Comments on fits to nucleon form-factor data

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Recent investigations of nucleon electromagnetic form factors are criticized, because the parametrizations belong to spectral functions which, in the neighborhood of threshold, are not compatible with the unitarity relation used in dispersion theory.

According to the simplest vector-dominance model the isovector parts of the nucleon form factors should be given by the ρ-exchange contributions. It is well known that this prediction is wrong not only at large momentum transfers, where the data decrease faster than $(m_\rho^2+k^2)^{-1}$, but also at small k^2; the experimental values for the slopes of G_E^V and G_M^V at $k^2=0$ are larger by a factor of about 2. In both cases the data are described much better by the "dipole fit," although there remain significant deviations.

Hammer *et al.*[1] attempted to show that the "dipole behavior" of the form factors can be derived from ρ exchange by a proper choice of the propagator for unstable particles. However, Goebel[2] and Hagen and Sudarshan[3] immediately published convincing arguments against their conclusions.

In a second paper[4] Hammer *et al.* investigated a specific ρ-meson propagator which is a special case of their earlier proposal. Their fit to the data up to $k^2=4$ (GeV/c)^2 is better than the dipole fit. However, there is an objection which was not discussed in Refs. 2 and 3: In the threshold region the spectral function of Hammer *et al.*[4] differs strongly (see Fig. 1) from the accurate result which can be obtained from the extended unitarity relation of the dispersion approach (Frazer and Fulco,[5] cited as FF). A similar objection applies to all vector-dominance calculations which do not incorporate the unitarity condition (see below). One should notice that, according to the dispersion relation, a correction to ImG near threshold leads to corrections to $G(t)$ not only at small negative t but in the whole physical region.

For simplicity we consider only the magnetic isovector form factor $G_M^V(t)$. (A similar argument applies to G_E^V.) According to FF its imaginary part is given by

$$\mathrm{Im}G_M^V(t)=\frac{q_t^{\,3}}{\sqrt{2t}}F_\pi(t)^*f_-^1(t)\equiv\frac{q_t^{\,3}}{\sqrt{2t}}|F_\pi|^2J_-(t)\,,\qquad t\geq 4\mu^2 \qquad (1)$$

where $t=-k^2$, $q_t^{\,2}=\frac{1}{4}t-\mu^2$, m = nucleon mass, μ = pion mass, $F_\pi(t)$ = pion form factor, $f_-^1(t)$ is one of the $J=T=1$ $\pi\pi N\bar{N}$ partial waves, $J_-(t)$ = numerator function in an N/D representation of $f_-^1(t)$, $D=F_\pi^{-1}$. Equation (1) is an exact consequence of unitarity for $t\leq 16\mu^2$.

In the t range of interest it is advantageous to decompose $f_-^1(t)$ into the known projection f_{-N}^1 of the nucleon exchange term and the rest: $f_-^1=f_{-N}^1+\tilde{f}_-^1$, where

$$f_{-N}^1(t)=\frac{16\sqrt{2}}{3}f^2m^2\,\frac{z}{t-2\mu^2}[Q_0(z)-Q_2(z)]\,,$$

$$z=\frac{t-2\mu^2}{4(m^2-\frac{1}{4}t)^{1/2}(\mu^2-\frac{1}{4}t)^{1/2}} \qquad (2)$$

and $Q_n(z)$ are the Legendre functions of the second kind. $\tilde{f}_-^1$ can be determined by continuation from the interval -0.5 (GeV/c)$^2\lesssim t\leq 0$, where it has been calculated from πN phase shifts and Regge amplitudes.[6-8] It turns out that f_-^1 is strongly dominated by f_{-N}^1 for t values between its singularity at $t=4\mu^2-\mu^4/m^2\approx 3.98\mu^2$ and about $t=0.4$ (GeV/c)^2.

FF also investigated the analytic structure of the pion form factor. In this case

$$\mathrm{Im}F_\pi=\frac{q_t^{\,3}}{\sqrt{t}}\,|F_\pi|^2N(t)\,, \qquad (3)$$

where the numerator function $N(t)$ of the $\pi\pi$ scattering amplitude is almost constant in the range of interest. Recent fits[9] to the data in the spacelike and timelike region are based on the assumption of FF for $N(t)$ or generalizations.

