Weak neutral currents and $\nu\,(\bar{\nu})$ disintegration of the deuteron

S. K. Singh
Department of Physics, Aligarh Muslim University, Aligarh (U.P.) India
(Received 24 October 1974)

A general formalism for calculating the cross section for neutrino (antineutrino) disintegration of the deuteron caused by the weak neutral currents predicted by the spontaneously broken gauge theories of Weinberg type is developed for moderate neutrino (antineutrino) energies in the MeV range. Numerical results are presented for the special case of low antineutrino energies relevant to the recent experiment of Gurr, Reines, and Sobel.

I. INTRODUCTION

Recent experimental results in deep-inelastic neutrino and antineutrino reactions[1] and certain other processes[2] suggest the existence of weak neutral currents predicted by the renormalizable spontaneously broken gauge theories (SBGT) of Weinberg type.[3] Some of these experiments seem to have achieved a stage where various model-independent as well as model-dependent tests[4] proposed to test quantitatively the existence of these currents can be performed. Apart from these high-energy processes considerable interest has recently developed in testing the existence of these currents in various phenomena involving nuclear physics[5] and atomic physics.[6] Attempts have been made to search experimentally for neutral-current effects in the process $\bar{\nu}_e + d \to \bar{\nu}_e + n + p$ at the Savannah River Plant fission-reactor neutrino facility.[7,8] The most recent experiment is due to Gurr *et al.*[8] and gives the following result:

$$\frac{\sigma_{\text{expt}}}{\sigma_{\text{theory}}} < 6\,. \tag{1}$$

The experimental observation of this process at low antineutrino energies (< 5 MeV) corresponding to the experiment of Gurr *et al.* provides clear evidence for the existence of the weak neutral current in a way independent of the Weinberg angle θ_W, the parameter which enters the theory. This is because at such low energies the final dinucleon state available is the singlet S state which can be achieved from the initial triplet S deuteron state through the axial-vector term (not involving the Weinberg angle θ_W) in the Lagrangian given by SBGT.[9] If we keep in mind the experimental result given by Eq. (1) it seems possible to make a definite statement regarding the existence of neutral-current effects through this process with the help of improved experiments which should be forthcoming in the near future. The aim of this paper is to provide the theoretical framework and numerical results which can be used for comparison with the results of the proposed experiment[10] and others done in this direction, as suggested in this note.

II. CALCULATION

The effective Lagrangian for a strangeness-conserving semileptonic process involving neutrinos is given by[11]

$$\mathcal{L} = \frac{G}{\sqrt{2}} \sum_{l=e,\mu} \left[\bar{l}\gamma_\mu (1+\gamma_5)\nu_l (J_1^\mu + iJ_2^\mu) + \text{H.c.} + \bar{\nu}_l \gamma_\mu (1+\gamma_5)\nu_l J_0^\mu \right], \tag{2}$$

where

$$J_0^\mu = A_3^\mu + xV_3^\mu + yJ_S^\mu\,. \tag{3}$$

$J_i = V_i + A_i$ is one of the isospin components of the usual $V-A$ currents and J_S is an isoscalar current. In the simple Weinberg model[12]

$$x = 1 - 2\sin^2\theta_W\,, \quad y = -2\sin^2\theta_W \tag{4}$$

and the neutral hadronic current J_0^μ is given by

$$J_0^\mu = J_3^\mu - 2\sin^2\theta_W\, J_{\text{em}}^\mu\,. \tag{5}$$

Now the matrix element for the process $\nu_e(\bar{\nu}_e) + d \to \nu_e(\bar{\nu}_e) + n + p$ can be written in the standard notation as

$$\mathfrak{M} = \frac{G}{\sqrt{2}}\, \bar{u}(k')\gamma_\mu (1+\gamma_5) u(k) \langle np | J_0^\mu | d\rangle\,, \tag{6}$$

where k and k' are the initial and final lepton momenta measured in the deuteron rest frame and $\langle np|J_0^\mu|d\rangle$ is the matrix element of the hadronic current derived in the impulse approximation to be[13,14]

$$\langle np | J_0^\mu | d\rangle = \int \phi_f^*(\vec{r}) [\Lambda_{0,p}^\mu e^{i\vec{q}\cdot\vec{r}/2} + \Lambda_{0,n}^\mu e^{-i\vec{q}\cdot\vec{r}/2}] \times \phi_i(\vec{r})\, d^3r\,, \tag{7}$$

where $\vec{q} = (\vec{k} - \vec{k}')$ is the three-momentum transfer. $\phi_i(\vec{r})$ is the deuteron wave function, $\phi_f(\vec{r})$ is the final dinucleon wave function, and $\Lambda_{0,i}^\mu$ $(i = p, n)$ is the nonrelativistic reduction[13] of the single-nucleon operator $\Gamma^{\mu}_{0,i}$ whose matrix element is defined as follows:

$$\langle i(p')|\Gamma^{\mu}_{0,i}|i(p)\rangle = \bar{u}(p')\left[F_1^{0,i}(q^2)\gamma^{\mu} + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2^{0,i}(q^2) - F_A^{0,i}(q^2)\gamma^{\mu}\gamma_5 + F_P^{0,i}(q^2)\gamma_5 q^{\mu}\right]u(p). \quad (8)$$

