluon propagator,

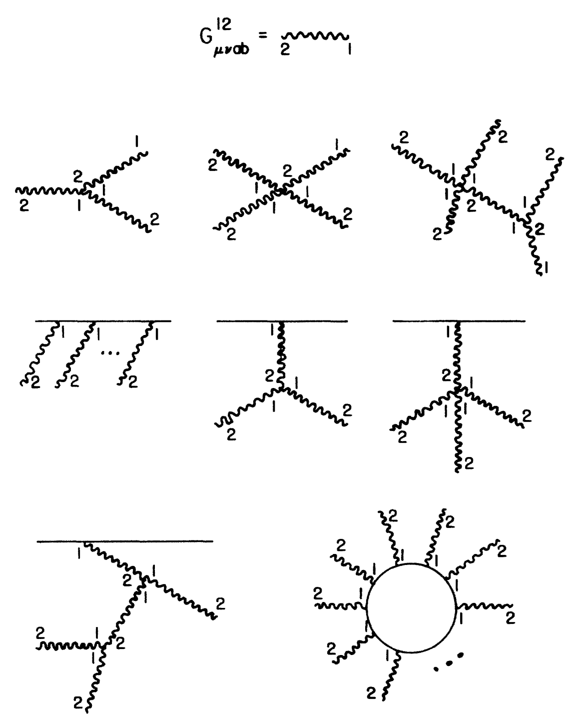

FIG. 5. Some examples of the surviving diagrams in the $\lambda^2 = 0$ limit of the non-Abelian model with principal-value gluon propagators. Except for the class of one-fermion-loop diagrams only tree diagrams exist in this limit. Note that there are no four (or more) external quark line diagrams and no two (or more) external $\underline{A}_\mu^1$ gluon "external" lines.

$$G(k) = \mathrm{P}\frac{1}{k^4}\,, \tag{99}$$

to be proportional to

$$\tilde{G}(x) = \theta(x^2)\,. \tag{100}$$

Since $\tilde{G}$ has a smooth finite limit as $x^2 \to 0$, the short-distance limit, arguments can be made[13] that low-mass bound states can occur in this model. In addition, Dalitz[14] has pointed out that the linearity of trajectories on the Chew-Frautschi plot would follow from a flat-bottomed, smooth interaction—a criterion which is met by Eq. (100). [It is interesting to note that had we used a Feynman propagator rather than principal value, then $\tilde{G}$ would have been $\ln(x^2)$ and thus the general criterion just stated would not have been met. This would appear to be another point in favor of our choice of principal-value propagators.]

Another property which is desirable in the bound-state solutions is nonrelativistic motion of the bound-state constituents.[15] Again an interaction of the form of Eq. (99) appears to realize this feature—even in the strong-binding limit. To see this we shall first take account of the Schwinger mechanism and in the spirit of Hartree-Fock theory modify the quark interaction to

$$G'(k) = \mathrm{P}\frac{1}{(k^2 - \mu^2)^2}\,. \tag{101}$$

If we now take Eq. (101) to be the Green's function for the effective gluon field and calculate the "Coulomb potential" of a static, point quark source located at the origin we find

$$\phi(r) = \frac{\phi_0}{\mu}\, e^{-\mu r}\,, \tag{102}$$

where $\phi_0$ is a constant independent of $\mu$. In the limit $\mu \to 0$ we find

$$\phi(r) \cong \phi_0\left(\frac{1}{\mu} - r + \cdots\right). \tag{103}$$

The first two terms of Eq. (103) correspond to choosing Eq. (99) rather than Eq. (101) as the gluon Green's function (in the limit $\mu \to 0$). Equation (102) includes vacuum polarization effects which damp the interaction at large distances. Thus Eq. (102) imperfectly reflects the possibility that a quark-antiquark pair can separate and induce another quark-antiquark pair to be created from the vacuum so that two color singlet mesons will result (presuming it is energetically favored). At shorter distances Eq. (102) appears to be a reasonable approximation. This exponential potential was studied within the framework of the Schrödinger equation in the strong-binding limit ($\phi_0/\mu$ large) by Greenberg.[16] He showed that the average momentum of the bound constituent in the $s$ state satisfied

$$\frac{p}{m} \sim \left(\frac{\mu}{m}\right)^{1/3} \tag{104}$$

with $m$ being the quark mass. Thus for $\mu/m$ small the quark motion is self-consistently nonrelativistic.

In conclusion, we have shown that a four-dimensional, Lorentz-invariant second-quantized field theory of hadron binding is possible with scaling electroproduction structure functions, only zero-triality physical particle states, and, apparently, linearly rising Regge trajectories and nonrelativistic constituents. A detailed study of the bound states is now in progress.

ACKNOWLEDGMENT

I am grateful to the members of the Newman Laboratory for interesting conversations.

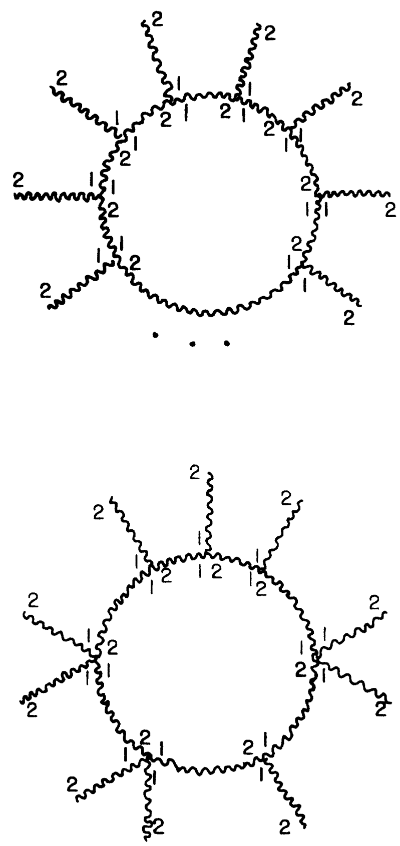

FIG. 6. Some additional diagrams which occur in the $\lambda^2 = 0$ limit of the non-Abelian model if Feynman gluon propagators are used. In addition, there will be Faddeev-Popov ghost-loop diagrams depending on the choice of gauge.

