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Nonlinear model for elastic nucleon-nucleon scattering outside the core

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A nonlinear pion-nucleon interaction, first applied to nucleon-nucleon scattering by Gupta and his collaborators, is reexamined in the context of a pion and meson-resonance model. It is shown that a pion-nucleon interaction, bilinear in the pion field operators, is a sensible correction to the usual one-pion-exchange, two-pion-exchange, and vector-resonance contributions in the intermediate range of the nucleon interaction ($L \geq 2$) provided the scalar resonance ϵ is strongly coupled to the nucleon.

I. INTRODUCTION

The two-pion-exchange (TPE) interaction has been studied exhaustively in the last decade, and it has been shown to improve the one-pion-exchange (OPE) interaction in the higher orbital-angular-momentum states ($L \geqslant 2$). Although there is no ambiguity in defining OPE, the TPE interaction seems to depend on the method of evaluation, and differences are especially noted in the S and P states.

The focus of this paper is on TPE defined by relativistic quantum field perturbation theory. The usual TPE contribution arises from the exact evaluation of all fourth-order diagrams from the pseudoscalar interaction by Gupta.[1] The corresponding phase parameters show that TPE corrects OPE for the set of phases with $L \geqslant 2$.[2-4] The vector mesons ω and ρ further inprove the agreement to $L=1$, but the agreement is only qualitative.[5] Moreover, the large size of TPE precludes the introduction of the scalar resonance ϵ with mass centered on 715 MeV. This effect if strongly coupled to the nucleon would destroy the reasonable agreement in the D state.

The TPE contribution has been evaluated in other ways. Among the most successful are those based on the dispersion-theoretical evaluation of the scattering matrix,[6,7] and those based on the quantum field theory and the approximate solution of the Bethe-Salpeter equation.[8,9] The same general conclusions follow—TPE is a beneficial correction to OPE, and the vector-meson resonances are needed to bring about better agreement for the lower-L states. In contrast, the scalar resonance strongly coupled to the nucleon is also beneficial to some of these models. The potential model described in the first of Refs. 8 uses an ϵ-N coupling constant greater than g_π^2, while the model of Binstock and Bryan[6] finds the value $g_\epsilon^2/4\pi c\hbar = 6.0$ to be most suitable. The model of Haracz and Thompson[9] requires $g_\epsilon^2/4\pi c\hbar = 20.0$. All these models produce results that are in good agreement with the phenomenological results for $L \geqslant 2$.

The TPE contribution derived in Refs. 1 contains the crossed and uncrossed diagrams, Figs. 1(a) and 1(b), and the fourth-order radiative corrections. The model of Binstock and Bryan[6] employs the $\pi\pi \to N\bar{N}$ amplitude represented by a nucleon pole and by a nucleon-plus-$\Delta(1236)$ pole with a width $\Gamma_\Delta = 120$ MeV. A cutoff in the momentum transfer variable is used with values $(3\mu)^2$ and $(4\mu)^2$. On comparing the TPE results from these two approaches, one finds large differences in the P state, with the field-theoretical result generally larger. For example, the phase shift ${}^3\delta_0^P$ at 300 MeV is $-8.145°$ for the Binstock model (nucleon pole only), while the field-theoretical result is $-31.7°$. The differences diminish for the higher partial waves. In the construction of the model of Haracz and Thompson,[9] it was found that the field-theoretical TPE is too large in the S and P states to be used along with the results arising from the reduced Bethe-Salpeter equation. On the other hand, the exact field-theoretical TPE contribution improved the model for the D state and above.

The P state is therefore strongly dependent on the methods used for calculating TPE in these models. Along with the obvious differences in techniques used and approximations made, these models differ in their means for identifying and excluding higher-order effects and for suppressing the TPE contributions from the crossed and uncrossed diagrams. The model of Refs. 6, for example, uses a cutoff that is kept small to avoid intrusion into the domain of the ρ resonance and the three-pion exchange. A larger cutoff is used in the dispersion-theoretical model of Clemtob and Riska,[7] and the TPE effect in this model is quite close to the field-theoretical result for $L \geq 2$. The P state is not included in Refs. 7 because of the large size of the TPE contribution to the phase shift ${}^3\delta_0^P$, even though TPE is reduced by S-wave subtractions generated from the helicity amplitudes for $N\bar{N} \to 2\pi$.

One might conclude that it is not as yet meaningful to take the P state seriously in theoretical models for nucleon-nucleon scattering. This leads us to speculate again about a nonlinear pion-nucleon interaction.