A comparison of Eqs. (1) and (3) shows that both spectral functions contain the experimentally known ρ contribution $|F_\pi|^2$. In the pion case its shape is only slightly modified by the factor $N(t)$ (and also by the ω contribution). But in ImG_M^V the nucleon exchange contribution in $J_-(t)$ causes a strong enhancement in the threshold region, which is responsible for instance for the difference between the nucleon and pion radii.[8,10] A quantitative prediction for the difference is possible from subtracted dispersion relations and Eqs. (1) and (3), since contributions from the t range beyond the ρ resonance are relatively small.

Figure 1 shows that the result of Hammer *et al.*[4] for $\mathrm{Im}G_M^V(t)$ in the threshold region is completely different from that derived according to the Frazer-Fulco theory (Refs. 6, 8). Therefore the model of Hammer *et al.* is not tenable, at least in applications, where the threshold behavior of the spectral function is relevant.

A second difficulty is related to the fact that Hammer *et al.*[4] essentially used a broad peak of the spectral function in order to fit the data in a range, where other authors need a dipole-like structure. Their figures show a good fit, but they had to pay a price: Since even a broad ρ peak gives a decrease slower than that of the data, they used a subtraction method and admitted a *"hard core."* Their result for $G_M^V(t)$ has a zero and goes to the value -0.95 as $t\rightarrow-\infty$.

We think that a more realistic fit should have negative values of $\mathrm{Im}G_M^V$ beyond $t_0=1\ \mathrm{GeV}^2$ instead of the hard core. Some information on the spectral function at $t>1\ \mathrm{GeV}^2$ can be obtained from the t dependence of

$$\frac{1}{\pi}\int_{t_0}^{\infty}\frac{dt'}{t'-t}\,\mathrm{Im}G_M^V(t')=G_M^V(t)-\frac{1}{\pi}\int_{4\mu^2}^{t_0}\frac{dt'}{t'-t}\,\mathrm{Im}G_M^V(t')\,, \tag{4}$$

where the right-hand side is calculated from data and from Eq. (1). Results will be reported elsewhere.[8]

A comparison of $F_\pi(t)$ from FF-type parametrizations[9] with the model of Hammer *et al.*[4] shows again that the result of these authors has a wrong behavior near $t=4\mu^2$. In this case it might be possible to modify their arbitrary assumption in Eq. (8) in such a way that a correct threshold behavior is obtained.[11] But this does not improve the situation in the case of the nucleon form factor, where it would not be reasonable to include in the ρ propagator the effects of the nucleon exchange term.

Since it was one of the aims of Hammer *et al.*[4] to determine new values for the $NN\rho$ coupling constants, we would like to point out that the notion of $NN\rho$ coupling constants has to be considered with care.[12] One possibility for the definition of $NN\rho$ coupling constants[13] is to approximate in dispersion integrals the ρ peak of $\mathrm{Im}f_\pm^1(t)$ by a δ function:

$$\begin{aligned}\mathrm{Im}f_-^1(t)&\equiv J_-(t)F_\pi(t)\\ &\approx\tfrac{1}{12}\sqrt{2}\,f_{\rho\pi\pi}(f_{\rho NN}-2g_{\rho NN})\delta(t-m_\rho^2)\,.\end{aligned} \tag{5}$$

This procedure is good in those cases where the numerator function is slowly variable and one has only the uncertainty resulting from the finite width. But in Eq. (5) the nucleon exchange singu-