In Eq. (8) the superscript 0 is to show that the forms factors refer to the charge-retention process and i refers to the proton and neutron. These form factors $F_1^{0,i}(q^2)$, $F_2^{0,i}(q^2)$, and $F_A^{0,i}(q^2)$ [the last term in Eq. (8) does not contribute when the leptons involved are neutrinos] can be related to the form factors occurring in the corresponding weak and electromagnetic process.[15] The matrix element defined in Eq. (6) can thus be calculated using suitable wave functions for the initial and final states.

We have used the following wave functions for the initial deuteron and final dinucleon states[16]:

$$\phi_i(\vec{r}) = \frac{1}{\sqrt{4\pi}}\frac{u(r)}{r}\chi_1^m, \quad (9)$$

$$\phi_f(\vec{r}) = \sqrt{4\pi}\sum i^L(2L+1)^{1/2}e^{i\delta_{JLS}}\langle JM|LOSM_S\rangle\langle JM'|LM_LSM_S\rangle D^J_{M'M}(R)Y_L^{M_L}(\hat{r})|SM_S\rangle\frac{F_{JLS}(p_c r)}{p_c r}, \quad (10)$$

where p_c is the relative momentum of the outgoing nucleons in their center-of-mass frame and is given by

$$\vec{p}_c^{\,2} = \tfrac{1}{4}q^2 + M(E_\nu - E'_\nu - B). \quad (11)$$

Here E_ν is the incident neutrino (antineutrino) energy, E'_ν is the final neutrino (antineutrino) energy, B is the deuteron binding energy, and

$$q^2 = -4E_\nu E'_\nu \sin^2(\theta/2),$$

where θ is the lepton scattering angle given by $\cos\theta = \hat{k}\cdot\hat{k}'$. δ_{JLS} is the phase shift in the channel (JLS) and $F_{JLS}(p_c r)$ is the solution of the Schrödinger equation solved with the appropriate potential in the state (JLS) and has the asymptotic form

$$\frac{F_{JLS}(p_c r)}{p_c r}\xrightarrow[p_c r\to\infty]{}\sin(p_c r - \tfrac{1}{2}l\pi + \delta_{JLS}). \quad (12)$$

A straightforward calculation gives the following result for the total cross section σ for this process:

$$\sigma = \frac{G^2M^2}{3\pi^2}\int_0^{q^2_{\max}} dq^2\int_{E_{\min}}^{E_{\max}} dE'_\nu\left(\frac{E'_\nu}{E_\nu}\right)\frac{p_c}{E_c}\left(\sum_{L,S}A_{LS}K_L^2 + \sum_{JLS}C_{JLS}\Delta_{JLS}\right), \quad (13)$$

where

$$q^2_{\max} = 4E_\nu(E_\nu - B)/(1+E_\nu/M), \quad E_{\min} = -q^2/4E_\nu, \quad E_{\max} = E_\nu - B + q^2/4M,$$

and

$$K_L = \int j_L(p_c r)j_L(qr/2)u(r)r\,dr, \quad (14)$$

$$\Delta_{JLS} = K_{JLS}^2 - K_L^2, \quad (15)$$

$$K_{JLS} = \frac{1}{p_c}\int F_{JLS}(p_c r)j_L(qr/2)u(r)\,dr. \quad (16)$$

The coefficients A_{LS} and C_{JLS} are given in Table I.

III. RESULTS AND DISCUSSION

To evaluate the total cross section σ one has to calculate numerically $K_{JLS}(p_c, q)$ from Eq. (16) for realistic two-nucleon potentials and perform the numerical integration involved in Eq. (13). To illustrate our results some plausible simplifying assumptions can, however, be made. We assume, on the basis of our earlier results in connection with the process $\nu_\mu + d \to \mu^- + p + p$,[13] that the second term in Eq. (13) when integrated over E'_ν is very small.[17] Under this assumption the calculation of σ involves a simple numerical integration which can be performed quite easily. In order to compare our results with the experimental results[7,8] and earlier theoretical results[18] on the process $\bar{\nu}_e + d \to \bar{\nu}_e + n + p$, we calculate σ when the final dinucleons are in the singlet S state. In this case the total cross section σ is given by