II. THE NONLINEAR MODEL

Nonlinear pion-nucleon interactions were studied by Gupta and Weihofen[10] with the requirement that the source functions in the pion field equations be expressible as a complete divergence. The interaction energy density

$$H_{\pi N} = i g_\pi : \bar{\psi}\gamma_5 \tau_i \psi U_{\pi i} : -(g_\pi^2/2M) : \bar{\psi}\psi U_{\pi i}^2 : \qquad (1)$$

is applied to nucleon-nucleon scattering, and it is shown that in addition to the usual OPE and TPE results the nonlinear term gives rise to a TPE contribution that is as large as the usual TPE result.[11] In Eq. (1), ψ is the nucleon and $U_{\pi i}$ the pion field operator, M is the average of the proton and neutron masses, and $c = \hbar = 1$. The contributions to TPE from the nonlinear term are derived exactly from the diagrams of Figs. 1(c) and 1(d). The partial-wave analysis of these results[12] indicates that the nonlinear TPE (TPE$'$) is especially large in the P state, where it generally worsens the agreement with the phenomenological phase parameters. On the other hand, it partially cancels TPE in the higher partial waves, and OPE + TPE + TPE$'$ converges more rapidly to OPE than does OPE + TPE for the higher values of L.

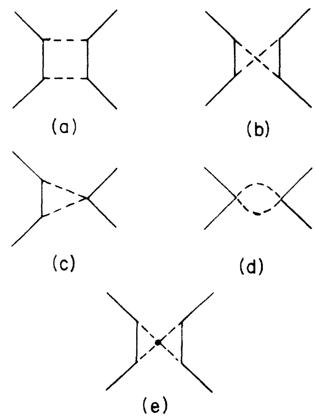

FIG. 1. Interaction diagrams. Diagrams (a) and (b) refer to the TPE effect from the usual pseudoscalar interaction, while diagrams (c) and (d) refer to the nonlinear TPE$'$ interaction. Diagram (e) refers to the π^4 interaction evaluated in Ref. 13.

The meson resonances are added through the interaction energy density

$$H_{RN} = i g_\eta : \bar{\psi}\gamma_5\psi U_\eta : + g_\omega : \bar{\psi}\gamma_\mu \psi U_{\omega\mu} : + i g_\rho : \bar{\psi}\gamma_\mu \tau_i \psi U_{\rho i\mu} : + \frac{f_\omega}{4M} : \bar{\psi}\sigma_{\mu\nu}\psi(U_{\omega\nu,\mu} - U_{\omega\mu,\nu}) + \frac{f_\rho}{4M} : \bar{\psi}\sigma_{\mu\nu}\tau_i\psi(U_{\rho i\nu,\mu} - U_{\rho i\mu,\nu}) : , \qquad (2)$$

where $\sigma_{\mu\nu} = (1/2i)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$, and U_η, $U_{\omega\mu}$, $U_{\rho i\mu}$ are the η pseudoscalar-isoscalar, ω vector-isoscalar, and ρ vector-isovector field operators, respectively. Following the model of Binstock and Bryan,[6] reasonable estimations of the experimental values for the coupling constants are

$$g_\eta^2/4\pi \cong 0,$$
$$g_\omega^2/4\pi = 5.0, \quad f_\omega/g_\omega = -0.12,$$
$$g_\rho^2/4\pi = 0.53, \quad f_\rho/g_\rho = 3.7,$$

and a nominal value of 14.0 is taken for the pion-nucleon coupling constant.

The scalar resonance is centered at 715 MeV with a width of 370 MeV. Binstock and Bryan,[6] present an interesting model for this resonance by taking a Breit-Wigner mass distribution and approximating this by a two-pole representation due to Gersten. They thus take

$$H_{\epsilon N} = g_1 : \bar{\psi}\psi U_{\epsilon 1} : + g_2 : \bar{\psi}\psi U_{\epsilon 2} : , \qquad (3)$$

where $g_1^2 = 0.275\, g_\epsilon^2$, $g_2^2 = 0.725\, g_\epsilon^2$, $m_1 = 508$ MeV, and $m_2 = 1180$ MeV. This representation is fascinating since the lower-mass pole contribution is nearly the same as that produced by the nonlinear pion-pion interaction,

$$H_{\pi^4} = 4\pi f : U_{\pi i}\, U_{\pi i}\, U_{\pi j} U_{\pi j} : . \tag{4}$$

The contribution from H_{π^4} corresponding to the diagram of Fig. 1(e) was evaluated[13] and found to closely resemble a scalar particle of mass 552 MeV provided

$$(g_\epsilon / g_\pi)^2 = -5.2 f .$$

Thus, the scalar resonance with broad width as depicted in Ref. 6 may also be described as a pion-pion resonance and a heavy scalar particle.