## APPENDIX

In Ref. 3 semiclassical arguments based on Dirac's theory of constraints were given to introduce the use of principal-value propagators. We will now describe a second-quantized realization of those arguments for the case of a scalar Klein-Gordon field $\phi(x)$ with the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}m^2\phi^2 . \tag{A1}$$

The generalization to vector gluons is immediate. The canonical equal-time commutation relations are

$$[\phi, \phi] = [\dot\phi, \dot\phi] = 0 , \tag{A2}$$

$$[\dot\phi(\vec{x}, t), \phi(\vec{y}, t)] = -i\delta^3(\vec{x} - \vec{y}) . \tag{A3}$$

If we expand $\phi(x)$ in plane waves,

$$\phi(\vec{x}, t) = \sum_{\vec{k}} (A_{\vec{k}} e^{-ik\cdot x} + A_{\vec{k}}^\dagger e^{ik\cdot x}) , \tag{A4}$$

then the $q$-number Fourier components $A_{\vec{k}}$ must satisfy

$$[A_{\vec{k}}, A_{\vec{k}'}] = [A_{\vec{k}}^\dagger, A_{\vec{k}}^\dagger] = 0 , \tag{A5}$$

$$[A_{\vec{k}}, A_{\vec{k}'}^\dagger] = \delta^3(\vec{k}' - \vec{k}) \tag{A6}$$

for consistency with Eqs. (A2) and (A3). Now the time-ordered product satisfies

$$T(\phi(x)\phi(y)) = \epsilon(x_0 - y_0)[\phi(x), \phi(y)] + \{\phi(x), \phi(y)\} , \tag{A7}$$

with $\epsilon(x_0) = \pm 1$ for $x_0 \gtrless 0$ and $\{A, B\} = AB + BA$. The first term on the right-hand side is a $c$ number completely determined by Eqs. (A5) and (A6). If the second $q$-number expression were zero, then we would obtain a principal-value propagator from Eq. (A7):

$$T(\phi(x)\phi(y)) \equiv i\int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot(x-y)}\, \mathrm{P}\, \frac{1}{(k^2 - m^2)} . \tag{A8}$$

We therefore require

$$\{\phi(x), \phi(y)\} = 0 , \tag{A9}$$

with the consequence

$$\begin{aligned}\{A_{\vec{k}}, A_{\vec{k}'}\} &= \{A_{\vec{k}}^\dagger, A_{\vec{k}'}^\dagger\} \\ &= \{A_{\vec{k}}, A_{\vec{k}'}^\dagger\} \\ &= 0 .\end{aligned} \tag{A10}$$

Equations (A5), (A6), and (A10) imply

$$A_{\vec{k}} A_{\vec{k}'} = A_{\vec{k}}^\dagger A_{\vec{k}'}^\dagger = 0 , \tag{A11}$$

$$A_{\vec{k}} A_{\vec{k}'}^\dagger = \tfrac{1}{2}\delta^3(\vec{k} - \vec{k}') , \tag{A12}$$

$$A_{\vec{k}}^\dagger A_{\vec{k}'} = -\tfrac{1}{2}\delta^3(\vec{k} - \vec{k}') \tag{A13}$$

for all $\vec{k}$ and $\vec{k}'$. Thus quadratic terms in $A$ and $A^\dagger$ are reduced to $c$ numbers. It should further be noted that the multiplication rule is not associative.[17] In fact, the multiplication rules of the $A$ and $A^\dagger$ operators in Eqs. (A11)–(A13) are realized by taking multiplication to be

$$UV = \tfrac{1}{2}[U, V] \tag{A14}$$

for $U$, $V$ being any $A_{\vec{k}}$ or $A_{\vec{k}'}^\dagger$. If we take an analogy to Lie-algebra theory seriously, where the adjoint representation of an algebra has a multiplication rule defined by commutators

$$\tilde{U} * \tilde{V} = [\tilde{U}, \tilde{V}] \tag{A15}$$

then we could call Eqs. (A11)–(A13) the adjoint representation of the Fourier components of $\phi$.

The $c$-number nature of $AA$, $A^\dagger A^\dagger$, or $AA^\dagger$ can be understood physically in the following manner. Since the $\phi$ field has principal-value propagators it is not associated with a particle but is merely the embodiment of an interaction between other objects (which we have suppressed in our Lagrangian). Consequently an emission of a $\phi$ field quantum must be directly correlated with a subsequent absorption—it cannot propagate into empty space. The $c$-number nature of $AA^\dagger$ reflects this correlation between emission and absorption.

Finally, it should be noted that the existence of a vacuum is inconsistent with Eqs. (A11)–(A13).

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[12]This does not preclude color-singlet states of the gluons from playing a role in the theory. They are not particles but can be exchanged between color-singlet quark states in scattering events. On naive dimensional grounds they should be most important in forward scattering. This leads to the possibility that the Pomeron might possibly be interpreted as a "two-gluon bound state". In the case of wide-angle scattering the predominant mechanism for large momentum transfer would appear to be constituent interchange due to the strong damping effects of $k^{-4}$ propagators on momentum transfer.

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