$$\sigma = \frac{G^2M^2}{3\pi^2}\int_0^{q^2_{\max}} dq^2 \int_{E_{\min}}^{E_{\max}} dE'_\nu \frac{E'_\nu}{E_\nu}\frac{p_c}{E_c}\left[F_A{}^2(q^2)\left(1-\frac{q^2}{4E_\nu E'_\nu}\right) - 2xF_{MV}(q^2)F_A(q^2)\frac{E_\nu+E'_\nu}{M}\frac{q^2}{4E_\nu E'_\nu}\right]K_0{}^2\,, \tag{17}$$

where $F_{MV}(q^2)=F_{M,p}(q^2)-F_{M;n}(q^2)$ is the isovector part of the magnetic form factor. If we use a Hulthén wave function for the deuteron and neglect D states,[13,19] K_0 is given by

$$K_0=\frac{[2\alpha\beta(\alpha+\beta)]^{1/2}}{\beta-\alpha}\frac{1}{2p_c q} \times \ln\left[\frac{(p_c+q/2)^2+\alpha^2}{(p_c-q/2)^2+\alpha^2}\,\frac{(p_c-q/2)^2+\beta^2}{(p_c+q/2)^2+\beta^2}\right], \tag{18}$$

with $\alpha=46$ MeV, $\beta=237$ MeV, and $q=|\vec{q}|$.

For the neutrino energies relevant to the experiment of Gurr *et al.*[8] the contribution of the second term in Eq. (17) is negligibly small, as it is suppressed by a factor proportional to E_ν/M. The main contribution thus comes from the first term alone which is independent of the Weinberg angle θ_W. We have plotted in Fig. 1(a) the cross sections calculated from Eq. (17) as a function of the incident neutrino energy E_ν, which should be used for comparison with the experimental result.[20] The results of the earlier calculation[18] are also shown in this figure [1(b)] for comparison.[21]

The large experimental uncertainty quoted in the experiment of Gurr *et al.*, which is due to the γ-ray signals received due to the capture of neutrons produced in the corresponding charge-exchange process $\bar{\nu}_e+d\to e^++n+n$, can be eliminated if the neutrino beam is used to look for the neutral-current effects in the process $\nu_e+d\to\nu_e+n+p$. This is because the allowed charge-exchange process with the neutrino beam, i.e., $\nu_e+d\to e^-+p+p$, does not give any neutrons in the final state, and hence no γ rays are produced which can compete with the γ rays produced in the charge-retention process.

TABLE I. The expressions for the coefficients A_{LS} and C_{JLS} quoted in Eq. (13) for different values of J, L, and S.

	Even L	Odd L
A_{L0}	$a_1(2L+1)$	$a'_1(2L+1)$
A_{L1}	$a'_3(2L+1)$	$a_3(2L+1)$
C_{LL0}	$a_1(2L+1)$	$a'_1(2L+1)$
$C_{L+1,L,1}$	$\frac{1}{2}(a'_2+a'_3)L+a'_3$	$\frac{1}{2}(a_2+a_3)L+a_3$
$C_{L,L,1}$	$(a'_3-a'_2)(L+\frac{1}{2})$	$(a_3-a_2)(L+\frac{1}{2})$
$C_{L-1,L,1}$	$\frac{1}{2}(a'_2+a'_3)L+\frac{1}{2}(a'_2-a'_3)$	$\frac{1}{2}(a_2+a_3)L+\frac{1}{2}(a_2-a_3)$

$$a_1=F_A{}^2(q^2)\left(1-\frac{q^2}{4E_\nu E'_\nu}\right)-2xF_{MV}(q^2)F_A(q^2)\frac{E_\nu+E'_\nu}{M}\frac{q^2}{4E_\nu E'_\nu},$$

$$a_2=x^2F_{1V}{}^2(q^2)\left(1+\frac{q^2}{4E_\nu E'_\nu}\right)-F_A{}^2(q^2)\frac{q^2}{2E_\nu E'_\nu}\left(1+\frac{2E_\nu E'_\nu}{\vec{q}^2}\right) -2xF_{MV}(q^2)F_A(q^2)\frac{E_\nu+E'_\nu}{M}\frac{q^2}{4E_\nu E'_\nu},$$

$$a_3=3x^2F_{1V}{}^2(q^2)\left(1+\frac{q^2}{4E_\nu E'_\nu}\right)+2F_A{}^2(q^2)\left(1-\frac{q^2}{4E_\nu E'_\nu}\right) -4xF_{MV}(q^2)F_A(q^2)\frac{E_\nu+E'_\nu}{M}\frac{q^2}{4E_\nu E'_\nu},$$

$$a'_1=O(1/M^2),$$

$$a'_2=\tfrac{1}{3}a'_3=y^2F_{1S}{}^2(q^2)\left(1+\frac{q^2}{4E_\nu E'_\nu}\right),$$

$F_{1V,S}(q^2)=F_{1,p}(q^2)\mp F_{1,n}(q^2)$ with $F_{1,p}(0)=1$ and $F_{1,n}(0)=0$,

$F_{MV}(q^2)=F_{M,p}(q^2)-F_{M,n}(q^2)$

with $F_{M,p}(0)=1+\mu_p$ and $F_{M,n}(0)=\mu_n$.