The nonlinear model is then based on the energy density

$$H = H_{\pi N} + H_{RN} + H_{\epsilon N} , \tag{5}$$

and it contains the one-boson-exchange, OPE, TPE, and TPE′ contributions.

Phase parameters are defined in a completely covariant manner by employing the method of K-matrix unitarization described by Bock and Haracz.[14] The partial-wave coefficients follow from H as a sum of the various effects,

$$\begin{aligned}\bar\alpha = {} & \bar\alpha(\mathrm{OPE}) + \bar\alpha(\eta) + \bar\alpha(\omega) + \bar\alpha(\rho) + \bar\alpha(\epsilon) \\ & + \bar\alpha(\mathrm{TPE}) + \bar\alpha(\mathrm{TPE}') ,\end{aligned} \tag{6}$$

and the partial-wave amplitudes are related to these as

$${}^S\alpha_J{}^{L,L'} = {}^S\bar\alpha_J{}^{L,L'} + i \sum_{\bar L = |J-S|}^{|J+S|} {}^S\bar\alpha_J{}^{L\bar L}\, {}^S\alpha_J{}^{\bar L L'} . \tag{7}$$

The phase parameters in the Yale notation follow as

$$\begin{aligned} {}^0\alpha_J{}^{J,J} &= \frac{1}{2i}\left[\exp(2iK_J) - 1\right] , \\ {}^1\alpha_J{}^{J,J} &= \frac{1}{2i}\left[\exp(2i\,{}^3\delta^J{}_J) - 1\right] , \\ {}^1\alpha_J{}^{J\pm1,J\pm1} &= \frac{1}{2i}\left\{(1-\rho_J{}^2)^{1/2}\left[\exp(2i\,{}^3\theta^{J\pm1}{}_J)\right] - 1\right\} , \\ {}^1\alpha_J{}^{J\pm1,J\mp1} &= \tfrac{1}{2}\exp\left[i({}^3\theta^{J+1}{}_J + {}^3\theta^{J-1}{}_J)\right] . \end{aligned} \tag{8}$$

III. RESULTS AND CONCLUSIONS

The phase shifts calculated from the nonlinear interaction energy density are shown in radians in Figs. 2, 3, and 4 for the P, D, and F states, respectively. The phase shift ${}^3\theta^D{}_1$ is excluded as it is coupled to the S state, and the coupling parameters ρ_J are not presented as neither TPE′ nor ϵ contribute significantly to these parameters. The

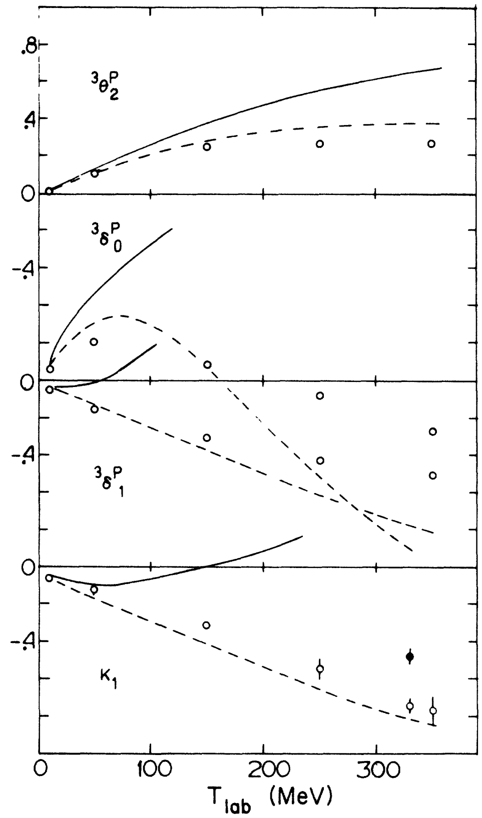

FIG. 2. The P-state phase shifts in radians versus the incident lab energy in MeV. The nonlinear model is shown as a solid line, while the linear model is a dashed line. The phenomenological points are those of Refs. 15 and 16.

Yale phase shifts are shown at 10, 150, 250, and 350 MeV with their parallel-shift uncertainties.[15] Also shown are the two phase-shift solutions of Signell and Holdeman[16] at 330 MeV, where solution 1 is shown as an open circle and solution 2 as a dot.

The nonlinear model is shown in Figs. 2–4 by a solid line. It includes the effects listed in Eq. (6) with the coupling constants given in Sec. II. The scalar resonance is included in the form of Eq. (3) with

$$g_\epsilon{}^2 = g_\pi{}^2 = 14.0 .$$

For reference, the model of Ref. 5 is shown in the figures by a dashed line. It includes the vector resonances, OPE, and TPE, but there is no TPE′ contribution and $g_\epsilon{}^2 = 0$. This model is called the linear model.