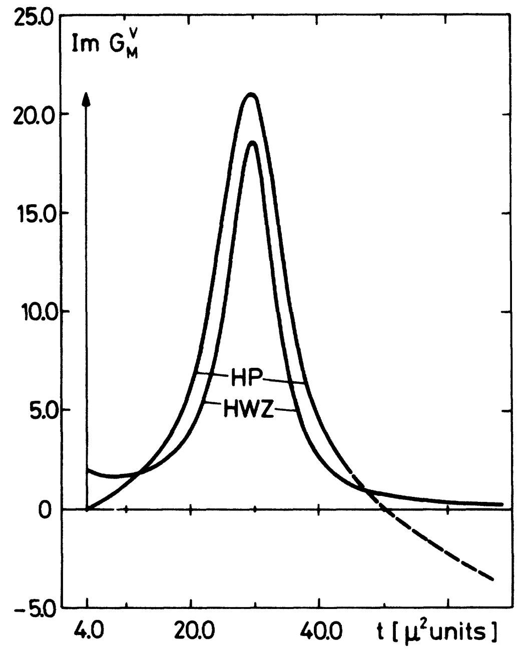

FIG. 1. $\mathrm{Im}G_M^V$ as a function of t. HWZ: Hammer *et al.* (Ref. 4). The curve starts with the value 0 and has an extremely large variation very near $t=4\mu^2$. HP: prediction from dispersion theory (Refs. 6,8). Because of the strong enhancement on the left wing of the ρ resonance the p-wave behavior $\mathrm{Im}G_M^V\sim q_t^3$ is practically not visible. Dashed line: estimation of the spectral function at larger t (Refs. 8, 20).

larity of $J_-(t)$ at $t=3.98\mu^2$ leads to a considerable variation not only near $t=4\mu^2$ but also in the region of the ρ peak.[6] As a consequence all attempts to approximate ρ exchange in πN scattering by a Feynman graph expression have a large additional uncertainty which is ignored in many applications. This effect should be taken into account in discussion of the universality relation $f_{\rho\pi\pi}=f_{\rho NN}$.[14] The ρ-dominance model can be adapted to the Frazer-Fulco theory only if one introduces a t-dependent $NN\rho$ vertex function in addition to the propagator of a broad ρ resonance.

Furthermore, one should notice that ρ dominance is *not* true in general for the $T=J=1$ $\pi\pi N\bar{N}$ amplitudes $f_\pm^1(t)$, even if the pseudovector nucleon Born term[14] is taken into account. For instance[15] it gives wrong predictions for $(d/dt)\mathrm{Re}f_+^1(t)$ at $t=0$ and $\mathrm{Re}f_-^1(0)$. Its validity for the differences of the πN s-wave scattering lengths [or $\mathrm{Re}f_+^1(0)$] was originally the main point in Sakurai's work on the vector-dominance hypothesis. But to our knowledge[16] there exists no derivation of this statement within the framework of dispersion theory or current algebra without further assumptions.

The results from the Frazer-Fulco approach have also been ignored in many other investigations on vector-dominance models, for instance in the recent work of Deo and Parida[17] (see also the discussion in Ref. 18). If the spectral function is expressed as a sum over narrow resonances,[19] one should treat the ρ contribution separately, since one has to correct for the enhanced left wing of this resonance. First results of a new analysis of all form-factor data from this point of view are given in Ref. 20.

Conclusion. The existing nucleon form-factor data can be fitted equally well by parametrizations which have qualitatively different spectral functions. Therefore it is important to take into account the additional restriction following from unitarity, which is usually ignored in vector-dominance models. The fit of Hammer *et al.*[4] has a wrong threshold behavior of the spectral function and its success is related to the fact that a hard core is admitted.

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[10]Since the error of $J_-(t)$ in the threshold region cannot be larger than several percent, one could think that the experimental result of G. T. Adylov *et al.* [Phys. Lett. 51B, 402 (1974)] $r_\pi^2/r_p^2 = 0.93 \pm 0.23$ leads to a serious difficulty. However, this ratio decreases to 0.82 ± 0.23 if one inserts the magnetic isovector nucleon radius of Ref. 8, which is based on new data at small $-t$. Furthermore, a recent determination of r_π from all other data for the pion form factor [Deo *et al.* (Ref. 9)] gave $r_\pi = 0.709 \pm 0.011$ fm, corresponding to 0.68 for the above ratio. An updated version [G. Nenciu, I. Raszillier, W. Schmidt, and H. Schneider, Nucl. Phys. B63, 285 (1973); H. D. Kiehlmann and W. Schmidt (unpublished)] of a bound derived from data in the timelike region gives $r_\pi^2 < 0.56$ fm^2, yielding $r_\pi^2/r_p^2 < 0.76$. One should notice that this ratio depends to some extent also on the behavior of the spectral functions beyond $t = m_\rho^2$.

[11]This point has recently been studied by Hammer *et al.*, who replaced Eq. (8) of their paper (Ref. 4) by $I^M(m'') = \alpha_M m'' (1 - \lambda_M^2/m''^2)^{3/2}$ for $m'' > \lambda_M$ (private communication).

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