(μ_p and μ_n are the magnetic moments of the proton and neutron, respectively.)

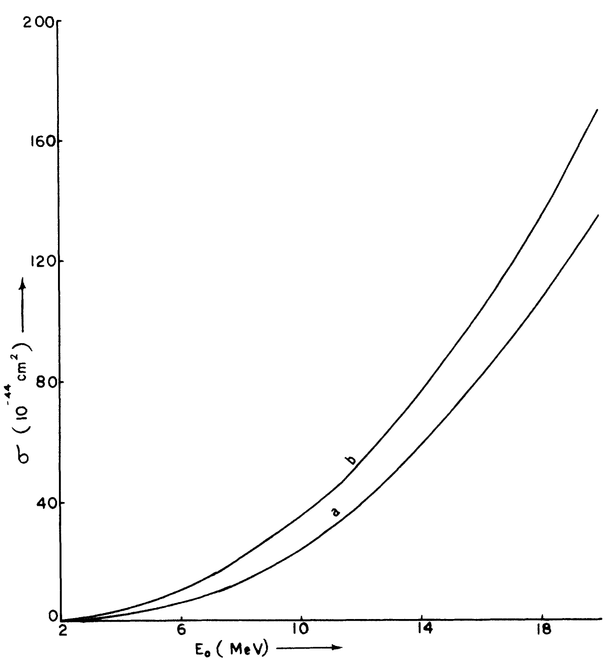

FIG. 1. Total cross section σ vs the neutrino energy E_0 measured from the threshold in units of electron mass, i.e., $E_0=(E_\nu-B)/m_e$ for (b) Gaponov's and Tytutin's result (Ref. 18) and (a) the present calculation.

Such an experiment can be performed at the Argonne National Laboratory (ANL) with the muon-neutrino beam. The muon-neutrino beam at ANL peaks around 0.5 GeV (see Ref. 22) and then final dinucleon states higher than the singlet S state will be produced. These results can then be used to determine the Weinberg angle and a theoretical analysis based on Eq. (13) will be desirable.

It should be emphasized that our present numerical results are only illustrative and the theoretical formulation developed here should prove useful in analyzing future experiments in this direction.

ACKNOWLEDGMENT

The author would like to thank Professor M. Z. R. Khan for his interest and encouragement, Professor H. S. Mani for his comments and suggestions, and Q. N. Usmani for helping us with the computer program.

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[9]The contribution of the weak magnetism term which depends upon the Weinberg angle θ_W is negligibly small at such low energies, as shown in Eq. (17) below.

[10]H. S. Gurr *et al.*, in their paper (Ref. 8), describe the proposed version of the experiment to be performed at the Savannah River Plant fission-reactor neutrino facility, and the results should be avilable in the near future.

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[15]This can be done following Weinberg (Ref. 11). We have used isospin considerations to derive our results. The isovector part of the current defined in Eq. (3) leads to the isotriplet dinucleon states, i.e., ^1S, ^3P, etc., while the isoscalar part leads to the isosinglet states, i.e., ^3S, ^1P, etc. Since an isoscalar deuteron target is used, there will be no interference terms and the total cross section will be the sum of the isoscalar and isovector contributions.

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[17]The contribution of the second term to the differential cross section $(d\sigma/dq^2)_{\nu d \to \mu^- + p + p}$ is $\sim 1\%$ for the singlet S diproton state. For the case of P and other higher states this contribution is still smaller (Ref. 13).

[18]Earlier attempts to calculate this process assume standard $V-A$ coupling for the neutral currents with some variations whose strength is determined from the corresponding charge-exchange process using isotopic spin invariance. These calculations are furthermore limited in their applicability as the final dinucleons are considered to be in the singlet S state alone. See, for example, T. Ahrens and T. P. Lang, Phys. Rev. C 3, 979 (1971); Yu. V. Gaponov and I. V. Tyutin Zh. Eksp. Teor. Fiz. 47, 1826 (1964) [Sov. Phys.—JETP 20, 1231 (1965)].

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[20]$F_A(q^2) \simeq F_A(0) = 1.2$ has been used in numerically evaluating the total cross section in Eq. (17).

[21]Gaponov and Tyutin (Ref. 18) do not mention the exact value of g_A used in their numerical computation. A possible over-all factor should therefore be kept in mind before comparing the curves 1(a) and 1(b).

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