We first note that the linear model gives a qualitative fit to the P-state phase shifts, while the nonlinear model only suggests the P state for the

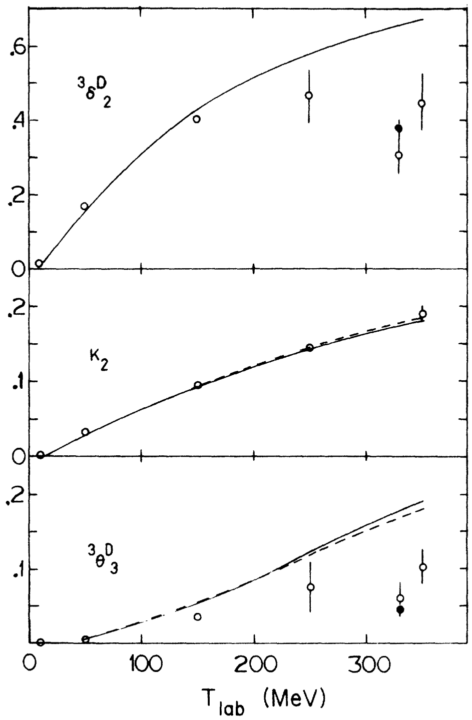

FIG. 3. The D-state phase shifts, in radians.

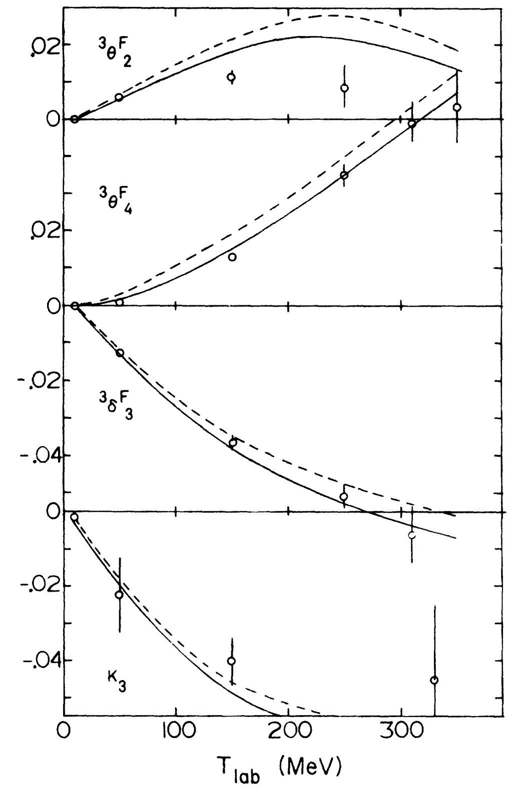

FIG. 4. The F-state phase shifts, in radians.

lowest energies. The D-state phase shifts from both models are nearly the same, and the fit is good from 10 to about 250 MeV. The F state is well established by both models. However, the nonlinear model is noticeably superior. This is especially evident for the phase shift $^3\theta^F_4$. The nonlinear model continues to be better for the higher values of L as TPE′ cancels TPE and provides accelerated convergence to OPE.[12]

We conclude that the nonlinear TPE′ contribution is a sensible correction to the usual OPE, TPE, and vector-resonance contributions provided the P state is only considered to be meaningful at low energies. It is also necessary for the ϵ resonance to be coupled strongly to the nucleon.

It seems reasonable to expect the P state at the higher energies to be affected significantly by many effects as yet uncalculated. Among these are the three-pion-exchange effect, the effect of the differing methods of unitarization, the effect of an exact inclusion of the $\Delta(1236)$ resonance, double-counting effects, and the infinite number of higher-order radiative corrections to TPE. One should note, however, that more recent TPE calculations[17] than those of Refs. 6 and 7 do, in fact, include the continuum of πN and $\pi\pi$ interactions, automatically including the experimental forms of such resonances as Δ and ϵ, and do apparently eliminate the problem of double counting. However, we can see no clear connection between such state-of-the-art dispersion TPE calculations and those reported here.

The nonlinear interaction given in Eq. (1) is regarded as a fundamental interaction for a special purpose. We wanted to show that it could be applied to elastic nucleon-nucleon scattering with some success once it is admitted that the core and near-core regions are really beyond prediction by theoretical methods that do not insert the core phenomenologically. The identification of fundamental and effective interactions cannot as yet be made as the nature of the meson resonances and their relationship to multipion processes is not clearly understood. It is not the purpose of this paper to claim that Eq. (1) is fundamental. We merely want to make the point that the state of the art has not progressed to the extent that one can unequivocally judge the validity of such an effect.

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